Exercise 10.1

Q1E

Consider the following curves:

$x = t^2 + t,\ y = t^2 - t$ and $-2 \le t \le 2$

The variables *x* and *y* are dependent of the variable '*t*'.

Substitute $t = -2$ in $x = t^2 + t,\ y = t^2 - t$.

The initial point is $(2,6)$.

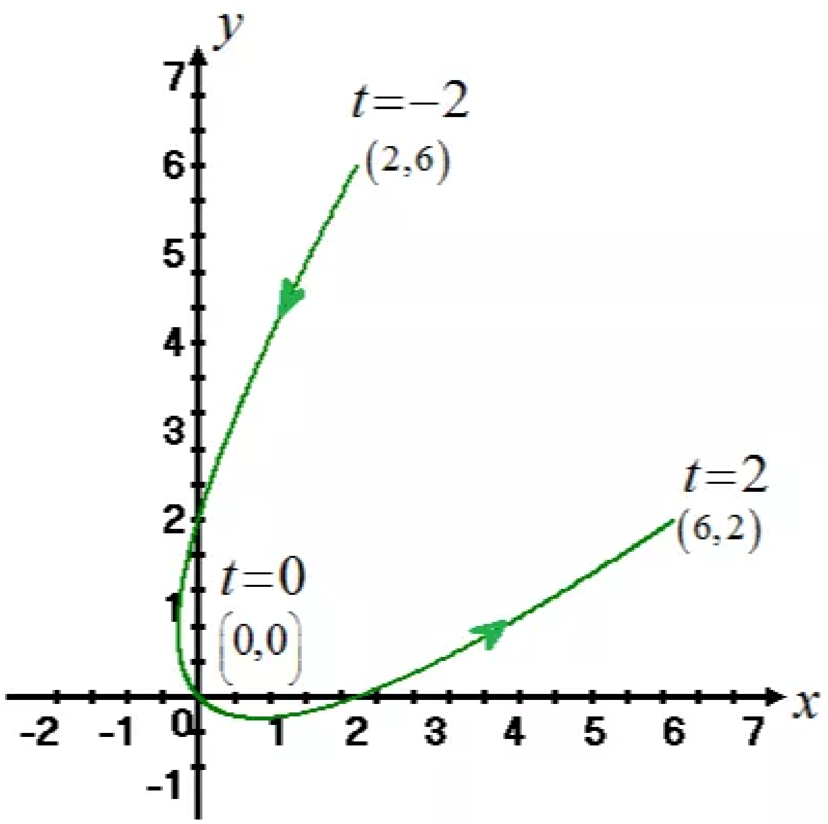
Substitute $t = 2$ in $x = t^2 + t,\ y = t^2 - t$.

The terminal point is $(6,2)$.

Substitute $t = 0$ in $x = t^2 + t,\ y = t^2 - t$.

The point is $(x,y) = (0,0)$.

The suitable graph is as follows:

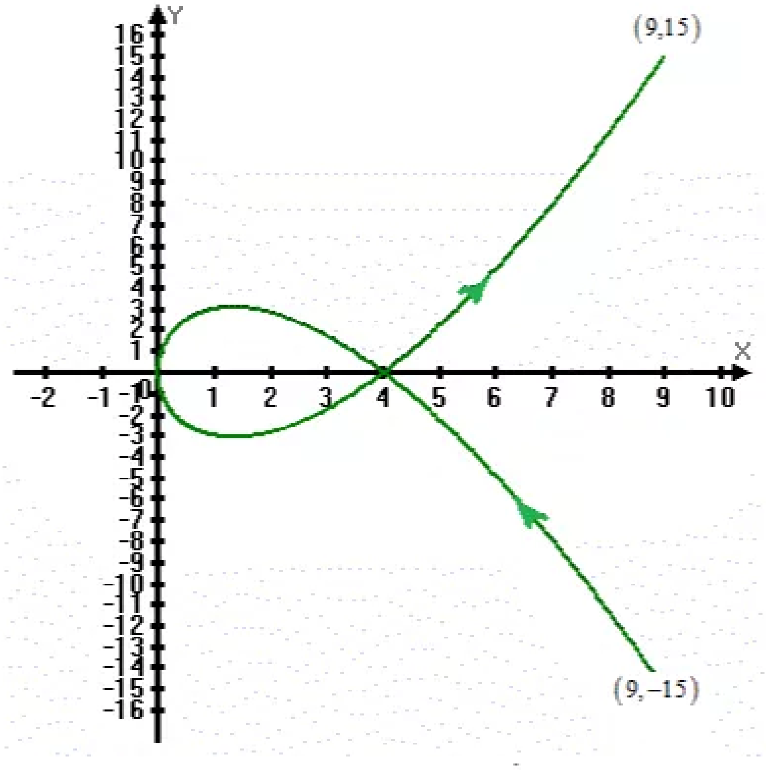


Q2E

Sol: Given $x = t^2, y = t^3 - 4t$ and $-3 \le t \le 3$

when $t = -3, x = 9, y = -15$
when $t = 0, x = 0, y = 0$
when $t = 3, x = 9, y = 15$
$\therefore$ the curve varies from $(9,-15)$ to $(9,15)$ through the origin.

The suitable graph is



Here t increases upward as shown in figure.

Q3E

Sol: Given $x = \cos^2 t, y = 1 - \sin t$ and $0 \le t \le \frac{\pi}{2}$

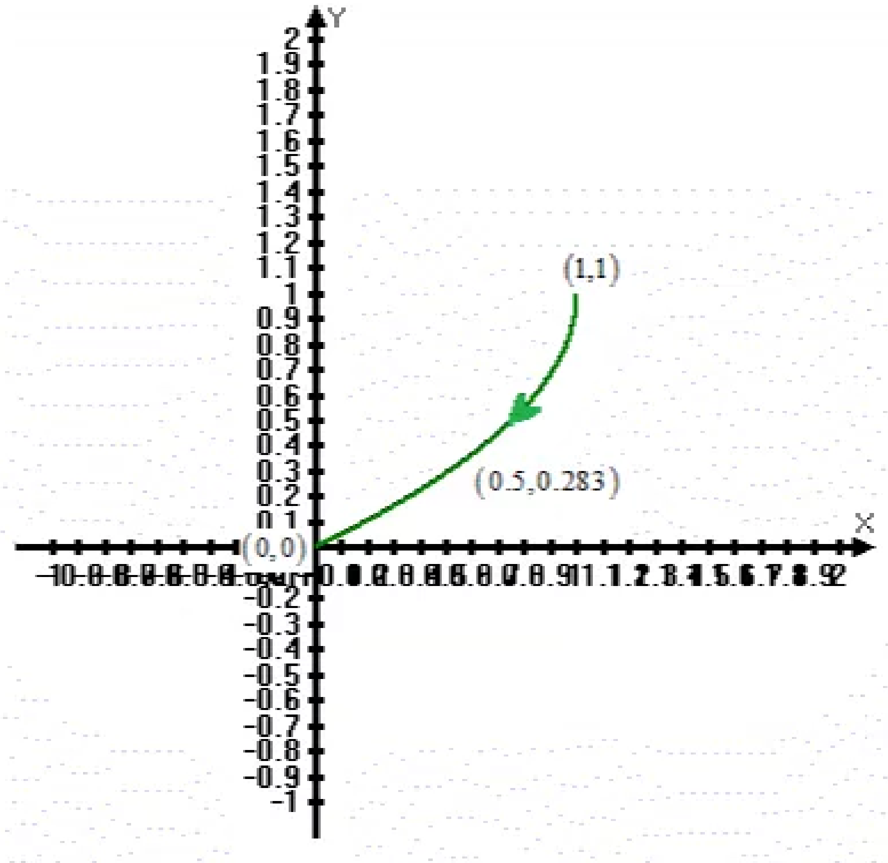
when $t = 0, x = 1, y = 1 \qquad = (1,1)$

when $t = \frac{\pi}{4}, x = \frac{1}{2} = 0.5, y = 1 - \frac{1}{\sqrt{2}} = 0.293 = (0.5, 0.293)$

when $t = \frac{\pi}{2}, x = 0, y = 0 \qquad (0,0)$

So, as $t \to \frac{\pi}{2}$, the curve proceeds from $(1,1)$ to $(0,0)$

The suitable graph is



Q4E

We have $x = e^{-t} + t, \quad y = e^{t} - t, \quad -2 \le t \le 2$

We calculate x and y for different value of t in the interval [-2, 2] and then Sketch the curve.

t	x	y
-2	$e^{2} - 2 \approx 5.39$	$e^{-2} + 2 \approx 2.14$
-1	$e - 1 \approx 1.72$	$e^{-1} + 1 \approx 1.37$
0	1	1
1	$e^{-1} + 1 \approx 1.37$	$e - 1 \approx 1.72$
2	$e^{-2} + 2 \approx 2.14$	$e^{2} - 2 \approx 5.39$

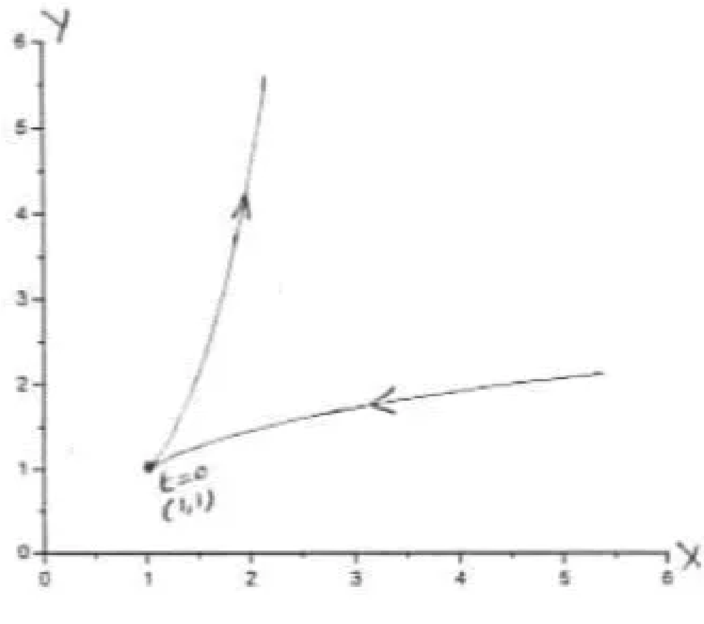


Fig. 1

Q5E

Sol: (a). Given $x = 3 - 4t, y = 2 - 3t$

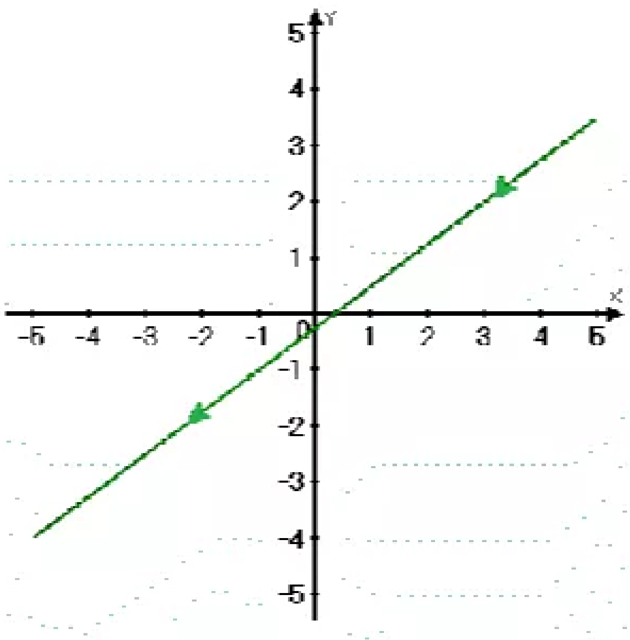
when $t = 2, x = -5, y = -4, (-5, -4)$

when $t = 3, x = -9, y = -7, (-9, -7)$

Observe that (-9,-7) is below (-5,-4).

So, as t increases, the curve decreases.

The suitable graph is



(b).

$$x=3-4t \Rightarrow -4t=x-3 \Rightarrow -t=\frac{x}{4}-\frac{3}{4}$$

$$y=2-3t=2+3\left(\frac{x}{4}-\frac{3}{4}\right)$$

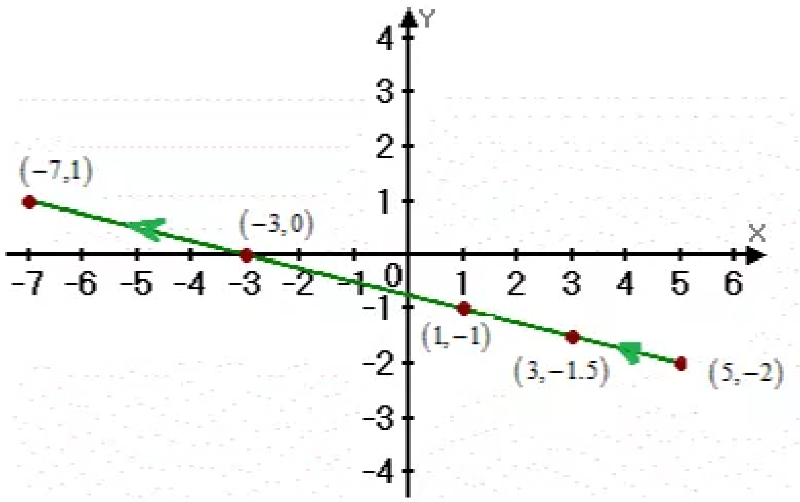
$$\Rightarrow \boxed{y=\frac{3}{4}x-\frac{1}{4}}$$

Q6E

Sol: Given $x=1-2t,\ y=\frac{1}{2}t-1$ and $-2\le t\le 4$

$t=-2 \Rightarrow x=5, y=-2$
$t=-1 \Rightarrow x=3, y=-1.5$
$t=0 \Rightarrow x=1, y=-1$
$t=2 \Rightarrow x=-3, y=0$
$t=4 \Rightarrow x=-7, y=1$

So, using the data, the graph is



(b)

$$y=\frac{1}{2}t-1 \Rightarrow t=2(y+1)$$

using this in $x=1-2t$, we get $x=1-4(y+1)$

or, the straight line shown in the grah is $y=\frac{-1}{4}(x+3)$

Q7E

Consider the parametric equations

$x = 1 - t^2, y = t - 2; \quad -2 \le t \le 2$

Now draw a table for plotting points.

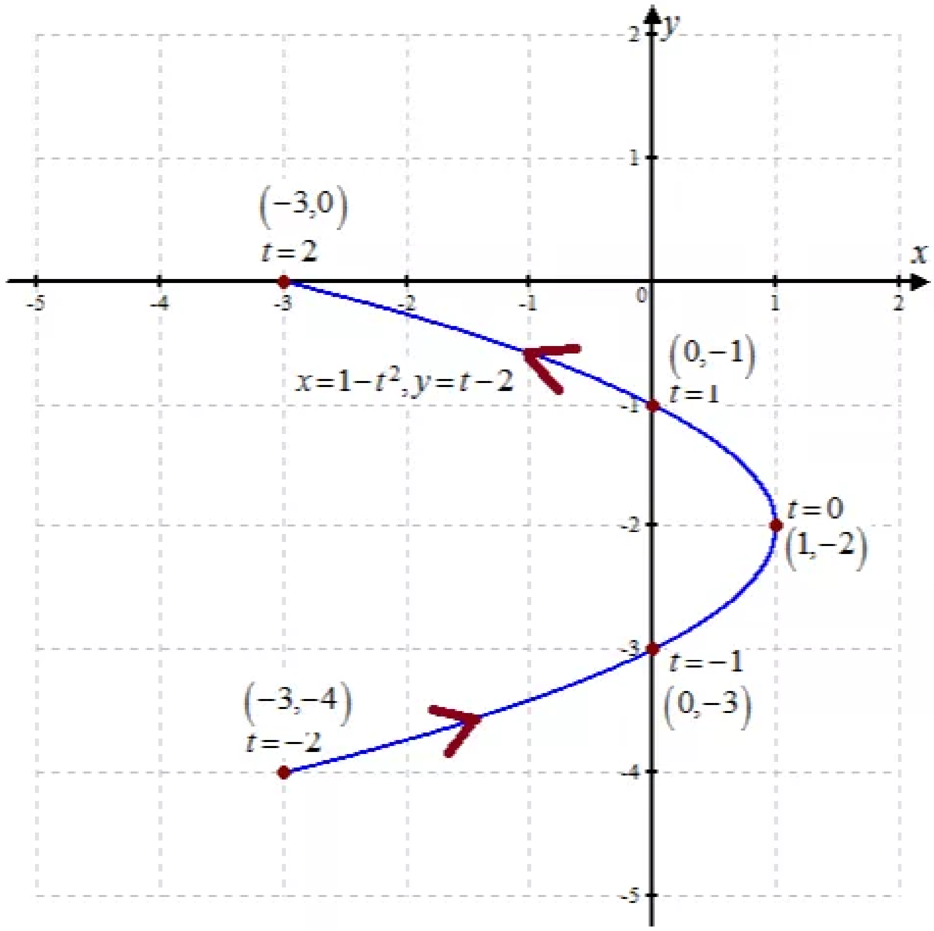
t	x	y
−2	−3	−4
−1	0	−3
0	1	−2
1	0	−1
2	−3	0

Now use *x*, *y* as coordinates to plot the graph.

As *t* increases from -2 to 2, *x* increases from -3 to 1 and *y* increases from -4 to 0.

So, the graph has upward direction.

The graph of the parametric equations is shown below:



(b)

Now to find Cartesian equation of the curve.

Since $y=t-2$, then $t=y+2$.

Now plugin $t=y+2$ in $x=1-t^2$, obtain that

$x=1-(y+2)^2,\ -4\le y\le 0$.

Thus, the Cartesian equation of the curve is

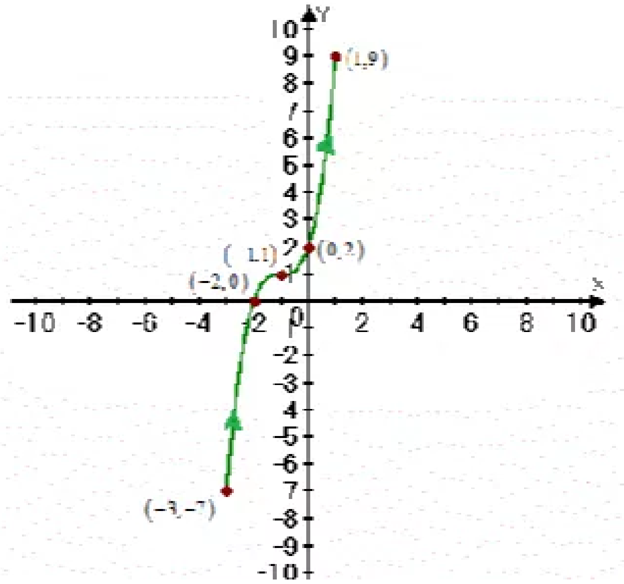
$x=1-(y+2)^2,\ -4\le y\le 0$

Q8E

Sol: Given $x=t-1,\ y=t^3+1$ and $-2\le t\le 2$

$t=-2\Rightarrow x=-3, y=-7$
$t=-1\Rightarrow x=-2, y=0$
$t=0\Rightarrow x=-1, y=1$
$t=1\Rightarrow x=0, y=2$
$t=2\Rightarrow x=1, y=9$

Using the x, y values as the coordinates, the we find the direction of the parametric curve.



(b)

$x = t - 1 \Rightarrow t = 1 + x$

using this in $y = t^3 + 1$, we get $y = (1+x)^3 + 1$

so, the given parametric equation in the coordinate form is

$y = x^3 + 3x^2 + 3x + 2$

Q9E

(A) We $x = \sqrt{t}$ we must have $t \ge 0$

$y = 1 - t$

We calculate x and y for different values of 't' where $t \ge 0$ and graph the curve where $x \ge 0$.

t	0	1	4	9	16	25	36	49
x	0	1	2	3	4	5	6	7
y	1	0	-3	-8	-15	-24	-35	-48

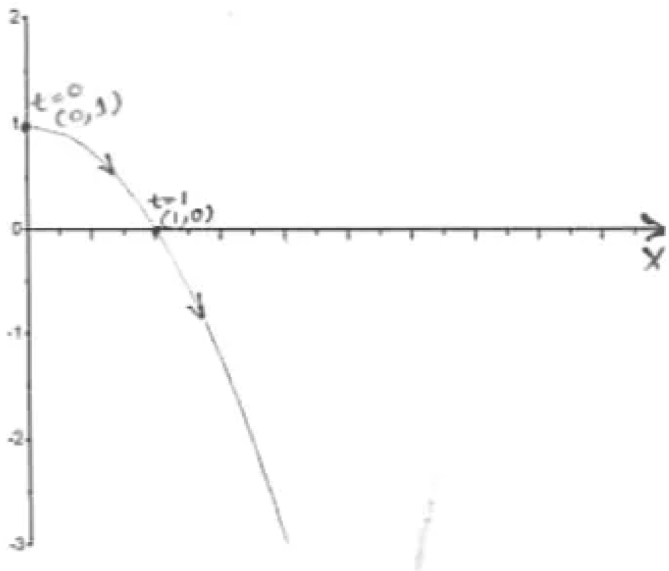


Fig. 1

(B) We have $x=\sqrt{t}$ --------- (1)

And $y=1-t$ --------- (2)

From (1) we have $x^2=t$

Putting the value of t in (2)

We have $\boxed{y=1-x^2}$ where $x\geq 0$.

Q10E

(A)

We have $x=t^2$

and $y=t^3$

We calculate the values of x and y for different values of t and sketch the curve.

t	-3	-2	-1	0	1	2	3
x	9	4	1	0	1	4	9
y	-27	-8	-1	0	1	8	27

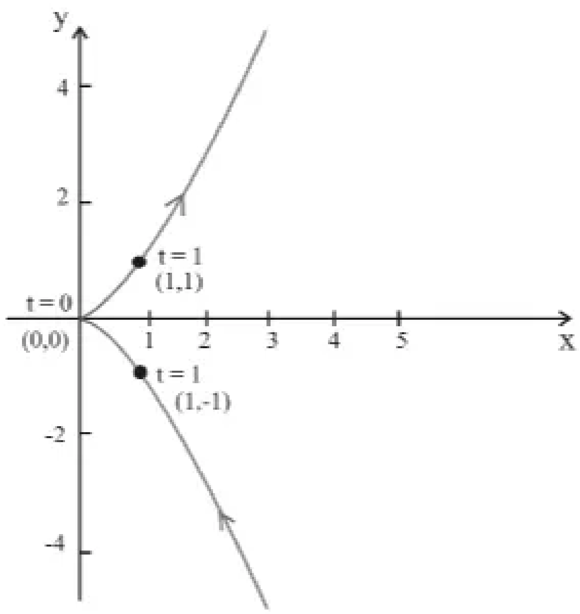


Fig. 1

(B) We have $x=t^2$ .....(1)

$y=t^3$ ---(2)

From (1) we have $t=\pm\sqrt{x}$

Putting the value of t in equation (2)

$\Rightarrow y=t^3$

$\Rightarrow y=\left(\pm\sqrt{x}\right)^3$

$\Rightarrow y=\pm x\sqrt{x}$

Squaring both sides

$\Rightarrow y^2=x^2.x$

$\Rightarrow \boxed{y^2=x^3}$ where $x\geq 0$.

Q11E

(a)

Consider the following curves:

$$x=\sin\frac{1}{2}\theta \text{ ...... } (1)$$

$$y=\cos\frac{1}{2}\theta \text{ ...... } (2)$$

Interval is $-\pi\le\theta\le\pi$.

Find the Cartesian equation of the curve.

