# 12

We know that, the position of a point in a plane can be determined, if the coordinates of the point with reference to two mutually perpendicular lines called $X$ and $Y$-axes, are known. In order to locate a point in space, two coordinate axes are insufficient. So, we need three coordinate axes called $X$, $Y$ and $Z$-axes.

The three numbers representing the three distances are called the coordinates of the point with reference to the three coordinate axes. So, a point in space has three coordinates. In this chapter, we shall study the basic concepts of geometry in three dimensional space.

# INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

## |TOPIC 1|

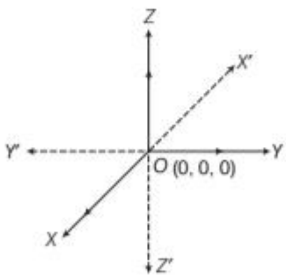
## Coordinate Axes and Coordinate Planes in Three Dimensional Space

**CHAPTER CHECKLIST**

- Coordinate Axes and Coordinate Planes in Three Dimensional Space
- Distance Formula and its Applications in Geometry

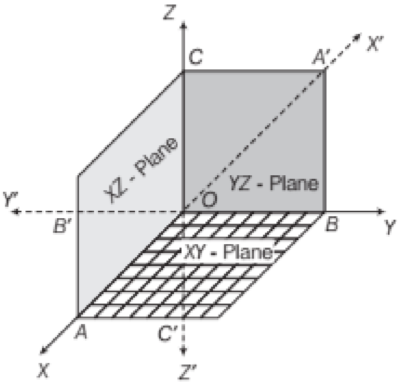
### COORDINATE AXES

Let $XOX'$, $YOY'$ and $ZOZ'$ be the three mutually perpendicular lines, intersecting at $O$. The point $O$ $(0,0,0)$ is called the **origin** and the lines ($XOX'$, $YOY'$ and $ZOZ'$) are called **rectangular coordinate axes** say $X, Y$ and $Z$, respectively. Thus in the given figure, $X'OX$ is called the $X$-axis, $Y'OY$ is called the $Y$-axis, and $Z'OZ$ is called the $Z$-axis.



# COORDINATE PLANES

The three coordinate axes define three mutually perpendicular planes $XOY$, $YOZ$ and $ZOX$ (or $XY$, $YZ$ and $ZX$) called **coordinate planes** which divide the space into eight parts called **octants**.




Thus, in the given figure,

$XOY$ is called the $XY$-plane.

$YOZ$ is called the $YZ$-plane.

$ZOX$ is called the $ZX$-plane.

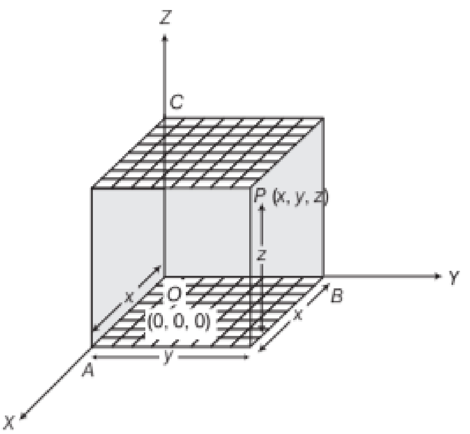
$XOYZ$, $X'OYZ$, $XOY'Z$,

$X'OY'Z$, $XOYZ'$, $X'OYZ'$, $XOY'Z'$

and $X'OY'Z'$ are called **octants.**

# COORDINATES OF A POINT IN SPACE

Let $P$ be a point in space. Through $P$, draw three planes parallel to the coordinate axes to meet the axes in $A$, $B$ and $C$, respectively. Let $OA = x$, $OB = y$ and $OC = z$. These three numbers taken in order are called **coordinates** of a point $P$ and written as $P(x, y, z)$.




Thus, **$x$-coordinate of a point $P$** = length of perpendicular from $P$ to $YZ$-plane with proper sign.

**$y$-coordinate of a point $P$** = length of perpendicular from $P$ to $ZX$-plane with proper sign.

**$z$-coordinate of a point $P$** = length of perpendicular from $P$ to $XY$-plane with proper sign.

## Sign of Coordinates

As we know that, three mutually perpendicular lines $X'OX$, $Y'OY$ and $Z'OZ$ divide the space into eight parts known as *octants*. Here, $OX$ is the positive direction and $OX'$ is the negative direction of $X$-axis. Similarly, $OY$ is the positive and $OY'$ is the negative direction of $Y$-axis and $OZ$ is the positive and $OZ'$ is the negative direction of $Z$-axis.

