16

Surface Area and Volume

16.01. Introduction

In the previous chapter we have studied the methods to find area of plane figures *e.g.* ,triangle, rectangle, circle, etc. In this chapter we will study about solid figures. The bricks, match-box, plane walls of house, water tanks, cricket ball etc. are different shapes of solid figures. Main difference between plane figure and solid figure is that plane figures are completely lie in plane whereas solid figures do not lie in a plane. Solid figures lie in space. Solid (cubical figures) are three dimensional.

The surface area of solid figures means sum of the area of all the surfaces and volume is the measure of the amount of space inside a solid figure. Area is measured is square units and volume in cubic units.

16.02. Surface Area and Volume of Cube and Cuboid

A match box, room, chalk box, brick etc., are examples of cuboid. In the figure 16.01 cuboid has shown, which has 6 faces, each face is rectangular in a plane, opposite faces are parallel and congruent and contains three pair of parallel faces. Two adjoining faces meets at a same line segment which is called edges. A cuboid has 12 edges.

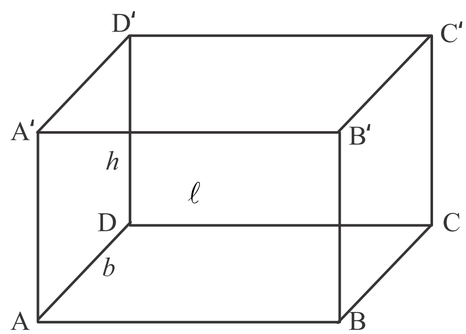

Fig. 16.01

Let $AB = A'B' = D'C' = DC = \ell$

$AD = A'D' = B'C' = BC = b$

$AA' = DD' = BB' = CC' = h$

Let we assume ℓ , b , h as length, breadth and height of the solid figurethen these perdendicular edges form a cuboid.

'If in six parallel faces, all are rectamgular, then it is called cuboid.

Cuboid is also called rectangular face solid *e.g.,* brick, box, room etc.

In cuboid, area of face $ABCD$ = Area of face $A'B'C'D'$ $= \ell \times b$

In cuboid, area of face $ADD'A'$ = Area of face $BCC'B'$ $= b \times h$

In cuboid, area of face $ABB'A'$ = Area of face $DCC'D' = h \times \ell$

Thus, total surface area of the cuboid

= 2 (Area of $ABCD$ + Area of $ADD'A$ + Area of $ABB'A'$)

$= 2(\ell \times b + b \times h + h \times \ell)$

$= 2(\ell b + bh + h\ell)$ square units

If height of cuboid is zero then it will take the form of rectangle.

16.03. Cube

When length, breadth and height of cuboid are same *i.e.,* $\ell = b = h \neq 0$ then it is called a cube. All the faces of cube are square and area of each face is same. Figure 16.02 shows a cube. If ℓ is the length of side of a cube then its total surface area

$= 2(\ell \times \ell + \ell \times \ell + \ell \times \ell)$

$= 2(\ell^2 + \ell^2 + \ell^2)$

$= 2 \times 3\ell^2$

$= 6\ell^2$ square units

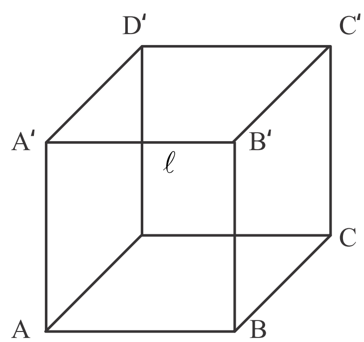

Figure 16.02

Now we will develop the method to find the volume of cube or cuboid.

According to fig. 16.03 take a cube of edge 1unit such cube is called unit cube. 1 cubic cm can be written as 1 cm^3.

Similarly volume of cuboid is measured in cube unit since length, breadth and height of cuboid and different so volume of cuboid will be their product.

Volume of cuboid *(V)* = $\ell \times b \times h$ cube unit

Where ℓ = length, b = breadth, h = height

Volume *(V)* = $\ell \times b \times h$ cube unit

$\because$ length, breadth and height of cube are same

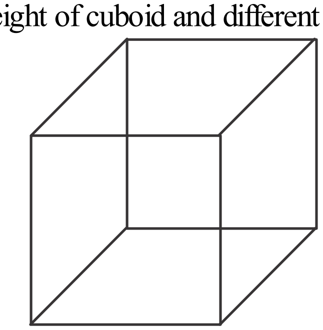

Figure 16.03

Volume of cube = $\ell \times \ell \times \ell = \ell^3$ cube unit

16.04. Diagonal of Cube and Cuboid

The line of adjoining the two opposite vertices of parallel face of cube or cuboid is called diagonal. So there are 4 diagonals.

Note : If ℓ, b, h are length, breadth and height of cuboid then,

Diagonal of cuboid = $\sqrt{\ell^2 + b^2 + h^2}$ units,

For cube, $\ell = b = h$

$\therefore$ diagonal of cube $= \ell\sqrt{3}$ units,

Units related to volume

(i) 1 litre = 1000 cubic cm.

(ii) 1 cubic cm = 10 × 10 × 10 cubic mm. = 1000 cubic mm

(iii) 1 cubic m = 100 × 100 × 100 cubic cm = 100,00,00 cubic cm

(iv) 1 cubic m = 1000 litre = 1 kilo litre

Example 1. The length, breadth and height of a closed wooden box are 90 cm, 50 cm, and 30 cm, respectively. Find the outer surface area of the box.