The Cartesian equation of the curve is as follows:

$$x^2+y^2=\left(\sin\frac{\theta}{2}\right)^2+\left(\cos\frac{\theta}{2}\right)^2 \text{ Squaring and adding}$$

$$=\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}$$

$$=1$$

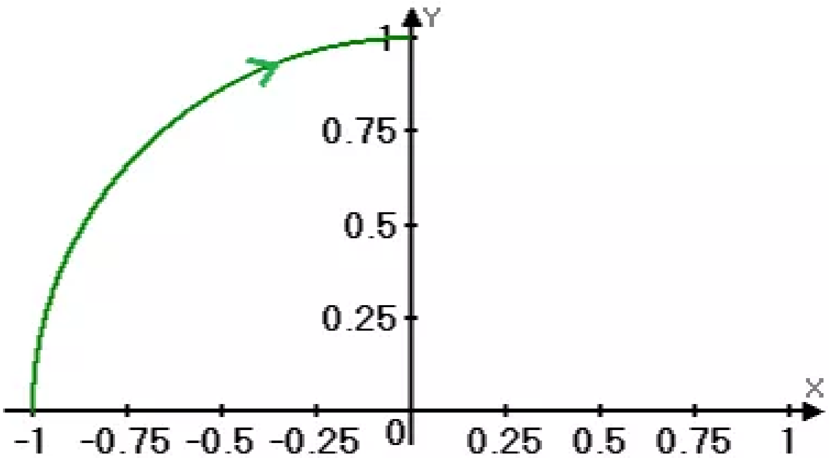
Therefore, the Cartesian equation of the curve is $\boxed{x^2+y^2=1,\ y\ge 0}$.

(b)

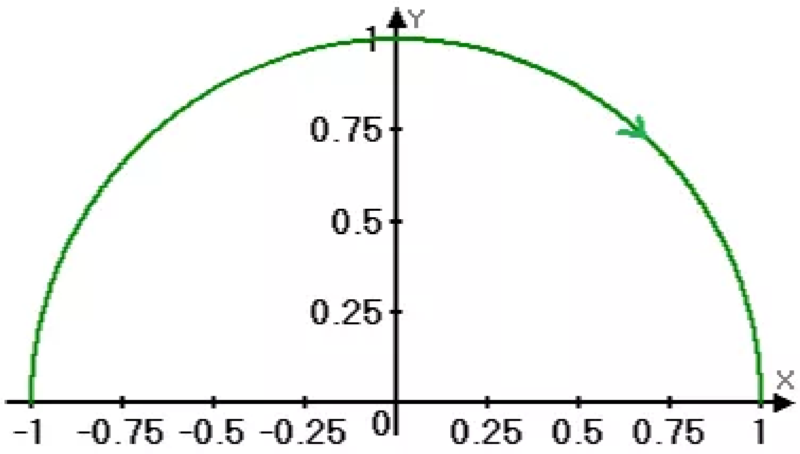
Find the direction in which the curve is traced as the parameter increases.

Sketch the curve and indicate with an arrow:

First diagram shows that the parametric curve is between $-\pi$ and 0 is as follows:



Second diagram shows the curve from $-\pi$ to π as follows:



Thus, the direction is confirmed.

Q12E

(a). Given $x = \frac{1}{2}\cos\theta, y = 2\sin\theta$ and $0 \le \theta \le \pi$

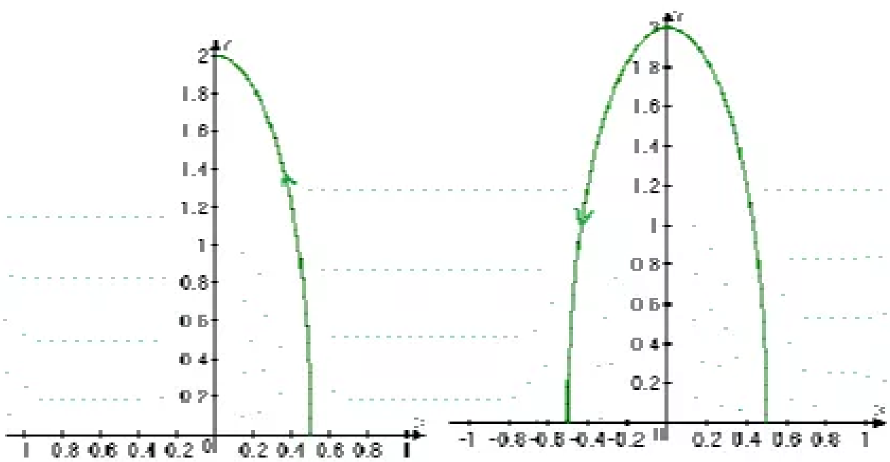
To change it to the Cartesian equation, we have

$$2x = \cos\theta, \frac{y}{2} = \sin\theta$$

$$\Rightarrow (2x)^2 + \left(\frac{y}{2}\right)^2 = \cos^2\theta + \sin^2\theta = 1$$

$$\Rightarrow \boxed{4x^2 + \frac{y^2}{4} = 1}$$

(b).



The first figure shows that the parametric curve from 0 to $\frac{\pi}{2}$ and the second is from 0 to π.

Thus the direction of tracing is confirmed.

The curve increases from right to left.

Q13E

Consider the following parametric equations of a curve:

$x = \sin t,\ y = \csc t,\ 0 < t < \frac{\pi}{2}$

(a)

To find the Cartesian equation of this curve, rewrite these equations as follows:

$x = \sin t,\ y = \frac{1}{\sin t}$

Put $x = \sin t$ in the equation $y = \frac{1}{\sin t}$ to get the following:

$y = \frac{1}{x}$

Or, rearrange to get the following:

$xy = 1$

So the Cartesian equation of the curve $x = \sin t,\ y = \csc t$ is $\boxed{xy = 1}$.

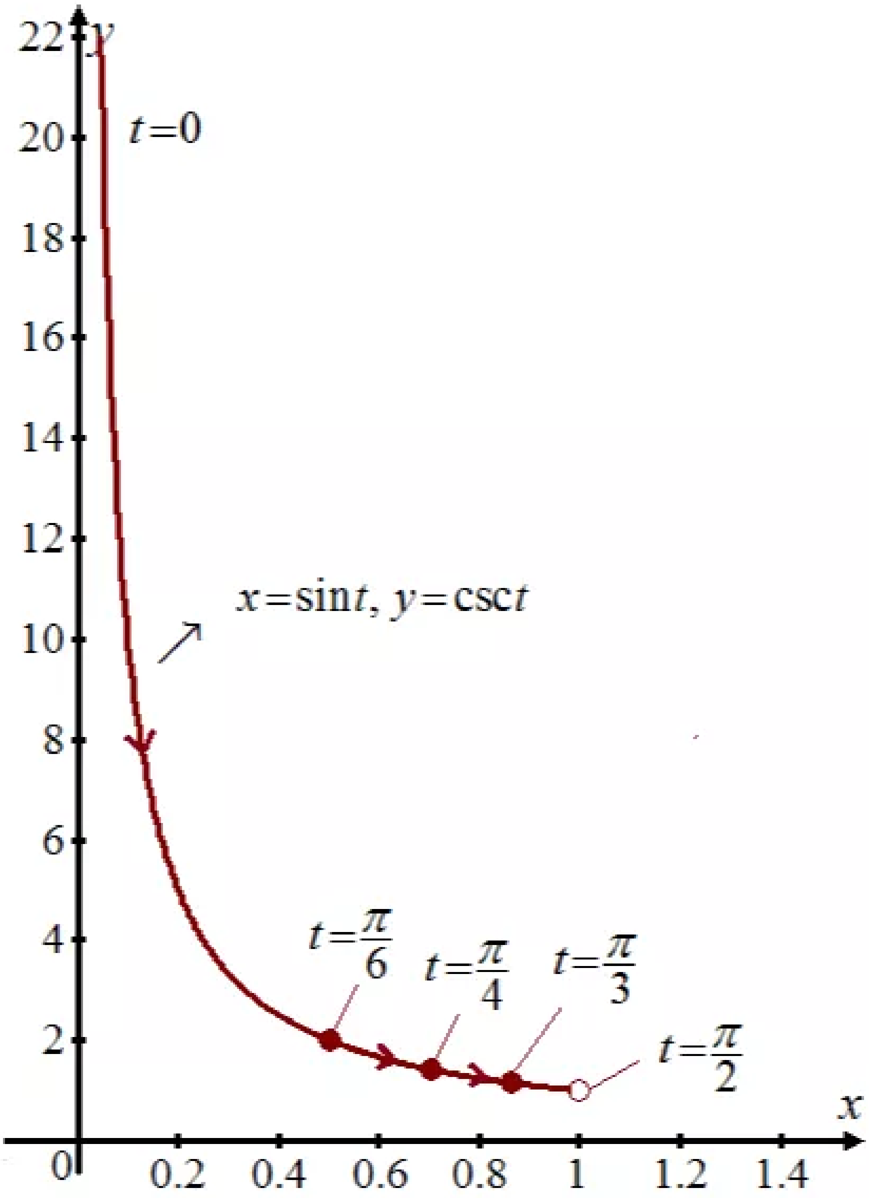
(b)

Make a table of the points (x, y) for different values of t to sketch the curve.

t	$x = \sin t$	$y = \csc t$
0	0	∞
$\frac{\pi}{6}$	0.5	2
$\frac{\pi}{4}$	0.707	1.414
$\frac{\pi}{3}$	0.866	1.154
$\frac{\pi}{2}$	1	1

Plot the above points, and join these, to get the graph of the parametric curve

$x=\sin t,\ y=\csc t,\ 0<t<\frac{\pi}{2}$, as follows:



Q14E

Consider the parametric equations of a curve,

$x=e^t-1,\ y=e^{2t}$, where $t\in\mathbb{R}$

(a) To find the Cartesian equation of this curve, rewrite these equations as follows.

$e^t=x+1,\ y=\left(e^t\right)^2$

Put $e^t=x+1$ in the equation $y=\left(e^t\right)^2$ to get,

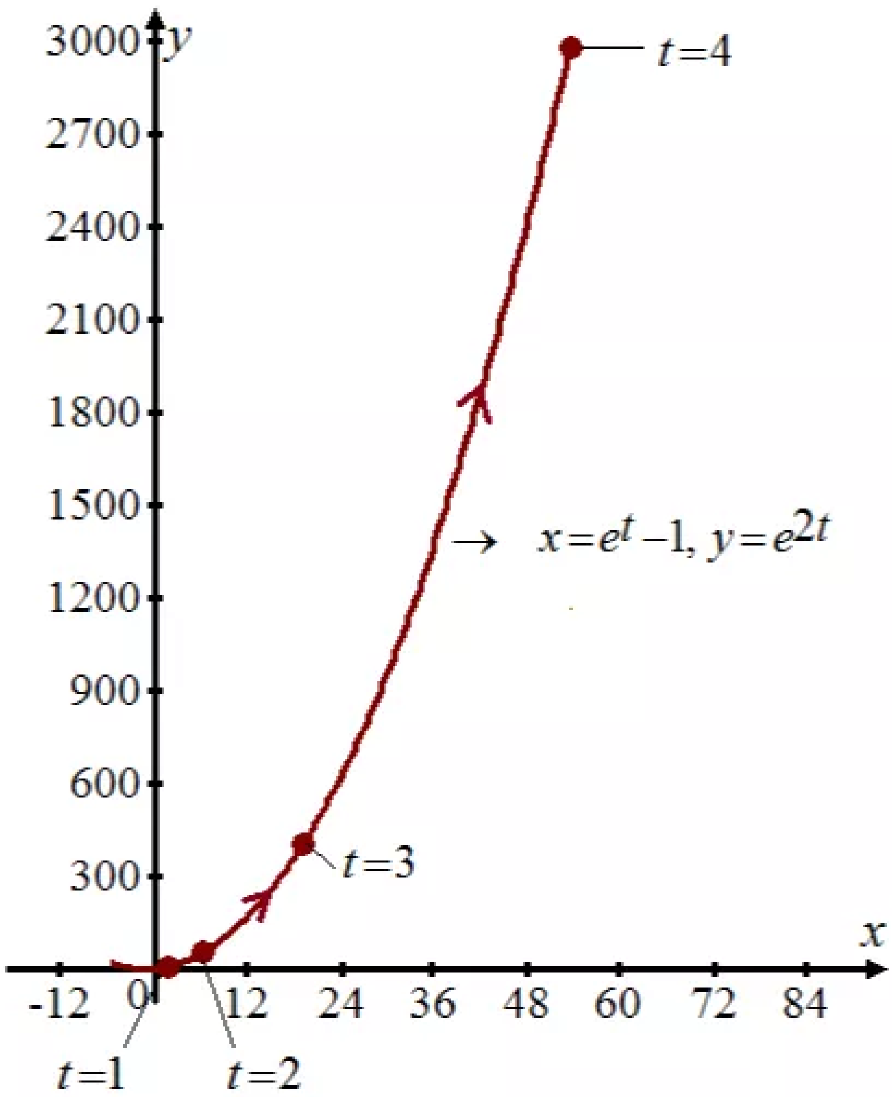
$y=(x+1)^2$

So the Cartesian equation of the curve $x=e^t-1,\ y=e^{2t}$ is $\boxed{y=(x+1)^2}$

(b) Make a table of the points (x, y) for different values of t to sketch the curve.

t	$x = e^t - 1$	$y = e^{2t}$
1	1.72	7.38
2	6.38	54.59
3	19.1	403.42
4	53.6	2980.95

By plotting the above points, and by join of these, we will get the graph of the parametric curve $x = e^t - 1,\ y = e^{2t}$, this shown below:



Q15E

Consider the parametric equations of a curve,

$x = e^{2t}$, $y = t + 1$, where $t \in \mathbb{R}$

(a) To find the Cartesian equation of this curve, rewrite these equations as follows.

$\ln x = \ln\left(e^{2t}\right), y = t + 1$

$\ln x = 2t, y = t + 1$

$t = \frac{1}{2}\ln x, y = t + 1$

Put $t = \frac{1}{2}\ln x$ in the equation $y = t + 1$ to get,

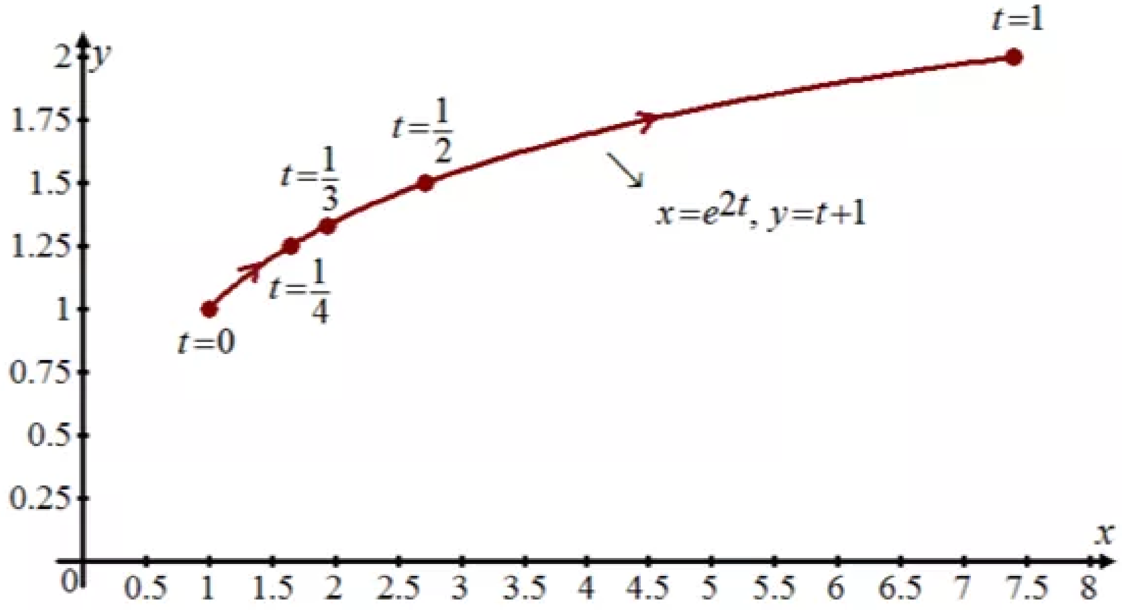
$y = \frac{1}{2}\ln x + 1$

So the Cartesian equation of the curve $x = e^{2t}$, $y = t + 1$ is $\boxed{y = \frac{1}{2}\ln x + 1}$

(b) Make a table of the points (x, y) for different values of t to sketch the curve.

t	$x = e^{2t}$	$y = t + 1$
0	1	1
$\frac{1}{4}$	1.65	1.25
$\frac{1}{3}$	1.94	1.33
$\frac{1}{2}$	2.72	1.5
1	7.39	2

By plotting the above points, and by join of these, we will get the graph of the parametric curve $x = e^{2t}$, $y = t + 1$, this shown below:



Q16E

(a)Consider the equations $x=\sqrt{t+1}, \quad \cdots\cdots(1)$

$y=\sqrt{t-1} \quad \cdots\cdots(2)$

To eliminate the parameter 't'.

Squaring and subtracting of (1) and (2), we get

$x^2 = t+1, y^2 = t-1$

$\Rightarrow x^2 - y^2 = t+1-t+1 = 2$

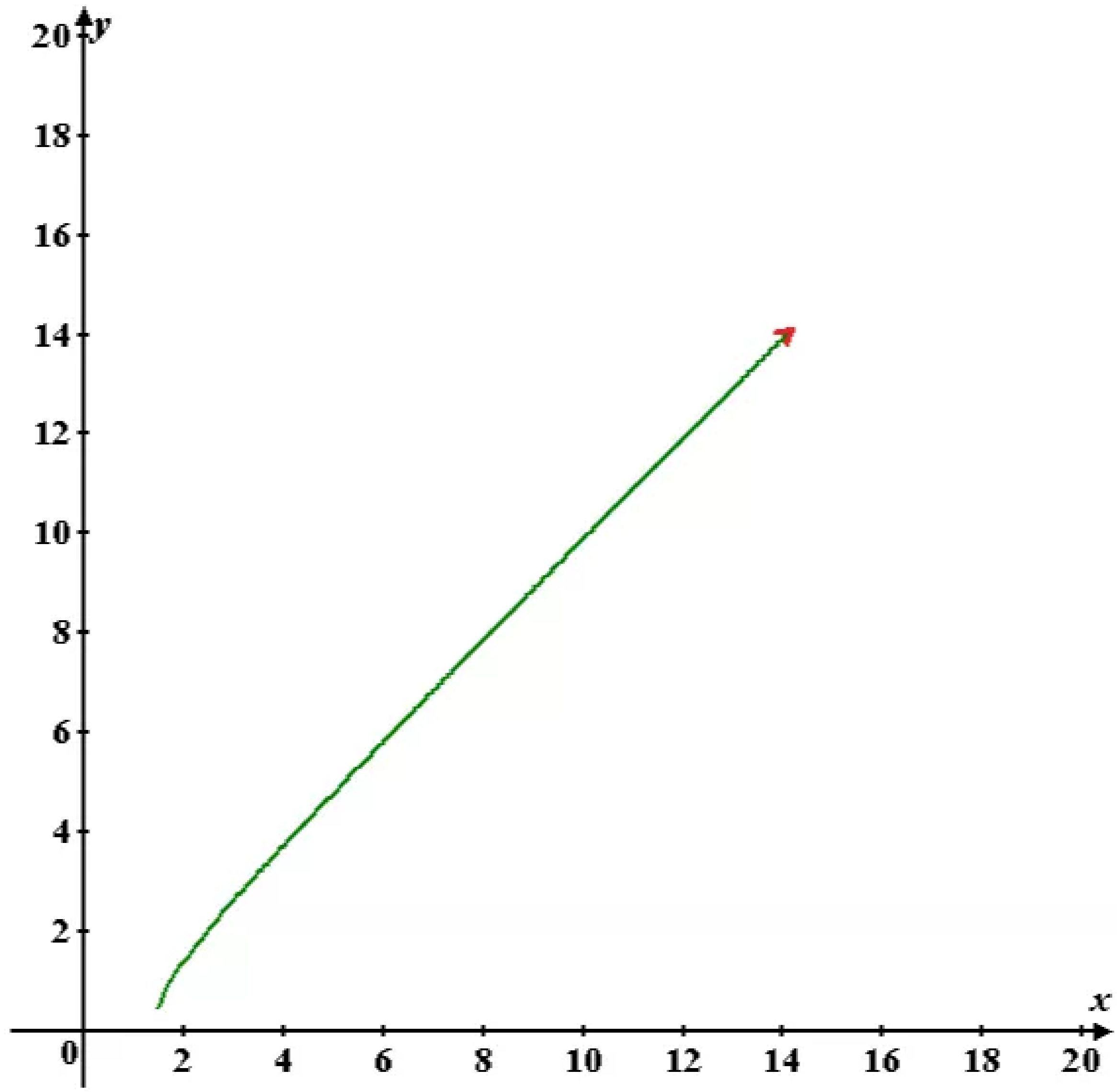
$\Rightarrow \boxed{x^2 - y^2 = 2}$

So, the above equation is the required Cartesian equation suitable to the parametric equation.

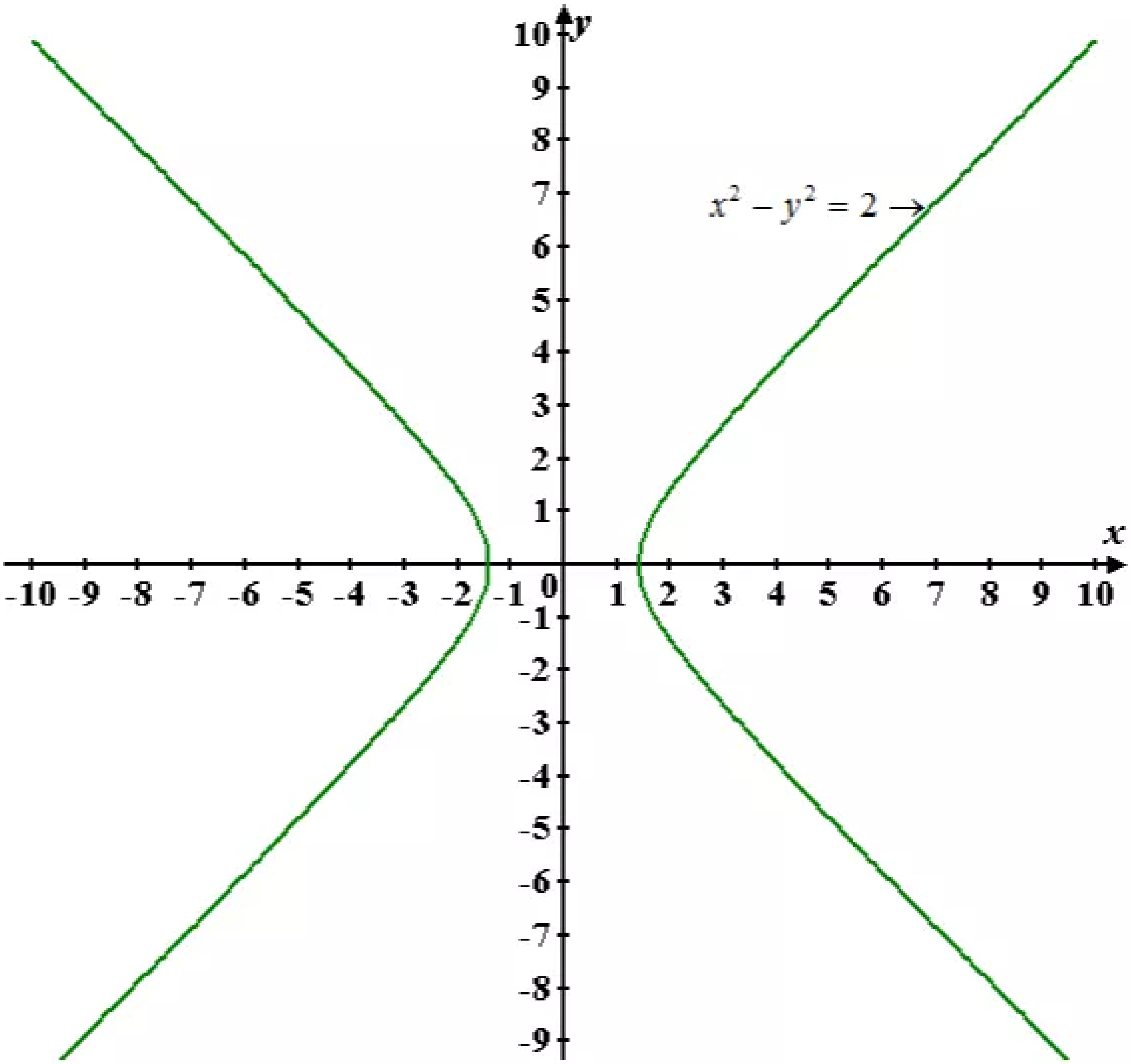
(b) The direction of the parametric curve is shown with the help of the red pointed arrow.

