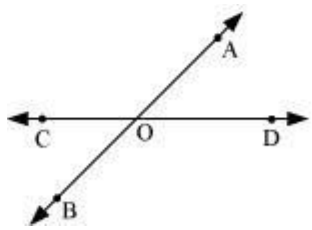
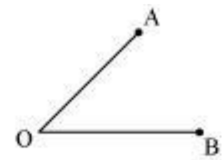
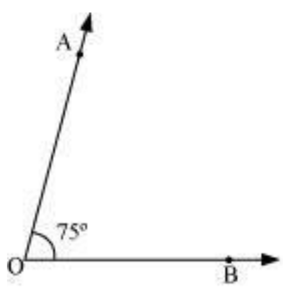
Lines and Angles

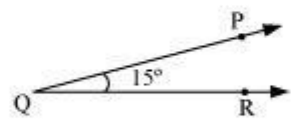
- An angle is made when two lines or line segments meet. For example:





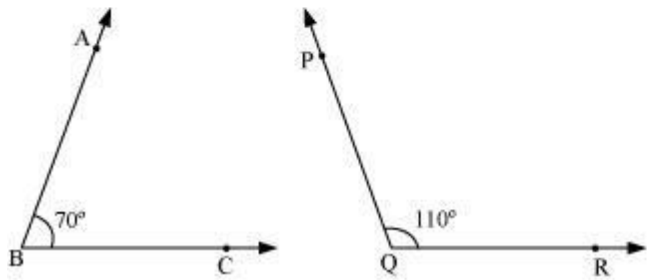
- When the sum of the measures of two angles is 90°, the angles are called **complementary angles**.





Here, ∠AOB and ∠PQR are complementary as (∠AOB + ∠PQR) = 75° + 15° = 90°.

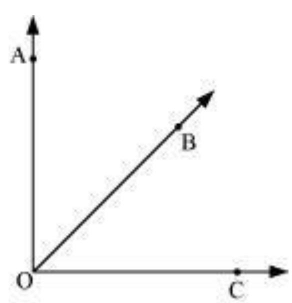
- When the sum of the measures of two angles is 180°, the angles are called **supplementary angles**.



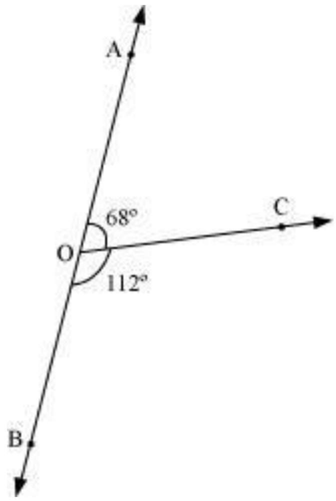
Here, ∠ABC and ∠PQR are supplementary as (∠ABC + ∠PQR) = 110° + 70° = 180°.

- A pair of angles are called adjacent angles, if:
  - they have a common vertex
  - they have a common arm
  - the non-common arms are on either side of the common arm

For example, ∠AOB and ∠BOC are adjacent angles as they have a common vertex O, common arm OB, and non-common arms OA and OC lie on either side of OB.

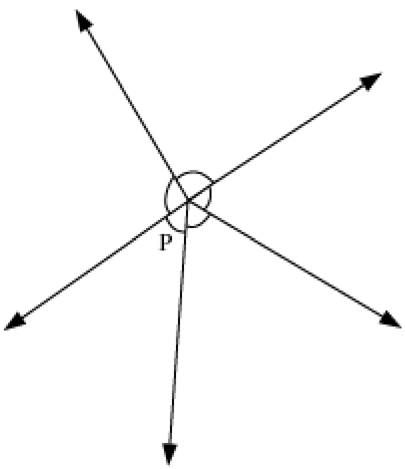


- A **linear pair** is a pair of adjacent angles whose non-common sides are opposite rays.
- The sum of the measures of the adjacent angles is 180°.



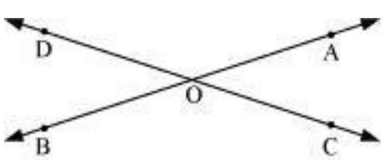
Here, ∠AOC and ∠BOC form a linear pair as ∠AOC + ∠BOC = 180°.