Solution : Length of box = 90 cm

Breadth of box = 50 cm

Height of box= 30 cm

Total outer surface area of box = 2 (ℓ ×b + b × h + h × ℓ)

$$= 2(90 \times 50 + 50 \times 30 + 30 \times 90) \text{ sq. cm.}$$

$$= 2(4500 + 1500 + 2700) \text{ sq. cm.}$$

$$= 2(8700) \text{ sq. cm.}$$

$$= \frac{17400}{10000} = 1.74 \text{ sq. cm.}$$

Example 2. Total surface area of a cube is 1014 m^2. Find the side of the cube.

Solution : Total surface area of a cube = 1014 sq.m.

Let side of cube $= x$ m

Total surface area of cube = 6 $(\text{side})^2$

$\therefore$ $6x^2 = 1014$

or $x^2 = \frac{1014}{6} = 169$

or $x = \sqrt{169} \text{ m} = 13$ m

Example 3. If length of cuboid is 12 m, breadth 9 m and height is 8 m then find its diagonal length.

Solution : Length of cuboid = 12 m

Breadth of cuboid = 9 m

Height of cuboid = 8 m

we know that diagonal of cuboid= $\sqrt{(\ell)^2+(b)^2+(h)^2}$

$$=\sqrt{(12)^2+(9)^2+(8)^2} \text{ m}$$

$$=\sqrt{144+81+64} \text{ m}$$

$$=\sqrt{289}=17 \text{ m}$$

Example 4. Length of a room is 5 m, breadth 3.5 m and height is 4 m. Find the cost of whitewash at four walls at the rate ₹20 per square meter.

Solution : Length of room = 5 m

Breadth = 3.5 m

Length = 4 m

Area of 4 walls of room $= 2\,(\ell + b)\,h$

$$=2(5+3.5)\times 4 \text{ sq. m}$$

$$=2\times 8.5\times 4 \text{ sq.m}$$

$$= 68 \text{ sq.m.}$$

Cost of whitewash at four walls = ₹ 68×20

or cost = ₹1360

Example 5. The perimeter of a surface of cube is 28 cm then find volume of the cube.

Solution : Preimeter of a face of cube = 28 cm.

∵ All sides of cube are equal

∴ Preimeter of 1 face = 4 × side

or 28 = 4 × side

or side $= \dfrac{28}{4} cm = 7$ cm.

Volume of cube = $(\text{face})^3 = (7)^3$

$$= 7\times 7\times 7 = 343 \text{ cube cm.}$$

Example 6. If ratio of length, breadth and height of a cuboid is 6 : 5 : 4 and its total surface area is 33300 sq. cm. then find the volume of the cuboid.

Solution : Let length, breadth and height of cuboid are $6x$, $5x$, $4x$ respectively ,

Area of cuboid = 33300 sq. cm.

∴ $2(\ell b+bh+h\ell)=33300$

or $2(6x\times 5x+5x\times 4x+4x\times 6x)=33300$

or $2(30x^2+20x^2+24x^2)=33300$

or $2\times 74x^2=33300$

or $x^2=\dfrac{33300}{2\times 74}$

or $x^2 = 225$

$\therefore$ $x = \sqrt{225} = 15$

or $x = 15$ cm

$\because$ ℓ ? 6? 15 cm = 90 cm, $b = 5 \times 15$ cm = 75 cm, $h = 4 \times 15$ cm = 60 cm.

Required volume $= 90 \times 75 \times 60$ cm^3

= 405000 cm^3

Example 7. The dimensions of a box are 3 m × 2 m × 1.80 m. Find the cost of varnishing at all the outer surfaces of the box at the rate of ₹ 12 per m^2.

Solution : Dimensions of box

ℓ = 3 m, b = 2 m, h = 1.80 m.

Total surface area of box = 2 [$\ell \times b + b \times h + h \times \ell$]

= 2 [3 × 2 + 2 × 1.80 + 1.80 × 3] m^2

= 2 [6 + 3.60 + 5.40] m^2

= 2 [6 + 9] m^2 = 2 × 15 m^2 = 30 m^2

Cost of varnishing at 30 m^2 area = ₹ 30 × 12 = ₹ 360

Example 8. Three metalic cubes are of edges 3 cm, 4 cm, and 5 cm. By melting these cubes a new cube is formed. Find the edge of this new cube.

Solution : Volume of cube of side 3 cm = (side)^3 = 3^3 cm^3 = 27 cm^3

Volume of cube of side 4 cm = (side)^3 = $(4)^3$ cm^3= 64 cm^3

Volume of cube of side 5 cm = (side)^3 = $(5)^3$ cm^3= 125 cm^3

Total volume of these cubes = (27 + 64 + 125) cm^3 = 216 cm^3

By melting these cubes, new cube is formed

$\therefore$ Volume of new cube = 216 cm^3

(side)^3 = 216

side $= \sqrt[3]{216} = (6 \times 6 \times 6)^{1/3} = 6$

$\therefore$ Edge of new cube is 6 cm.

Exercise 16.1

1. The length of a cube is 12 cm, breadth is 2 cm and height is 5 cm. Find the total surface area of cuboid and volume of the cuboid.
2. The edges of three cubes are 8 cm, 6 cm, and 1 cm respectively. After melting these cubes a new cube is formed. Find the total surface area of the new cube.
3. The dimensions of a box are 50 cm × 36 cm × 25 cm. How much sq. cm. cloth will be required for making cover of this box ?
4. Each surface area of a cube is 100 cm^2. The cube is cut into two equal parts by a plane which is parallel to the base then find total surface area of equal parts. Then find total surface area of equal part.
5. A box without lid is made by wood of thickness 3 cm. Its outer length is 146 cm, breadth is 116 cm and height is 83 cm. Find the cost of painting the internal surface of the box at the rate ₹ 2 per 1000 sq. cm.
6. The sum of length, breadth and height of cuboid is 19 cm and length of its diagonal is 11 cm. Find the total surface area of the cuboid.
7. A room with square floor of side 6 m contains 180 m^3 air. Find the height of the room.

