CHAPTER 13

Trigonometric Levelling

13.1 Introduction

- Trigonometric levelling is the indirect method of levelling wherein the elevation of a point is determined from the vertical angle measured with a theodolite and horizontal distance measured with a tape or a chain.

  Trigonometric levelling is of two types viz.:

  (a) **Plain trigonometric levelling** : In plain trigonometric levelling, the principles of plain surveying are employed. It is assumed that the distances measured are small compared to the radius of earth and thus the effect of earth's curvature and refraction can be ignored.

  (b) **Geodetic trigonometric levelling** : In geodetic trigonometric levelling, the distances between the points are comparable to that of earth's radius and thus the simple principles of plain surveying cannot be applied. It requires the correction due to earth's curvature and refraction.
- However trigonometric levelling is not as accurate as direct levelling. With the advent of tacheometers, trigonometric levelling has now become simple because the horizontal distances also can be computed directly.

13.2 Determination of Height of Top Level of a High Object when its Base is not Accessible

- When the base of a high object (like chimney, cooling tower, transmission line tower etc.) is not accessible then it is not possible to determine the horizontal distance between the object base and the instrument station.
- Thus for doing so, two settings of the instrument are required wherein it is assumed that the high object and the two instrument settings lie in the same vertical plane.
- Depending on the levels of the two instrument axes, there can be four possible cases as described below.

Case I: The Instrument Axes are at the Same Level

With a theodolite, take both face readings i.e. face left and face right readings.

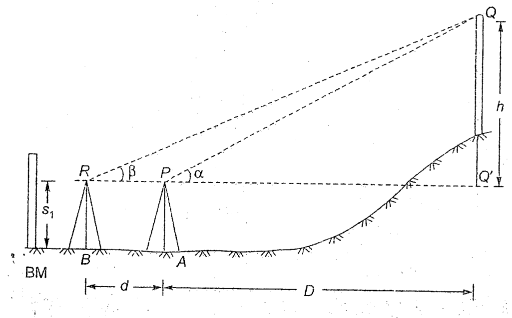


Fig. 13.1 *Instrument axes at the same level*

Let α = Angle of inclination of the top of object when the instrument is at A

β = Angle of inclination of the top of object when the instrument is at B

D = Horizontal distance between A and Q

In $\Delta PQQ'$, $h = D\tan\alpha$

In $\Delta RQQ'$, $h = (D + d)\tan\beta$

Thus from these two equations,

$$D\tan\alpha = (D + d)\tan\beta$$

$$D = \frac{d\tan\beta}{(\tan\alpha - \tan\beta)} \quad ...(13.1)$$

Substituting this value of D in any of the above equations,

$$h = \frac{d\tan\alpha\tan\beta}{(\tan\alpha - \tan\beta)}$$

$$= \frac{d\sin\alpha\sin\beta}{\sin(\alpha - \beta)} \quad ...(13.2)$$

Thus RL of Q = RL of $P + h$

= RL of BM + $s_1 + h$

The problem with this method is that it is very difficult set up the instrument axis of B exactly at the same level as that of A.

Case II Instrument Axis at B is Higher than at A

Let s_1 = Staff reading on BM when the instrument is at A

s_2 = Staff reading on BM when the instrument is at B

Q' = Projection of Q on horizontal line through P

Q'' = Projection of Q on horizontal line through R

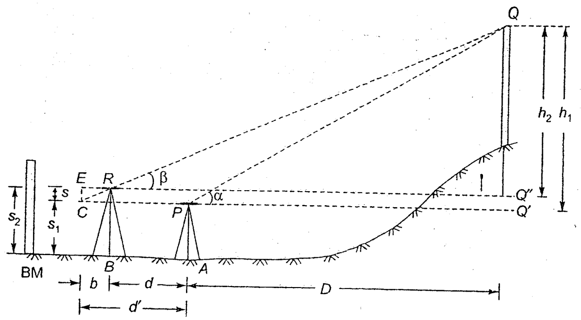


Fig. 13.2 *Instrument axis at B higher than at A*

The line of sight *QR* when extended back meets the horizontal line through *P* at *C*.