*The sign of coordinates of the points in the octants in which the space is divided are given in the following table*

| Octants / Coordinates | I | II | III | IV | V | VI | VII | VIII |
|---|---|---|---|---|---|---|---|---|
| | $OXYZ$ | $OX'YZ$ | $OX'Y'Z$ | $OXY'Z$ | $OXYZ'$ | $OX'YZ'$ | $OX'Y'Z'$ | $OXY'Z'$ |
| $x$ | + | − | − | + | + | − | − | + |
| $y$ | + | + | − | − | + | + | − | − |
| $z$ | + | + | + | + | − | − | − | − |

**EXAMPLE |1|** Name the octants in which the following points lie.

(1, 2, 3), (4, − 2, 3), (4, − 2, − 5), (4, 2, − 5), (−4, 2, − 5), (−4, 2, 5), (−3, − 1, 6), (2, − 4, − 7) **[NCERT]**

***Sol.*** The name of octants in which given points lie, are given in the following table

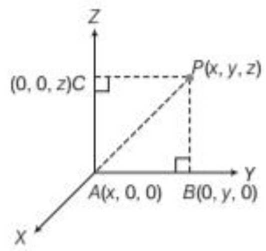
| Point | Octant | Name |
|---|---|---|
| (1, 2, 3) | I (all the coordinates are positive) | $XOYZ$ |
| (4, −2, 3) | IV ($y$-coordinate is negative) | $XOY'Z$ |
| (4, −2, −5) | VIII ($y$ and $z$-coordinates are negative) | $XOY'Z'$ |
| (4, 2, −5) | V ($z$-coordinate is negative) | $XOYZ'$ |
| (−4, 2, −5) | VI ($x$ and $z$-coordinates are negative) | $X'OYZ'$ |
| (−4, 2, 5) | II ($x$-coordinate is negative) | $X'OYZ$ |
| (−3, −1, 6) | III ($x$ and $y$-coordinates are negative) | $X'OY'Z$ |
| (2, −4, −7) | VIII ($y$ and $z$-coordinates are negative) | $XOY'Z'$ |

**EXAMPLE |2|** Find the octant in which the points (−3, 4, 2) and (−3, 1, − 4) lie.

***Sol.*** The point (−3, 4, 2) lies in II octant and the point (−3, 1, −4) lies in VI octant.

## Coordinates of a Point on Coordinate Axes

Let $P(x, y, z)$ be any point on a space and we draw three perpendicular lines from $P$ to the coodinate axes meet at points $A$, $B$ and $C$. The coordinates of a point $P$ on $X$-axis will be of the form $(x, 0, 0)$. Here, $x$ is the distance of foot of perpendicular from $P$ to $X$-axis from the origin with suitable sign.




The sign of $x$ is positive or negative, according as the foot of perpendicular lies on positive or negative direction of $X$-axis. Thus, coordinates of a point on

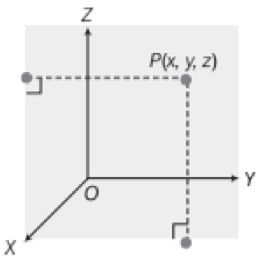
(i) $X$ axis will be of the form $(x, 0, 0)$
(ii) $Y$-axis will be of the form $(0, y, 0)$
(iii) $Z$-axis will be of the form $(0, 0, z)$.

**EXAMPLE |3|** A point is on the $X$-axis. What are its $y$ and $z$-coordinates?

***Sol.*** Coordinates of any point on the $X$-axis is $(x, 0, 0)$.
Because at $X$-axis, both $y$ and $z$-coordinates are zero.
So, $y$ and $z$-coordinates of point are zero.

## Coordinates of a Point in Coordinate Planes

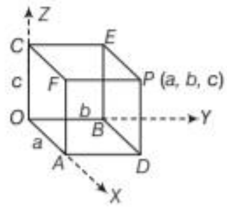
Let $P(x, y, z)$ be any point on a space and we draw three perpendicular lines from $P$ to the coordinate planes $XY$, $YZ$ and $ZX$, respectively.




Then, coordinates of a point in

(i) $XY$-plane will be of the form $(x, y, 0)$.
[$\because$ In $XY$-plane, the $z$-coordinate of a point is zero]
(ii) $YZ$-plane will be of the form $(0, y, z)$ and
(iii) $ZX$-plane will be of the form $(x, 0, y)$.

**EXAMPLE |4|** In the given figure, if the coordinates of point $P$ are $(a, b, c)$, then write the coordinates of $A$, $D$, $B$, $C$ and $E$.

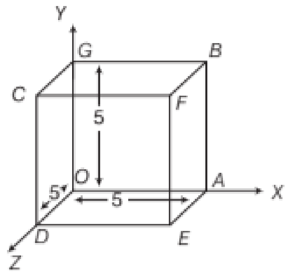



***Sol.*** Given, the coordinates of point $P$ are $(a, b, c)$.
Which shows that, $OA = a$, $OB = b$ and $OC = c$.
Now, point $A$ lies on $X$-axis, so its coordinates are $(a, 0, 0)$. Point $D$ lies in $XY$-plane, so its coordinates are $(a, b, 0)$. Point $B$ lies on $Y$-axis, so its coordinates are $(0, b, 0)$.
Point $C$ lies on $Z$-axis, so its coordinates are $(0, 0, c)$ and point $E$ lies in $YZ$-plane, so its coordinates are $(0, b, c)$.
Hence, the coordinates of required points are
$A(a, 0, 0)$, $D(a, b, 0)$, $B(0, b, 0)$, $C(0, 0, c)$ and $E(0, b, c)$.

**EXAMPLE |5|** What are the coordinates of the vertices of a cube whose edge is 5 units, one of whose vertices coincides with the origin and three edges passing through the origin coincides with the positive direction of the axes through the origin?