8. How many bricks are required to make a wall of dimensions 44 m long, 1.5 m height and 35 cm broad. If dimension of 1 brick is 22cm × 10 cm × 7 cm.
9. Find the maximum length of rod which can be kept in a room of size 10 m × 8 m × 6 m.
10. The volume of a cube is 512 cubic metre. Find its edge.
11. 5 m, 30 cm and 3 m are length, breadth and height of a wall respectively. How many bricks of dimensions 20 cm × 10 cm × 7.5 cm will be required to make the wall.
12. The ratio of length, breadth and height of a cubid is 5 : 3 : 2. If total surface area of cuboid is 558 cm^2, then find their edges.

16.05. Surface Area and Volume of Cylinder

You have seen the measuring jar, test tube, round pipe. etc. The objects which have one lateral curved surface and congruent circular cross section is called circular cylinder. The line, joining the centres of circular cross-sections is called axes of cylinder. If the axis of cylinder is perpendicular to circular cross-section then cylinder is called right circular cylinder. In this chapter , whenever the word cylinder is used it means right circular cylinder.

If $ABCO$ is a rectangular area it rotates about line OA then we get a solid cylinder of radius r and height h as shown in fig. 16.04.

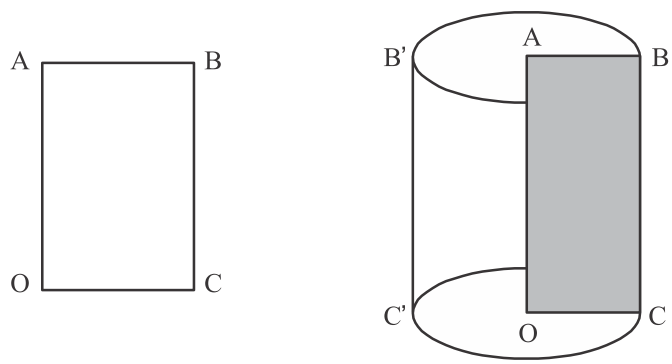

Fig. 16.04

The lines which are parallel to OA and lies on the curved surface of the cylinder are called generators. Here lines BC, B'C' are generators. On placing the cylinder vertically then the downward circular end is called base of the cylinder. In the figure 16.04 CB is the height of the cylinder. The radius of the circular ends is called radius of the cylinder.

In hollow cylinder both the ends are open whereas in solid cylinder they are closed. To find curved surface area of cylinder of radius r and height h (fig. 16.05) , if we cut curved surface along the line segment BC and spread it then it will take shape of a rectangle $BCDE$

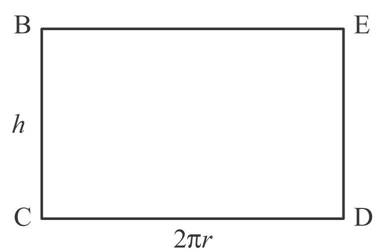

Fig. 16.05

When sheet is in the form of curved surface of cylinder then line segment *BC* and *ED* coincides so

∴ Area of curved surface of cylinder

$$= \text{Area of rectangle BCDE}$$
$$= 2\pi r \times h$$
$$= 2\pi rh$$

Area of base if cylinder $= \pi r^2$

Total surface area of cylinder = Curved surface+ 2 Area of base

$$= 2\pi rh + 2\pi r^2$$
$$= 2\pi r(h+r)$$

The space occupied by cylinder is called its volume. If radius of cylinder is *r* and height is *h* then

Volume of cylinder = Area of base × height

$$= \pi r^2 \times h$$
$$= \pi r^2 h$$

If outer radius of a hollow cylinder is r_1 and inner radius is r_2 also height is h then in fig. 16.06, volume of hollow cylinder.

$$= \pi r_1^2 h - \pi r_2^2 h$$
$$= \pi\left(r_1^2 - r_2^2\right)h$$

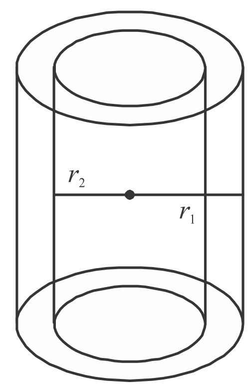

Fig. 16.06

Total surface area of hollow cylinder

$$= 2\pi r_1 h + 2\pi r_2 h + 2\pi r_1^2 - 2\pi r_2^2$$
$$= 2\pi h(r_1 + r_2) + 2\pi\left(r_1^2 - r_2^2\right)$$
$$= 2\pi h(r_1 + r_2) + 2\pi(r_1 + r_2)(r_1 - r_2)$$
$$= 2\pi(r_1 + r_2)(h + r_1 - r_2)$$

Illustrative Examples

Example 1. The area of base of a cylinder is 154 cm^2 and height is 21 cm. Find its volume and curved surface area.

Solution : Area of base of cylinder = 154 cm^2

Height of cylinder (h) = 21 cm

Volume of cylinder = Area of base × height

= 154 × 21 cm^3

= 3234 cm^3

∴ Area of base $= \pi r^2$

or $$154 = \frac{22}{7} \times r^2$$

or $$154 \times 7 = 22 \times r^2$$

or $$r^2 = \frac{154 \times 7}{22} = 49$$

or $$r = \sqrt{49} \text{ cm} = 7 \text{ cm}$$

Curved surface area of cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 21 \text{ cm}^2$$

$$= 2 \times 22 \times 21 \text{ cm} = 924 \text{ sq. cm.}$$

Example 2. The height of a cylinder is 11 cm amd its curved surface area is 968 cm^2. Find the radius of cylinder.