Thus $EC = s_2 - s_1 = s$ (say)

$ER = b = (s_2 - s_1)\cot\beta$

Thus $d' = d + b$

$= d + (s_2 - s_1)\cot\beta$

$= d + s\cot\beta$

In $\Delta PQQ'$ $h_1 = D\tan\alpha$

In $\Delta RQQ''$ $h_2 = (D + d)\tan\beta$

From above these two equations

$$h_1 - h_2 = s = D\tan\alpha - (D + d)\tan\beta$$

$$D(\tan\alpha - \tan\beta) = s + d\tan\beta$$

$$D = \frac{(s + d\tan\beta)}{(\tan\alpha - \tan\beta)}$$

$$= \frac{(d + s\cot\beta)\cos\alpha.\sin\beta}{\sin(\alpha - \beta)} \quad ...(13.3)$$

Now $h_1 = D\tan\alpha$

$$= \frac{(d + s\cot\beta)\sin\beta\sin\alpha}{\sin(\alpha - \beta)} \quad ...(13.4)$$

Thus RL of Q = RL of BM + $s_1 + h_1$

Case III Instrument Axis at B is Lower than at A

Proceeding as per Case II

$d' = d - b$

$= d - s\cot\beta$

$\therefore$ $h_1 = D\tan\alpha,\ h_2 = (D + d)\tan\beta$

$\therefore$ $h_2 - h_1 = D(\tan\beta - \tan\alpha) + d.\tan\beta$

$\Rightarrow$ $s - d\tan\beta = D(\tan\beta - \tan\alpha)$

$\Rightarrow$ $$D = \frac{(d - s\cot\beta)\tan\beta}{(\tan\alpha - \tan\beta)} \quad ...(13.5)$$

and $$h_1 = \frac{(d - s\cot\beta)\tan\beta\tan\alpha}{(\tan\alpha - \tan\beta)}$$

$$= \frac{(d - s\cot\beta)\sin\beta\sin\alpha}{\sin(\alpha - \beta)} \quad ...(13.6)$$

Thus RL of Q = RL of BM + $s_1 + h_1$

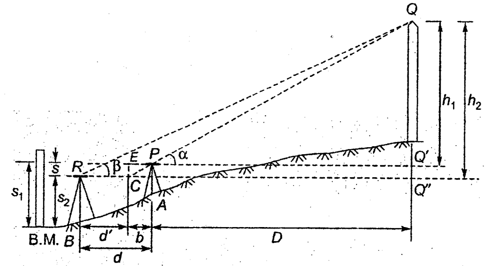


Fig. 13.3 *Instrument axis B lower than at A*

Case IV. Instrument Axes having Large Difference in Levels

In case the instrument axes at stations *A* and *B* have large difference in levels then it is not possible to have staff readings s_1 and s_2. In this case the following procedure is adopted:

1. Measure the angle α to the top of the object when the instrument is at station *A* after making all the necessary adjustments for the theodolite.
2. Plunge the telescope and mark a point *B* at a distance ***d*** from *A* in the vertical plane through *A* and *Q*.
3. Shift the instrument to station *B* and measure angle β to the top of the object when the instrument is at station *B* after making all the necessary adjustments for the theodolite.
4. Keeping the instrument at *B* in this position only, hold a staff at station *A*. Keep the height of the target on the staff as ***r*** or mark a graduation ***r*** on the staff. (Fig. 13.5)
5. Measure angle γ to the target or the staff reading at graduation ***r***.

Let s = Difference in the levels of instrument axes between *A* and *B* (Fig. 13.4)

When this difference ***s*** is known then this case is similar to **Case III** above.