As t increases, the value of y increases and so, the curve is upward.

But, the Cartesian equation do not show this direction while it is a hyperbola.



Cartesian equation:



Q17E

Consider the parametric equations of the following curve:

$x = \sinh t,\ y = \cosh t.$

(a)

To find the Cartesian equation of this curve, use the following result:

$\cosh^2 t - \sinh^2 t = 1$.

Substitute $x = \sinh t,\ y = \cosh t$ in the equation, $\cosh^2 t - \sinh^2 t = 1$, to get the following equation:

$y^2 - x^2 = 1$

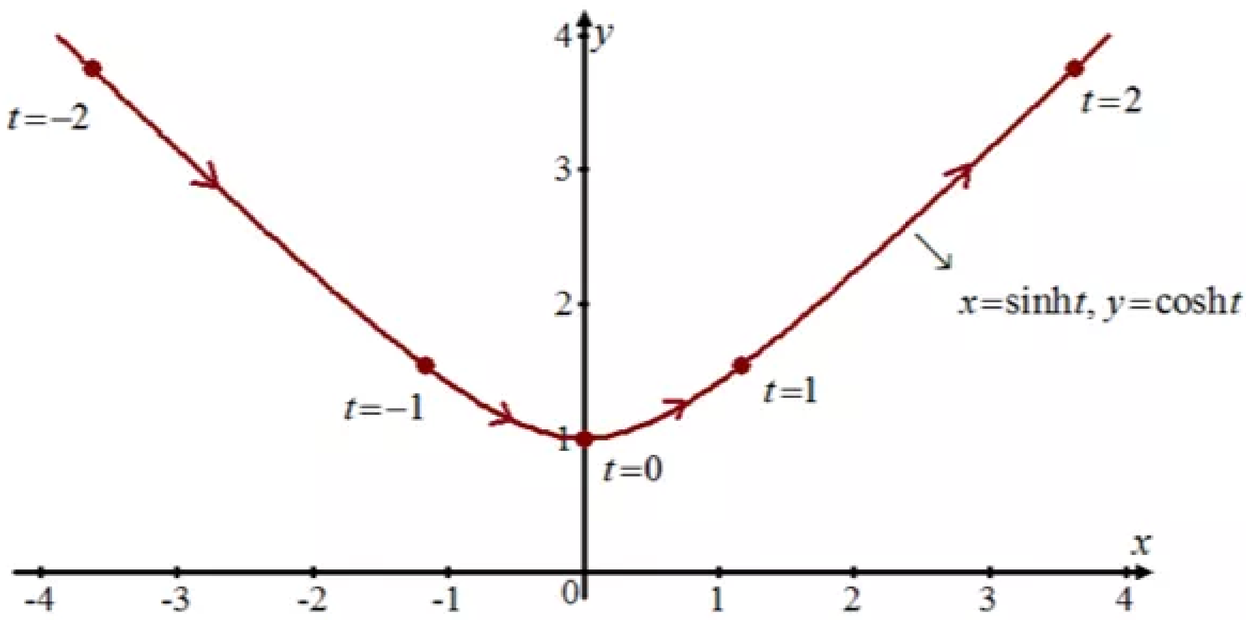
So the Cartesian equation of the curve $x = \sinh t,\ y = \cosh t$ is $\boxed{y^2 - x^2 = 1}$.

(b)

Make a table of the points (x, y) for different values of t to sketch the curve.

t	$x = \sinh t$	$y = \cosh t$
-2	-3.62	3.76
-1	-1.17	1.54
0	0	1
1	1.17	1.54
2	3.62	3.76

Plot the above points, and join these, to obtain the graph of the parametric curve $x = \sinh t,\ y = \cosh t$, this is depicted as follows:



Q18E

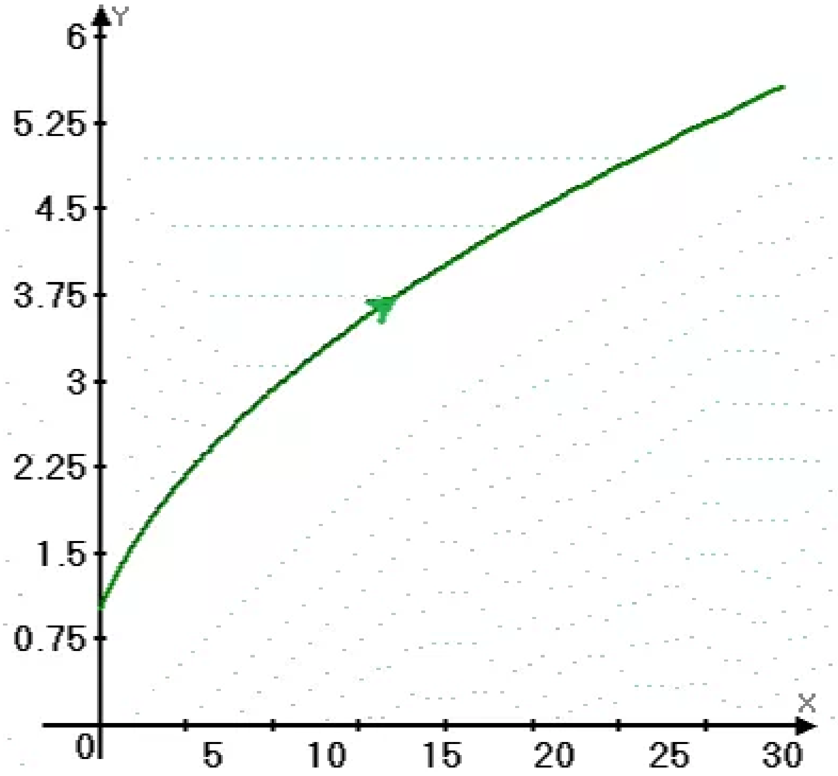
Sol: (a). Given $x = \tan^2\theta, y = \sec\theta$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

we know $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow 1 + x = y^2$

or $\boxed{y^2 - x = 1}$

(b).



The curve increases along with the increase of t from 6.5

Q19E

Consider the position of a particle, which is given by the parametric equations as,

$$x = 3 + 2\cos t,\ y = 1 + 2\sin t,\ \frac{\pi}{2} \le t \le \frac{3\pi}{2}$$

To write this equation into Cartesian form, rewrite these equations as,

$$x - 3 = 2\cos t,\ y - 1 = 2\sin t$$

$$(x-3)^2 = (2\cos t)^2,\ (y-1)^2 = (2\sin t)^2$$

$$(x-3)^2 + (y-1)^2 = (2\cos t)^2 + (2\sin t)^2$$

$$(x-3)^2 + (y-1)^2 = 4(\cos^2 t + \sin^2 t)$$

$$(x-3)^2 + (y-1)^2 = 4$$

And, at $t=\frac{\pi}{2}$, the values of x and y are given as,

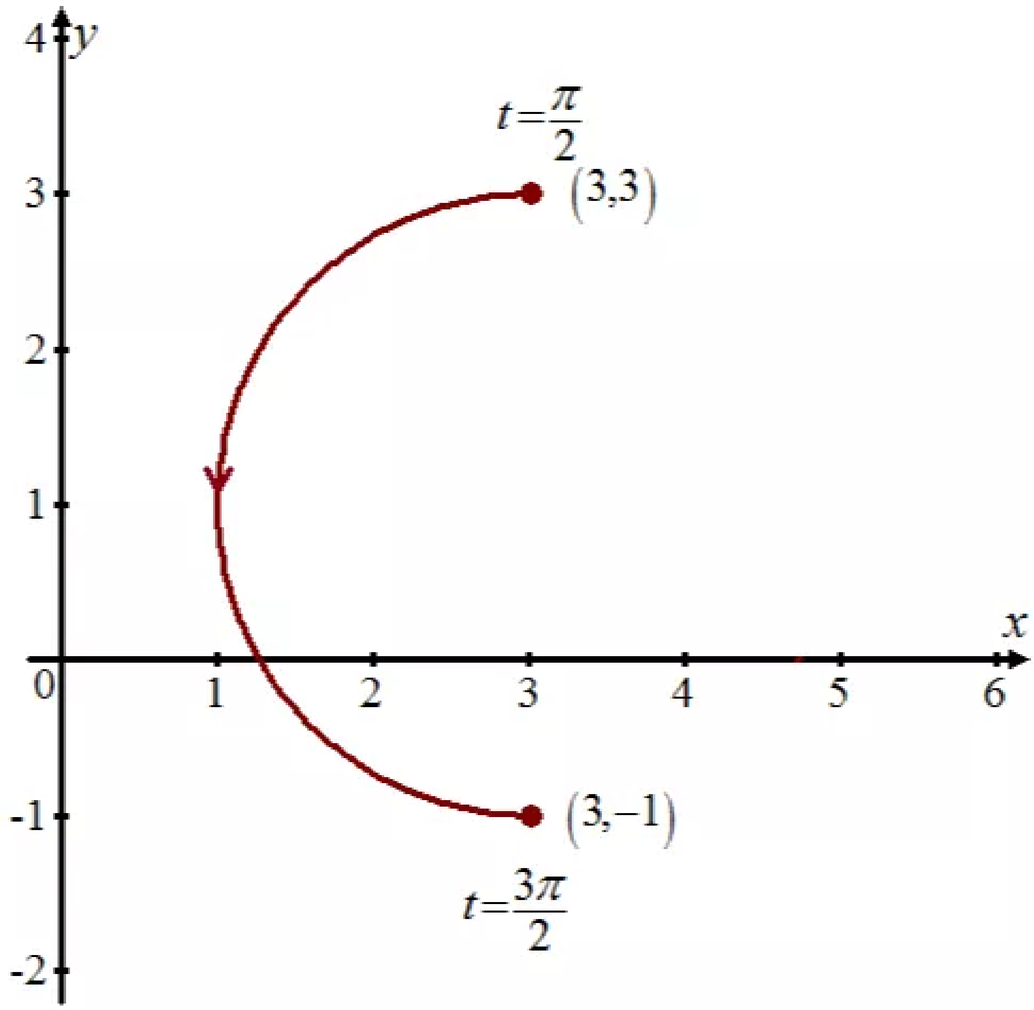
$$x=3+2\cos\frac{\pi}{2},\ y=1+2\sin\frac{\pi}{2}$$
$$x=3+2(0),\ y=1+2(1)$$
$$x=3,\ y=3$$

Also at $t=\frac{3\pi}{2}$, the values of x and y are given as,

$$x=3+2\cos\frac{3\pi}{2},\ y=1+2\sin\frac{3\pi}{2}$$
$$x=3+2(0),\ y=1+2(-1)$$
$$x=3,\ y=-1$$

So the path of the particle is a circle from $(3,3)$ to $(3,-1)$ in counter-clockwise direction.

The path of the given particle $(x-3)^2+(y-1)^2=4$, from $(3,3)$ to $(3,-1)$ is shown below:



Q20E

Consider the position of a particle, which is given by the parametric equations as,

$$x = 2\sin t,\ y = 4 + \cos t,\ 0 \le t \le \frac{3\pi}{2}$$

To change the equations into Cartesian form, rewrite these equations as,

$$\frac{x}{2} = \sin t,\ y - 4 = \cos t$$

$$\left(\frac{x}{2}\right)^2 = (\sin t)^2,\ (y-4)^2 = (\cos t)^2$$

$$\frac{x^2}{4} + (y-4)^2 = \sin^2 t + \cos^2 t$$

$$\frac{x^2}{2^2} + \frac{(y-4)^2}{1^2} = 1$$

For $t = 0$, the values of x, y are given as follows.

$$x = 2\sin 0,\ y = 4 + \cos 0$$

$$x = 2(0),\ y = 4 + 1$$

$$(x, y) = (0, 5)$$

For $t = \frac{3\pi}{2}$, the values of x, y are given as follows.

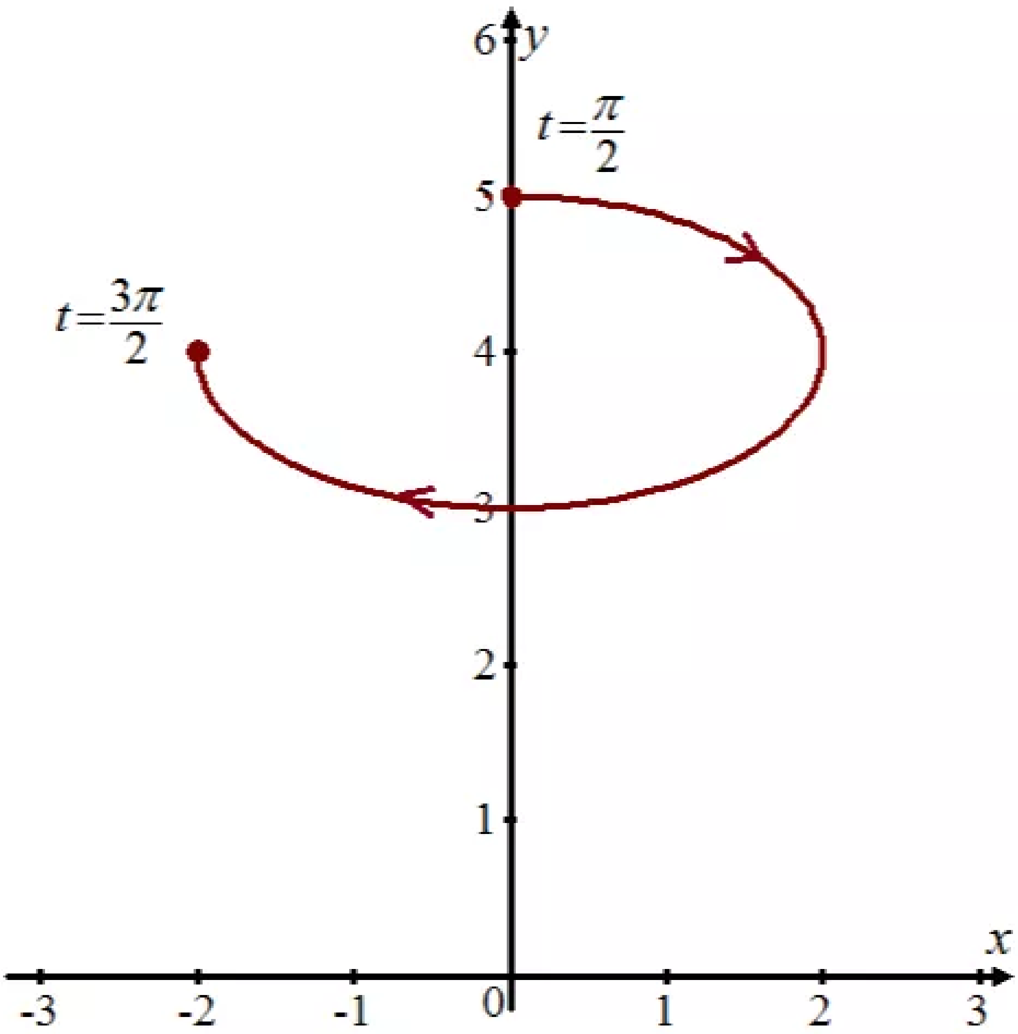
$$x = 2\sin\frac{3\pi}{2},\ y = 4 + \cos\frac{3\pi}{2}$$

$$x = 2(-1),\ y = 4 + 0$$

$$(x, y) = (-2, 4)$$

So the given curve represents an ellipse from $(0,5)$ to $(-2,4)$ in the direction of clockwise.

The position of the given particle from $(0,5)$ to $(-2,4)$ in the direction of clockwise is shown below:



Q21E

Consider the motion of a particle, which is given by the parametric equations as,

$x = 5\sin t,\ y = 2\cos t,\ -\pi \le t \le 5\pi$

To change the equations into Cartesian form, rewrite these equations as,

$\frac{x}{5} = \sin t,\ \frac{y}{2} = \cos t$

$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = \sin^2 t + \cos^2 t$

$\frac{x^2}{25} + \frac{y^2}{4} = 1$ ...... (1)

For $t=-\pi$, the values of x, y are given as follows.

$x=5\sin t,\ y=2\cos t$

$x=5\sin(-\pi),\ y=2\cos(-\pi)$

$x=-5\sin\pi, y=2\cos\pi$

$x=-5(0), y=2(-1)$

$(x,y)=(0,-2)$

For $t=5\pi$, the values of x, y are given as follows.

$x=5\sin t,\ y=2\cos t$

$x=5\sin(5\pi),\ y=2\cos(5\pi)$

$x=5\sin(2(2\pi)+\pi),\ y=2\cos(2(2\pi)+\pi)$

$x=5\sin\pi, y=2\cos\pi$

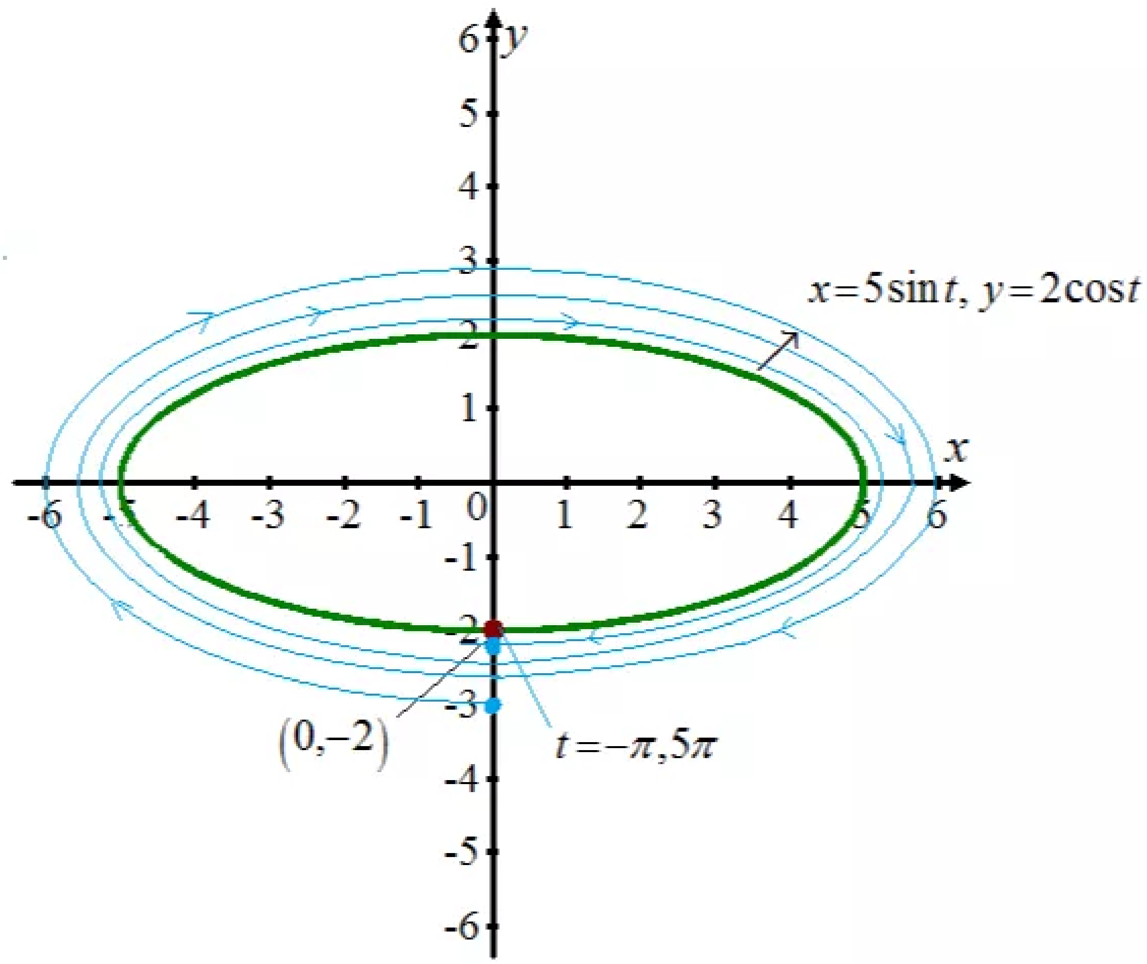
$x=5(0), y=2(-1)$

$(x,y)=(0,-2)$

And, we have $-\pi+3(2\pi)=5\pi$

This mean, moves 3 times clockwise around the ellipse (1), starting and ending at $(0,-2)$.

The motion of the given particle is shown below:



Q22E

Consider the motion of a particle, which is given by the parametric equations as,

$x = \sin t,\ y = \cos^2 t,\ -2\pi \le t \le 2\pi$

To change the equations into Cartesian form, rewrite these equations as,

$x = \sin t,\ y = 1 - \sin^2 t$

Substitute $x = \sin t$ in the equation $y = 1 - \sin^2 t$ to get,

$y = 1 - x^2$ ...... (1)

This equation represents a parabola.

For $t = -2\pi$, the values of x, y are given as follows.

$$x = \sin t,\ y = \cos^2 t$$
$$x = \sin(-2\pi),\ y = \cos^2(-2\pi)$$
$$x = -\sin(2\pi),\ y = (\cos(2\pi))^2$$
$$x = 0,\ y = (1)^2 = 1$$
$$(x, y) = (0, 1)$$

For $t = 2\pi$, the values of x, y are given as follows.

$$x = \sin t,\ y = \cos^2 t$$
$$x = \sin(2\pi),\ y = \cos^2(2\pi)$$
$$x = 0,\ y = (1)^2 = 1$$
$$(x, y) = (0, 1)$$

Now we find the position (x, y) of the particle in each interval of length $\frac{\pi}{2}$, where

$-2\pi \le t \le 2\pi$.

At, $t = -2\pi$.

$(x, y) = (0,1)$

At, $t = -2\pi + \frac{\pi}{2} = -\frac{3\pi}{2}$.

$(x, y) = (1,0)$

At, $t = -\frac{3\pi}{2} + \frac{\pi}{2} = -\pi$.

$(x, y) = (0,1)$

At, $t = -\pi + \frac{\pi}{2} = -\frac{\pi}{2}$.

$(x, y) = (-1,0)$

At, $t = -\frac{\pi}{2} + \frac{\pi}{2} = 0$.

$(x, y) = (0,1)$

At, $t = 0 + \frac{\pi}{2} = \frac{\pi}{2}$.

$(x, y) = (1,0)$

Continue the above, we have

At, $t = \frac{\pi}{2} + \frac{\pi}{2} = \pi$.