Solution : Height of cylinder $= 11$ cm

Let radius of cylinder $= r$

Curved surface area of cylinder $= 2\pi rh = 968$

or $$2 \times \frac{22}{7} \times r \times 11 = 968$$

$$r = \frac{968 \times 7}{2 \times 22 \times 11} \text{ cm} = 14 \text{ cm.}$$

Example 3. If volume of cylinder is 448 π cm^3 and height is 7 cm then find curved surface area of cylinder.

Solution : Volume of cylinder $= \pi r^2 h$

∴ $$448\pi = \pi \times r^2 \times 7$$

or $$448 = 7r^2$$

or $$r^2 = \frac{448}{7} = 64$$

or $$r = \sqrt{64} = 8 \text{ cm}$$

Curved surface area of cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 8 \times 7 \text{ sq.}$$

$$= 44 \times 8 = 352 \text{ sq. cm.}$$

Total surface area of cylinder $= 2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 8(7 + 8) \text{ sq.}$$

$$= 2 \times \frac{22}{7} \times 15 \times 8 \text{ sq.cm} = \frac{5280}{7} \text{ sq. cm} = 754.28 \text{ sq. cm.}$$

Example 4. A hollow cylinder is of height 21 decimetre. Its outer and inner diameter are 10 cm and 6 cm respectively. Find its volume.

Solution : Height of hollow cylinder $= 21$ dm. [$\because$ 10 cm = 1 dm]

$$= 21 \times 10 = 210 \text{ cm.}$$

Outer diameter of hollow cylinder $= 10$ cm.

$\therefore$ Outer radius $(r_1) = \dfrac{\text{diameter}}{2} = \dfrac{10}{2}\text{cm} = 5\text{cm.}$

Inner diameter of hollow cylinder $= 6$ cm.

Inner radius $(r_2) = \dfrac{6}{2}\text{cm} = 3\text{cm.}$

Volume of hollow cylinder $= \pi\left(r_1^2 - r_2^2\right)h$

$$= \frac{22}{7}\left[(5)^2 - (3)^2\right] \times 210 \text{ cm}^3$$

$$= \frac{22}{7}[25 - 9] \times 210 \text{ cm}^3$$

$$= \frac{22}{7} \times 16 \times 210 \text{ cm}^3 = 10560 \text{ cm}^3$$

Example 5. The ratio of radius and height of a cylinder is 1 : 3. If volume of cylinder is 3234 cm^3 then find total surface area of cylinder.

Solution : Let r is radius of cylinder and height is 3 r

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times r^2 \times 3r = 3234$$

$$r^3 = \frac{3234 \times 7}{22 \times 3}$$

or $r^3 = 343$

or $r^3 = (7)^3$

or $r = 7$

Thus, height of cylinder $3 \times 7\text{cm} = 21\text{cm.}$

Total surface area of cylinder $= 2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 7(21 + 7) \text{ cm}^2$$

$$= 2 \times 22 \times 28 \text{ cm}^2$$

$$= 1232 \text{ cm}^2.$$

Example 6. Length of a roller is 2 m and its diameter is 1.4 m. Find, how much field will be flat by roller in 5 revolutions ?

Solution : Length of roller (h) = 2 m

diameter of roller = 1.4 m

Radius of roller $= \frac{1.4}{2} = 0.7$ m

Area of flat surface by roller in 1 revolution = Curved surface area

$$\therefore \quad 2\pi rh = 2\times\frac{22}{7}\times 0.7\times 2 \text{ m}^2$$

$$= 2\times 22\times\frac{1}{10}\times 2 \text{ m}^2$$

$$= 8.8 \text{ m}^2$$

$\therefore$ In 5 revolution required flat area = 8.8 × 5 = 44 m^2

Exercise 16.2

1. The diameter of a cylinder is 14 cm and height is 15cm. Find the total surface area of cylinder and its volume.
2. Find the curved surface area, total surface area and volume of a right circular cylinder having radius of the base 3 cm and height 7 cm.
3. Find the curved surface area and volume of cylinder whose height is 21 cm and area of its one end is 154 cm^2 .
4. Find the ratio of curved surface area and volume of two right circular cylinder whose radius are in ratio 2: 3 and heights are in the ratio 5 :4.
5. The total surface area of a solid cylinder is 462 cm^2 . Its curved surface area is one third of total surface area. Find the volume of cylinder.
6. Find the volume of cylinder whose curved surface area is 660 cm^2 and height is 15 cm.
7. If the volume of a cylinder is $30\pi cm^3$ and area of base is $6\pi cm^2$, then find height of cylinder.
8. The volume and curved surface area of a cylinder are 1660 cm^3, and 660 cm^2 respectively. Find the radius and height of cylinder.
9. Find the ratio of total surface area and curved surface area of a cylinder whose height and radius are 7.5 cm and 3.5 cm respectively.
10. A 20 m deep welll with diameter 7 m is dug and the earth from digging is evenly spread out to form a plateform 22 m $\times$ 14 m. Find the height of plateform.
11. 30800 cm^3 water can be filled in a cylindrical vessel. If its internal radius is 14 cm, then find its internal curved surface area.
12. If width of hollow cylinder is 2 cm. Its internal diameter is 14 cm and height is 26 cm. Both the ends of cylinder are open, then find the total surface area of hollow cylinder.
13. If both ends of a hollow cylinder are open. Its height is 20 cm and internal and external radius are 26 cm and 30 cm respectively. Find the volume of this hollow cylinder.

16.06 Cone

You have seen joker's cap, icecream cone. These objects are like as cone. In the given figure 16.07, VO is a line segment, other line segment VA makes an angle θ with VO revolves about side VO then form a cone.