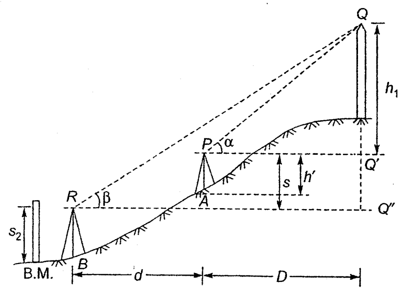


Fig. 13.4 *Large difference in levels of instrument axes*

Thus, $$D = \frac{(d - s\cot\beta)\tan\beta}{(\tan\alpha - \tan\beta)} \quad ...(13.7)$$

$$h_1 = \frac{(d - s\cot\beta)\tan\beta\tan\alpha}{(\tan\alpha - \tan\beta)}$$

$$= \frac{(d - s\cot\beta)\sin\beta\sin\alpha}{\sin(\alpha - \beta)} \quad ...(13.8)$$

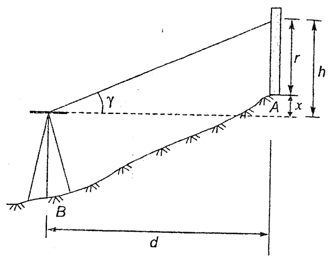


Fig. 13.5 *Angle measurement at target staff*

But in this case, the critical thing is to determine '*s*'.

Let x = Height of station A from instrument axis of B

$= h - r$

where h = Height of the target above instrument axis at B

h' = Height of instrument axis at A above the base of station A

From Fig. 13.5

$h = d\tan\gamma$

$s = x + h'$

$= (h - r) + h'$

$(D\tan\gamma - r) + h'$

Thus after knowing s, the difference of elevations h_1 can be determined from the above equations.

Now, RL of Q = RL of instrument axis at $A + h_1$

= RL of instrument axis at $B + s + h_1$

= (RL of BM + s_2) + $s + h_1$

13.3 Determination of the Height of Object when the Two Instrument Stations are not in the same Vertical Plane

As shown in Fig. 13.6 the instrument stations A and B are not in the same vertical plane. The following procedure is employed:

1. Set up the instrument at A and level it properly with respect to the altitude level.
2. Sight the top of object Q and measure the vertical angle α_1.
3. Sight the station B and measure the horizontal $\angle BAQ_1$ (= θ_1).
4. Shift the instrument to station B and level it properly.
5. Sight the top of the object Q and measure the vertical angle α_2.
6. Sight the station A and measure the horizontal $\angle ABQ_1$ (= θ_2).
7. Determine the RL of Q as:
   From triangle PQQ'

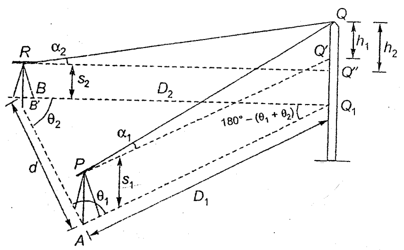


Fig. 13.6 *Two instrument stations not in the same vertical plane*

$QQ' = h_1 = D_1\tan\alpha_1$

where, D_1 = Horizontal distance between A and Q

From triangle RQQ''

$QQ'' = h_2 = D_2\tan\alpha_2$

where D_2 = Horizontal distance between B and Q

Let vertical projection of Q is Q_1 on the horizontal plane through A and the vertical projection of B is B' that too on the horizontal plane through A.

Let $\angle B'AQ_1 = \theta_1$

and $\angle AB'Q_1 = \theta_2$

From sine law in triangle $AB'Q_1$

$$\frac{\sin\theta_2}{AQ_1} = \frac{\sin\theta_1}{B'Q_1} = \frac{\sin[180° - (\theta + \theta_2)]}{AB'}$$

$$\frac{\sin\theta_2}{D_1} = \frac{\sin\theta_1}{D_2} = \frac{\sin[180° - (\theta_1 + \theta_2)]}{d}$$

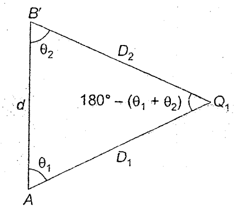


Fig. 13.7 *Horizontal plane through A, B' and Q_1*

Thus, $$D_1 = \frac{d\sin\theta_2}{\sin(\theta_1 + \theta_2)} \quad ...(13.9)$$

and $$D_2 = \frac{d\sin\theta_1}{\sin(\theta_1 + \theta_2)} \quad ...(13.10)$$

Thus, $$h_1 = D_1\tan\alpha_1 = \frac{d\sin\theta_2.\tan\alpha_1}{\sin(\theta_1 + \theta_2)} \quad ...(13.11)$$

Thus RL of Q = RL of instrument axis at $A + h_1$

= RL of $BM + s_1 + h_1$

where s_1 = Staff reading at BM when the instrument is at A

For checking purpose, take a staff reading s_2 on the staff held at BM when the instrument is at B.