- The sum of angles around a point is equal to 360°.



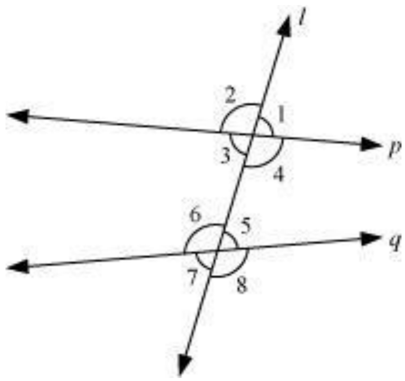
In this figure, five angles have a common vertex, which is point P. In other words, the five angles make a complete turn and therefore the sum of these five angles will be equal to 360°. This is true no matter how many angles make a complete turn.

- When two lines intersect, the vertically opposite angles so formed are equal.



Here, ∠AOC = ∠BOD and ∠AOD = ∠BOC.

- A line which intersects two or more lines at distinct points is called **transversal** to the lines.

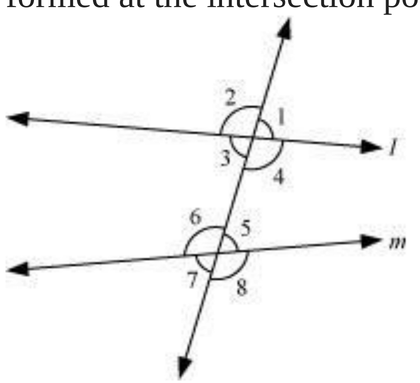


Here, line *l* is a transversal with respect to lines *p* and *q*.

1.
1. ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8 are pairs of corresponding angles.
2. ∠3 and ∠5, ∠4 and ∠6 are pairs of alternate interior angles.
3. ∠1 and ∠7, ∠2 and ∠8 are pairs of alternate exterior angles.
4. ∠3 and ∠6, ∠4 and ∠5 are pairs of interior angles on the same side of the transversal.
5. ∠1 and ∠8, ∠2 and ∠7 are pairs of exterior angles on the same side of the transversal.

- **Corresponding angles**

  When a transversal intersects two lines *l* and *m*, the **c**orresponding angles so formed at the intersection points are named as follows:



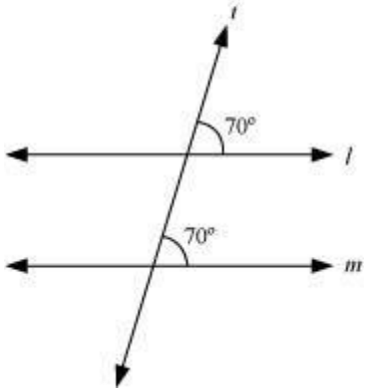
  ∠1 and ∠5, ∠2 and ∠6
  ∠3 and ∠7, ∠4 and ∠8

- **Corresponding angles axiom**

  If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.
  In the above figure, if lines *l* and *m* become parallel then we will have following pair of equal angles:
  ∠1 =∠5, ∠2 =∠6, ∠3 =∠7 and ∠4 = ∠8

- **Converse of corresponding angles axiom**

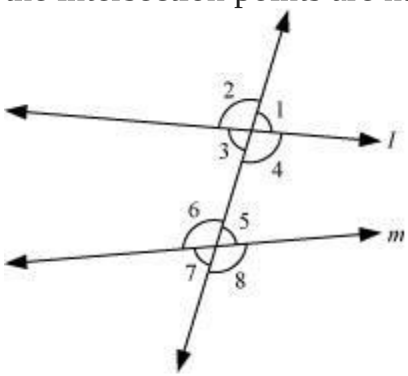
If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.



In the figure, the corresponding angles are equal. Therefore, the lines *l* and *m* are parallel to each other.