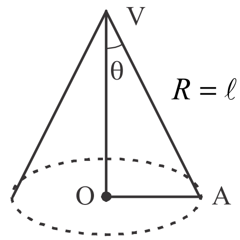

Fig. 16.07

Length of line segment VO is called height of cone and is generally denoted h. Line segment VO is axis of cone. Point V is called vertex of cone and side VA is called slant height of cone which is generally expressed by l. Base of cone is a circle whose centre is O and radius $OA = r$ (say). If line VO is perpendicular to base then cone is called right circular cone. In this chapter we will study about right circular cone.

The solid obtained on revolving a right angled triangle about one of its sides (other than hypotenuse) is called a cone or a right cicular cone.

$$VA^2 = VO^2 + OA^2$$

or $$\ell^2 = h^2 + r^2$$

$$\ell = \sqrt{h^2 + r^2}\text{ , VA is called slant height of cone}$$

Take sector OAB of circle joins sides OA and OB such that we can obtained a cone, perimeter of its base is equal to length of arc AB and radius of sector is slant height of cone. According to fig. 16.08.

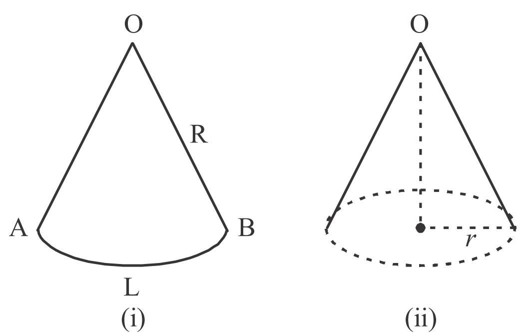

Fig. 16.08

= Curved surface area of cone.

$$= \frac{1}{2} L \times R$$

$$= \frac{1}{2}\text{ Perimeter of base of cone} \times \text{slant height}$$

$$= \frac{1}{2} \times 2\pi r \times \ell$$

Slant surface of cone $=\pi r\ell$

Total surface area of cone = curved surface of cone + area of base of cone

$$=\pi r\ell+\pi r^2$$

$$=\pi r(\ell+r)$$

If r and h are radius and height of cone, then

Slant height of cone $=\ell=\sqrt{r^2+h^2}$

Total surface area of cone $=\pi r(\ell+r)$

Volume of cone $=\frac{1}{3}\pi r^2 h$

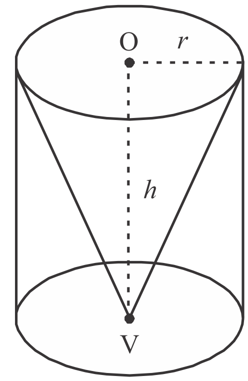

Fig. 16.09

We know that $\pi r^2 h$ is volume of cylinder whose radius is r and height h. If radius and height of cone are r and h respectively then volume of cone will be $\frac{1}{3}\pi r^2 h$. It can be verified by an experiment that volume of cone is one third the volume of cylinder of same radius and same height.

Take a measuring jar of radius r and h. Its volume is $\pi r^2 h$. According to fig, 16.09. Take a cone of same radius and height. Let volume of cone is V. Fill the cone with water and drop in the cylinder you will see that after repeating this process three times then jar will fill with water completely. So it concludes that volume of cone $V=\frac{1}{3}\pi r^2 h$.

Illustrative Examples

Example 7. The diameter of ther base and height of a cone are 12 m and 10 m. Find the total surface area of cone.

Solution : Given : Diameter of base of cone = 12 m

Radius of cone $=\frac{\text{diameter}}{2}=\frac{12}{2}\text{m}=6\text{m}$

and slant height of cone $(\ell)=10$ m

Total surface area of cone $=\pi(\ell+r)r$

$$=\frac{22}{7}(10+6)\times 6=\frac{22}{7}\times 16\times 6$$

$$=\frac{2112}{7}=301.71$$

Thus, total surface area of cone = 301.71 m^2

Example 8. The curved surface area of a cone is 2035 cm^2 and diameter of its base is 35 cm. Find the slant height of cone.

Solution : Given : Curved surface area of a cone = 2035 cm^2

$\therefore$ Radius of cone $(r) = \frac{\text{diameter}}{2} = \frac{35}{2}\text{cm} = 17.5\text{cm}$

$\therefore$ $2035 = \frac{22}{7} \times 17.5 \times \ell$

or $2035 = 22 \times 2.5 \times \ell$

or $\ell = \frac{2035}{22 \times 2.5} = \frac{2035}{55}$

or $\ell = 37$

Thus, slant height of cone will be 37 cm.

Example 9. The volume of cone of height 9 cm is 16632 cm^3. Find radius of its base.

Solution : Given : volume of cone = 16632 cm^3

height of cone (h) = 9 cm

volume of cone $= \frac{1}{3}\pi r^2 h$

$\therefore$ $16632 = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 9$

or $16632 = \frac{22}{7} \times r^2 \times 3$

or $16632 \times 7 = 22 \times r^2 \times 3$

or $r^2 = \frac{16632 \times 7}{22 \times 3} = \frac{756 \times 7}{3}$

or $r^2 = 252 \times 7$

or $r = \sqrt{36 \times 7 \times 7}$

or $r = 6 \times 7 = 42$

Thus radius of cone = 42 cm.