Thus, $$h_2 = D_2\tan\alpha_2 = \frac{d\sin\theta_1.\tan\alpha_1}{\sin(\theta_1 + \theta_2)} \quad ...(13.12)$$

RL of Q = RL of instrument axis at $B + h_2$

= RL of $BM + s_2 + h_2$

The two values giving the level of Q should be the same. In case there is the difference in the values then take mean of the two values.

13.4 Indirect Levelling on a Steep Slope

The following procedure is adopted to assess the difference of elevations between the points P and Q lying on a step slope:

1. Select a suitable turning point.
2. Set up the instrument at any convenient station say O_1 on the line PR.
3. Make the line of collimation approximately parallel to the ground surface. Clamp the telescope in this position.
4. Take back sight PP' on the staff held at P and measure the vertical angle α_1 to P'. Determine the RL of P' as:

   RL of P' = RL of $P + PP'$

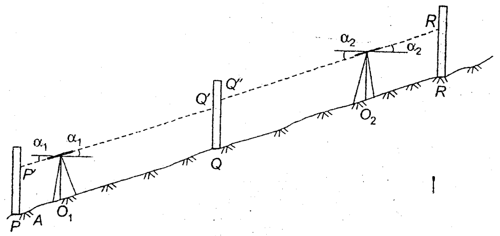


Fig. 13.8 *Indirect levelling on steep slope*

5. Take foresight *QQ'* on the staff held at turning point *Q* without changing the vertical angle α_1. Measure distance between *P* and *Q* i.e. *PQ*.

   Now $\qquad RL$ of $Q = RL$ of $P' + PQ \sin\alpha_1 - QQ'$

   RL of $Q = RL$ of $P + PP' + PQ \sin\alpha_1 - QQ'$

6. Now shift the instrument to station O_2 somewhere midway between *Q* and *R*.

   Make the line of collimation approximately parallel to the slope of the ground surface. Clamp the telescope in this position.

7. Take back sight *QQ"* on the staff held at turning point *Q* and measure the vertical angle α_2.

   Now $\qquad RL$ of $Q'' = RL$ of $Q + QQ''$

8. Take foresight *RR'* on the staff held at point *R* without changing the vertical angle α_2. Measure the sloping distance *QR*.

   Now $\qquad RL$ of $R = RL$ of $Q'' + QR \sin\alpha_2 - RR'$

   RL of $R = (RL$ of $P + PP'\ PQ \sin\alpha_1 - QQ') + QQ'' + QR \sin\alpha_2 - RR'$

In case there are some intermediate points whose elevations are required then a slightly different procedure is followed. Let there be an intermediate point *A* at a sloping distance *PA* from *P*. The *RL* of point *A* can be determined from the setting of the instrument at O_1.

$$RL \text{ of } A = RL \text{ of } P' + PA \sin\alpha_1 - AA'$$

where $\qquad AA'$ = Intermediate sight on the staff held at *A*.

13.5 Effect Due to Refraction

- When stations in the trigonometric leveling are far apart then it is required to correct the observed readings for refraction and curvature of earth.
- These corrections may be applied either on the observed vertical angles or in the linear measurements to the computed apparent difference of elevations of the stations.
- Refraction makes the object to appear higher than what it actually is. This effect decreases the staff reading. In case of plane surveying, the correction is applied linearly to all the staff readings while in geodetic trigonometric surveying, this correction is applied to the observed angles.

In Fig. 13.9 the two points under consideration viz. *P* and *Q* are quite far apart and thus the line of sight *PQ* is not a straight line but is curved. This line is concave towards the ground surface. Lines *PP'* and *QQ'* indicate the apparent sights from *P* and *Q* respectively.

Now, it is well known that vertical angles are measured from the horizontal plane, and thus the observed vertical angles at *P* and *Q* are $\angle P'PO'$ and $\angle Q'QO''$ respectively being denoted as α_1 and β_1 respectively.

