Chapter 9

Clocks and Calendars

CHAPTER HIGHLIGHTS

- ☞ *Clocks*
- ☞ *Calendar*
- ☞ *Leap and Non-leap Year*
- ☞ *Counting the Number of Odd Days*

Clocks

The hour hand and the minute hand of a clock move in relation to each other continuously, and at any given point of time, they make an angle between 0° and 180° with each other.

If the time shown by the clock is known, the angle between the hands can be calculated. Similarly, if the angle between two hands is known, the time shown by the clock can be found out.

When we say angle between the hands, we normally refer to the acute/obtuse angles (upto 180°) between the two hands and not the reflex angle (> 180°).

For solving the problems on clocks, the following points will be helpful.

- Minute hand covers 360° in 1 hour, i.e. in 60 minutes. Hence, MINUTE HAND COVERS 6° PER MINUTE.
- Hour hand covers 360° in 12 hours. Hence, hour hand covers 30° per hour. Hence, HOUR HAND COVERS 1/2° PER MINUTE.

The following additional points also should be remembered. In a period of 12 hours, the hands make an angle of

- 0° with each other (i.e. they coincide with each other), 11 times.
- 180° with each other (i.e., they point exactly in opposite directions), 11 times.
- 90° or any other angle with each other, 22 times.

NOTE

We can also solve the problems on clocks using the method of 'Relative Velocity'.

In 1 minute, minute hand covers 6° and hour hand covers 1/2°.

Therefore, relative velocity = $6 - 1/2 = 5\frac{1}{2}°$ per minute.

Alternately, in 1 hour, the minute hand covers 60 minute divisions, whereas the hour hand covers 5 minute divisions.

$\therefore$ Relative speed = 60 – 5 = 55 minutes per hour.

However, adopting the approach of actual angles covered is by far the simplest and does not create any confusion.

Points to Note

- Any angle other than (0° and 180°) is made 22 times in a period of 12 hours.
- In a period of 12 hours, there are 11 coincidences of the two hands, when the two hands are in a straight line facing opposite directions.
- The time gap between any two coincidences is 12/11 hours or $65\frac{5}{11}$ minutes.
- If the hands of a clock (which do not show the correct time) coincide every p minutes, then

If $p > 65\frac{5}{11}$, then the watch is going slow or losing time.

If $p < 65\frac{5}{11}$, then the watch is going fast or gaining time.

To calculate the angle 'θ' between the hands of a clock, we use the following formula (where m = minutes and h = hours)

1. $\theta = \frac{11}{2}m - 30h$ $\left(\text{when } \frac{11}{2}m > 30h\right)$

2. $\theta = 30h - \frac{11}{2}m$ $\left(\text{when } 30h > \frac{11}{2}m\right)$

Solved Examples

Example 1

What is the angle between the minute hand and the hour hand of a clock at 3 hours 40 minutes?

(A) 20° (B) 70° (C) 90° (D) 130°

Solution

The angle between the hands can be calculated by $\theta = \left|\frac{11}{2}m - 30h\right|$, where m is minutes and h is hours. Here, $m = 40$ and $h = 3$

$$\therefore \quad \theta = \left|\frac{11}{2} \times 40 - 30 \times 3\right| = |220 - 90| = 130°$$

The angle between the two hands is 130°.

Example 2

Find the time between 2 and 3 O'clock at which the minute hand and the hour hand make an angle of 60° with each other.

Solution

In the formula $\theta = \left|\frac{11}{2}m - 30h\right|$,

$$\theta = 60° \text{ and } h = 2$$

$$\therefore \quad 60 = \frac{11}{2}m - 30 \times 2$$

$$\frac{11}{2}m = 120$$

$$m = \frac{240}{11} = 21\frac{9}{11}m \text{ past } 2$$

(or) $$60 = 30 \times 2 - \frac{11}{2}m$$

$$\therefore \quad \frac{11}{2}m = 0$$

$$m = 0$$

Therefore, the angle between the hour hand and the minute hand is 60° at 2 O'clock and at $21\frac{9}{11}$ minutes past 2 O'clock.

Example 3

Find the time between 2 and 3 O'clock at which the minute hand and the hour hand overlap.

Solution

When the two hands overlap, the angle between them is 0°.

$$\theta = \left|\frac{11}{2}m - 30h\right|$$

$$\therefore \quad \theta = 0° \quad \text{and} \quad h = 2$$

$$\frac{11}{2}m = 30 \times 2$$

$$m = \frac{120}{11} = 10\frac{10}{11} \text{ min past } 2.$$

Example 4

Find the time between 2 and 3 O'clock at which the minute hand and the hour hand are perpendicular to each other.