Example 10. The ratio of radius and height of a cone is 5 : 12 and its volume is 2512 cm^3. Find slant height and radius of base of cone (take $\pi = 3.14$)

Solution : Ratio of radius and height of cone = 5 : 12

Radius of cone $(r) = 5x$ cm

Height of cone $(h) = 12x$ cm

Volume of cone = 2512 cm^3

$\therefore$ $\frac{1}{3}\pi r^2 h = 2512$

or $\frac{1}{3} \times 3.14 \times (5x)^2 \times 12x = 2512$

or $\frac{1}{3}\times3.14\times25x^2\times12x=2512$

or $3.14\times25x^2\times4x=2512$

or $314x^3=2512$

or $x^3=\frac{2512}{314}=8$

or $(x)^3=(2\times2\times2)$

or $x^3=2^3$

or $x=2$

Thus, radius of cone $=5\times2=10$ cm.

Height of cone $=12\times2=24$ cm.

Example 11. A conical tent that is 14 metres high and its base area is 346.5 metre^2. This tent is made of 1.5 metre wide canvas, then find out its length.

Solution : Height of conical tent h = 14 m.

Radius $=r$ m

Area of base of cone $=\pi r^2$

Area of base = 346.5 m^2

$$\frac{22}{7}\times r^2=346.5$$

or $r^2=\frac{346.5\times7}{22}$

or $r^2=110.25$

or $r=10.5$ m

Slant height of tent $\ell=\sqrt{r^2+h^2}$

$$=\sqrt{(10.5)^2+(14)^2}$$

$$=\sqrt{110.25+196}$$

$$=\sqrt{306.25}=17.5 \text{ m.}$$

Area of canvas = Curved surface area of tent

$=\pi r\ell$ m^2

$=\frac{22}{7}\times10.5\times17.5\,\text{m}^2$

$=577.5$ m^2

$$\text{Length of canvas} = \frac{\text{Area}}{\text{Width}}$$

$$= \frac{577.5}{1.5} \text{ m} = 385 \text{ m}$$

Exercise 16.3

1. Find the curved surface area, total surface area, and volume of a cone whose height is 28 cm and radius of base is 21 cm.
2. Find the slant height of the right circular cone whose volume is 1232 cm^3 and height is 24 cm.
3. Find the total surface area of cone whose diameter of base is 14 m, and slant height is 25 m.
4. Find the curved surface area and total surface area of right circular cone whose radius of base is 14 cm and slant height is 50 cm.
5. Find the volume of right circular cone whose radius of base is 6 cm and height is 8 cm.
6. Find the radius of base of a cone whose curved surface area is 1884.4 m^2 and its slant height is 12 cm.
7. Find the height of the right circular cone of slant height is 25 cm and area of its base is 154 cm^2.
8. The base of two cones are of same diameter. Ratio of their slant height is 5 : 4.If curved surface area of smaller cone is 400 cm^2 then find the curved surface area of bigger one.
9. The ratio of slant height and radius of a cone 7 : 4. If its curved surface area is 792 cm^2 , then find its radius.
10. The circumference of base of a conical tent is 9 m and height is 44m. Find the volume of air inside it.
11. The radius and height of a conical vessel are 10 cm and 18 cm respectively, which is filled with water to the brim. It is poured in a cylindrical vessel of radius 5 cm. Find the height of water level in cylindrical vessel.
12. A cone of maximum height is cut from a cube of edge14 cm. Find the volume of cone.
13. Find the slant height, curved surface area and total surface area and volume of a cone whose base radius and heights are 7 cm and 24cm respectively.
14. The radius of a sector is 12 cm and angle is 120°. By coinciding its straight sides a cone is formed. Find the volume of that cone.

16.07. Sphere

A sphere or semi-sphere is a solid obtained on revolving full or half respectively a circle about any diameter of it.

We can define sphere as set of all the points in space are called sphere which are at equal distant from a fixed point. Fixed point is called centre of the sphere. Distance from centre to any point is called centre of the sphere. Distance from centre to any point of this set is called radius, cricket ball, football are examples of sphere.

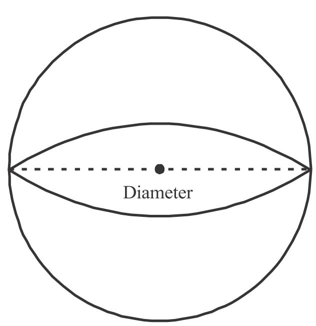

Fig. 16.10

The line segment which passes through the centre of sphere and whose two end points lie on sphere is called diameter of sphere. All the diameter of sphere are of same length. Diameter of sphere is twice of its radius. The space occupied by sphere is called its volume.

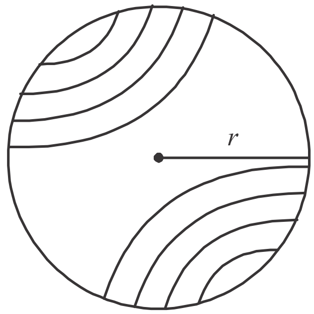

Fig. 16.11

Following formula can be given without proof.

If r is radius of sphere then

volume of sphere $= \frac{4}{3}\pi r^3$

The surface area of sphere $= 4\pi r^2$

Volume of hemisphere of radius $= \frac{2}{3}\pi r^3$

Total surface area of hemisphere $= 3\pi r^2$

If r_1 and r_2 are external radius of spherical shell

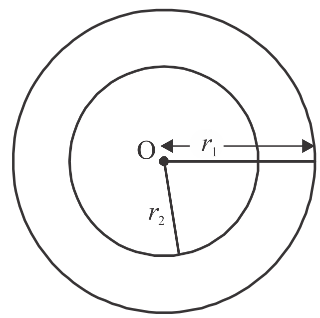

Fig. 16.12

Volume of spherical shell $= \frac{4}{3}\pi\left(r_1^3 - r_2^3\right)$

Total surface area of spherical shell $= 4\pi\left(r_1^2 + r_2^2\right)$

Illustrative Examples

Example 12. Find the surface area of sphere of radius 7 cm.