In case there is no refraction then the correct angle of elevation at $P = \angle QPO'$ and therefore the correction due to refraction (r) is $\angle P'PQ$.

Thus correct angle $\alpha = \alpha_1 - \angle P'PQ = \alpha_1 - r$

Here the angle r is called as the angle of refraction.

Similarly in case of depression angles, the correct angle of depression in the absence of refraction is $\angle PQO''$ and therefore the correction for refraction (r) is $\angle PQQ'$.

Thus correct angle $\beta = \beta_1 + \angle PQQ'' = \beta_1 + r$

This correction for refraction is positive for depression angles and negative for elevation angles.

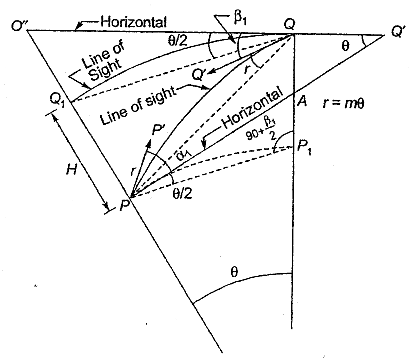


Fig. 13.9 *Effect of refraction*

13.5.1 Coefficient of Refraction

The coefficient of refraction (m) is the ratio of angle of refraction (r) and the angle (θ) subtended at the earth's center by the distance over which observations of the stations were made.

Thus, $\qquad m = \dfrac{r}{\theta}$

$$r = m\theta = \frac{mD}{R}\text{radians} = \frac{mD}{R\sin 1''}\text{ seconds} \qquad ...(13.13)$$

where $\qquad R$ = radius of earth (= 6367 km)

The value of m varies between 0.06 to 0.08 and usually a mean value of 0.07 is taken.

13.5.2 Refraction Correction

Here the value of angle *r* depends on the relative magnitudes of distance *D* and the difference in elevation *H*.

Case-I *D* is small but *H* is very large

In this particular case, the angle of elevation is α_1 and the angle of depression is β_1.

From Fig. 13.9 $\qquad \angle PQO'' = \theta$

From $\Delta PQO'$, ext. $\angle PQO''$ is given as

$$\angle PQO'' = \angle QPO'' + \angle QO'P$$

$$\beta_1 + r = (\alpha_1 + r) + \theta$$

$$\Rightarrow \quad 2r = \theta + \alpha_1 + \beta_1$$

$$\Rightarrow \quad 2r = \theta - (\beta_1 - \alpha_1)$$

$$\Rightarrow \quad r = \frac{\theta}{2} - \left(\frac{\beta_1 - r_1}{2}\right) \qquad ...(13.14)$$

$$\Rightarrow \quad m\theta = \frac{\theta}{2} - \left(\frac{\beta_1 - r_1}{2}\right)$$

$$\Rightarrow \quad \beta_1 = \alpha_1 + \theta(1 - 2m)$$

$$\Rightarrow \quad \beta_1 - \alpha_1 = \theta(1 - 2m) \qquad ...(13.15)$$

This implies that observed angle of depression β_1 is larger than observed angle of elevation α_1 by an amount equal to β_1.

Case-II *D* **is very large but** *H* **is small**

Here in this case, both the angles α_1 and β_1 are the angles of depression.

Thus substitute $-\alpha_1$ in place of α_1 in the expression of **Case-I** we have,

$$r = \theta/2 - (\beta_1 + \alpha_1)/2$$
$$m\theta = \theta/2 - (\beta_1 + \alpha_1)/2$$
$$\beta_1 + \alpha_1 = \theta(1 - 2m) \qquad ...(13.16)$$

13.6 Effect Due to Curvature of Earth

- The curvature effect makes the object to appear lower than the object actually is.
- In plane surveying, it increases the staff reading and correction is therefore negative. In geodetic trigonometric leveling, as the various points are widely distributed, the curvature correction is applied directly to the observed angles.

In **Fig. 13.9** the angle α_1 was being measured with respect to the horizontal line *PO'*. The same angle with respect to level surface is $\angle P'PP_1$. This can be measured with respect to chord PP_1 where P_1 is the vertical projection of *Q* on a level line passing through *P*. The curvature correction for earth is given by,

$$C_c = \angle O'PP_1 = \frac{\theta}{2}$$

For elevation angles, this correction is positive.