***Sol.*** Given, edge of a cube is 5 unit. It is clear that

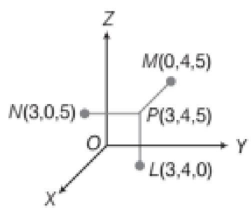
| | |
|---|---|
| Coordinate of $O = (0, 0, 0)$ | Coordinate of $A = (5, 0, 0)$ |
| Coordinate of $G = (0, 5, 0)$ | Coordinate of $D = (0, 0, 5)$ |
| Coordinate of $B = (5, 5, 0)$ | Coordinate of $F = (5, 5, 5)$ |
| Coordinate of $E = (5, 0, 5)$ | Coordinate of $C = (0, 5, 5)$ |




**EXAMPLE |6|** Let $L$, $M$, $N$ be the feet of the perpendiculars drawn from the point $P(3, 4, 5)$ on the $XY$, $YZ$ and $ZX$-planes, respectively. Find the distance of these points $L$, $M$, $N$ from the point $P$. **[NCERT Exemplar]**

***Sol.*** $L$ is the foot of perpendicular drawn from the point $P(3, 4, 5)$ to the $XY$-plane.

Therefore, the coordinates of the point $L$ are $(3, 4, 0)$. The distance between the points $(3, 4, 5)$ and $(3, 4, 0)$ is 5 units. Similarly, the lengths of the foot of perpendiculars on $YZ$ and $ZX$-planes are 3 and 4 units, respectively.



**EXAMPLE |7|** $L$ is the foot of the perpendicular drawn from a point $P$ (5, 4, 6) on the $XY$-plane. Find the coordinates of point $L$.

**Sol.** Since, in $XY$-plane, $z$-coordinate will be zero. Hence, the coordinates of the foot of the perpendicular $L$ (5, 4, 0).

### Method to Locate a Point P (x, y, z) in Space

To locate a point $P(x, y, z)$ in space, *we use the following steps*

**Step I** First, take a point $A$ in suitable direction of $X$-axis (i.e. positive or negative) at a distance $|x|$ from origin $O$.

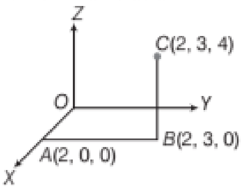
**Step II** Now, from point $A$, draw a line perpendicular to $X$-axis which will be parallel to $Y$-axis. Then, take point $B$ at this line at a distance $|y|$ from $A$.

**Step III** Through point $B$, draw a line perpendicular to $XY$-plane which will be parallel to $Z$-axis. Then, take point $C$ at this line at a distance $|z|$ from $B$.

Thus, point $C$ will represent the given point in space.

**EXAMPLE |8|** Locate the point (2, 3, 4) in space.

**Sol.** Given point is (2, 3, 4). Here, $x$-coordinate i.e. 2 is positive. So, we take a point $A$ in positive direction of $X$-axis at a distance 2 from $O$. Thus, point $A$ be (2, 0, 0). From point $A$, we draw a line perpendicular to $X$-axis which will be parallel to $Y$-axis and take point $B$ at a distance 3 from $A$ in positive direction. Thus, point $B$ be (2, 3, 0). Draw a line parallel to $Z$-axis from point $B$, which is perpendicular to $XY$-plane and take point $C$ at this line in positive direction at a distance 4 units from $B$. Thus, point $C$ (2, 3, 4) is the required location of given point in space, which is shown in the figure.




# TOPIC PRACTICE 1

## OBJECTIVE TYPE QUESTIONS

**1** The three numbers representing the perpendicular distances of the point from three mutually perpendicular planes are called the

(a) coordinates of the point with respect to the two coordinate planes
(b) coordinates of the origin with respect to the three coordinate planes
(c) coordinates of the point with reference to the three coordinate planes
(d) None of the above

**2** The octants in which the points (–3, 1, 2) and (–3, 1, –2) lie, are respectively

(a) II and VI (b) III and V
(c) I and IV (d) II and V

**3** A point is on the $X$-axis. Its $y$-coordinate and $z$-coordinate are respectively

(a) $y, z$ (b) $y$, 0 (c) $z$, 0 (d) 0, 0

**4** If a point is on the $ZX$-plane, then its coordinates will be

(a) $(x, y, 0)$ (b) $(0, y, z)$
(c) $(x, 0, z)$ (d) $(x, y, z)$

**5** If a point is in $XZ$-plane, then its $y$-coordinate is

(a) $x$ (b) $y$ (c) 0 (d) $z$

## VERY SHORT ANSWER Type Questions

**6** Name the octant in which each of the following points lies. (Each part carries 1 marks) **[NCERT Exemplar]**

(i) (4, –2, 3) (ii) (4, –2, –5)
(iii) (4, 2, –5) (iv) (– 3, –1, 6)

**7** $L$ is the foot of the perpendicular drawn from a point $P$(6, 7, 8) on the $XY$-plane. What are the coordinates of point $L$? **[NCERT Exemplar]**

## SHORT ANSWER Type Questions

**8** Let $L, M, N$ be the feet of the perpendiculars drawn from a point $P$(3, 4, 5) on the $X, Y$ and $Z$-axes respectively. Find the coordinates of $L$, $M$ and $N$. **[NCERT Exemplar]**