$(x, y) = (0,1)$

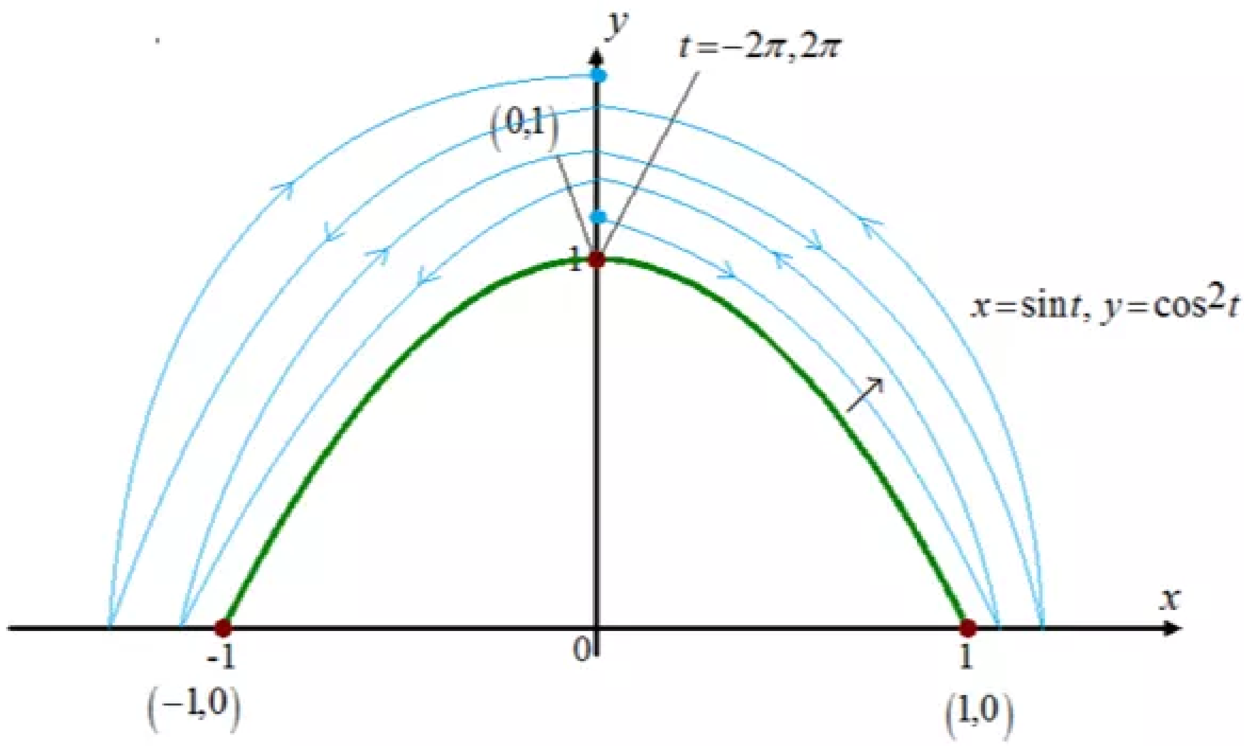
At, $t = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$.

$(x, y) = (-1,0)$

At, $t = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$.

$(x, y) = (0,1)$

The motion of the particle between the above potions, given by the equations $x = \sin t,\ y = \cos^2 t,\ -2\pi \le t \le 2\pi$, is shown below:



Q23E

We have $x = f(t)$ and $y = g(t)$

range of $f = [1,4]$ so $1 \le x \le 4$

range of $g = [2,3]$ so $2 \le y \le 3$

The curve is contained in a rectangle described by $1 \le x \le 4$ and $2 \le y \le 3$.

Q24E

(A) By the given curves we see that $1 \le x \le 2$ for $0 \le t \le 1$

and $-1 \le y \le 1$ for $0 \le t \le 1$

So parametric curve must be a loop between x = 1 and x = 2 starting and ending at (1, 0)

So parametric curve for the given curve is $\boxed{\text{labeled}(\text{III})}$

(B) By the given curves we see that curve starts from (0, 0)

And $-2 \le x \le 2$ for $0 \le t \le 1$

$-2 \le y \le 2$ for $0 \le t \le 1$

Both the graphs are periodic

The value of x moves from -2 to 2 four times and the value of y moves from -2 to 2, six times

So we can conclude that for the given curves, the parametric curve is the curve $\boxed{\text{labeled}(\text{I})}$

(C) By the given curves we see that

$-2 \le x \le 2 \quad for\ -2 \le t \le 2$

$and \quad 0 \le y \le 2 \quad for \quad -2 \le t \le 2$

The value of x moves from -2 to 2 three times and the value of y moves from 0 to 2 once.

Parametric curve for the given curves is $\boxed{\text{labeled}(IV)}$

(D) From the given curves we see that

$-2 \le x \le 2 \quad for \quad -2 \le t \le 2$

And $\quad -2 \le y \le 2 \quad for \quad -2 \le t \le 2,$

Curve passes through the points (2, 2), (2,-2) and (-2,-2)

So parametric curve for the given curves is $\boxed{\text{labeled}(II)}$

Q25E

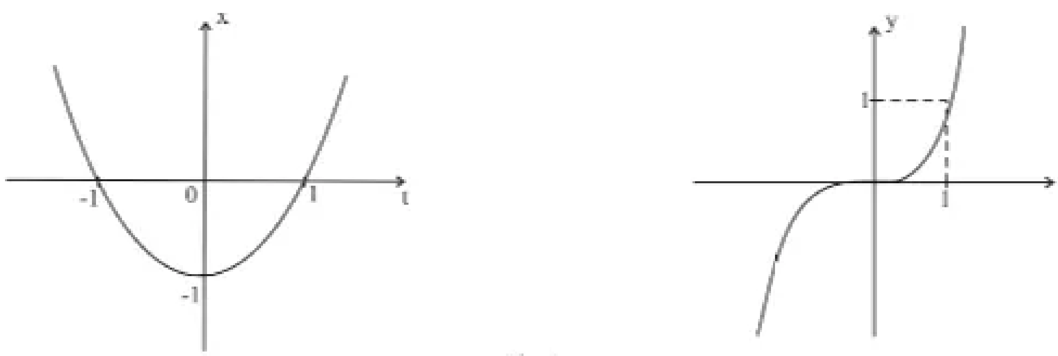


Fig. 1

From the graphs we see that

At $t = 0, \quad x = -1$ and $y = 0$

At $t = -1, \quad x = 0$ and $y = -1$

At $t = 1, \quad x = 0$ and $y = 1$

For parametric curve we have following conclusions

(1) Curve passes from the points (-1,0) , (0,-1) and (0,1)

(2) Direction of the curve is from (0, -1) to (0, 1) through the point (-1, 0)

Now we sketch the parametric curve

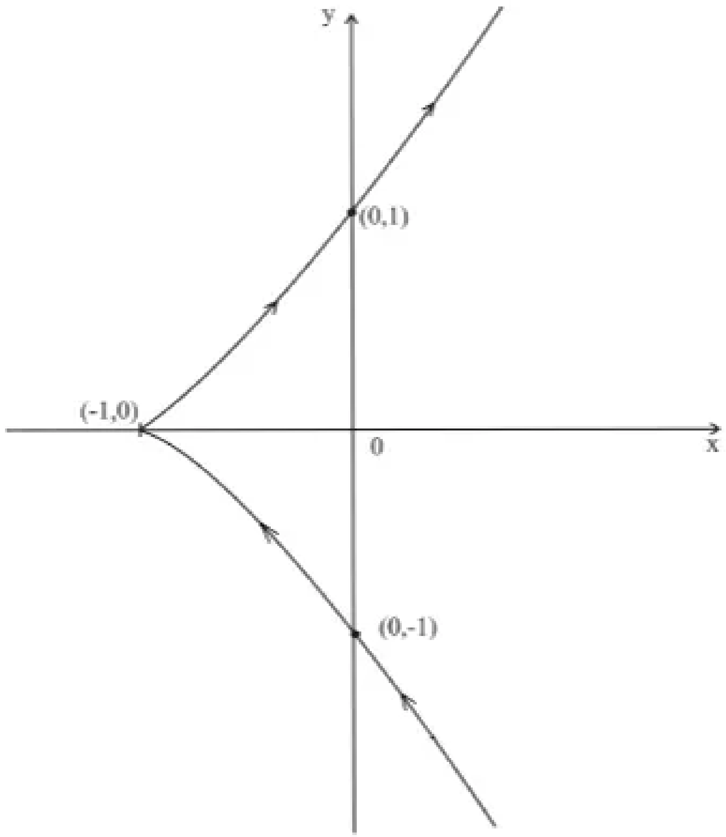


Fig. 2

Q26E

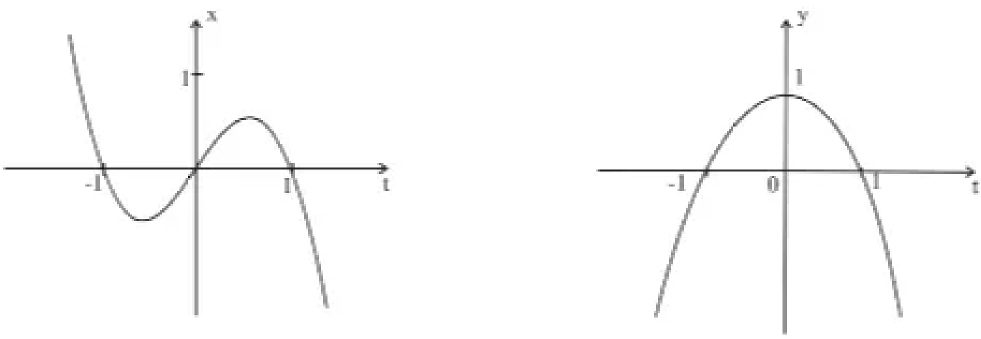


Fig. 1

$at\, t = -1, \quad x = 0 \quad and \quad y = 0$

$at\, t = 0, \quad x = 0 \quad and \quad y = 1$

$at\, t = 1, \quad x = 0 \quad and \quad y = 0$

For t =-1 and t = 1, the curve passes through (0, 0), which means that the curve forms a loop between (0, 0) and (0, 1)

Parametric curve passes through (0, 0) and (0, 1)

Direction of the curve is positive x-axis to negative x-axis

Now we can sketch the curve

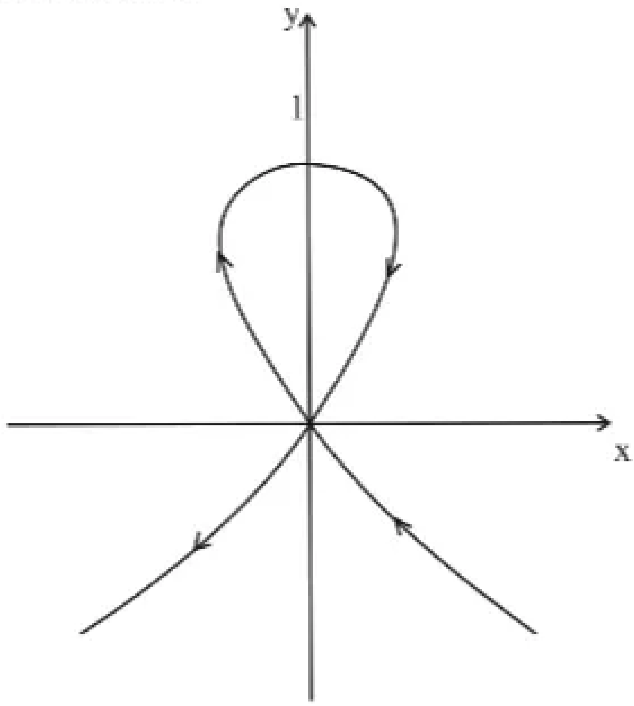


Fig. 1

Q27E

From the graphs we see that
At $t = 0$, $x = 0$, and $y = 0$
At $t = 1/2$, $x = 0$ and $y = 1$
At $t = 1$, $x = 0$ and $y = 0$
We have $-1 \le x \le 1$ and $0 \le y \le 1$
We conclude that
Curve lies between $x = -1$ to $x = 1$
Curve passes through the origin and (0, 1)
Curve is starting from (0, 0) and ending at (0, 0)
As t t increases the value of x oscillates between -1 and 1.
For $0 < t < 1/2$, the value of y increase up to 1 and then for $1/2 < t < 1$, the value of y decreases up to 0.

We sketch the parametric curve

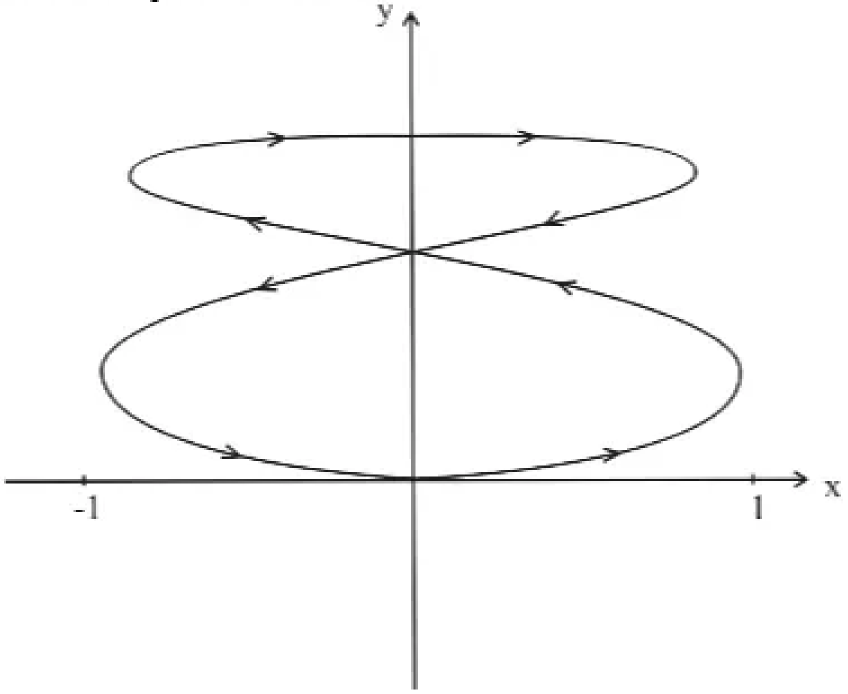
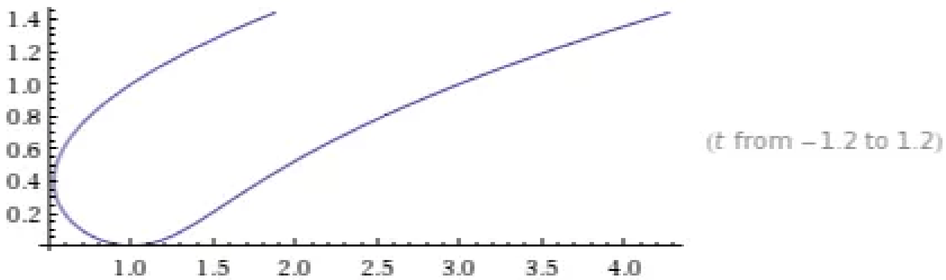


Fig. 1

Q28E

(a) Given parametric equations are:

$x = t^4 - t + 1, y = t^2$

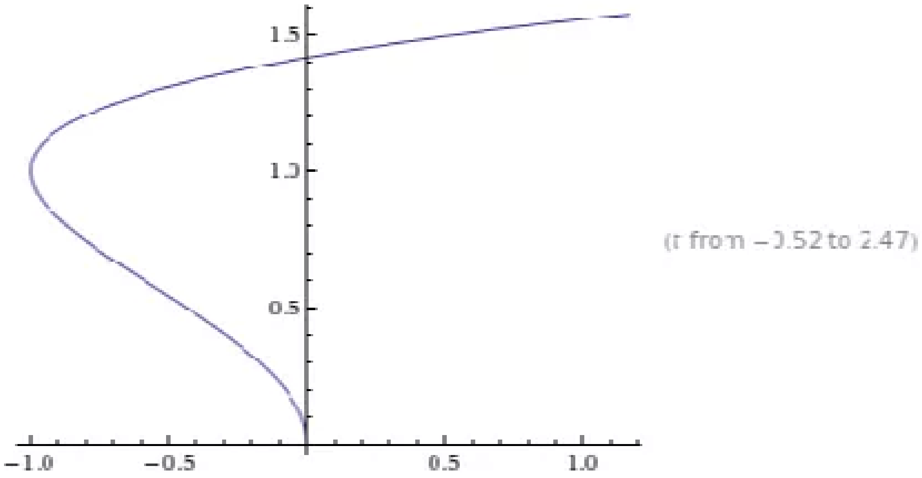


This equation's graph matches with that of (V).

As t goes to infinity, both x and y go to infinity. The only graphs that have this behavior are (V), (I), and (IV). The graph must also contain the point (1,0) achieved at $t = 0$. Graph (V) and (IV) satisfy this requirement. However, you know that this function should not have any oscillatory behavior. Therefore, the parametric equations match graph (V).

(b) Given parametric equations are:

$x = t^2 - 2t, y = \sqrt{t}$

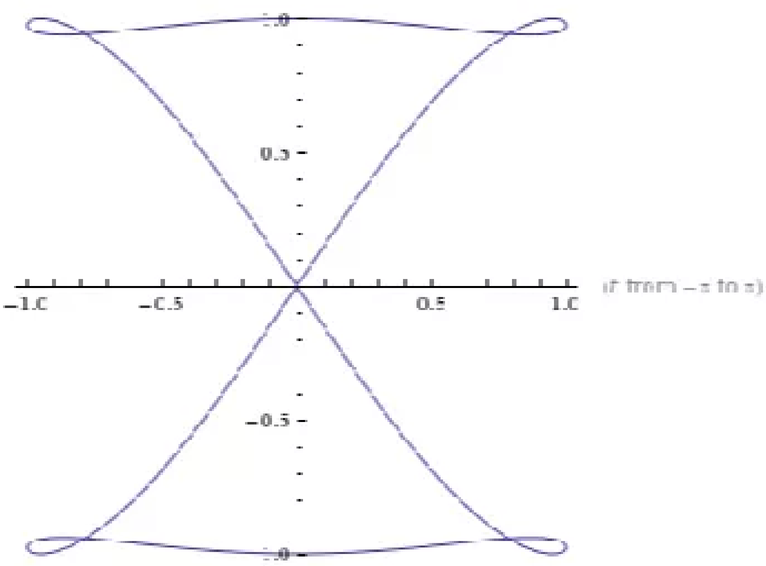


This equation's graph matches with that of (I).

As t goes to infinity, both x and y go to infinity. The only graphs that have this behavior are (V), (I), and (IV). The graph must also contain the point (0,0) achieved at $t = 0$. Only graph (I) satisifies this requirement.

(c) Given parametric equations are:

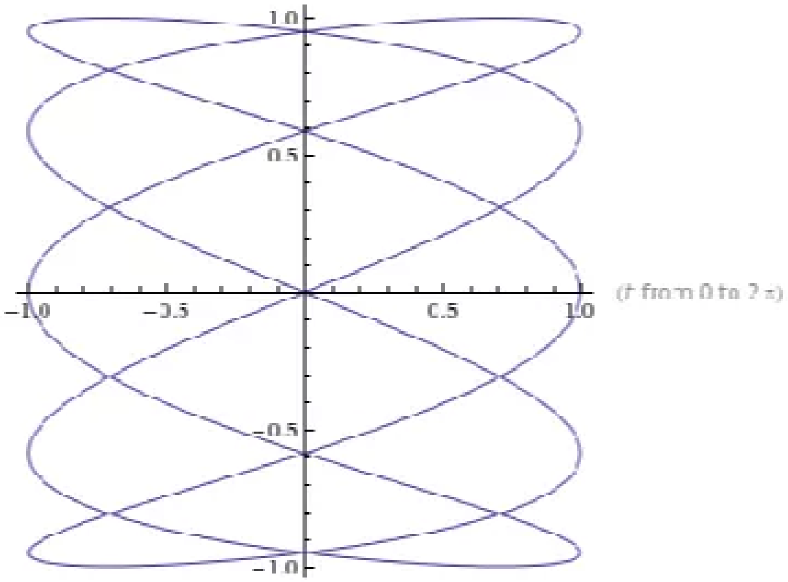
$x = \sin 2t, y = \sin(t + \sin 2t)$



This equation's graph matches with that of (II).

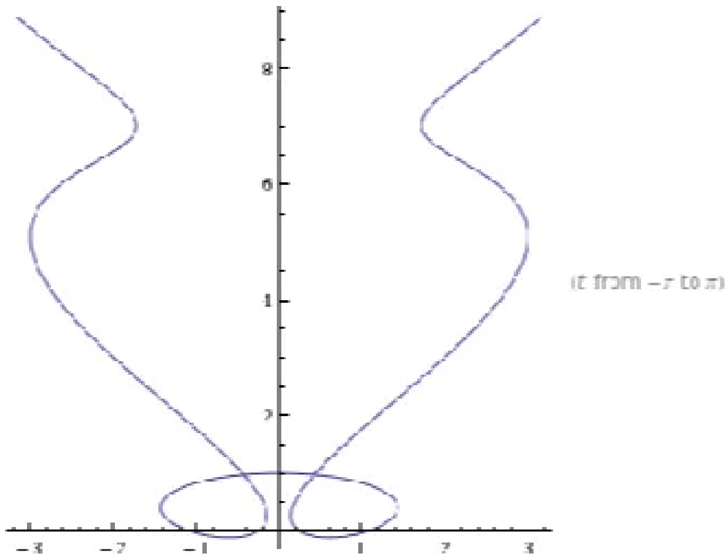
The graph must contain the point (0,1) achieved at $t = \dfrac{\pi}{2}$. Graph (II) is the only graph that satisifies this.

(d)Given parametric equations are:
$x = \cos 5t, y = \sin 2t$



This equation's graph matches with that of (VI).
The graph must contain the point (1,0) achieved at $t = 0$. Graph (VI) is the only graph that satisifies this.

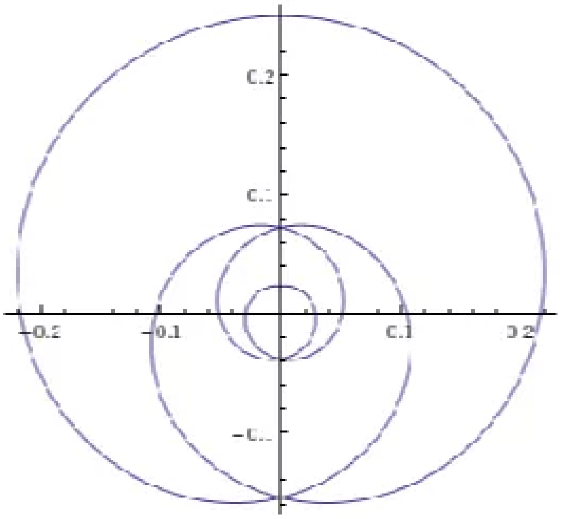
(e) Given parametric equations are:
$x = t + \sin 4t, y = t^2 + \cos 3t$



This equation's graph matches with that of (IV).
As t goes to infinity, both x and y go to infinity. The only graphs that have this behavior are (V), (I), and (IV). However, graph (IV) is the only one that shows any oscillatory behavior.

(f) Given parametric equations are:

$$x = \frac{\sin 2t}{4+t^2}, y = \frac{\cos 2t}{4+t^2}$$



This equation's graph matches with that of (III).

The only graphs that exhibit oscillatory behavior are graphs (III), (II), (IV), and (VI). The graph must also satisfy the fact that the point (0,0) cannot be a point on the graph. Graph (III) and (IV) satisfy this requirement. However, for graph (IV), as t goes to infinity, both x and y go to infinity. This is not the case for these equations.