Solution : Radius of sphere $= 7$ cm.

The surface area of sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$= 22 \times 28 \text{ cm}^2$$

$$= 616 \text{ cm}^2$$

Example 13. The radius of a hemisphere is 3.5 cm. Find its volume and total surface area.

Solution : Radius of hemisphere $(r) = 3.5$ cm.

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times (3.5)^3 \text{ cm}^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 \text{ cm}^3$$

$$= \frac{2}{3} \times 22 \times 0.5 \times 12.25 \text{ cm}^3$$

$$= \frac{269.5}{3} \text{cm}^3 = 89.83 \text{ cm}^3$$

Thus, total surface area of hemishpere $= 3\pi r^2$

$$= 3 \times \frac{22}{7} \times (3.5)^2 \text{ cm}^2$$

$$= 3 \times \frac{22}{7} \times 3.5 \times 3.5 \text{ cm}^2$$

$$= 3 \times 22 \times 0.5 \times 3.5 \text{ cm}^2$$

$$= 115.5 \text{ cm}^2$$

Example 14. The radius of a lead sphere is 5 cm. How many small spheres of 5 mm radius can be formed form this sphere?

Solution : Radius of lead sphere is $(r) = 5$ cm

Volume of big lead sphere $(v) = \frac{4}{3}\pi (5)^3 \text{ cm}^3$

$$= \frac{4}{3}\pi \times 5 \times 5 \times 5 \text{ cm}^3$$

Radius of small lead sphere $(r_1) = 5$ mm $= \frac{5}{10}$ cm $= 0.5$ cm

Volume of 1 small lead sphere $= \frac{4}{3} \times \pi \times 0.5 \times 0.5 \times 0.5 \text{ cm}^3$

No. of small lead sphere $= \frac{\text{Volume of big shere}}{\text{Volume of small sphere}}$

$$= \frac{\frac{4}{3} \times \pi \times 5 \times 5 \times 5}{\frac{4}{3} \times \pi \times 0.5 \times 0.5 \times 0.5} = 1000 \text{ small spheres}$$

Example 15. The surface area of a ball is 1386 cm^2. Find the radius of ball.

Solution : Given : surface area of ball (sphere) = 1386 cm^2

or $$4\pi r^2 = 1386$$

or $$4 \times \frac{22}{7} \times r^2 = 1386$$

or $$r^2 = \frac{1386 \times 7}{4 \times 22} = 110.25$$

or $$r = \sqrt{110.25} = 10.5$$

Thus radius of ball will be 10.5 cm.

Example 16. The ratio of surface areas of two sphere is 4 : 9 find the ratio of their surface areas and volumes.

Solution : Let r_1 and r_2 are radius of two spheres and $4\pi r_1^2$ and $4\pi r_2^2$ are their surface areas

Ratio of surface areas of two sphere $\frac{\pi r_1^2}{\pi r_2^2} = \frac{4}{9}$

or $$\frac{r_1^2}{r_2^2} = \frac{4}{9}$$

or $$\frac{r_1}{r_2} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

Ratio of volumes of two sphere $= \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3} r_2^3} = \frac{r_1^3}{r_2^3}$

$$= \left(\frac{r_1^3}{r_2^3}\right) = \left(\frac{2}{3}\right)^3 = 8 : 27$$

Exercise 16.4

1. Find the surface area and volume of a sphere of radius 1.4 cm.
2. Find the volume of the sphere whose surface area is 616 cm^2.
3. Find the surface area and volume of a hemisphere of radius 4.5 cm.
4. Find the surface area of a sphere whose volume is 38808 cm^3.
5. A cylinder is made of lead whose radius is 4 cm and height is10 cm. By melting this, how many spheres of radius is 2 cm, can be formed?
6. A hollow spherical shell is 2 cm, thick. If its outer radius is 8 cm, then find the volume of metal used in it.
7. How many cones of 3 cm. radius and 6 cm in height are formed by melting a metallic sphere of radius 9 cm?
8. Eight spheres of same radius from a metallic sphere of 10 cm radius, are formed. Find the surface area of each sphere so obtained.

9. Find the volume of sphere if its surface area is 5544 cm^2.
10. The dimensions of a solid rectangular slab of lead is 66 cm, 42 cm and 21 cm respectively. Find by melting this, how many sphere of diameter 4.2 cm can be formed?
11. A sphere of 6 cm diameter is dropped into cylindrical vesselof diameter 12 cm. Find the rise in water in vessel.
12. A hemispherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 4 cm. How many bottles are necessary to empty the bowl?
13. The diameter of a sphere is 0.7 cm. If 3000 spheres, completely filled with water, are drawn out from a water tank then find volume of water drawn out.
14. A hollow hemispherical vessel has external and internal radius as 43 cm and 42 cm. respectively. If cost of colouring it, is 7 paisa per square cm. then find cost of painting the vessel.