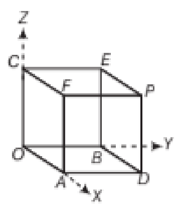
**9** If $A, B, C$ are the feet of perpendiculars from a point $P$ on the $XY$, $YZ$ and $ZX$-planes respectively, then find the distance of $A, B$ and $C$ from $P$, where the point $P$ is

(i) (3, 4, 2) (ii) (– 5, 3, 7)

**10** If $A, B$ and $C$ are the feet of perpendiculars from a point on $XY$, $YZ$ and $ZX$-planes respectively, then find the coordinates of $A, B$ and $C$ in each of the following, where the point $P$ is

(i) (–5, 3, 7) (ii) (4, – 3, –5)

**11** In the given figure, if the coordinates of the point $P$ are (3, 4, 6), then write the coordinates of $A$, $D$, $B$, $C$ and $E$.

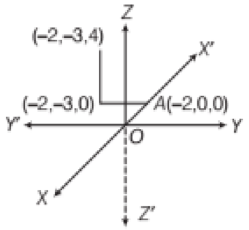



**12** Locate the point (– 2, – 3, 4) in space.

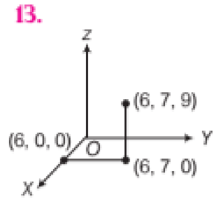
**13** Locate the point (6, 7, 9) in space.

## HINTS & ANSWERS

1. (c) The given three numbers are called the coordinates of the point with reference to the three coordinate planes.
2. (a) The point (−3, 1, 2) lies in second octant (i.e. $X'OYZ$) and the point (−3, 1, −2) lies in VI octant (i.e. $X'OYZ'$).
3. (d) Coordinates of any point on the $X$-axis is $(x, 0, 0)$. Therefore, both $y$ and $z$-coordinates are zero.
4. (c) Coordinates of a point in $ZX$-plane will be $(x, 0, z)$ because in $ZX$-plane, $y$-coordinate is zero.
5. (c) Any point on the $XZ$-plane will have the coordinates $(x, 0, z)$, then its $y$-coordinate is 0.
6. (i) Fourth octant (ii) Eighth octant (iii) Fifth octant (iv) Third octant
7. (6, 7, 0)
8. $L$ (3, 0, 0), $M$ (0, 4, 0) and $N$ (0, 0, 5)
9. (i) 2 units, 3units, 4 units (ii) 7units, 5 units, 3 units
10. (i) $A(-5, 3, 0)$, $B(0, 3, 7)$, $C(-5, 0, 7)$
    (ii) $A(4, -3, 0)$, $B(0, -3, -5)$, $C(4, 0, -5)$
11. (3, 0, 0), (3, 4, 0), (0, 4, 0), (0, 0, 6), (0, 4, 6)
12.




13.




# TOPIC 2

# Distance Formula and Its Applications in Geometry

## DISTANCE BETWEEN TWO POINTS

The distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

or $$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

**Remark**

*If O is the origin and P (x, y, z) is a point in space, then*

$$OP = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2} = \sqrt{x^2 + y^2 + z^2}$$

**EXAMPLE |1|** Find the distance between the points $A(2, 3, 1)$ and $B(1, -2, 0)$.

**Sol.** Given points are $A(2, 3, 1)$ and $B(1, -2, 0)$.

Here, $x_1 = 2,\ y_1 = 3,\ z_1 = 1$

and $x_2 = 1,\ y_2 = -2,\ z_2 = 0$

∴ The distance between the points $A$ and $B$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(1-2)^2 + (-2-3)^2 + (0-1)^2}$$
$$= \sqrt{(-1)^2 + (-5)^2 + (-1)^2}$$
$$= \sqrt{1 + 25 + 1}$$
$$= \sqrt{27} = 3\sqrt{3} \text{ units}$$

**EXAMPLE |2|** Find the values of $x$, if the distance between two points $(x, -8, 4)$ and $(3, -5, 4)$ is 5.

**Sol.** Given points are $(x, -8, 4)$ and $(3, -5, 4)$.

and distance between these points = 5

$$\therefore \sqrt{(x-3)^2 + (-8+5)^2 + (4-4)^2} = 5$$
$$\Rightarrow \sqrt{(x-3)^2 + (-3)^2 + 0} = 5$$
$$\Rightarrow (x-3)^2 + 9 + 0 = 25$$

[squaring both sides]

$$\Rightarrow (x-3)^2 = 16 \Rightarrow (x-3)^2 = (4)^2$$
$$\Rightarrow x - 3 = \pm 4$$

[taking square root both sides]

$$\therefore x = 7 \text{ or } -1$$

**EXAMPLE |3|** Find the distance of point $P$ (3, 6, 9) from the $YZ$-plane using distance formula.

**Sol.** When we draw a perpendicular line from the point $P(3, 6, 9)$ on the $YZ$-plane, the $x$-coordinate of foot of perpendicular will be zero and the other coordinates ($y$ and $z$) will be 6 and 9, i.e. coordinates of a point on $YZ$-plane (which is the foot of perpendicular drawn from $P$ to $YZ$ plane) be $Q$ (0, 6, 9).