Solution

When two hands are perpendicular, $\theta = 90°$ and $h = 2$

$$\therefore \quad \theta = \left(\frac{11}{2}m - 30h\right) \quad \text{or} \quad \left(30h - \frac{11}{2}m\right)$$

$$90 = \frac{11}{2}m - 30 \times 2; \quad \frac{11}{2}m = 150$$

$$m = \frac{300}{11} = 27\frac{3}{11} \text{ minutes past } 2$$

(or) $$90 = 30 \times 2 - \frac{11}{2}m$$

$$\Rightarrow \quad \frac{11}{2}m = -30$$

As m cannot be negative, this case is not possible.
So, the hands are perpendicular to each other only once i.e. at $27\frac{3}{11}$ minutes past 2 O'clock.

Example 5

Find the time between 2 and 3 O'clock at which the minute hand and the hour hand are on the same straight line but are pointing in opposite directions.

Solution

When two hands are pointing opposite directions and are on a straight line the angle between them would be 180°. i.e. $\theta = 180°$ and $h = 2$.

$$180° = \frac{11}{2}m - 30h; \quad \frac{11}{2}m = 180 + 60 = 240$$

$$m = \frac{480}{11} = 43\frac{7}{11}$$

So, at $43\frac{7}{11}$ minutes past 2 O'clock the hands will be at 180°.

Calendar

Suppose you are asked to find the day of the week on 30th June, 1974, it would be a tough job to find it if you do not know the method. The method of finding the day of the week lies in the number of 'odd days'.

NOTE

Every 7th day will be the same day count wise, i.e. if today is Monday, then the 7th day counting from Tuesday onwards will once again be Monday. Odd days is the days remaining after completion of an exact number of weeks. Odd days is the reminder obtained on dividing the total number of days with seven.

Example: 52 days ÷ 7 = 3 odd days.

Leap and Non-leap Year: A non-leap year has 365 days, whereas a leap year has one extra day because of 29 days in the month of February. Every year which is divisible by 4 is called a leap year. Leap year consists of 366 days (52 complete weeks + 2 days), the extra two days are the odd days. So, a leap year has two odd days.

An non-leap year consists of 365 days (52 complete weeks + 1 day). The extra one day is the odd day.

NOTE

Every century, year which is a multiple of 400, is a leap year. A century year which is not divisible by 400 is a non-leap year.

Example: 400, 800, 1200, 1600 … are leap years.
500, 700, 900, 1900 … are non-leap years.

Counting the number of Odd Days: 100 years consist of 24 leap years + 76 ordinary years. (100 years when divided by 4, we get 25. But at the 100th year is not a leap year, hence only 24 leap years).

$= 2 \times 24$ odd days $+ 1 \times 76$ odd days $= 124$ days

$= 17$ weeks $+ 5$ days

The extra 5 days are the odd days.

So, 100 years contain 5 odd days.

Similarly, for 200 years we have 10 extra days (1 week + 3 days).

∴ 200 years contains 3 odd days.

Similarly, 300 years contain 1 odd day and 400 years contain 0 odd days.

Counting of number of odd days, when only one date is given: Here, we take January 1st 1 AD as the earlier date and we assume that this day is a Monday. We take its previous day, i.e. Sunday as the reference day. After this the above-mentioned method is applied to count the number of odd days and find the day of the week for the given date.

Counting number of odd days, when two dates are given: Any month which has 31 days has 3 odd days.

(∵ 31 ÷ 7 leaves 3 as remainder) and any month which has 30 days has 2 odd days (30 ÷ 7 leaves 2 as remainder).

Then, the total number of odd days are calculated by adding the odd days for each month. The value so obtained is again divided by 7 to get the final number of odd days. The day of the week of the second date is obtained by adding the odd days to the day of the week of the earlier date.

Example 6

If you were born on 14th April, 1992, which was a Sunday, then on which day of the week does your birthday fall in 1993?

(A) Monday (B) Tuesday
(C) Wednesday (D) Cannot be determined

Solution

14th April 1992 to 14th April 1993 is a complete year, which has 365 days. Hence, the number of odd days from 14th April 1992 to 14th April 1993 is 1.

Hence, 14th April 1993 is one day after Sunday, i.e. Monday.

Example 7

If 1st Jan, 1992, is a Tuesday then on which day of the week will 1st Jan, 1993, fall?

(A) Wednesday (B) Thursday
(C) Friday (D) Saturday

Solution

Since 1992 is a leap year there are 2 odd days.

Hence, 1st January 1992 is two days after Tuesday, i.e., Thursday.