Similarly, the angle β_1 should be measured with respect to chord QQ_1.

Thus curvature correction is,

$$C_c = \angle O''QQ_1 = \frac{\theta}{2}$$

For depression angles, this correction is negative.

Now, $$\frac{\theta}{2} = \frac{D}{2R} \text{ radians}$$

Also, $$1 \text{ radian} = 206265 \text{ seconds}$$

Thus, $$\frac{\theta}{2} = \frac{D}{2R\sin 1''} \text{ seconds} \qquad ...(13.17)$$

13.7 Total Correction Due to Curvature and Refraction

The total or the combined correction due to curvature and refraction is given by,

$$C = C_c + C_r$$
$$= \frac{\theta}{2} - r$$
$$= \frac{D}{2R\sin 1''} - \frac{mD}{R\sin 1''}$$
$$= \frac{(1-2m)D}{2R\sin 1''} \qquad ...(13.18)$$

This combined correction is positive for elevation angles and negative for depression angles.

13.8 Axis Signal Correction

- If the height of the signal or the target is not equal to that of the instrument axis then the measured angles are to be corrected by a correction called as axis signal correction which is also known as eye object correction.
- Let h_1 and h_2 be the heights of the instrument at *P* and *Q* respectively and s_1 and s_2 be the heights of the targets at *P* and *Q* respectively.

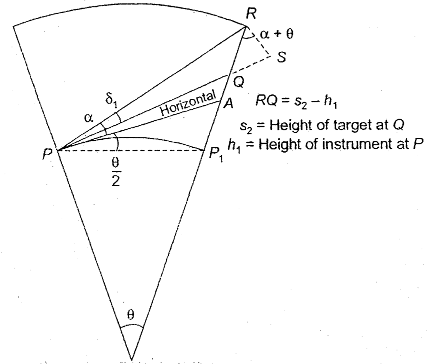


Fig. 13.10 *Axis signal correction*

Let *R* be the top of signal at *Q*.

Thus $RQ = s_2 - h_1$

Let $\alpha = \angle RPA$ = Observed angle from *P* to *Q*

$\delta_1 = \angle RPQ$ = Axis signal correction at *P*

Draw *RS* perpendicular to *PR* at *R* to meet *PQ* produced at *S*.

O is the center of earth in ΔPRO

$\therefore \quad \angle RPO = \angle RPO + \angle APO = \alpha + 90°$

$\angle PRO = 180° - \angle RPA - \theta$

$= 180° - (\alpha + 90°) - \theta = 90° - (\alpha + \theta)$

Thus, $\quad \angle QRS = 90° - \angle PRS = 90° - [90° - (\alpha + \theta)] = (\alpha + \theta)$

$\because \delta_1$ is very small, $\angle RSQ \simeq 90°$

$\therefore \quad RS = RQ\cos(\alpha + \theta) = (s_2 - h_1)\cos(\alpha + \theta)$

In $\angle PRP_1 \quad \angle PRP_1 = \alpha + \theta/2$

$= 180° - (\alpha + \theta/2) - 90° + (\alpha + \theta)$

$= 90° + \theta/2$

From sine rule,

$$\frac{\sin\angle PP_1R}{PR} = \frac{\sin\angle PRP_1}{PP_1}$$

$$\Rightarrow \quad PR = PP_1 \cdot \frac{\sin\angle PP_1R}{\sin\angle PRP_1} = D \cdot \frac{\sin(90° + \theta/2)}{\sin[90° - (\alpha + \theta)]}$$

$$= \frac{D \cdot \cos\theta/2}{\cos(\alpha + \theta)} \qquad ...(13.19)$$

In ΔPRS, $$\tan\delta_1 = \frac{PS}{PR} = \frac{(s_2 - h_1)\cos(\alpha + \theta)}{\dfrac{D\cos\theta/2}{\cos(\alpha + \theta)}} = \frac{(s_2 - h_1)\cos^2(\alpha + \theta)}{D \cdot \cos\theta/2} \qquad ...(13.20)$$