∴ Distance between $P$ and $Q$,

$$QP = \sqrt{(3-0)^2 + (6-6)^2 + (9-9)^2}$$

$$[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}]$$

$$= \sqrt{3^2 + 0^2 + 0^2} = 3 \text{ units}$$

**EXAMPLE |4|** If a parallelopiped is formed by planes drawn through the points (5, 8, 10) and (3, 6, 8) parallel to the coordinate planes, then find the length of edges and diagonal of the parallelopiped by using distance formula.

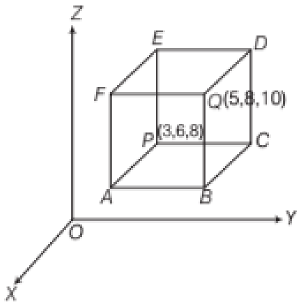
**Sol.** Let $P \equiv (3, 6, 8)$ and $Q \equiv (5, 8, 10)$

Now,

$PE$ = Distacne between parallel planes $ABCP$ and $FQDE$

$= |10 - 8|$ [$\because$ these planes are perpendicular to $Z$-axis]

= 2 units




$PA$ = Distance between parallel planes $ABQF$ and $PCDE$

$= |5 - 3|$ [$\because$ these planes are perpendicular to $X$-axis]

= 2 units

$PC$ = Distance between parallel planes $APEF$ and $BCDQ$

$= |8 - 6|$ [$\because$ these planes are perpendicular to $Y$-axis]

= 2 units

$\therefore$ Length of diagonal = Distance between $P$ and $Q$

$$= \sqrt{(5-3)^2 + (8-6)^2 + (10-8)^2}$$

[using the distance formula]

$$= \sqrt{2^2 + 2^2 + 2^2}$$

$$= \sqrt{4 + 4 + 4}$$

$$= \sqrt{12}$$

$$= 2\sqrt{3} \text{ units}$$

Hence the length of each edge of parallelopiped is 2 units and the length of its diagonal is $2\sqrt{3}$ units.

**EXAMPLE |5|** Find the equation of the curve formed by the set of all points whose distances from the points (3, 4, − 5) and (−2, 1, 4) are equal.

**Sol.** Let $P(x, y, z)$ be any point on the given curve and let $A(3, 4, -5)$ and $B(-2, 1, 4)$ be the given points.

Then, $PA = PB \Rightarrow PA^2 = PB^2$

$$\Rightarrow (x-3)^2 + (y-4)^2 + (z+5)^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$$

$$\Rightarrow x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 + 10z = x^2 + 4 + 4x + y^2 + 1 - 2y + z^2 + 16 - 8z$$

$$\Rightarrow 10x + 6y - 18z - 29 = 0$$

Hence, the required curve is

$$10x + 6y - 18z - 29 = 0$$

**EXAMPLE |6|** Find the equation of set of point $P$ such that $PA^2 + PB^2 = 2k^2$, where $A$ and $B$ are the points (3, 4, 5) and (−1, 3, − 7), respectively. **[NCERT]**

**Sol.** Given points are $A(3,4,5)$ and $B(-1, 3, -7)$.

Let the coordinates of point $P$ be $(x, y, z)$.

Then, $PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$

[$\because$ distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$]

and $PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$

By the given condition $PA^2 + PB^2 = 2k^2$,

$$(x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = 2k^2$$

$$\Rightarrow x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z + x^2 + 2x + 1 + y^2 + 9 - 6y + z^2 + 49 + 14z = 2k^2$$

$$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = 2k^2 - 109$$

which is the required equation.

**EXAMPLE |7|** Find the coordinates of a point equidistant from the four points $O(0, 0, 0)$, $A(l, 0, 0)$, $B(0, m, 0)$ and $C(0, 0, n)$. **[NCERT Exemplar]**

**Sol.** Let $P(x, y, z)$ be required point.

Then, $OP = PA = PB = PC$

Now, $OP = PA \Rightarrow OP^2 = PA^2$

$$\Rightarrow (0-x)^2 + (0-y)^2 + (0-z)^2 = (x-l)^2 + (y-0)^2 + (z-0)^2$$

$$\Rightarrow x^2 + y^2 + z^2 = x^2 - 2lx + l^2 + y^2 + z^2$$

$$\Rightarrow 2lx = l^2 \Rightarrow x = \frac{l}{2}$$

Similarly, $OP = PB \Rightarrow y = \frac{m}{2}$ and $OP = PC \Rightarrow z = \frac{n}{2}$

Hence, the coordinates of the required point are $\left(\frac{l}{2}, \frac{m}{2}, \frac{n}{2}\right)$

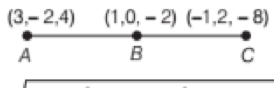
## Condition for Collinearity

Three points $A$, $B$ and $C$ are said to be collinear, if

$$AB + BC = AC$$

**EXAMPLE |8|** Verify that the points (3, − 2, 4), (1, 0, − 2) and (−1, 2, − 8) are collinear.

**Sol.** Let $A(3, -2, 4)$, $B(1, 0, -2)$ and $C(-1, 2, -8)$ be the given points.




Then, $AB = \sqrt{(1-3)^2 + (0+2)^2 + (-2-4)^2}$

[using distance formula]

$$= \sqrt{4 + 4 + 36} = \sqrt{44} = 2\sqrt{11} \text{ units}$$

$$BC = \sqrt{(-1-1)^2 + (2-0)^2 + (-8+2)^2}$$
[using distance formula]
$$= \sqrt{4+4+36}$$
$$= \sqrt{44} = 2\sqrt{11} \text{ units}$$

and $$CA = \sqrt{(3+1)^2 + (-2-2)^2 + (4+8)^2}$$
[using distance formula]
$$= \sqrt{16+16+144}$$
$$= \sqrt{176} = 4\sqrt{11} \text{ units}$$

We observe that,
$$AB + BC = 2\sqrt{11} + 2\sqrt{11}$$
$$= 4\sqrt{11} = CA$$

Hence, $A$, $B$ and $C$ are collinear.