Q29E

We have to graph the curve $x = y - 3y^3 + 5y^5$

Let $y = t$ then $x = t - 3t^3 + t^5$

Now we calculate x and y for different values of t and graph the curve.

t	-3	-2	-1	0	1	2	3
x	-165	-10	1	0	-1	10	165
y	-3	-2	-1	0	1	2	3

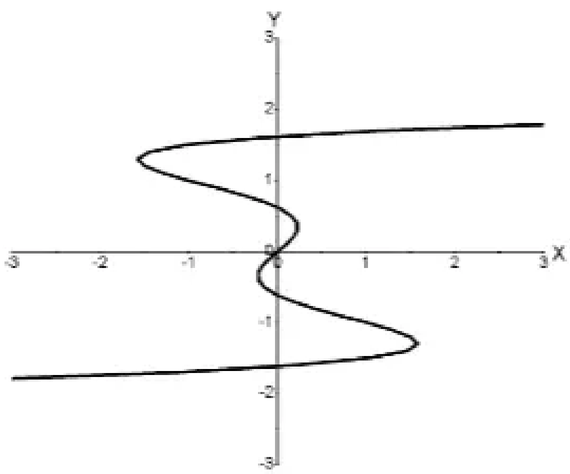


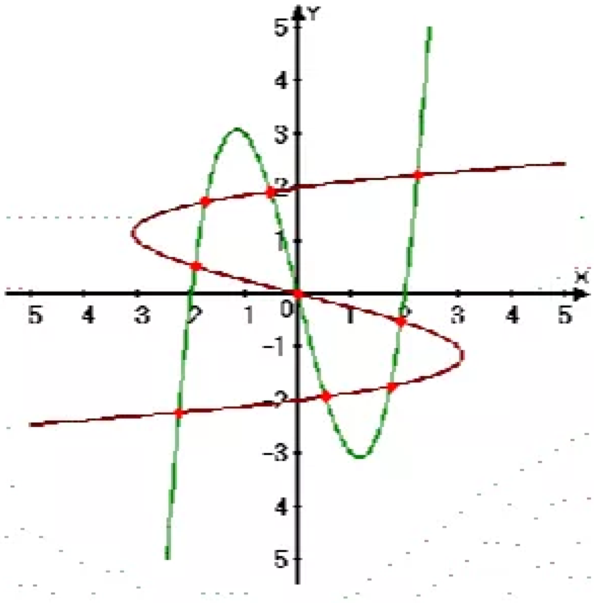
Fig.1

Q30E

Sol: Given

$x = y^3 - 4y, y = x^3 - 4x$

The curves are shown in the following graph.



The vermillion color dots show the points of intersection.

There are nine points of intersection

(-2.24,-2.24), (0.53,-1.93), (1.74,-1.74), (1.93,-0.53), (-1.74,1.74), (-0.53, 1.93), (0, 0), (-1.93, 0.53), (2.24,2.24)

Q31E

(A) We have $x = x_1 + (x_2 - x_1)t$ ......(1)

$y = y_1 + (y_2 - y^2)t$ ......(2) where $0 \le t \le 1$

From equation (1) we have $t = \frac{(x - x_1)}{(x_2 - x_1)}$

Putting the values of t in equation (2)

$$y = y_1 + (y_2 - y_1)\frac{(x - x_1)}{(x_2 - x_1)}$$

$$\Rightarrow (y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

This is the equation of line segment joining the points (x_1, y_1) and (x_2, y_2)

(B) Here $P_1 = (-2, 7)$ and $P_2 = (3,-1)$

So $x_1 = -2,\ x_2 = 3,\ y_1 = 7$ and $y_2 = -1$

Putting these values in equations (1) and (2) respectively

$$x = -2 + (3+2)t \quad \Rightarrow \boxed{x = 5t - 2} \quad ....(3)$$

$$y = 7 + (-1-7)t \quad \Rightarrow \boxed{y = 7 - 8t} \quad ......(4)$$

Equations (3) and (4) are the parametric equations to represent the line segment from (-2, 7) to (3,-1)

Q32E

Let ABC be the triangle with vertices A (1, 1), B (4, 2), and C (1, 5)

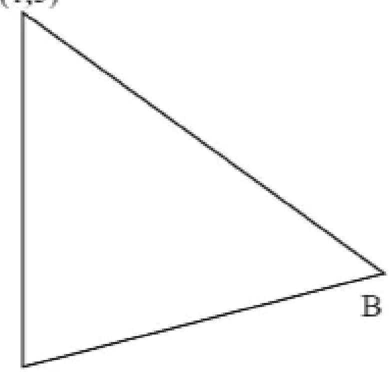


Fig. 1

The parametric equations to represent the line segment from (x_1, y_1) to (x_2, y_2)

$$x = x_1 + (x_2 - x_1)t \quad and \quad y = y_1 + (y_2 - y_1)t \quad where\ 0 \le t \le 1$$

Parametric equations to represent line segment from (1, 1) to (4, 2)

$$x = 1 + (4-1)t \ \ and \quad y = 1 + (2-1)t$$

or $x = 1 + 3t$ and $y = 1 + t$

So $\boxed{x = 1 + 3t}$ .....(1)

$\boxed{y = 1 + t}$ .....(2) Where $0 \le t \le 1$

Now Parametric equations to represent line segment from (4, 2) to (1, 5)

$$x = 4 + (1-4)t \quad and \quad y = 2 + (5-2)t$$

$\Rightarrow \boxed{x = 4 - 3t}$ ....(3) where $0 \le t \le 1$

And $\boxed{y = 2 + 3t}$ ......(4)

Parametric equations to represent line segment line from (1, 5) to (1, 1)

$$x = 1 + (1-1)t \ \ and \quad y = 1 + (5-1)t$$

Or $\boxed{x = 1}$ ....(5) where $0 \le t \le 1$

$\boxed{y = 1 + 4t}$ ....(6)

Now using these parametric equations and with the help of computer we graph the triangle

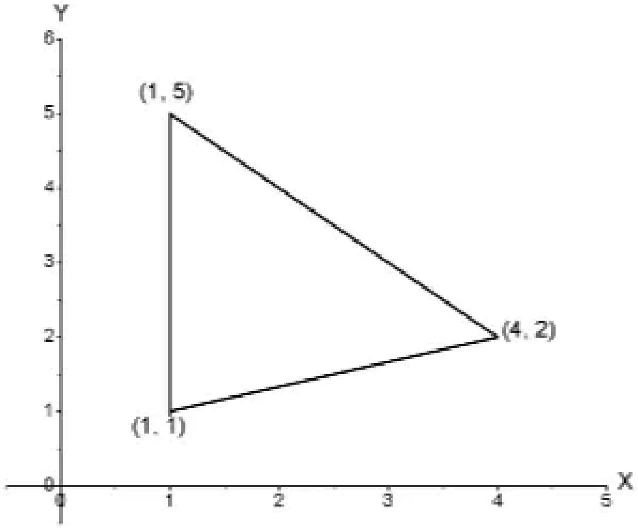


Fig. 2

Q33E

Equation is $x^2 + (y-1)^2 = 4$.

$\Rightarrow \quad x^2 + (y-1)^2 = 4.1$

$\Rightarrow \quad x^2 + (y-1)^2 = 4(\cos^2 t + \sin^2 t)$

$\Rightarrow \quad x^2 + (y-1)^2 = 4\cos^2 t + 4\sin^2 t$

$\Rightarrow \quad x^2 = 4\cos^2 t \quad \text{and} \quad (y-1)^2 = 4\sin^2 t$

If we take positive square roots.

Then $x = 2\cos t$ and $y - 1 = 2\sin t$.

$$\Rightarrow \left.\begin{aligned} x &= 2\cos t \\ \text{and } y &= 1 + 2\sin t \end{aligned}\right\}$$

There parametric equations represent the motion of a particle around the circle $x^2 + (y-1)^2 = 4$, anticlockwise.

(A) We want the motion around the circle clockwise so, we have to change these equations as,

$X = 2\cos t$

$Y = 1 - 2\sin t$

Since sin t and cos t are periodic functions with period of 2π so, for only once around clockwise and starting at (2,1) we can restrict these parametric equation as

$$\boxed{\begin{aligned} x &= 2\cos t \\ y &= 1 - 2\sin t \end{aligned}} \text{ where } 0 \le t \le 2\pi$$

(B) We want to move the particle three times around the circle anticlockwise. So we take the interval $[0, t\pi]$ in the original equations.

$x = 2\cos t.$ where $0 \le t \le 6\pi$

$y = 1 + 2\sin t.$

(C) Now we want to start at (0, 3) and move the particle anti-clock wise halfway around the circle, for this me must have.

$x_1 = 0$

$\Rightarrow \quad 2\cos t = 0 \quad \Rightarrow t = \pi/2$

So, we use the parametric equations.

$$\boxed{\begin{array}{l} x = 2\cos t \\ x = 1 + 2\sin t \end{array}} \quad \text{where } \frac{\pi}{2} \le t \le \frac{3\pi}{2}$$

Q34E

(A) We have $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Given, equation of ellipse)

Using the identity

$\sin^2 t + \cos^2 t = 1 \qquad 0 \le t \le 2\pi$

We have $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \sin^2 t + \cos^2 t$

By comparison we have

$\frac{x^2}{a^2} = \sin^2 t$ and $\frac{y^2}{b^2} = \cos^2 t$

$\Rightarrow \boxed{x = a\sin t} \qquad \boxed{y = b\cos t}$ where $0 \le t \le 2\pi$

These equations are the parametric equations for the ellipse.

(B) Now taking $a = 3$

We graph the parametric equations $x = 3\sin t$ and $y = b\cos t$

Where b = 1, 2 ,4 and 8 respectively.

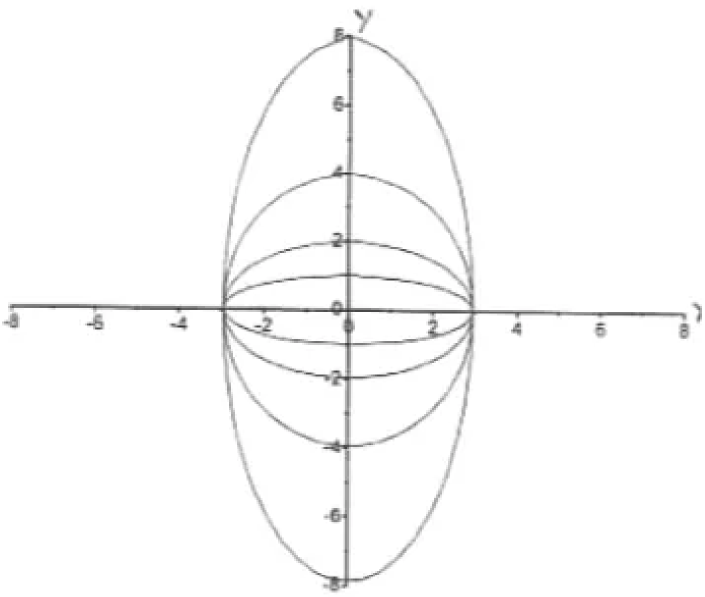


Fig. 1

(C) As b increases, the ellipse stretches vertically

Q35E

To produce the figure of a laughing face, use the advanced grapher as follows:

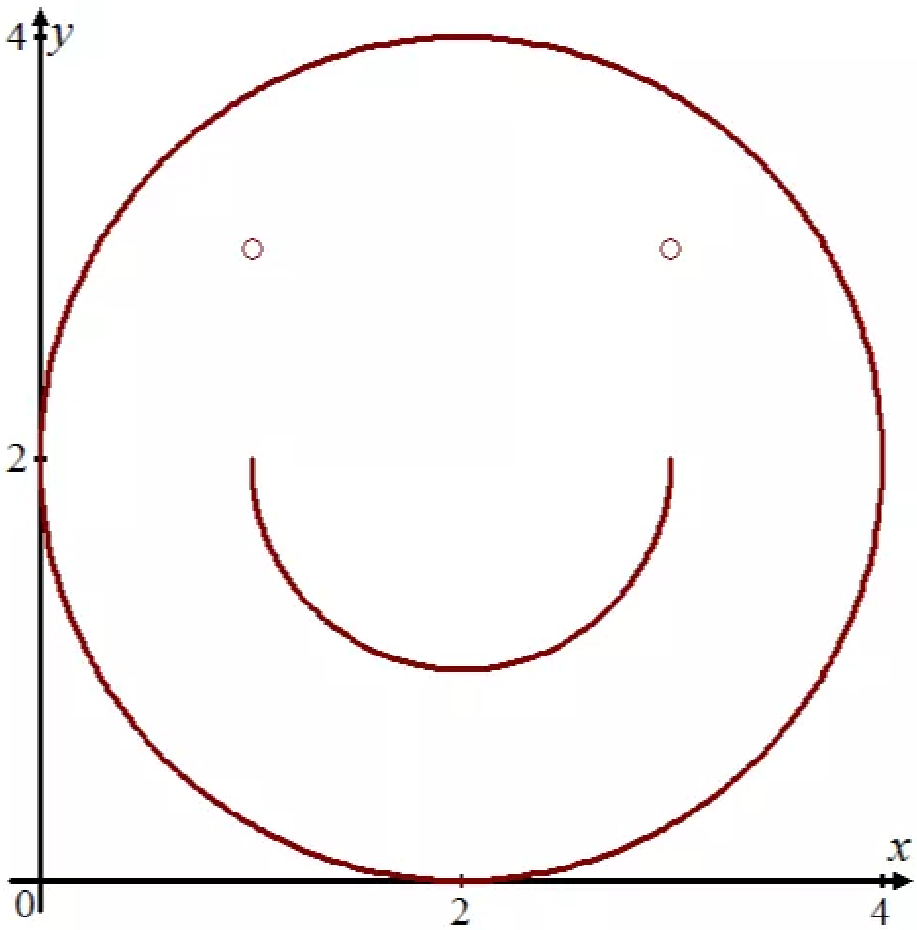
First sketch the equation of the following semi-circle:

$(x-2)^2+(y-2)^2=4$

Then, sketch the inner circle $(x-2)^2+(y-2)^2=1$ for $1\le y\le 2$.

Finally plot the points $(1,3),(3,3)$.

The sketch of the laughing face using a grapher is as follows:



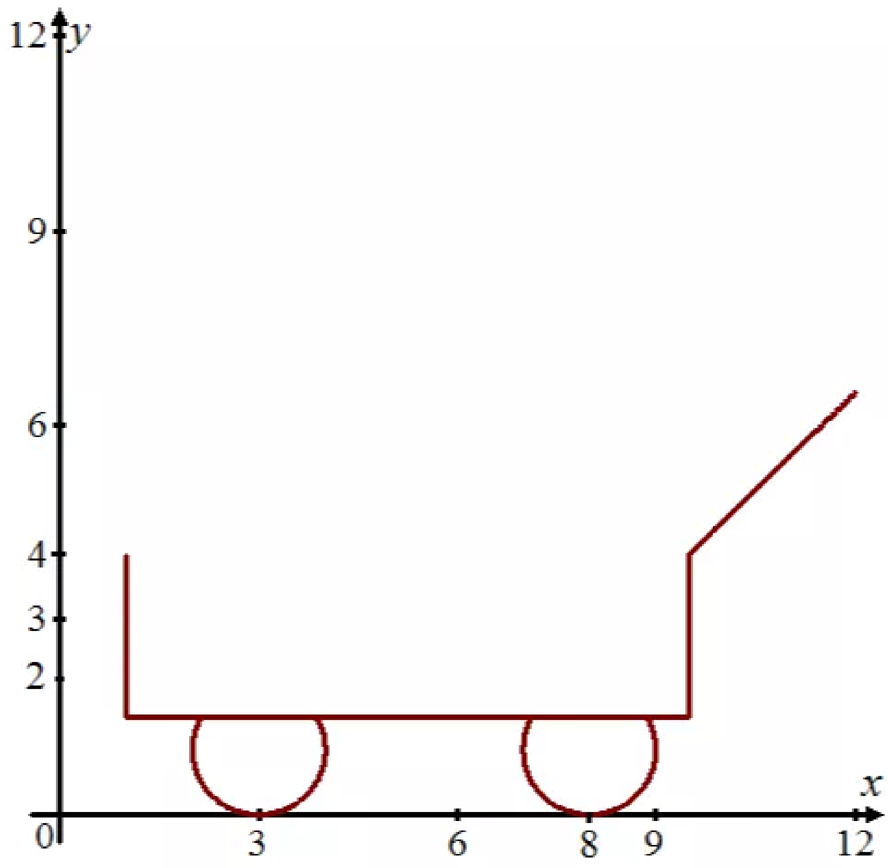
Q36E

It is need to produce the picture of a trolley.

To do this, use advanced grapher as follows.

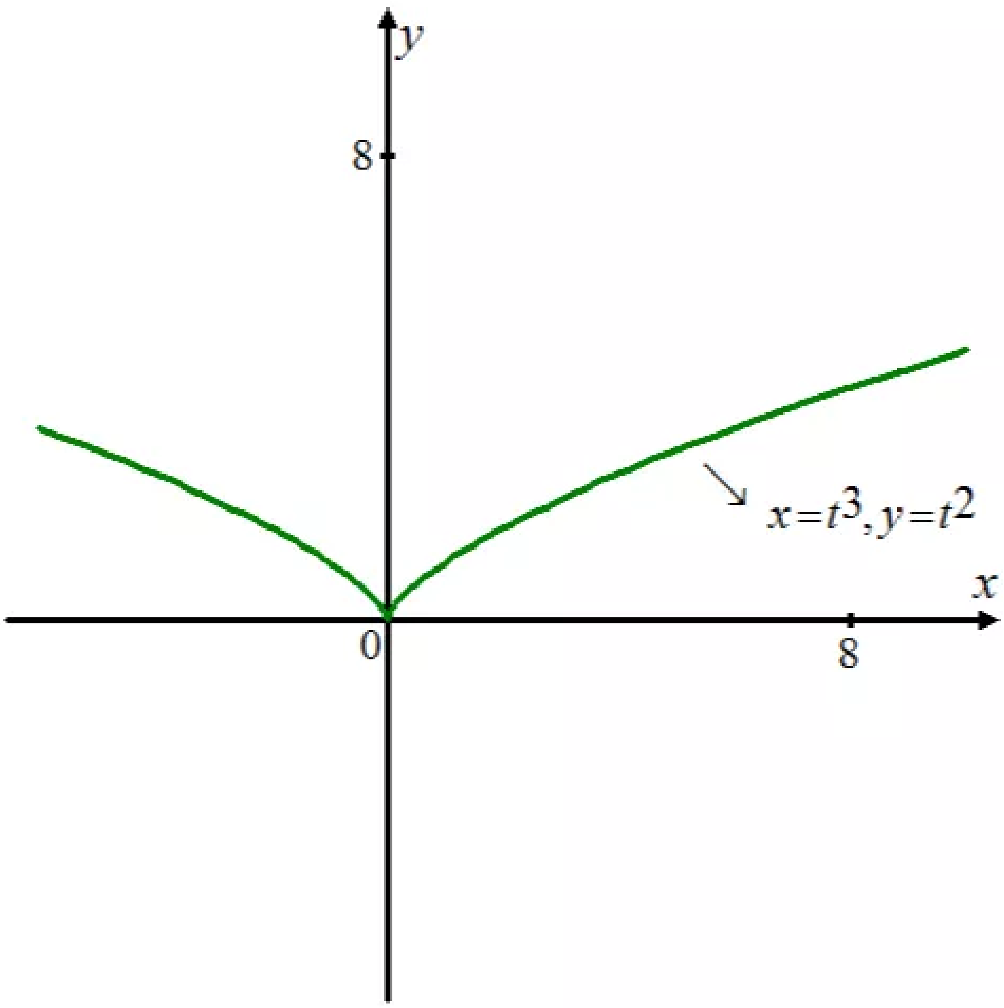
1. Sketch the graph of a part of the circle $(x-8)^2+(y-1)^2=1$ for $y\le 1.5$
2. Sketch the graph of a part of the circle $(x-3)^2+(y-1)^2=1$ for $y\le 1.5$
3. Sketch the graph of a part of the horizontal line $y=1.5$ for $1\le x\le 9.5$
4. Sketch the graph of a part of the vertical line $x=1$ for $1.5\le y\le 4$
5. Sketch the graph of a part of the vertical line $x=9.5$ for $1.5\le y\le 4$
6. Sketch the graph of a part of the line $y=x-5.5$ for $9.5\le x\le 12$

Then the sketch of the trolley using the above information is shown below:

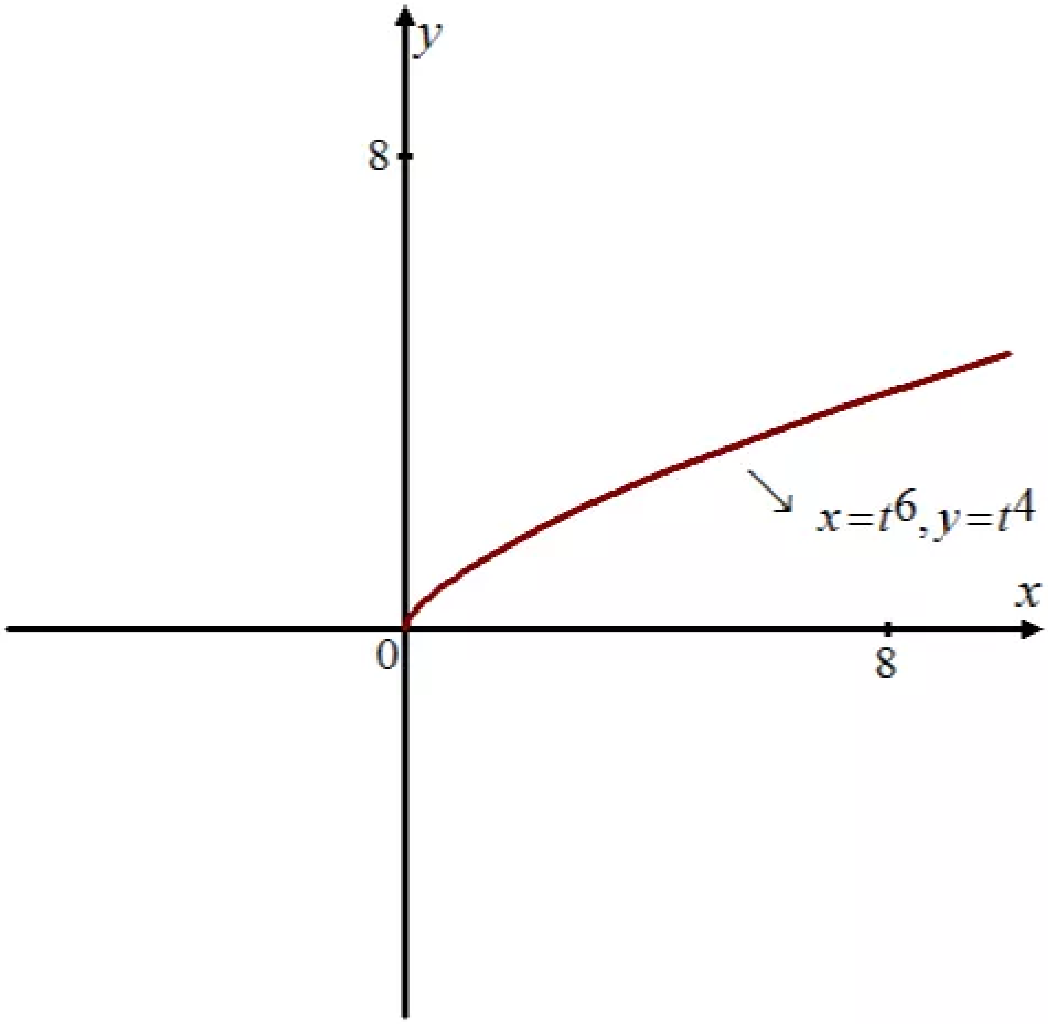


Q37E

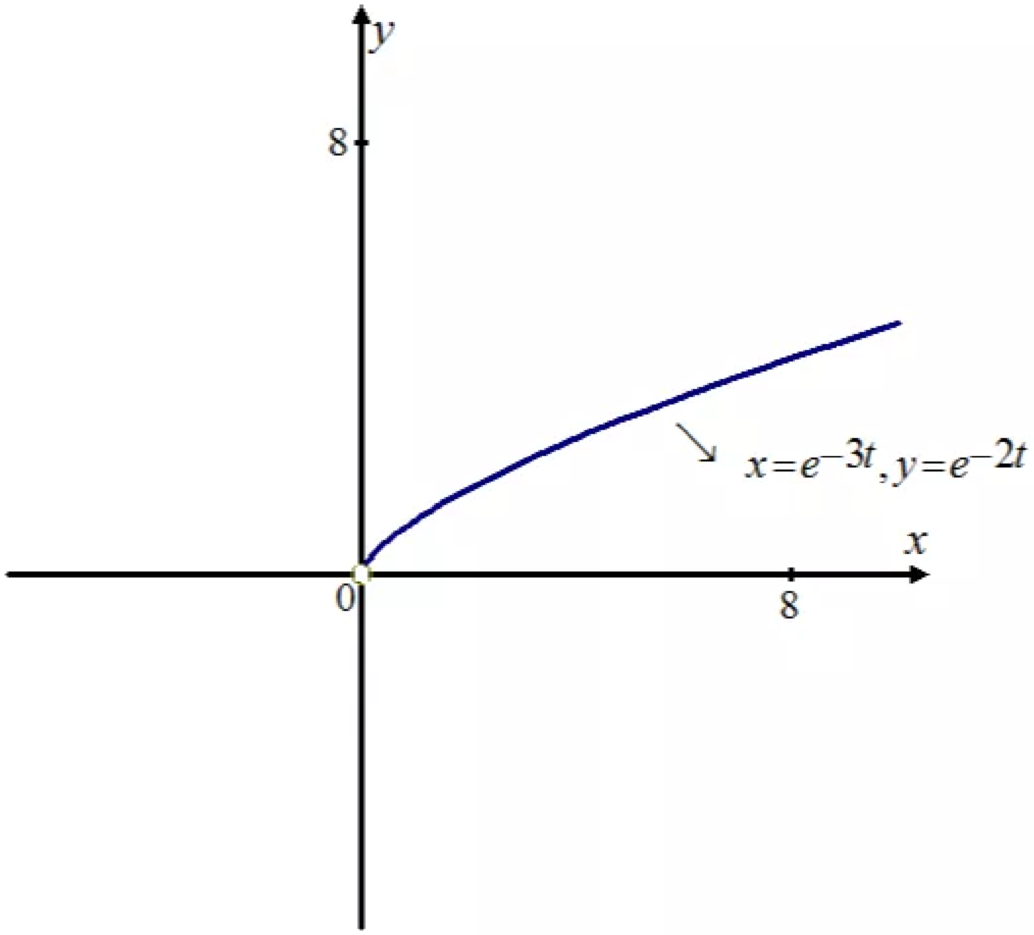
(a) The graph of the parametric curve given by the equations, $x = t^3, y = t^2$, is shown below:



(b) The graph of the parametric curve given by the equations, $x = t^6, y = t^4$, is shown below:



(c) The graph of the parametric curve given by the equations, $x = e^{-3t}, y = e^{-2t}$, is shown below:



From the three figures, observe that the curve is generated in part (a) for $x < 0$, and $x \geq 0$.

In part (b), only the portion of the curve generated with $x \geq 0$.

The curve in part (c), does not passes through the origin.

Because $e^{-3t} \neq 0, e^{-2t} \neq 0$ for $x = 0, y = 0$

So, in part (c), the portion of the curve is generated with $x > 0$.

(a) Consider the parametric equations of a curve,

$x = t, y = t^{-2}$

Rewrite these equations as,

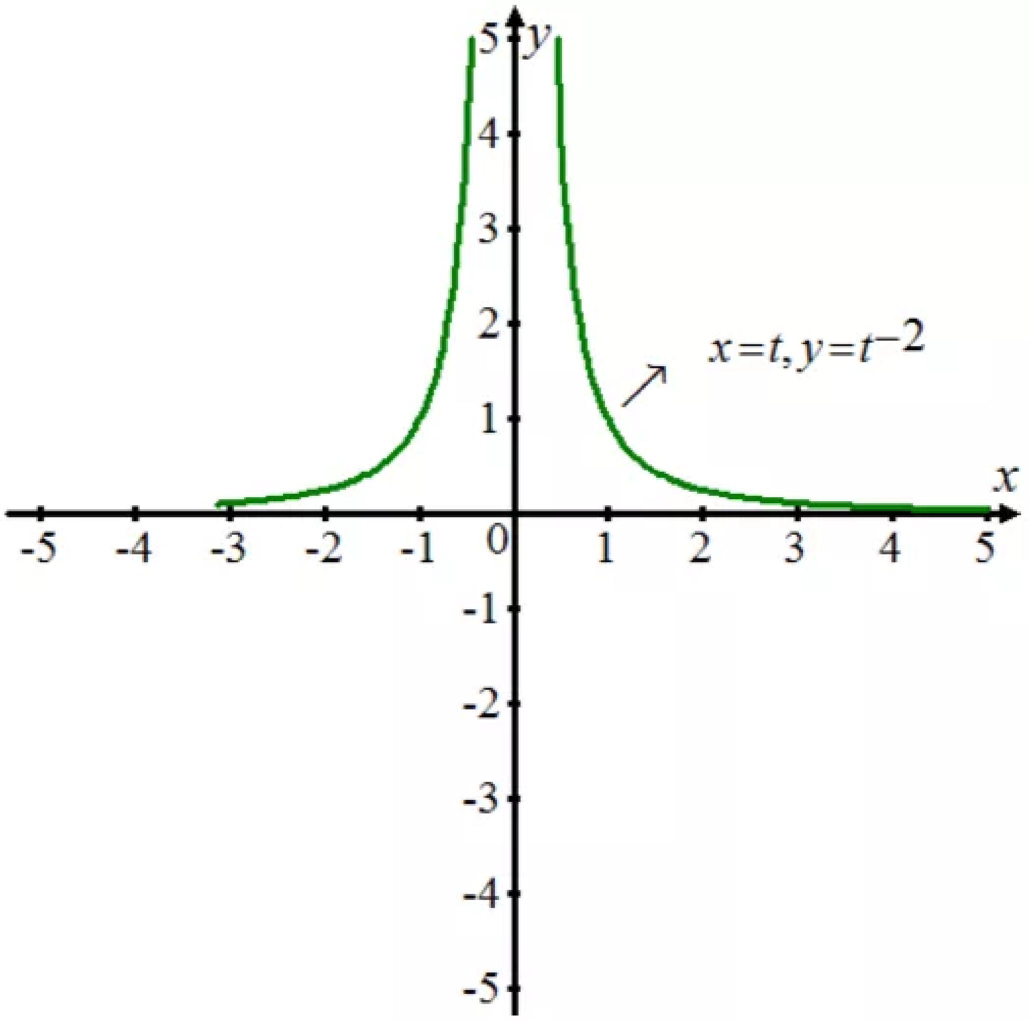
$$x = t, y = \frac{1}{t^2}$$

For any value of $t \neq 0$, the value of x may be either positive or negative.

And, as $t^2 > 0$, the value of y is always positive.

So the graph spreads in 1st and 2nd quadrants.

The graph of the curve $x = t, y = t^{-2}$ is shown below:



(b) Consider the parametric equations of a curve,

$x = \cos t, y = \sec^2 t$

Rewrite these equations as,

$$x = \cos t, y = \frac{1}{\cos^2 t}$$

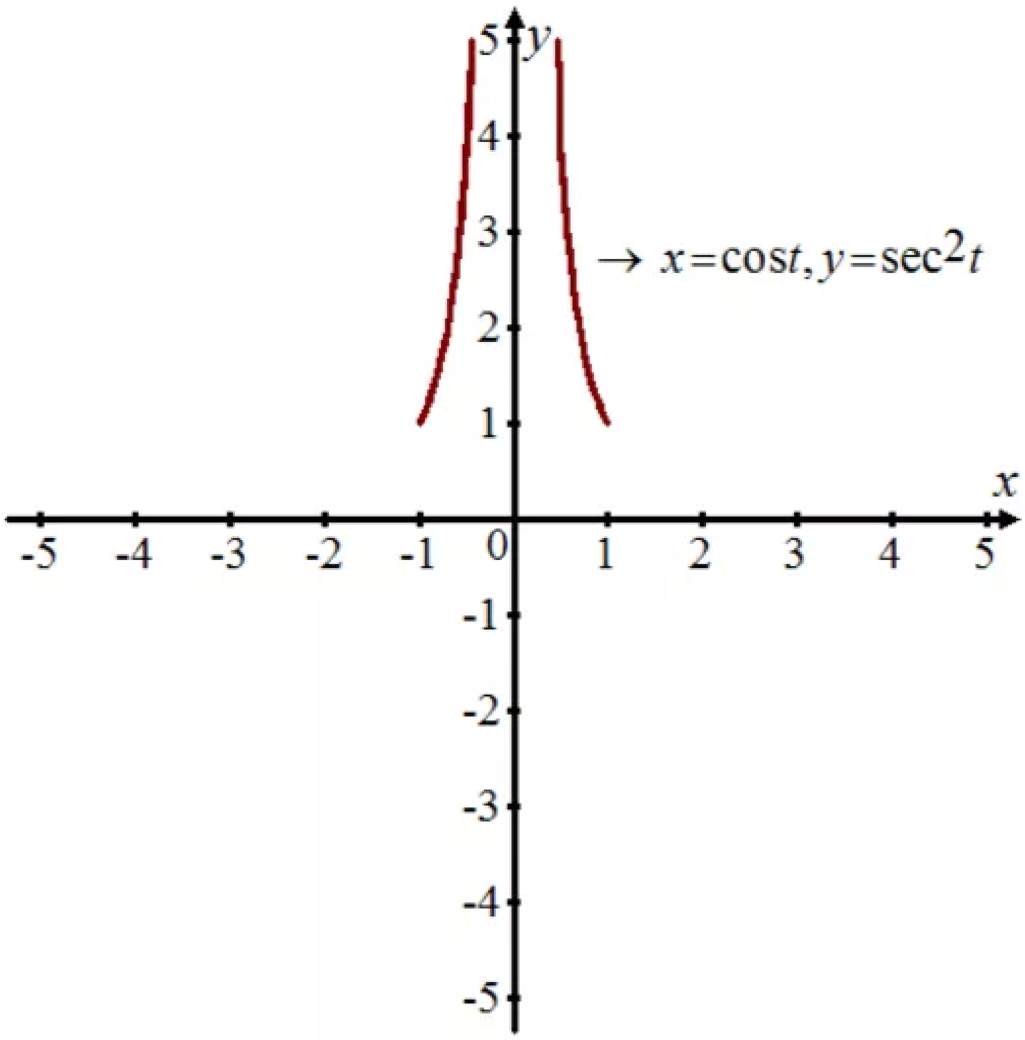
For all $t \neq (2n+1)\frac{\pi}{2}, n \in Z$, the value of $\cos^2 t > 0$

And, $\cos^2 t \leq 1$ for all $t \neq (2n+1)\frac{\pi}{2}, n \in Z$

Therefore, $y = \frac{1}{\cos^2 t} \geq 1$

So the graph of the curve $x = \cos t, y = \sec^2 t$ shifts one unit above the *x*-axis.

The graph of the curve $x = \cos t, y = \sec^2 t$ is shown below:



(c) Consider the parametric equations of a curve,

$x = e^t, y = e^{-2t}$

Rewrite these equations as,

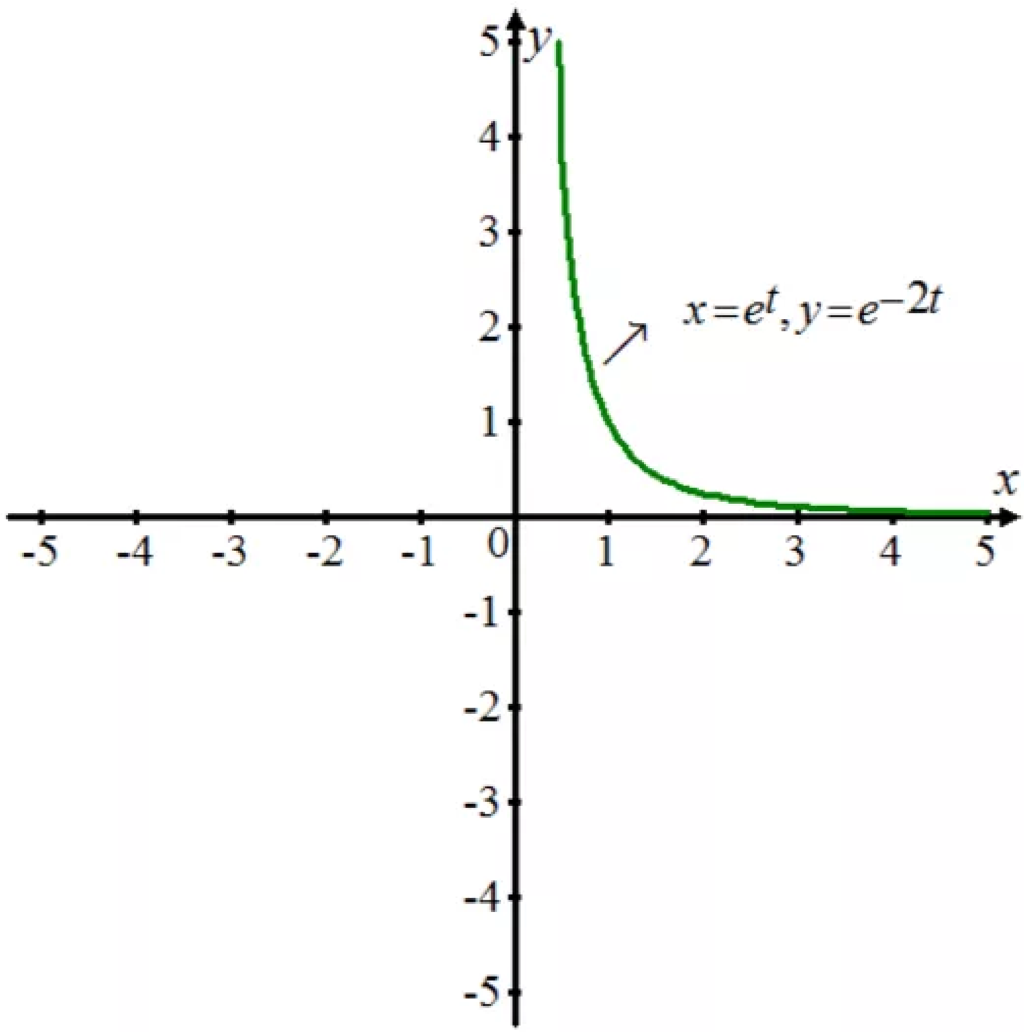
$$x = e^t, y = \frac{1}{\left(e^t\right)^2}$$

For any value of t, the exponential function is positive.

So, $x > 0, y > 0$

Therefore, the graph of this curve lies in 1st quadrant only.

The graph of this function is shown below:



And the Cartesian form of the curve in part (a), $x=t, y=\frac{1}{t^2}$ is,

$$y=\frac{1}{x^2}$$

The Cartesian form of the curve in part (b), $x=\cos t, y=\frac{1}{\cos^2 t}$ is,

$$y=\frac{1}{x^2}$$

The Cartesian form of the curve in part (c), $x=e^t, y=\frac{1}{\left(e^t\right)^2}$ is,

$$y=\frac{1}{x^2}$$

The portions of the curve $y=\frac{1}{x^2}$ represented by 3 different parametric forms are shown in three subparts (a), (b), and (c).

Q39E

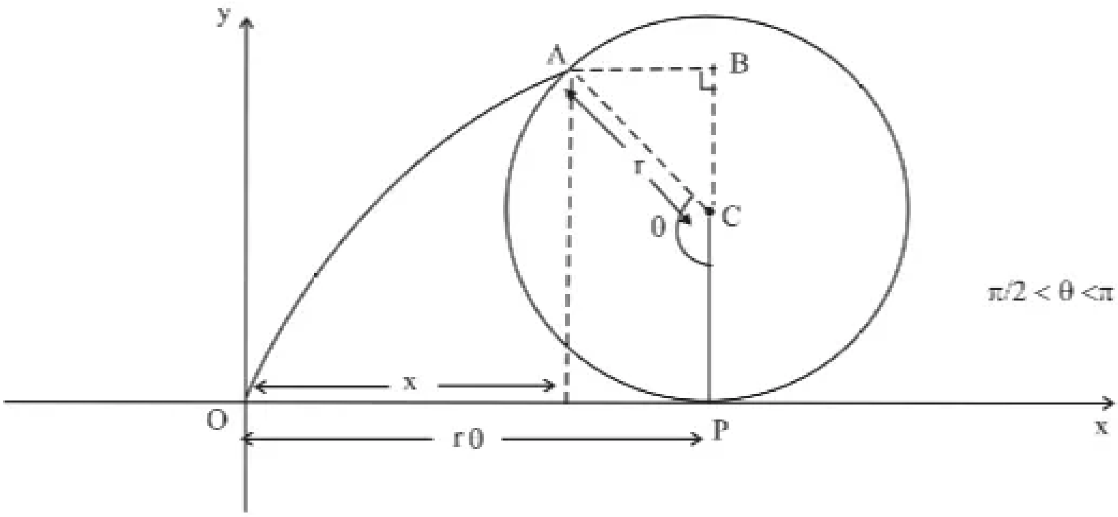


Fig. 1

It triangle ABC, angle ACP = θ radians then angle $ABC = \pi - \theta$

So $|AB| = r\sin(\pi - \theta) = r\sin\theta$ Since $[\sin(\pi - \theta) = \sin\theta]$

and $|BC| = r\cos(\pi - \theta) = -r\cos\theta$ Since $[\cos(\pi - \theta) = -\cos\theta]$

Let the coordinates of the point A be (x,y) , then

$$x = |OP| - |AB|$$

$$= r\theta - r\sin\theta = r(\theta - \sin\theta)$$

And $y = r + |BC|$

$$\Rightarrow y = r + (-r\cos\theta) = r(1 - \cos\theta)$$

Thus parametric equations of the cycloid are

$$\boxed{x = r(\theta - \sin\theta)}, \quad \boxed{y = r(1 - \cos\theta)} \quad \text{where } \frac{\pi}{2} < \theta < \pi$$

Q40E

Case1: when d < r

Let P be a point at the distance d from the center of a circle C of radius r., where d < r. Point P makes an angle θ with center C.

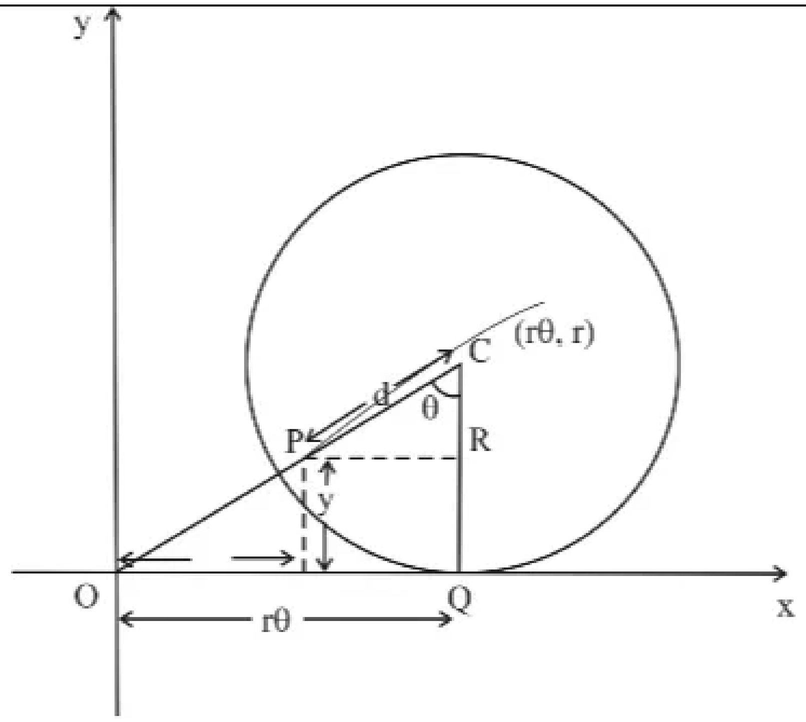


Fig. 1

Distance between centre of the circle and origin is $= r\theta$

In triangle PCR, $\angle C = \theta$, $|PC| = d$

then $|PR| = d\sin\theta$

and $|CR| = d\cos\theta$

Let the co- ordinates of P be (x, y)

Then $x = r\theta - |PR|$

$\Rightarrow x = r\theta - d\sin\theta$

And $y = r - |CR| = r - d\cos\theta$

There fore parametric equations of the trashed are

$$\boxed{x = r\theta - d\sin\theta \quad \text{and} \quad y = r - d\cos\theta}$$

Case2: for $d > r$,

Derivation and equations will be same as for $d < r$, Now we sketch torched for d < r and d > r