Miscellaneous Exercise-16

1. The total surface area of a cube is 486 cm^2 , edge of cube is
(a) 6 cm (b) 8 cm (c) 9 cm (d) 7 cm
2. The length, breadth and height of cuboid are 3 m, 2 m and 1 m respectively. Total surface area of cuboid will be :
(a) 12 m^2 (b) 11 m^2 (c) 21 m^2 (d) 22 m^2
3. The diameter of a sphere is 6 cm, the volume of sphere will be
(a) 16 π cm^3 (b) 20 π cm^3 (c) 36 π cm^3 (d) 30 π cm^3
4. A cylinder has radius of its base 14 cm and 10 cm in height. Curved surface area of cylinder will be :
(a) 810 cm^2 (b) 880 cm^2 (c) 888 cm^2 (d) 890 cm^2
5. The volume and height of a cone is 308 cm^2 and 6 cm respectively. Radius of its base will be :
(a) 7 cm (b) 8 cm (c) 6 cm (d) none of these
6. A solid metallic hemisphere has diameter 42 cm. Find the cost of polishing the total surface at the rate 20 paise per cm^2.
7. A cone, a hemisphere and a cylinder are formed by same radius and same height. Write ratio of their volumes.
8. The left part of a solid body is cylindrical and right part is conical. If diameter of cylindrical is 14 cm and length is 40 cm and of diameter cone is 14 cm and height is 12 cm, then find the volume of solid.
9. A metallic sphere of radius 9 cm is melted and then recast into small cones of radius 3 cm and height 6 cm. Find the number of cones thus formed.
10. The population of a village is 4000. There is requirement of 150 *l* water per person daily. There is a water tank of dimensions 20m × 15 m × 6 m in village. For how many days water of this tank will suficient.
11. Three solid metallic sphere of radii 6 cm, 8 cm and 10 cm respectively are melted and then recast into a big sphere. Find the radius of this sphere.
12. A conical vessel has radius 10 cm and height 18 cm and is completely filled with water which is transferred in a cylindrical vessel of radius 5 cm. Find the level of water (height) in cylindrical vessel.
13. A candle of diameter 2.8 cm is formed from a cuboid of dimensions 11 cm × 3.5 cm × 2.5 cm. find the length of candle.
14. The diameter of a metallic sphere is 6 cm. It is melted and drawn into a wire of length 36 m and of uniform circular cross-section.. Find its radius.

Important Points

1. Area of 4 walls of room = 2 height (length + breadth)
   $= 2 \times (l + b) \times h$
2. Total surface area of cuboid $= 2(\ell b + bh + h\ell)$
3. Volume of cuboid $= \ell \times b \times h =$ Area of base × height
4. Area of 4 walls of cube $= 4\ell^2$
5. Total surface area of cube $= 6\ell^2$
6. Volume of cube $= \ell^3$
7. Diagonal of cuboid $= \sqrt{\ell^2 + b^2 + h^2}$
8. Diagonal of cube $= \sqrt{3}\ell$
9. Curved surface area of cylinder $= 2\pi rh$
10. Total surface area of cylinder $= 2\pi r(r + h)$
11. Volume of cylinder $= \pi r^2 h =$ Area of base × height
12. Total surface area of hollow cyclinder $= 2\pi(r_1 + r_2)(h + r_1 - r_2)$
13. Volume of hollow cylinder $= \pi(r_1^2 - r_2^2)h$
14. Curved surface area of cone $= \pi r\ell$
15. Slant height of cone $\ell = \sqrt{h^2 + r^2}$
16. Total surface area of cone $= \pi r(r + \ell)$
17. Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times$ Area of base × height
18. Surface area of solid sphere $= 4\pi r^2$
19. Volume of soild sphere $= \frac{4}{3}\pi r^3$
20. Curved surface area of hemisphere $= 2\pi r^2$
21. Total surface area of hemisphere $= 3\pi r^2$
22. Volume of hemisphere $= \frac{2}{3}\pi r^3$
23. Total surface area of spherical shell $= 4\pi(r_1^2 + r_2^2)$
24. Volume of spherical shell $= \frac{4}{3}\pi(r_1^3 - r_2^3)$ where $r_1 > r_2$
25. 1 m^3 = 1000 litre = 1 kilo letre
    1 litre = 1000 cm^3
    1 acre = 100 m^2
    1 cm^3 = 1000 mm^3
    1 m^3 = 100,00,00 cm^3

Answer Sheet

Exercise 16.1

1. 426 cm^2, 540 cm^2 2. 486 cm^2 3. 7900 cm^2 4. 400 cm^2
5. ₹ 110.80 6. 240 cm^2 7. 5 m 8. 15000 bricks
9. $10\sqrt{2}$ m 10. 8 m 11. 3000 bricks 12. 15 cm, 9 cm, 6 cm

Exercise 16.2

1. 968 cm^2, 2310 cm^3 2. 132 cm^2, 188.57 cm^3, 198 cm^3
3. 3234 cm^3, 924 cm^2 4. 5 : 6 and 5 : 9 5. 539 cm^3 6. 2310 cm^3
7. 5 cm 8. 5 cm, 21 cm 9. 22 : 15 10. 2.5 m
11. 4400 cm^2 12. 2794 cm^2 13. 3520 cm^3

Exercise 16.3

1. 2310 cm^2, 3696 cm^2, 12936 cm^3 2. 25 cm 3. 704 m^2
4. 2200 cm^2, 2816 cm^2 5. 301.71 cm^3 6. 5 cm approx
7. 24 cm 8. 320 cm^2 9. 12 cm 10. 462 m^3
11. 24 cm 12. 718.67 cm^3 13. 25 cm, 550 cm^2, 704 cm^2, 1232 cm^3
14. 189.61 cm^3

Exercise 16.4

1. 24.64 cm^2, 11.5 cm^3 2. 1437.33 cm^3
3. 190.93 cm^2, 190.93 cm^3 4. 5544 cm^2
5. 15 6. 1240.38 cm^3 7. 54 8. 100 π cm^2
9. 38808 cm^3 10. 1500 11. 1 cm 12. 54
13. 539 cm^3 14. ₹ 397.43

Miscellaneous Exercise- 16

1. (c) 2. (d) 3. (c) 4. (b) 5. (a)
6. ₹ 831.60 7. 1 : 2 : 3 8. 6776 cm^3 9. 54 10. 3 days 11. 12 cm
12. 24 cm 13. 15.6 cm 14. $\frac{1}{10}$ cm = 0.1 cm

❑