# APPLICATIONS OF DISTANCE FORMULA IN GEOMETRY

Sometimes to identify the given geometric figure, we use the distance formula. For this type of problems, we show that given points satisfies the property related to given geometrical figure by using distance formula.

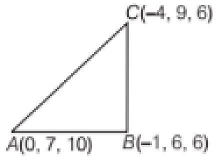
## Properties of Some Geometrical Figures

1. **Properties of Triangles**
   - (i) **Scalene triangle** All three sides are unequal.
   - (ii) **Right angled triangle** The sum of squares of any two sides of a triangle is equal to the square of the third side.
   - (iii) **Isosceles triangle** Any two sides of a triangle are equal.
   - (iv) **Equilateral triangle** All three sides of a triangle are equal.
2. **Properties of Quadrilaterals**
   - (i) **Rectangle** Opposite sides are equal and diagonals are equal.
   - (ii) **Parallelogram** Opposite sides are equal and diagonals are unequal. Also, diagonals bisect each other.
   - (iii) **Rhombus** All four sides are equal and diagonals are unequal.
   - (iv) **Square** All four sides are equal and diagonals are equal.

**EXAMPLE |9|** Show that the points (0, 7, 10), (−1, 6, 6) and (−4, 9, 6) are the vertices of a right angled isosceles triangle.

**Sol.** Let $A(0,7,10)$, $B(-1, 6, 6)$ and $C(-4, 9, 6)$ be the given points.

Then, $$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$$
[using the distance formula]
$$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2} \text{ units}$$




$$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2}$$
$$= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

and $$AC = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2} = \sqrt{16+4+16}$$

$$\Rightarrow AC = \sqrt{36} = 6 \text{ units} \quad \text{...(i)}$$

Now, $AB^2 + BC^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 18 + 18 = 36$ units

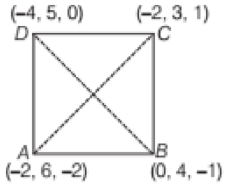
$\therefore \quad AB^2 + BC^2 = AC^2$ [from Eq. (i)]

Also, $AB = BC = 3\sqrt{2}$ units

Hence, $ABC$ is a right angled isosceles triangle.

**EXAMPLE |10|** Show that the points (−2, 6, −2), (0, 4, −1), (−2, 3, 1) and (−4, 5, 0) are the vertices of a square.

**Sol.** Let $A(-2, 6, -2)$, $B(0, 4, -1)$, $C(-2, 3, 1)$ and $D(-4, 5, 0)$ be the given points.




$$\therefore AB = \sqrt{(0+2)^2 + (4-6)^2 + (-1+2)^2}$$
[using the distance formula]
$$= \sqrt{4+4+1} = \sqrt{9} = 3 \text{ units}$$

$$BC = \sqrt{(-2-0)^2 + (3-4)^2 + (1+1)^2}$$
$$= \sqrt{4+1+4} = \sqrt{9} = 3 \text{ units}$$

$$CD = \sqrt{(-4+2)^2 + (5-3)^2 + (0-1)^2}$$
$$= \sqrt{4+4+1} = \sqrt{9} = 3 \text{ units}$$

$$AD = \sqrt{(-4+2)^2 + (5-6)^2 + (0+2)^2}$$
$$= \sqrt{4+1+4} = \sqrt{9} = 3 \text{ units}$$

Here, $AB = BC = CD = DA$.

So, $ABCD$ is a square or a rhombus.

Now, $$AC = \sqrt{(-2+2)^2 + (3-6)^2 + (1+2)^2}$$
$$[\because \text{distance} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}]$$
$$= \sqrt{0+9+9} = \sqrt{18} \text{ units}$$

and $$BD = \sqrt{(-4-0)^2 + (5-4)^2 + (0+1)^2}$$
$$= \sqrt{16+1+1} = \sqrt{18} \text{ units}$$

Since, diagonal $AC$ = diagonal $BD$

Hence, $ABCD$ is a square.