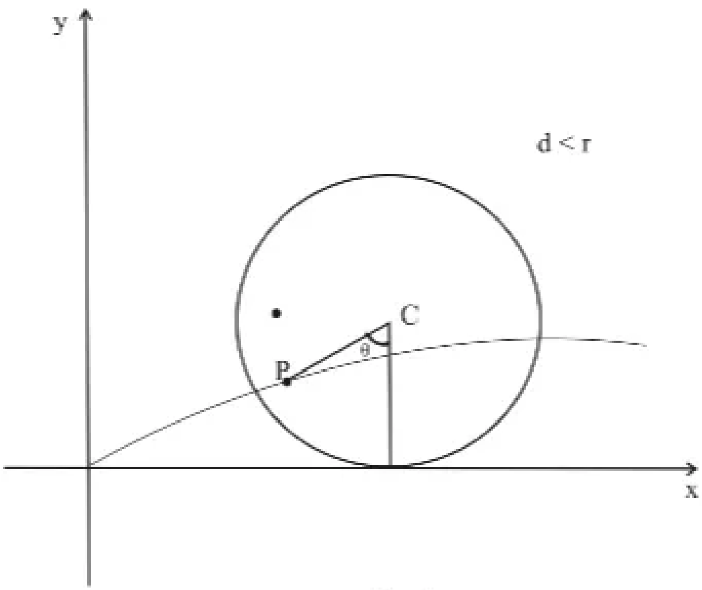


Fig. 2

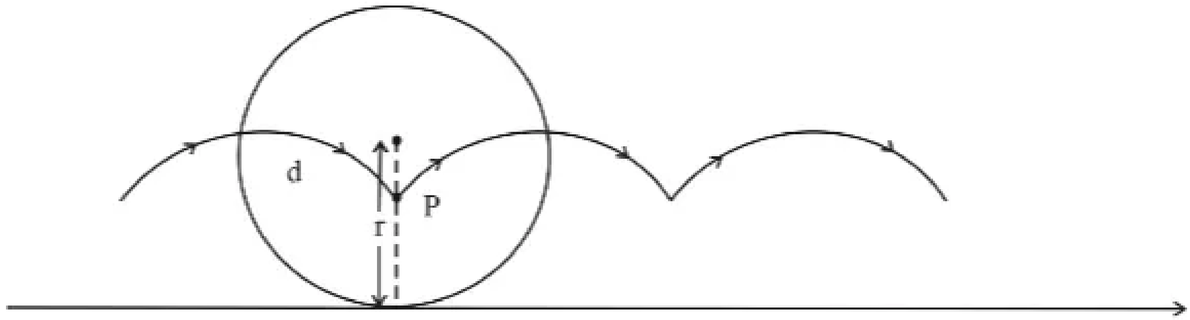


Fig. 3

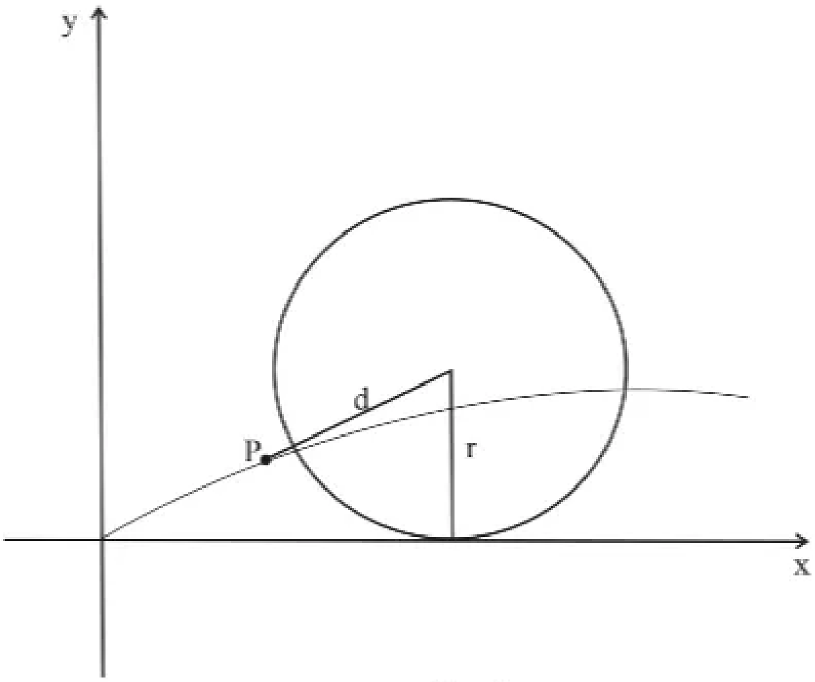


Fig. 4

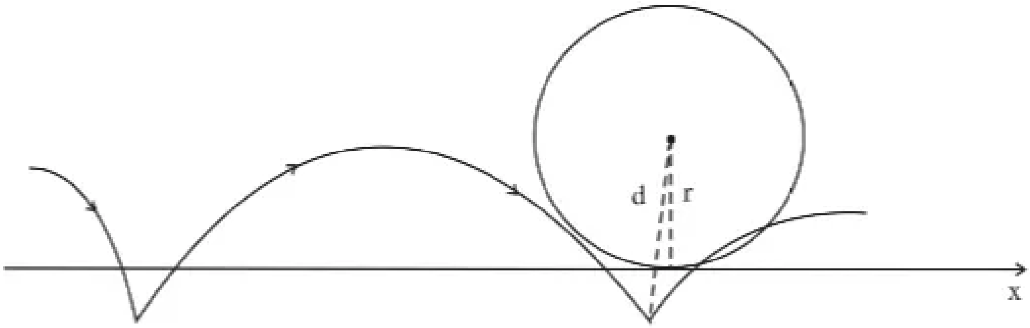


Fig. 5

Q41E

Radius of small circle is b

And radius of bigger circle is a clearly a > b

We have to find coordinates of point P in terms of θ, say(x, y)

From figure $|OQ| = \text{a}$ and $|ON| = \text{b}$

Then $|NQ| = \text{a} - \text{b}$

Since angle $QOM = \theta$ so by property of similar triangle angle $QNP = \theta$

By the triangle QOM we have

$|OM| = \text{a}\cos\theta$ and $|QM| = \text{a}\sin\theta$

$\Rightarrow x = \text{a}\cos\theta$ .....(1)

And by the triangle QNP

$|QP| = (\text{a}-\text{b})\sin\theta$

then $|PM| = |QM| - |QP|$

$= \text{a}\sin\theta - (\text{a}-\text{b})\sin\theta$

$= \text{a}\sin\theta - \text{a}\sin\theta + \text{b}\sin\theta$

Or $|PM| = \text{b}\sin\theta$

$\Rightarrow y = \text{b}\sin\theta$ ......(2)

Thus the co-ordinates of point P are $(\text{a}\cos\theta, \text{b}\sin\theta)$

Thus the parametric equations are

And $\boxed{x = \text{a}\cos\theta}$ ....(1)

$\boxed{y = \text{b}\sin\theta}$ ....(2)

From (1) and (2)

$\frac{x}{a} = \cos\theta$ .....(3)

$\frac{y}{6} = \sin\theta$ .....(4)

Adding the squares of the equations (3) and (4)

$$\frac{x^2}{a^2} + \frac{y^2}{6^2} = Cos^2\theta + \sin^2\theta$$

$$\Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \qquad (cos^2\theta + \sin^2\theta = 1)$$

This is an equation of an ellipse.

Q42E

a is the radius outer circle and b is the radius of inner circle. Since AB is tangent to the lager circle at A then by property of tangent of a circle.

We have angle OAB $= \pi/2$ (right angle)

Then by the property of right triangle

$$|OB| = |OA|\sec\theta$$

$$\Rightarrow |OB| = a\sec\theta$$

If the coordinates of P are (x, y) then $x = a\sec\theta$

Since $|PB| = |CB'|$

and $|CB'| = |OC|\sin\theta$

$\Rightarrow |PB| = b\sin\theta = y$

Then parametric equations are $\boxed{\begin{array}{l} x = a\sec\theta \\ y = b\sin\theta \end{array}}$

Q43E

We have to find the coordinates of point P.

Since x-coordinate of P = x co-ordinate of Point C

We have $|CD| = 2a$

and $\angle COD = \theta$

then $\dfrac{|OD|}{|CD|} = \cot\theta$

$\Rightarrow |OD| = 2a\cot\theta$

So x –co-ordinate of P= $2a\cot\theta$

Now $\angle EAO = \pi/2$

$and\ \angle EOA = \pi/2 - \theta$

Then $|OA| = |OE|.\cos(\pi/2 - \theta)$

$|OA| = 2a\sin\theta$ $[\cos(\pi/2 - \theta) = \sin\theta$

Then By the triangle OAB,

$$|AB| = |OA|\sin\theta$$
$$= 2a\sin\theta.\sin\theta$$
$$= 2a\sin^2\theta$$

Then by y-co-ordinate if of P = 2a $\sin^2\theta$

Thus the parametric equations are $\boxed{x = 2a\cot\theta}$ and $\boxed{y = 2a\sin^2\theta}$

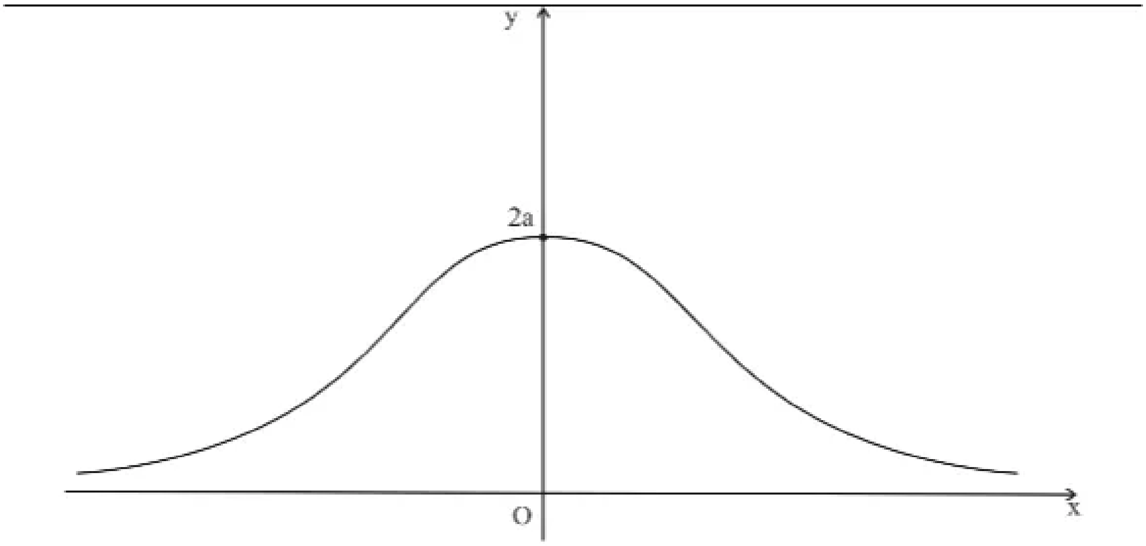


Fig. 2

Q45E

(A) The position of one particle at time t is given by

$$x_1 = 3\sin t, \quad y_1 = 2\cos t \quad 0 \le t \le 2\pi$$

And the position of second particle at time t is given by

$$x_2 = -3 + \cos t,\ y_2 = 1 + \sin t \quad 0 \le t \le 2\pi$$

Now we sketch the curves with the help of computer.

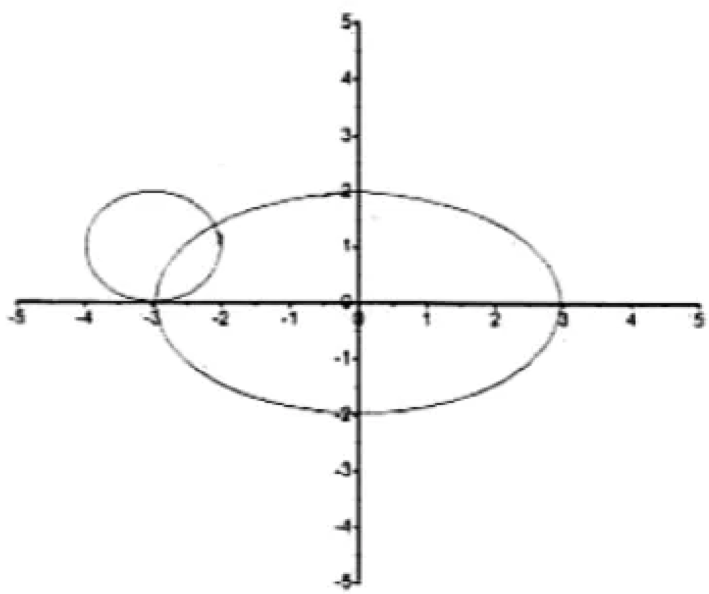


Fig. 1

We see that there are two points of intersection

(B) We see that points of intersections are (-3, 0) and $\approx(-2.1,\ 1.4)$

For collision we calculate time for both particles at these points

At (-3, 0) $\rightarrow$ (first particle) $x_1 = 3\sin t$

$$\Rightarrow 3\sin t = -3$$
$$\Rightarrow \sin t = -1$$
$$\Rightarrow t = 3\pi/2$$

And $x_2 = -3 + \cos t$

$$\Rightarrow -3 = -3 + \cos t$$
$$\Rightarrow \cos t = 0 \quad \Rightarrow t = \pi/2, 3\pi/2, 5\pi/2$$

We see that at $t = 3\pi/2$, both the curve intersect at (-3, 0)

at $\approx(-2.1, 1.4)$

For first particle $\Rightarrow x_1 = 3\sin t$

$$\Rightarrow -2.1 = 3\sin t$$
$$\Rightarrow \sin t = -(2.1)/3$$
$$\Rightarrow t = \sin^{1}\left(-\frac{2.1}{3}\right) \approx -0.78$$

For second particle

$$x_2 = -3 + \cos t$$
$$\Rightarrow -2.1 = -3 + \cos t$$
$$\Rightarrow \cos t = 0.9$$
$$\Rightarrow t = \cos^{-1}(0.9) \approx 0.45$$

So time for both particle at (-2.1, 1.4) is not same then there is only one collision point $\boxed{(-3,\ 0) \text{ at } t = 3\pi/2}$

(C) It the path of the second particle is

$$x_2 = 3 + \cos \quad \text{and} \quad y_2 = 1 + \sin t$$

Then

Center of the circle will be (3, 1) in place of (-3, 1)

And points of intersections will be (3, 0) and (2.1, 1.4)

But there is no collision point.

Q46E

(A) Parametric equations for the position of projectile at time t are

$$x = (v_0 \cos\alpha)t, \qquad y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2$$

We have

Initial velocity $v_0 = 500\ \text{m/s}$ *and* $\alpha = 30° = \pi/6$ radians

And $g = 9.8\ \text{m/s}^2$ (Acceleration due to gravity)

The equation becomes.

$$x = \left(500\cos(\pi/6)\right)t \quad \text{and} \quad y = \left(500\sin(\pi/6)\right)t - \frac{1}{2}gt^2$$

Or $\quad x = \frac{500\sqrt{3}}{2}t \quad$ and $\quad y = \frac{500}{2}t - \frac{1}{2}gt^2$

Or $\quad x = 250\sqrt{3}\,t \quad$ and $\quad y = 250t - \frac{1}{2}gt^2$

The bullet will hit the ground when y = 0

So $\quad 250t - \frac{1}{2}gt^2 = 0$

$$\Rightarrow 250 - \frac{1}{2}gt = 0$$

$$\Rightarrow t = \frac{250\times 2}{g} = \frac{500\text{ m/s}}{9.8\text{ m/s}^2} \approx 51\text{seconds}$$

So bullet will hit the ground after about $\boxed{51\text{ seconds}}$

Distance from the gun after time t is $250\sqrt{3}\,t = x$

Distance from the gun after 51 seconds $= 250\sqrt{3}\times 51$

≈ 22083.65 m.

At a distance about $\boxed{22083.65\text{meters}}$, from the gun, bullet will hit the ground.

For finding the maximum height we have to

Maximize $y = 250t - \frac{1}{2}gt^2$

Differencing with respect to t

$$\frac{dy}{dt} = 250 - gt$$

$\frac{dy}{dt} = 0$, *when* $\quad 250 - gt = 0 \quad$ or $\quad gt = 250$

Or $\quad t = \frac{250}{g}$ seconds

(C) $\quad$ We have $\quad x = (v_0\cos\alpha)t \quad$ .....(1)

and $\quad y = (v_0\sin\alpha)t - \frac{1}{2}gt^2 \quad$ ....(2)

From (1) we have $t = \frac{x}{(v_0\cos\alpha)} = \frac{x}{v_0}\sec\alpha$

We see that $\frac{dy}{dt} < 0$ for $t > \frac{250}{g}$ and $\frac{dy}{dt} > 0$ for $t < \frac{250}{g}$

So y has maximum at t = $250/g$ seconds

So maximum height

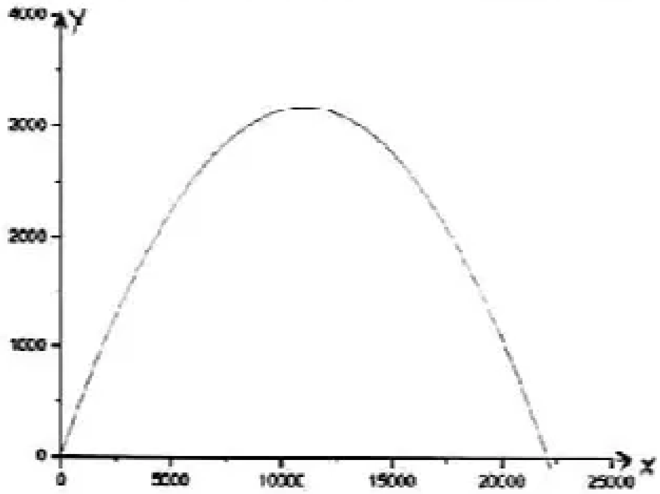
$$y = 250 \times \frac{250}{g} - \frac{1}{2} g . \frac{250 \times 250}{g^2}$$

$$\Rightarrow y = \frac{62500}{g} - \frac{62500}{2g}$$

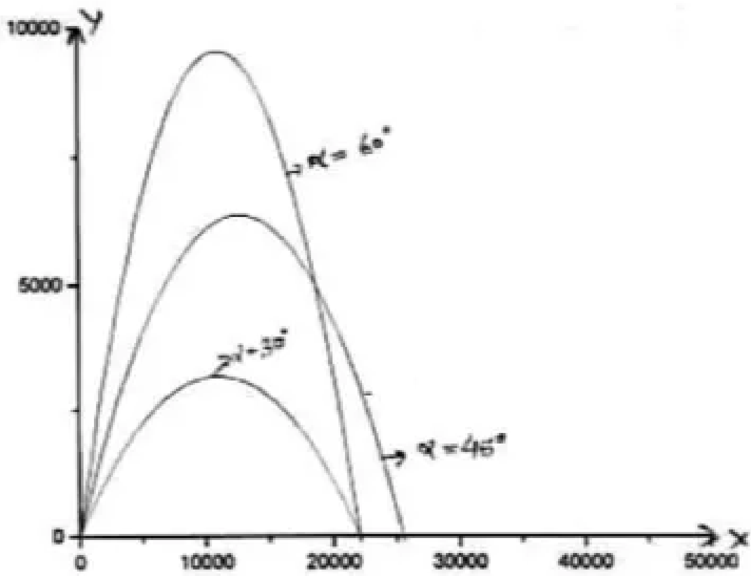
$$\Rightarrow y = \frac{62500}{2g} \text{ meters}$$

$$\boxed{\approx 3188.77 \text{ meters}}$$

(B) We sketch the curve and see that our answers in part (a) are reasonable



Now we sketch the curves for $\alpha = 30°, 45°, 60°$, we see that for $\alpha = 45°$, the distance between gun and bullet is maximum when bullet touches the ground. And height of the bullet increases as α increases.



Putting the values of t in equation (2)

$$y = (v_0 \sin\alpha)\frac{x}{v_0}\sec\alpha - \frac{1}{2}g\frac{x^2}{v_0^2}\sec^2\alpha$$

$$\Rightarrow \boxed{y = x\tan\alpha - \frac{1}{2}g\frac{\sec^2\alpha}{v_0^2}x^2}$$

This is the equation of a parabola
So path of is parabolic

Q47E

We have $\quad x = t^2 \quad$ ....(1)

and $\quad y = t^3 - ct \quad$ ......(2)

From (1) we have

$t = \pm\sqrt{x} \quad$ where $x > 0$

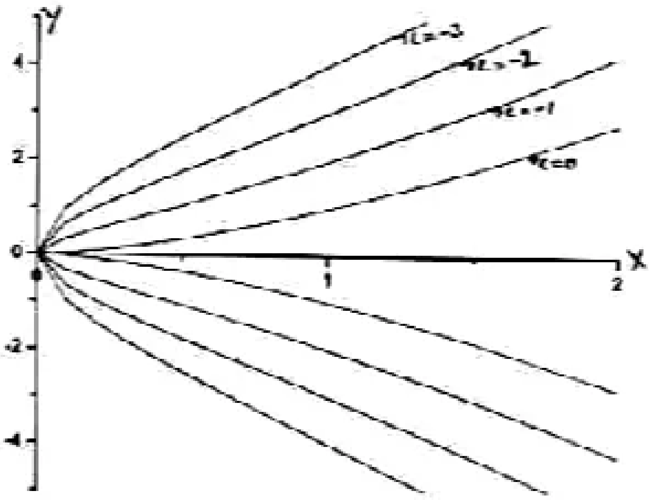
So from (2), putting the values of t

$$y = t^3 - ct$$
$$= t(t^2 - c)$$

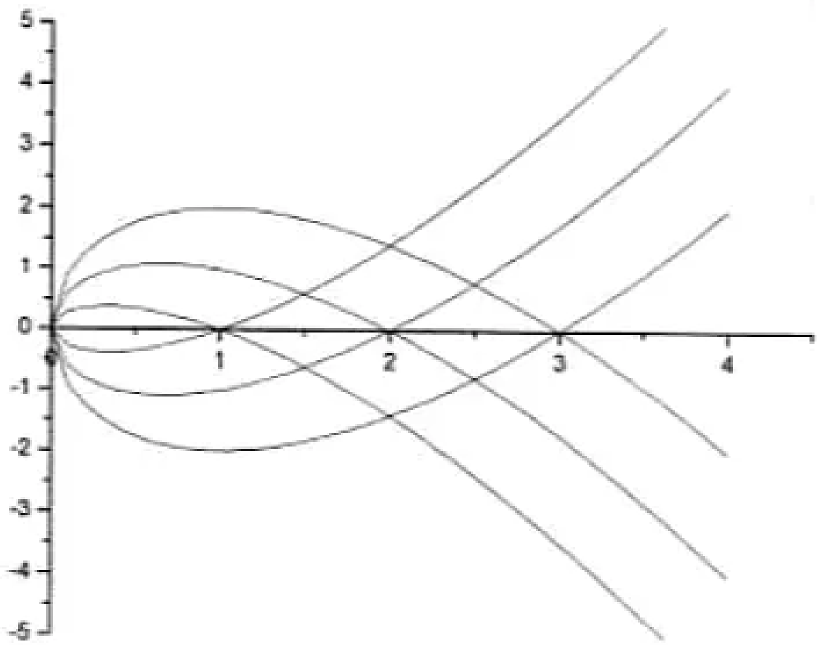
$$\Rightarrow \boxed{y = \pm\sqrt{x}(x-c)} \quad \text{or} \quad \boxed{y^2 = x(x-c)^2}$$

This the equation of family of the curve

Now we sketch the curves for $c \le 0$
We see that for $c \le 0$, graph is in cusp form and when c decreases, size of cusp increases



Now we sketch the curves for $c > 0$, we see that for $c > 0$ graph has a loop and size of the loop increases as c increases.



Q48E

Consider the parametric equations:

$x = 2ct - 4t^3, y = -ct^2 + 3t^4$.

Graph the family of curves for different values of c.

For $c = 1$, the equations are, $x = 2t - 4t^3, y = -t^2 + 3t^4$

For $c = 2$, the equations are, $x = 4t - 4t^3, y = -2t^2 + 3t^4$

For $c = 3$, the equations are, $x = 6t - 4t^3, y = -3t^2 + 3t^4$.