Example 8

If 1st April, 2003 was Monday, then which day of the week will 25th December of the same year be?

(A) Tuesday (B) Wednesday
(C) Thursday (D) Friday

Solution

The number of days from 1st April to 25th December

(29 + 31 + 30 + 31 + 31 + 30 + 31 + 30 + 25) days = 268 days

$$= \frac{268}{7} = 38 + 2 \text{ odd days.}$$

Hence, 25th December is two days after Monday, i.e. Wednesday.

Example 9

Which year will have the same calendar as that of 2005?

(A) 2006 (B) 2007
(C) 2008 (D) 2011

Solution

Year: 2005 + 2006 + 2007 + 2008 + 2009 + 2010
Odd days : 1+ 1+ 1+ 2+ 1 + 1

Total number of odd days from 2005 to 2010 are $7 \cong 0$ odd days.

Hence, 2011 will have the same calendar as that of 2005.

Example 10

What day of the week was 18th April 1901?

(A) Monday (B) Tuesday
(C) Wednesday (D) Thursday

Solution

18th April 1901 ⇒ (1600 + 300) years + 1st January to 18th April 1901.

1600 years have – 0 odd days
300 years have – 1 odd day
The number of days from 1st January, 1901 to 18th April 1901 is (31 + 28 + 31 + 18) days
108 days ≅ 3 odd days
∴ Total number of odd days = 3 + 1 = 4
Hence, 18th April 1901 is Thursday.

Exercises

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. What is the angle covered by the minute hand in 22 minutes?
(A) 66° (B) 110° (C) 132° (D) 220°

2. By how many degrees will the minute hand move, in the same time, in which the hour hand moves 6°?
(A) 54° (B) 84° (C) 72° (D) 60°

3. What is the angle between the hands of the clock, when it shows 40 minutes past 6?
(A) 40° (B) 70° (C) 80° (D) 90°

4. What is the angle between the two hands of a clock when the time is 25 minutes past 7 O'clock?
(A) $62\frac{1}{2}°$ (B) $66\frac{1}{2}°$
(C) $72\frac{1}{2}°$ (D) $69\frac{1}{2}°$

5. At what time between 9 and 10 O'clock, will the two hands of the clock coincide?
(A) $43\frac{3}{11}$ minutes past 9 O'clock
(B) $45\frac{6}{11}$ minutes past 9 O'clock
(C) $49\frac{1}{11}$ minutes past 9 O'clock
(D) $49\frac{6}{11}$ minutes past 9 O'clock

6. At what time between 4 and 5 O'clock are the two hands of a clock in the opposite directions?
(A) $52\frac{3}{11}$ minutes past 4 O'clock
(B) $54\frac{6}{11}$ minutes past 4 O'clock
(C) $51\frac{7}{11}$ minutes past 4 O'clock
(D) $53\frac{9}{11}$ minutes past 4 O'clock

7. The angle between the two hands of a clock is 20° and the hour hand is in between 2 and 3. What is the time shown by the clock?
(A) $7\frac{3}{11}$ minutes past 2
(B) $14\frac{6}{11}$ minutes past 2
(C) $15\frac{5}{11}$ minutes past 2
(D) Both (A) and (B)

8. Which of the following can be the time shown by the clock, when the hour hand is in between 4 and 5 and the angle between the two hands of the clock is 60°?
(A) $16\frac{4}{11}$ min past 4 (B) $18\frac{9}{11}$ min past 4
(C) $32\frac{8}{11}$ min past 4 (D) $36\frac{5}{11}$ min past 4

9. How many times, the two hands of a clock will be at 30° with each other in a day?
(A) 36 (B) 40 (C) 44 (D) 48

10. If the time in a clock is 10 hours 40 minutes, then what time does its mirror image show?
(A) 2 hours 20 minutes
(B) 1 hour 15 minutes
(C) 1 hour 10 minutes
(D) 1 hour 20 minutes

11. There are two clocks on a wall, both set to show the correct time at 5:00 p.m. The clocks lose 2 minutes and 3 minutes respectively in an hour. When the clock which loses 2 minutes in one hour shows 9:50 p.m. on the same day, then what time does the other clock show?
(A) 9:30 p.m. (B) 9:40 p.m.
(C) 9:45 p.m. (D) 10:15 p.m.

12. A watch that gains uniformly was observed to be 1 minute slow at 8:00 a.m. on a day. At 6:00 p.m. on the same day it was 1 minute fast. At what time did the watch show the correct time?
(A) 12:00 noon (B) 1:00 p.m.
(C) 2:00 p.m. (D) 3:00 p.m.