# TOPIC PRACTICE 2

## OBJECTIVE TYPE QUESTIONS

**1** Points $P(2, 4, 6)$, $Q(-2, -2, -2)$ and $R(6, 10, 14)$ are

(a) vertices of a triangle (b) collinear
(c) non-collinear (d) Both (a) and (b)

**2** Distance of the point (3, 4, 5) from the (0, 0, 0) is **[NCERT Exemplar]**

(a) $\sqrt{50}$ (b) 3 (c) 4 (d) 5

**3** The point on $Y$-axis which is at a distance $\sqrt{10}$ from the point (1, 2, 3), is

(a) (0, 2, 0) (b) (0, 0, 2)
(c) (0, 0, 3) (d) None of these

**4** The length of the foot of perpendicular drawn from the point $P(3, 4, 5)$ on $Y$-axis is

(a) 10 (b) $\sqrt{34}$ (c) $\sqrt{113}$ (d) $5\sqrt{2}$

**5** If the distance between the points $(a, 0, 1)$ and $(0, 1, 2)$ is $\sqrt{27}$, then the value of $a$ is

(a) 5 (b) $\pm 5$
(c) $-5$ (d) None of these

## VERY SHORT ANSWER Type Questions

**6** Find the distance from the origin to (6, 6, 7). **[NCERT Exemplar]**

**7** How far apart are the points (2, 0, 0) and (−3, 0, 0)?

**8** Find the distance between the following pairs of points. (Each part carries 1 mark)
(i) (−1, 3, −4) and (1, −3, 4) (ii) (2, −1, 3) and (−2, 1, 3)

**9** Show that, if $x^2 + y^2 = 1$, then the point $(x, y, \sqrt{1 - x^2 - y^2})$ is at a distance 1 unit from the origin.

**10** What is the length of foot of perpendicular drawn from the point $P(3, 5, 6)$ on $Y$-axis by using distance formula?

## SHORT ANSWER Type I Questions

**11** Find the point on $X$-axis which is equidistant from the points $A(3, 2, 2)$ and $B(5, 5, 4)$. **[NCERT Exemplar]**

**12** Determine the point in $YZ$-plane which is equidistant from three points $A(2,0,3)$, $B(0, 3, 2)$ and $C(0, 0, 1)$.

**13** Find the equation of the set of the point $P$, the sum of whose distance from $A(4, 0, 0)$ and $B(-4, 0, 0)$ is equal to 10.

**14** Find the equation of the set of points which are equidistance from the points (1, 2, 3) and (3, 2, −1). **[NCERT]**

**15** Using distance formula, show that the points $A(1, -1, 3)$, $B(2, -4, 5)$ and $C(5, -13, 11)$ are collinear.

**16** Prove that the triangle formed by joining the three points whose coordinates are $A(1, 2, 3)$, $B(2, 3, 1)$ and $C(3, 1, 2)$, is an equilateral triangle.

**17** Show that $D(-1, 4, -3)$ is the circumcentre of $\Delta ABC$ with vertices $A(3, 2, -5)$, $B(-3, 8, -5)$ and $C(-3, 2, 1)$.

## SHORT ANSWER Type II Questions

**18** Are the points $A(3, 6, 9)$, $B(10, 20, 30)$ and $C(25, -41, 5)$, the vertices of a right angled triangle?

**19** Show that $\Delta ABC$ with vertices $A(0, 4, 1)$, $B(2, 3, -1)$ and $C(4, 5, 0)$ is right angled. **[NCERT Exemplar]**

**20** Show that the points $P(0, 7, 10)$, $Q(-1, 6, 6)$ and $R(-4, 9, 6)$ form a right angled isosceles triangle. **[NCERT Exemplar]**

# HINTS & ANSWERS

**1.** (b) Clearly, $PQ = \sqrt{(-2-2)^2 + (-2-4)^2 + (-2-6)^2}$
$= \sqrt{16 + 36 + 64} = \sqrt{116} = 2\sqrt{29}$
$QR = \sqrt{(6+2)^2 + (10+2)^2 + (14+2)^2}$
$= \sqrt{64 + 144 + 256} = \sqrt{464} = 4\sqrt{29}$
$PR = \sqrt{(6-2)^2 + (10-4)^2 + (14-6)^2}$
$= \sqrt{16 + 36 + 64} = \sqrt{116} = 2\sqrt{29}$

Since, $QR = PQ + PR$. Therefore, given points are collinear.

**2.** (a) Required distance $= \sqrt{(3-0)^2 + (4-0)^2 + (5-0)^2}$
$= \sqrt{9 + 16 + 25} = \sqrt{50}$

**3.** (a) Let $P$ be the point on $Y$-axis. Then, it is of the form $P(0, y, 0)$. Since, the point (1, 2, 3) is at a distance $\sqrt{10}$ from $(0, y, 0)$, therefore $\sqrt{(1-0)^2 + (2-y)^2 + (3-0)^2} = \sqrt{10}$

$\Rightarrow \quad y^2 - 4y + 4 = 0 \Rightarrow (y-2)^2 = 0 \Rightarrow y = 2$

Hence, the required point is (0, 2, 0).

4. (b) Let $L$ be the foot of perpendicular from point $P$ on the $Y$-axis.
Then the coordinates of $L$ be (0, 4, 0).
Now, the distance between the points
(0, 4, 0) and (3, 4, 5) is $\sqrt{9+25}$ i.e. $\sqrt{34}$.