Now $\frac{\theta}{2}$ is very small

$\Rightarrow \quad \cos\theta/2 \simeq 1$

$$\therefore \quad \tan\delta_1 \simeq \frac{(s_2 - h_1)\cos^2\alpha}{D} \qquad ...(13.21)$$

For deviation angles, connection δ_1 is negative

Similarly if observations are taken from Q towards P,

$$\tan s_2 = \frac{(s_2 - h_1)\cos^2\beta}{D}$$

For depression angles, correction δ_2 is positive

Now if angles α and β are very small then

$$\tan \delta_1 \simeq \frac{s_2 - h_1}{D} \text{radians}$$

$$\Rightarrow \quad \delta_1 = \frac{s_2 - h_1}{D \sin 1''} \text{seconds}$$

$\therefore$ Corrected angles for axis-signal correction are,

$$\alpha_1 = \alpha - \delta_1$$

and

$$\beta_2 = \beta + \delta_2$$

where β = observed angle of depression α = observed angle of deviation.

Illustrative Examples

Example 13.1 A theodolite was set-up at station A and staff at station B. The distance between A and B is 2.8 km. If RL of instrument axis was 171.45 m and angle of elevation at point 3.5 m above the foot of staff was 10° 45′ then determine the RL of station B.

Solution:

Height of vane above the line of collimation (H)

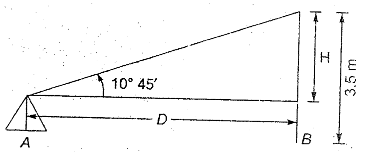
$= D\tan\theta$

$= 2800 \tan 10° 45' = 531.6$ m

$\therefore$ RL of B = RL of instrument axis + $H - 3.5$

$= 171.45 + 531.6 - 3.5$

$= 699.55$ m



Example 13.2 Find the RL of the top of the flag post located over a building. The angles of elevation from a point on the ground to the top and bottom of flag post are 7° and 15° respectively. The height of the flag post is 3 m and a reading of 2.685 m was taken at a BM of RL 110 m.

Solution:

From ΔP_1TR, $\quad \tan 7° = \dfrac{RT}{x}$

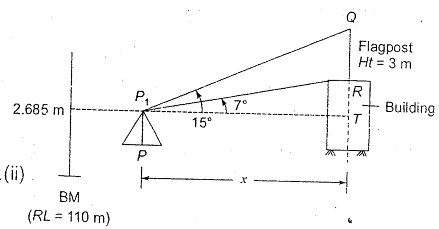
$\Rightarrow \quad RT = x\tan 7°$ ...(i)

From ΔP_1TQ, $\quad \tan 7° = \dfrac{RT}{x}$

$\Rightarrow \quad \tan 15° = \dfrac{QT}{x} = \dfrac{QR + RT}{x} = \dfrac{3 + RT}{x}$ ...(ii)

Form (i) and (ii)

$x \tan 7° = x \tan 15° - 3$



$\Rightarrow \quad x = 20.67$ m

$\therefore \quad RT = x\tan 7° = 2.54$ m

$\therefore$ RL of flag-post top, $\quad Q = 110 + 2.685 + RT + 3$

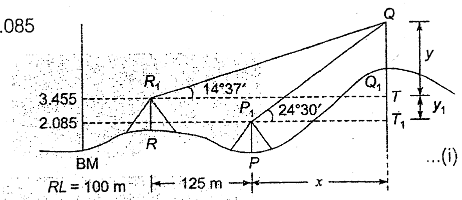
$= 110 + 2.685 + 2.54 + 3 = 118.225$ m

Example 13.3 There is a signalling tower on the top of a hill. In order to determine the elevation of the tower top Q, observations were made from two stations P and R. Points P, Q and R lie in the same plane. If angles of elevation of the top point Q from P and R are respectively 24° 30′ and 14° 37′, determine the elevation of foot of tower of signal tower height is 4 m. The staff readings on BM of RL 100 m are 2.085 m and 3.455 m respectively from instrument located at P and R. Distance between P and R is 125 m.