13. A watch, which gains uniformly, was observed to be 6 minutes slow at 9:00 a.m. on a Tuesday and 3 minutes fast at 12:00 noon on the subsequent Wednesday. When did the watch show the correct time?
(A) 9:00 p.m. on Tuesday
(B) 12:00 a.m. on Wednesday
(C) 3:00 a.m. on Wednesday
(D) 6:00 a.m. on Wednesday

14. The number of odd days in a non-leap year is
(A) 0 (B) 1 (C) 2 (D) 3

15. What will be next leap year after 2096?
(A) 2100 (B) 2101 (C) 2104 (D) 2108

16. If 21st March 2000 was a Monday, what day of the week will 21st March 2003 be?
(A) Tuesday (B) Friday
(C) Thursday (D) Wednesday

17. If 5th January 2001 was a Friday then what day of the weak will 25th December 2001 be?
(A) Monday (B) Tuesday
(C) Wednesday (D) Thursday

18. If 14th February 2001 was a Wednesday, then what day of the week will 14th February 2101 be (i.e. after a century)?
(A) Friday (B) Saturday
(C) Sunday (D) Monday

19. If 8th February 1995 was a Wednesday, then what day of the week will 8th February 1994 be?
(A) Wednesday (B) Thursday
(C) Tuesday (D) Monday

20. If holidays are declared only on Sundays and in a particular year 12th March is a Sunday, is 23rd September in that year a holiday?
(A) Yes
(B) No
(C) Yes, if it is a leap year.
(D) No, if it is a leap year.

21. Which day of the week was 1601, Jan 15?
(A) Monday (B) Tuesday
(C) Wednesday (D) Thursday

22. In a year, if 23rd November is a Friday then what day of the week will 14th March in that year be?
(A) Monday
(B) Wednesday
(C) Sunday
(D) Cannot be determined

23. The calendar of which of the following years is the same as that of the year 2001?
(A) 2005 (B) 2006 (C) 2007 (D) 2008

24. Pankaj met his friend three days ago. He told his friend that he has his last exam five days later. He met his friend again, three days after the last exam. Six days after he met his friend after the last exam, they left for a vacation. The day on which they left for a vacation is a Saturday. What is today?
(A) Saturday (B) Tuesday
(C) Sunday (D) Cannot be determined

25. Five days ago Shweta lost her phone. Two days after loosing the phone she lodged a complaint with the police. Six days after lodging the complaint she bought a new phone. Four days after buying a new phone, i.e. on a Thursday she found her old phone. On which day did she loose her phone?
(A) Friday (B) Saturday
(C) Thursday (D) None of these

Previous Years' Questions

1. The Palghat Gap (or Palakkad Gap), a region about 30 km wide in the southern part of the Western Ghats in India, is lower than the hilly terrain to its north and south. The exact reasons for the formation of this gap are not clear. It results in the neighbouring regions of Tamil Nadu getting more rainfall from the South West monsoon and the neighboring regions of Kerala having higher summer temperatures. **[GATE, 2014]**
What can be inferred from this passage?
(A) The Palghat gap is caused by high rainfall and high temperatures in Southern Tamil Nadu and Kerala.
(B) The regions in Tamil Nadu and Kerala that are near the Palghat Gap are near the low–lying.
(C) The low terrain of the Palghat Gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala.
(D) Higher summer temperatures result in higher rainfall near the Palghat Gap area.

2. Geneticists say that they are very close to confirming the genetic roots of psychiatric illnesses such as depression and Schizophrenia, and consequently, that doctors will be able to eradicate these diseases through early identification and gene therapy.
On which of the following assumptions does the statement above rely? **[GATE, 2014]**
(A) Strategies are now available for eliminating psychiatric illnesses.
(B) Certain psychiatric illnesses have a genetic basis.
(C) All human diseases can be traced back to genes and how they are expressed.
(D) In the future, genetics will become the only relevant field for identifying psychiatric illness.

3. The old city of Koenigsberg, which had a German majority population before World War 2, is now called Kaliningrad. After the events of the war, Kaliningrad is now a Russian territory and has a predominantly Russian population. It is bordered by the Baltic Sea on the north and the countries of Poland to the South and West Lithuania to the east respectively. Which of the statements below can be inferred from this passage? **[GATE, 2014]**

(A) Kaliningrad was historically Russian in its ethnic make up.
(B) Kaliningrad is part of Russia despite it not being contiguous with the rest of Russia.
(C) Koenigsberg was renamed Kaliningrad, as that was its original Russian name.
(D) Poland and Lithuania are on the route from Kalinigrad to the rest of Russia.