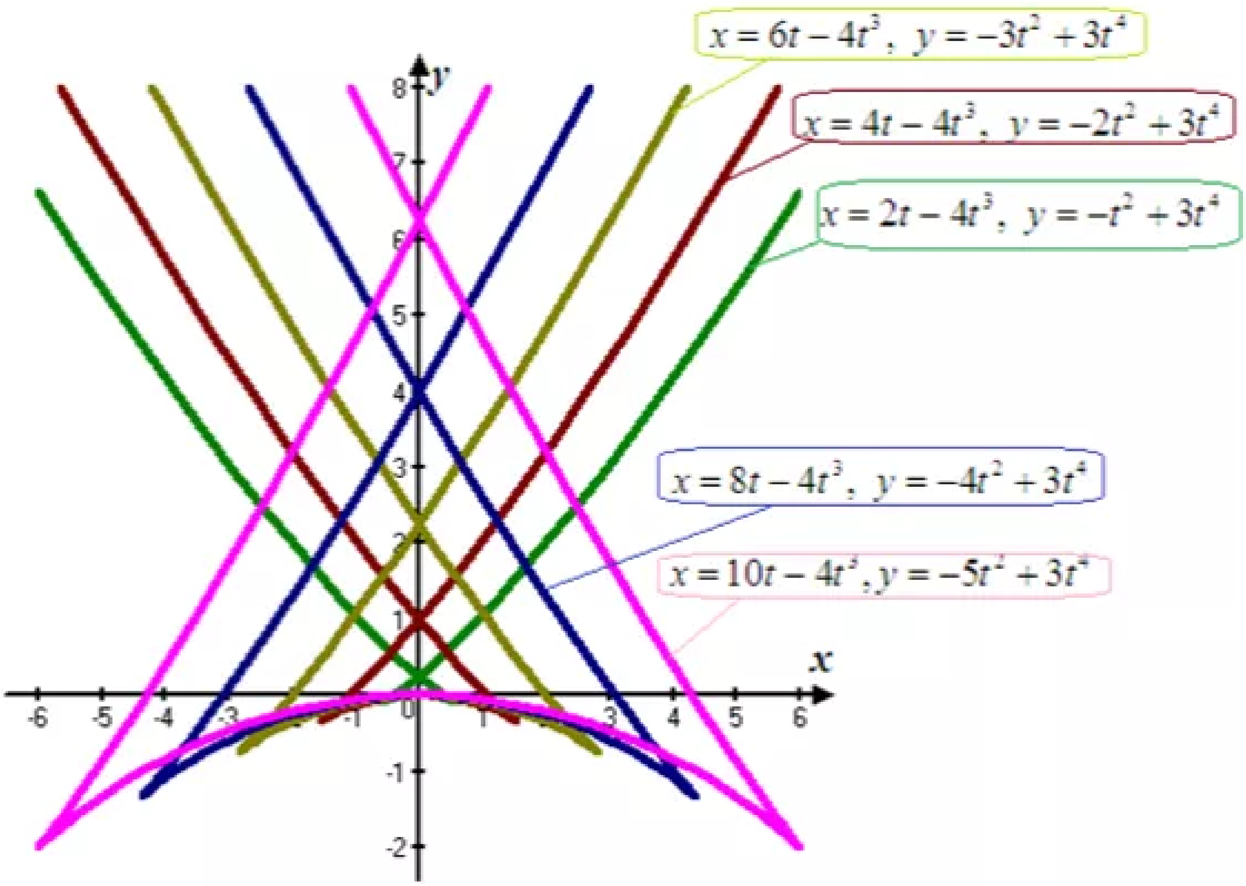
For $c = 4$, the equations are, $x = 8t - 4t^3, y = -4t^2 + 3t^4$.

For $c = 5$, the equations are, $x = 10t - 4t^3, y = -5t^2 + 3t^4$.

The following are the graphs of,

$$x = 2t - 4t^3,\ y = -t^2 + 3t^4$$
$$x = 4t - 4t^3,\ y = -2t^2 + 3t^4$$
$$x = 6t - 4t^3,\ y = -3t^2 + 3t^4$$
$$x = 8t - 4t^3,\ y = -4t^2 + 3t^4$$
$$x = 10t - 4t^3, y = -5t^2 + 3t^4$$

Using Advanced Grapher:

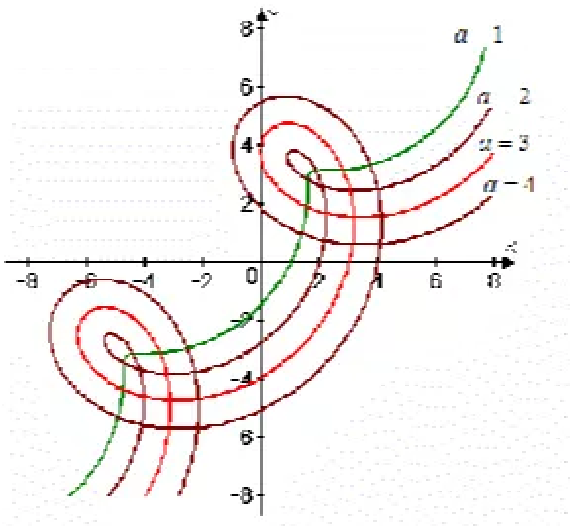


From the above figure, it is observed that, the common property that the curves have that they contain the same shape only the size increases as *c* increases.

As *c* increases from 1 to 5, the width and the heights of the curves are increases.

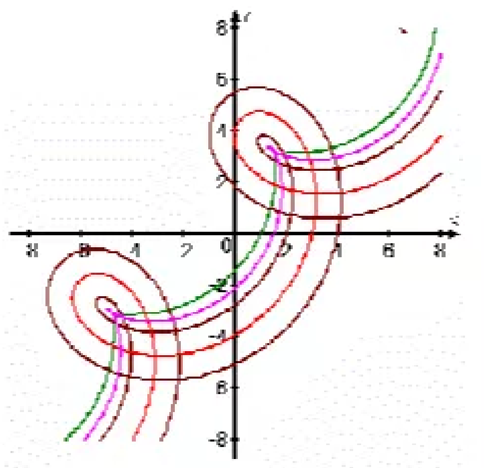
Q49E

Sol: Given $x = t + a\cos t, y = t + a\sin t$ and $a > 0$



as 'a' increases from 1, the loop size is increasing.

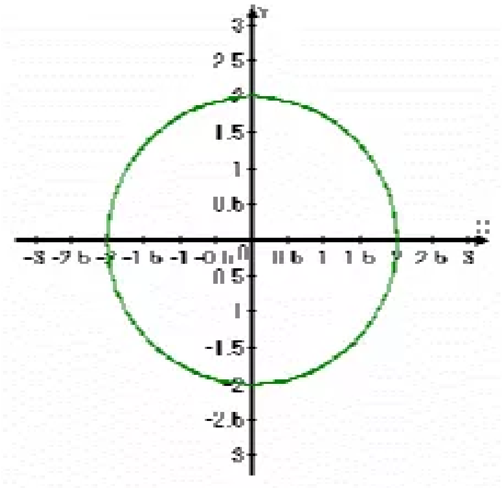
For a = 1.5, the given polar curve has the loop.



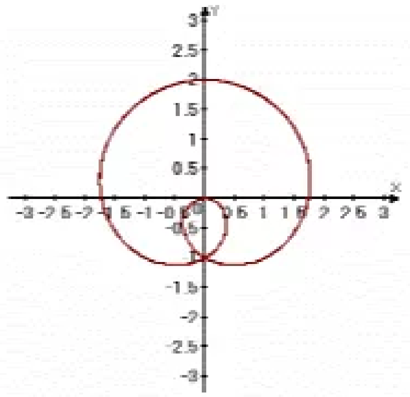
Q50E

Sol: Given $x = \sin t + \sin nt$, $y = \cos t + \cos nt$

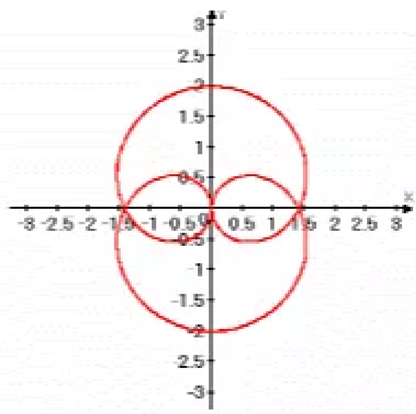
When n =12, there is no loop and just a circle of radius 2 units.



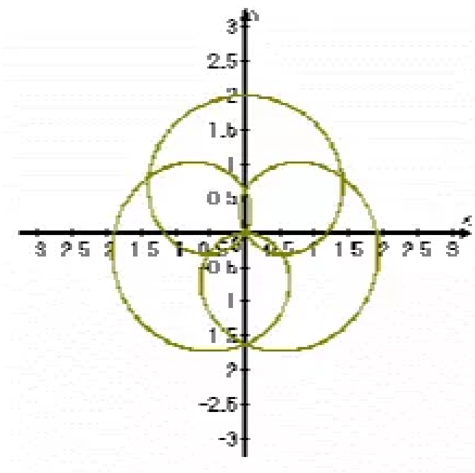
When n = 2, there is 1 loops (cardoids) passing through origin



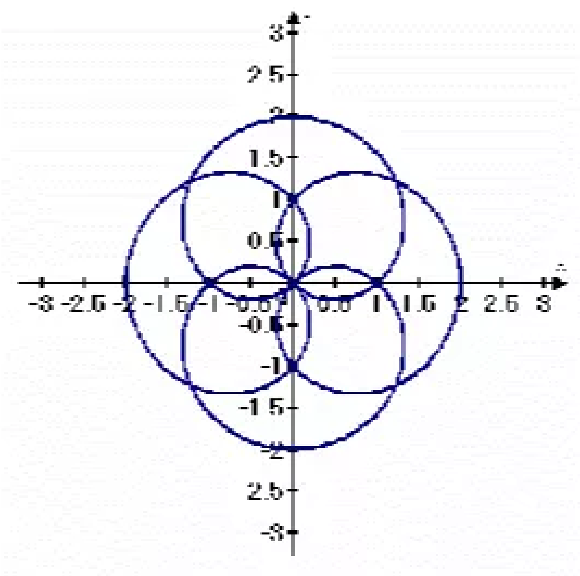
When n = 3, there are 2 loops (cardoids) passing through origin



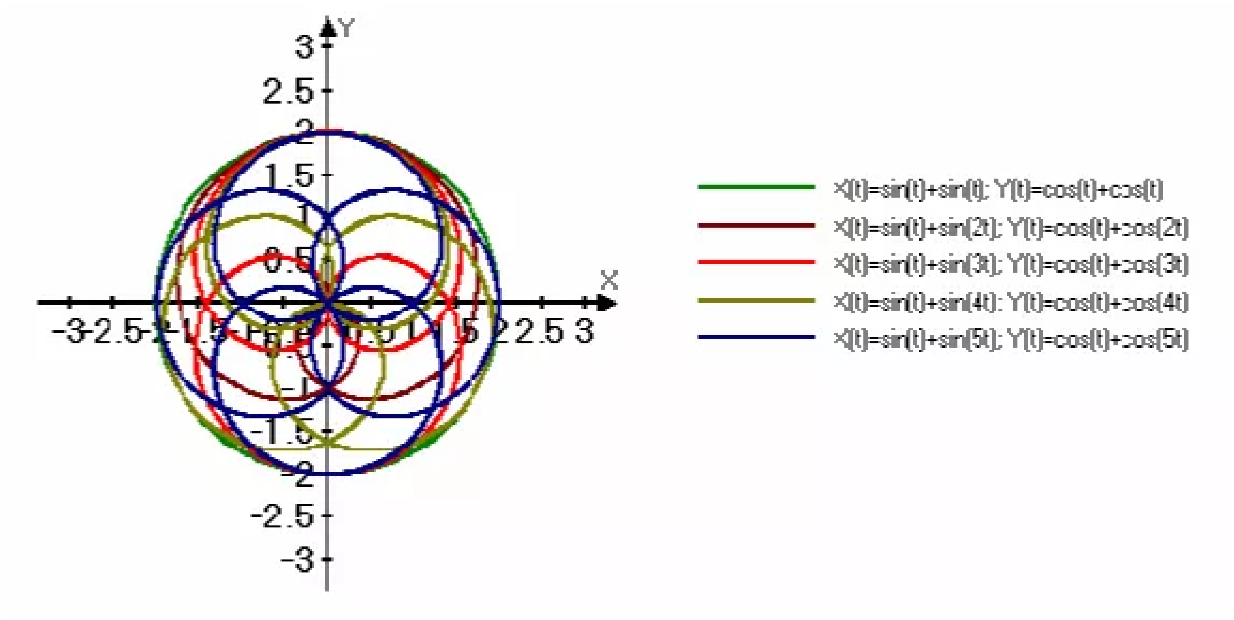
When n = 4, there are 3 loops (cardoids) passing through origin



When n = 5, there are 4 loops (cardoids) passing through origin



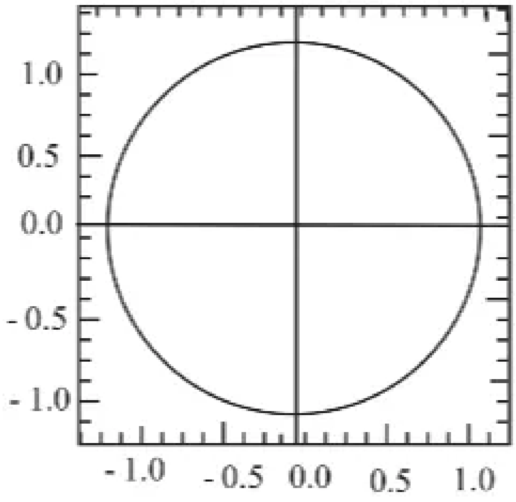
In general, there will be n-1 loops for the given parametric equations all passing through origin and included in the circle when n = 1.



Q51E

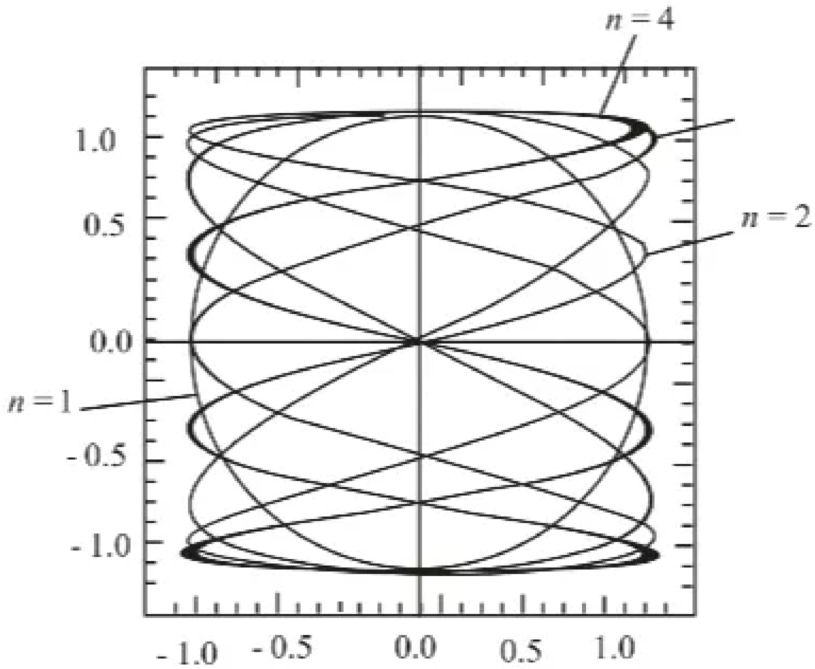
We need to investigate how the curves $x = a \times \sin(nt)$ and $y = b \times \cos(t)$ vary when a, b, and n vary. With $n = 1, 2, 3, \ldots$

For $a = b = n = 1$



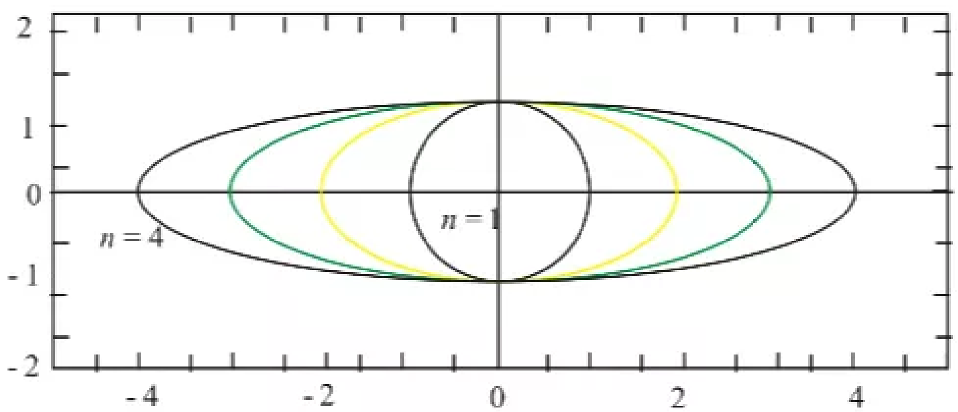
The graph is simply circle with radius 1 and center (0,0). This graph is symmetric about the x-axis, y-axis and the origin.

If we variy n and let $a = b = 1$

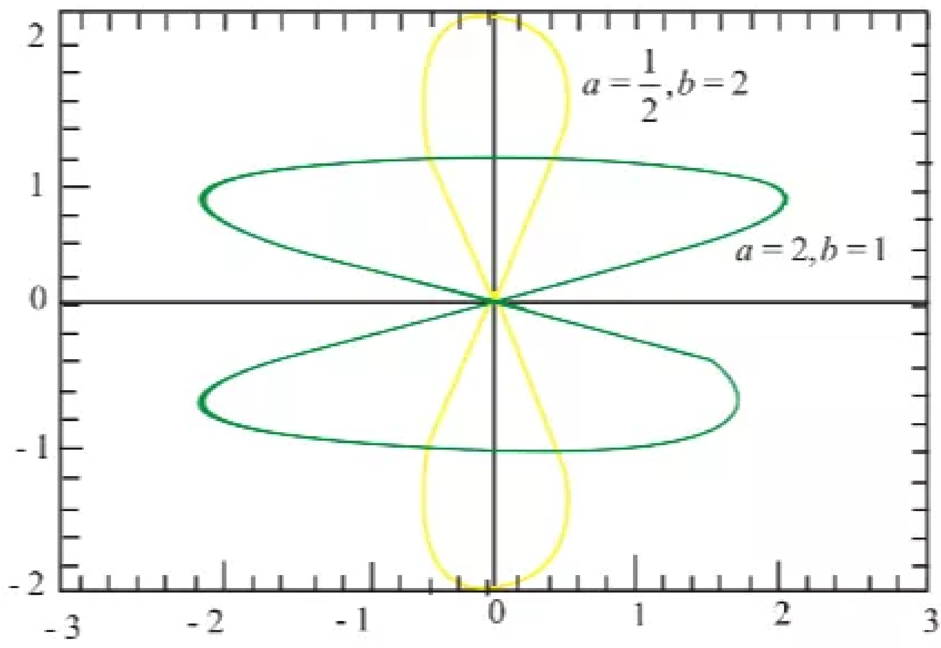


For $n=2$ the graph crosses istself at the origin and there are loops above and below the x-axis. In general, the figures have n-1 points of intersection, all of which are on the y-axis, and a total of n closed loops. Notice that all the Lissojaous are symmetric about the x-axis.

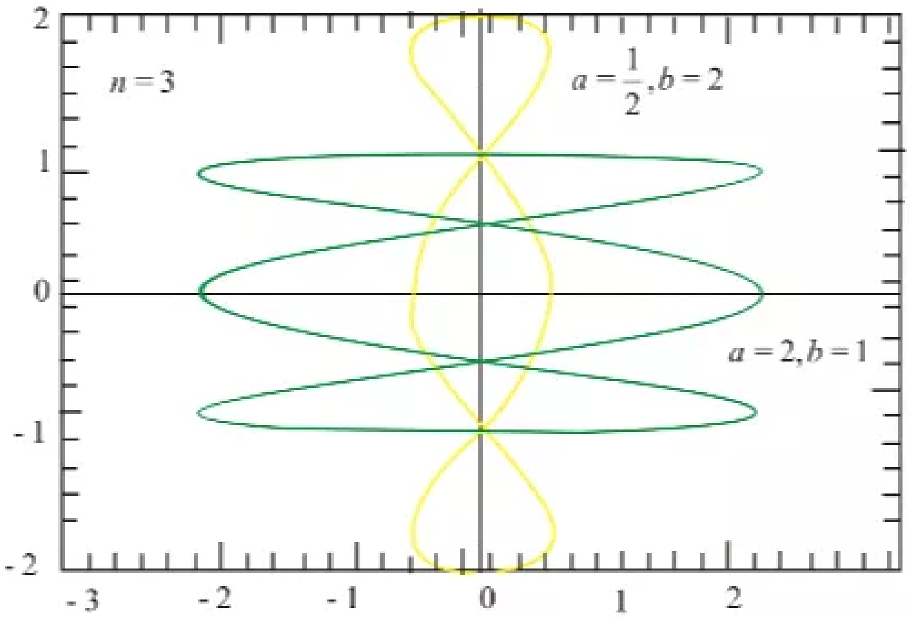
With $b = n = 1$ and a = 1, 2, 3and 4



$n = 2$ $a = 1/2$ if $b = 2$ and $a = 2$ if $b = 1$



$n = 3$

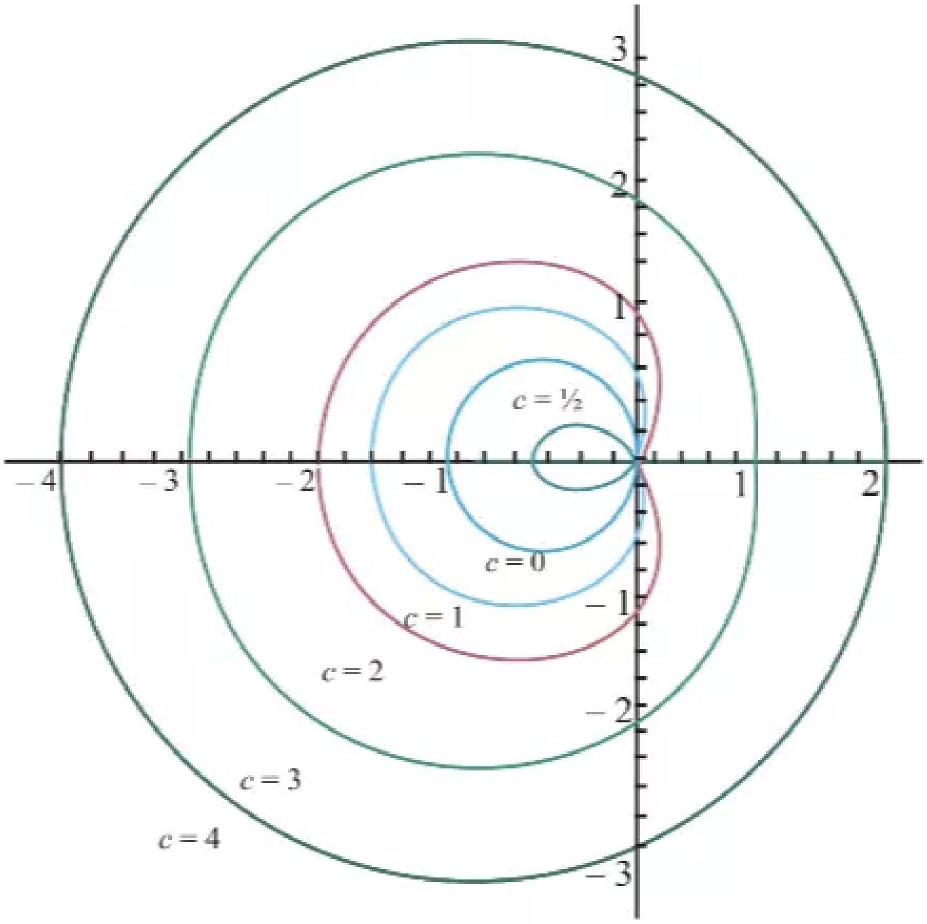


Q52E

We need to investigate the family of curves defined by the parametric equations:

$x = \sin t\left(c - \sin t\right), y = \cos t\left(c - \sin t\right)$

We can use $0 \le t \le 2\pi$ or $-\pi \le t \le \pi$



We first observe that for $c = 0$, we obtain a circle with center $(-1/2, 0)$ and radius $1/2$. As the value of c increases, there is a larger outer loop and a smaller inner loop until $c = 1$, when we obtain a curve with a dent. As c increases, we get a curve with a dimple until $c = 2$. For $c > 2$, we have convex limacons. For negative values of c, we obtain the same graphs as for positive c, but with different values of t corresponding to the points on the curve.