Solution:

$TT_1 = y_1 = 3.455 - 2.085$

$= 1.37$ m



In $\Delta P_1 T_1 Q$, $\quad \tan 24° 30' = \dfrac{y + y_1}{x}$

$\Rightarrow \quad 0.45573 = \dfrac{y + 1.37}{x}$ ...(i)

In $\Delta R_1 T_1 Q$, $\quad \tan 14° 37' = \dfrac{y + y_1}{x}$

$\Rightarrow \quad 0.26079 = \dfrac{y}{125 + x}$ ...(ii)

Solving (i) and (ii)

$x = 174.25$ m

$y = 78.04$ m

But $\quad y = QQ_1 + Q_1T = 4 + Q_1T$

$\Rightarrow \quad 78.04 = 4 + Q_1T$

$\Rightarrow \quad Q_1T = 74.04$ m

$\therefore$ RL of foot of signal tower = RL of BM + staff reading when the + Q_1T instrument was at R

$= 100 + 3.455 + 74.04 = 177.495$ m

Example 13.4 Find the RL of a tower (C) from the following data obtained from stations A and B which are 60 m apart.

$\angle BAC = 60°$ $\qquad \angle ABC = 50°$

Angle of elevation from A to top of tower = 32°

Angle of elevation from B to top of tower = 28°

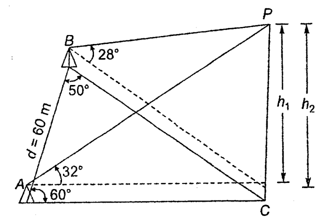
Staff reading from A on BM of RL 80.00 = 2.350 m

Staff reading from B on the same BM = 2.850 m

Solution:

$$h_1 = \frac{d\sin 50° \tan 32°}{\sin(50° + 60°)} = \frac{60 \sin 50° \tan 32°}{\sin 110°} = 30.56 \text{ m}$$

$$h_2 = \frac{60 \sin 60° \tan 28°}{\sin(50° + 60°)} = 29.4 \text{ m}$$



$\therefore$ RL of tower top P = RL of instrument axis at $A + h_1$

$= (80.00 + 2.35) + 30.56 = 112.91$ m

Alternatively, RL of tower top P = RL of instrument axis at $B + h_2$

$= (80.00 + 2.850) + 29.4 = 112.25$ m

Thus there is a small difference in the two values.

$$\therefore \quad \text{Average RL of tower top P} = \frac{112.91+112.25}{2} = 112.58 \text{ m}$$

Objective Brain Teasers

Q.1 In trigonometric leveling, which of the following correction is subtracted from the observed vertical angle?

(i) Refraction correction
(ii) Curvature correction
(iii) Axis signal correction

(a) (i) only (b) (i) and (ii)
(c) (ii) and (iii) (d) (iii) only

Q.2 Which of the following correction is relevant with regards to trigonometric levelling?

(a) Correction for dip
(b) Correction for parallax
(c) Correction for curvature
(d) Axis signal correction

Q.3 Trigonometric levelling is ________ method of levelling

(a) Direct (b) Indirect
(c) Precise (d) Both (a) and (b)

Q.4 The following instruments are used in trigonometric levelling

(a) Theodolite, tape (b) Sextant, tape
(c) Theodolite, chain (d) Clinometer, chain

Q.5 Trigonometric levelling is most suitable for

(a) Plain terrain
(b) sloping terrain
(c) undulating and rugged terrain
(d) All of the above

Answers

1. (b) 2. (d) 3. (b) 4. (a) 5. (c)

Student's Assignments

Ex.1 A considerable different levels, stations A and B being in plane with top of the chimney. The angles of elevation to the top of chimney from A and B are 37° 22′ and 22° 18′ respectively. The angle of elevation from B to a vane 2 m above the foot of staff held at A is 15° 30′. The instrument heights at A and B are 1.59 m and 1.37 m respectively. Given AB = 135 m and RL of B = 115.380 m. Determine the RL of top of chimney and the horizontal distance of chimney from A.

Ex.2 Determine the RL of top of a tower from the following observations.

Inst. Station	Vertical angle of top of tower	Staff reading on BM	RL of BM	Remarks
A	18° 35′	2.835 m	100.00 m	AB = 70 m
B	11° 15′	1.395 m		

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