- **Alternate angles**

  When a transversal intersects two lines *l* and *m*, the alternate angles so formed at the intersection points are named as follows:

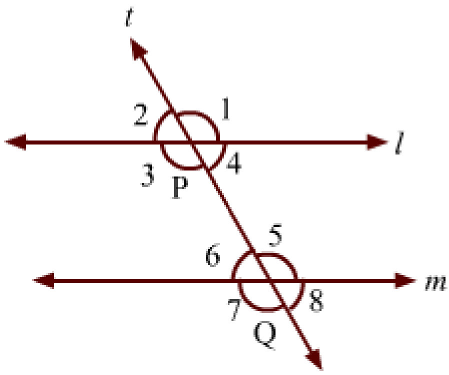


Alternate interior angles
∠3 and ∠5, ∠4 and ∠6

Alternate exterior angles
∠1 and ∠7, ∠2 and ∠8

- **Alternate angles axiom**

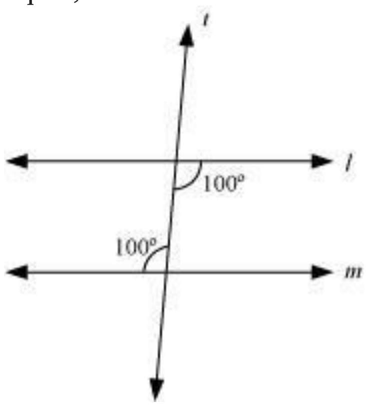
  If a transversal intersects two parallel lines, then the angles in each pair of alternate angles are equal.



In the above figure, lines *l* and *m* are parallel. So, by using the alternate angles axiom, we can say that:
∠1 = ∠7, ∠2 = ∠8, ∠3 = ∠5 and ∠4 = ∠6

- **Converse of alternate angles axiom**

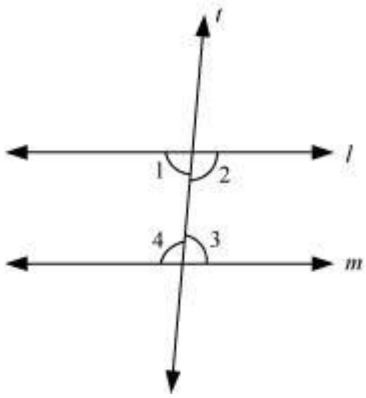
  If a transversal intersects two lines such that the angles in a pair of alternate angles are equal, then the two lines are parallel.



In the above figure, alternate interior angles are equal (100°) and thus, lines *l* and *m* are parallel.

- **Property of interior angles on the same side of a transversal:**

  If a transversal intersects two parallel lines, then the angles in a pair of interior angles on the same side of the transversal are supplementary.
  For example,



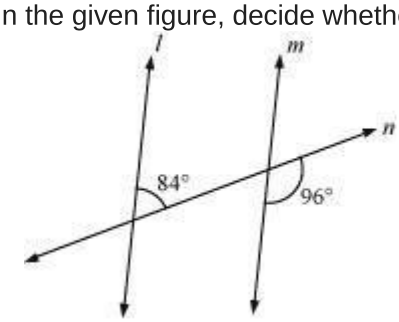
In the given figure, if lines *l* and *m* are parallel to each other then ∠1 + ∠4 = 180° and ∠2 + ∠3 = 180°.

- **Converse of the property of interior angles on the same side of a transversal:**

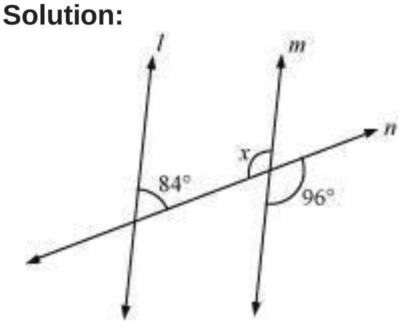
If a transversal intersects two lines such that the interior angles on the same side of the transversal are supplementary, then the lines intersected by the transversal are parallel.

Example:

In the given figure, decide whether *l* is parallel to *m* or not.



Solution:



∠*x* = 96° (Vertically opposite angles)

∠*x* + 84 = 96° + 84° = 180°

i.e., Sum of the interior angles on the same side of the transversal is supplementary.

Therefore, *l*||*m.*