4. The number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in north India is twice the number diagnosed last year. Municipal authorities have concluded that measures to control the mosquito population have failed in this region.
Which one of the following statements, if true, does not contradict this conclusion? **[GATE, 2014]**

(A) A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent.
(B) More cases of dengue are now reported because of an increase in the Municipal Office's administrative efficiency.
(C) Many more cases of dengue are being diagnosed this year since the introduction of a new and effective diagnostic test
(D) The number of people with malarial fever (also contracted from mosquito bites) has increased this year.

5. At what time between 6 am and 7 am, will the minute hand and hour hand of a clock make an angle closest to 60°? **[GATE, 2014]**

(A) 6:22 am (B) 6:27 am
(C) 6:38 am (D) 6:45 am

6. Which number does not belong in the series below?
2, 5, 10, 17, 26, 37, 50, 64 **[GATE, 2014]**

(A) 17 (B) 37
(C) 64 (D) 26

7. A dance programme is scheduled for 10:00 am. Some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be accompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is **[GATE, 2014]**

(A) Students should come at 9:00 am and parents should come at 10:00 am.
(B) Participating students should come at 9:00 am accompanied by a parent, and other parents and students should come by 10:00 am.
(C) Students who are not participating should come by 10:00 am and they should not bring their parents. Participating students should come at 9:00 am.
(D) Participating students should come before 9:00 am. Parents who accompany them should come at 9:00 am. All others should come at 10:00 am.

8. By the beginning of the 20th century, several hypotheses were being proposed, suggesting a paradigm shift in our understanding, of the universe. However, the clinching evidence was provided by experimental measurements of the position of a star which was directly behind our sun.
Which of the following inference(s) may be drawn from the above passage?

I. Our understanding of the universe changes based on the positions of stars.
II. Paradigm shifts usually occur at the beginning of centuries.
III. Stars are important objects in the universe.
IV. Experimental evidence was important in confirming this paradigm shift. **[GATE, 2014]**

(A) I, II and IV (B) III only
(C) I and IV (D) IV only

9. After several defeats in wars, Robert Bruce went in exile and wanted to commit suicide. Just before committing suicide, he came across a spider attempting tirelessly to have its net. Time and again, the spider failed but that did not deter it to refrain from making attempts. Such attempts by the spider made Bruce curious. Thus, Bruce started observing the near-impossible goal of the spider to have the net. Ultimately, the spider succeeded in having its net despite several failures. Such act of the spider encouraged Bruce not to commit suicide. And then, Bruce went back again and won many a battle, and the rest is history.
Which one of the following assertions is best supported by the above information? **[GATE, 2013]**

(A) Failure is the pillar of success.
(B) Honesty is the best policy.
(C) Life begins and ends with adventures.
(D) No adversity justifies giving up hope.

10. Given the sequence of terms AD, CG, FK, JP, ? the next term is **[GATE, 2012]**

(A) OV (B) OW
(C) PV (D) PW

11. Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.
Based on the above passage which topic would not be included in a unit on bereavement? **[GATE, 2012]**
(A) How to write a letter of condolence
(B) What emotional stages are passed through in the healing process
(C) What the leading causes of death are
(D) How to give support to a grieving friend.

12. 25 persons are in a room. 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. The number of persons playing neither hockey nor football is _____. **[GATE, 2010]**
(A) 2 (B) 17
(C) 13 (D) 3

13. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.
Unemployed : Worker **[GATE, 2010]**
(A) Fallow : Land
(B) Unaware : Sleeper
(C) Wit : Jester
(D) Renovated : House

14. Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e., brothers and sisters). All were born on 1st January, in different years. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts:
I. Hari's age + Gita's age > Irfan's age + Saira's age
II. The age difference between Gita and Saira is 1 year. However Gita is not the oldest and Saira is not the youngest.
III. There are no twins.
Which of the following in a possible order in which they were born? **[GATE, 2010]**
(A) HSIG
(B) SGHI
(C) IGSH
(D) IHSG

Answer Keys

Exercises

1. C	**2.** C	**3.** A	**4.** C	**5.** C	**6.** B	**7.** D	**8.** C	**9.** C	**10.** D
11. C	**12.** B	**13.** C	**14.** B	**15.** C	**16.** C	**17.** B	**18.** D	**19.** C	**20.** B
21. A	**22.** B	**23.** C	**24.** B	**25.** B					

Previous Years' Questions

1. C	**2.** B	**3.** B	**4.** D	**5.** A	**6.** C	**7.** B	**8.** D	**9.** D	**10.** A
11. C	**12.** D	**13.** A	**14.** B						