5. (b) Given, $\sqrt{(a-0)^2+(0-1)^2+(1-2)^2}=\sqrt{27}$
$\Rightarrow \sqrt{27}=\sqrt{a^2+1+1} \Rightarrow a^2=25 \Rightarrow a=\pm 5$

6. Origin = (0, 0, 0) **Ans.** 11 units

7. 5 units

8. (i) $2\sqrt{26}$ units (ii) $2\sqrt{5}$ units

9. Distance of the point $(x, y, \sqrt{1-x^2-y^2})$ from origin is given as $d=|\sqrt{x^2+y^2+(\sqrt{1-x^2-y^2})^2}|$

10. **Hint** Point on $Y$-axis is (0, 5, 0). **Ans.** $\sqrt{45}$ units

11. **Hint** Let the point on $X$-axis be $P(x, 0, 0)$.
Then, $(x-3)^2+(0-2)^2+(0-2)^2$
$=(x-5)^2+(0-5)^2+(0-4)^2$ **Ans.** $\left(\frac{49}{4}, 0, 0\right)$

12. Let $P(0, y, z)$ be a point on the $YZ$-plane such that $PA=PB=PC$.
Now, $PA=PB \Rightarrow (0-2)^2+(y-0)^2+(z-3)^2$
$=(0-0)^2+(y-3)^2+(z-2)^2 \Rightarrow z-3y=0$ ...(i)
and $PB=PC$
$\Rightarrow (0-0)^2+(y-3)^3+(z-2)^2$
$=(0-0)^2+(y-0)^2+(z-1)^2$
$\Rightarrow 3y+z=6$ ...(ii)
Now, solve Eqs. (i) and (ii). **Ans.** (0, 1, 3)

13. $PA+PB=10$ **Ans.** $9x^2+25y^2+25z^2-225=0$

14. $x-2z=0$

15. **Hint** Show that $AB+BC=CA$.

16. **Hint** Show that $AB=BC=CA$

17. **Hint** Show that $AD=BD=CD$

18. **Hint** $CA^2+AB^2 \neq BC^2$ **Ans.** No.

19. **Hint** Show that $AB^2+BC^2=AC^2$

20. **Hint** $PQ=QR$ and $PQ^2+QR^2=PR^2$

# SUMMARY

- In three dimensions, the coordinate axes of a rectangular cartesian coordinate system are three mutually perpendicular lines, which are $X$, $Y$ and $Z$-axes.
- The pair of axes formed the coordinate planes, which are $XY$, $YZ$ and $ZX$-planes.
- The three coordinate planes divide the space into eight octants.

| Octants | I | II | III | IV | V | VI | VII | VIII |
|---|---|---|---|---|---|---|---|---|
| $X$ | + | − | − | + | + | − | − | + |
| $Y$ | + | + | − | − | + | + | − | − |
| $Z$ | + | + | + | + | − | − | − | − |

- The coordinate of a point in the $XY$-plane is the form $(x, y, 0)$. Similarly, the coordinates of the $YZ$ and $ZX$-planes are $(0, y, z)$ and $(x, 0, z)$ respectively.
- The coordinates of the point in the $X$, $Y$ and $Z$-axes are the form $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$, respectively.
- The distance between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- **Properties of Triangles**
  - (i) **Scalene triangle** All three sides are unequal.
  - (ii) **Right angled triangle** The sum of squares of any two sides of a triangle is equal to the square of the third side.
  - (iii) **Isosceles triangle** Any two sides of a triangle are equal.
  - (iv) **Equilateral triangle** All three sides of a triangle are equal.
- **Properties of Quadrilaterals**
  - (i) **Rectangle** Opposite sides are equal and diagonals are equal.
  - (ii) **Parallelogram** Opposite sides are equal and diagonals are unequal. Also, diagonals bisect each other.
  - (iii) **Rhombus** All four sides are equal and diagonals are unequal.
  - (iv) **Square** All four sides are equal and diagonals are equal.

# CHAPTER PRACTICE

## OBJECTIVE TYPE QUESTIONS

**1.** Which of the following statements is correct?

(a) The coordinates of the origin $O$ are $(0, 0, 0)$

(b) The coordinates of any point on the $X$-axis will be as $(0, y', z)$

(c) The coordinates of any point in the $YZ$-plane will be $(x, 0, 0)$

(d) All of the above are correct

**2.** $L$ is the foot of the perpendicular drawn from a point $P(6, 7, 8)$ on the $XY$-plane, then the coordinates of point $L$ are **[NCERT Exemplar]**

(a) $(6, 0, 0)$ (b) $(6, 7, 0)$

(c) $(6, 0, 8)$ (d) None of these

**3.** $L$ is the foot of the perpendicular drawn from a point $(6, 7, 8)$ on $X$-axis. The coordinates of $L$ are

(a) $(6, 0, 0)$ (b) $(0, 7, 0)$

(c) $(0, 0, 8)$ (d) None of these

**4.** What is the locus of a point for which $y = 0$ and $z = 0$?

(a) Equation of $X$-axis (b) Equation of $Y$-axis

(c) Equation of $Z$-axis (d) None of these

**5.** Which of the following statement is correct?

(a) The $X$-axis and $Y$-axis taken together determine a plane known as $XY$-plane

(b) The coordinates of a point in the $XY$-plane are of the form $(0, 0, z)$

(c) Coordinate planes divide the space into six octants

(d) All the above are correct

**6.** Which of the following statements is incorrect?

(a) The equation of the plane $z = 6$ represents a plane perpendicular to the $XY$-plane, having a $z$-intercept of 6 units

(b) The equation of the plane $x = 0$ represents the $YZ$-plane

(c) The point on the $X$-axis with $x$-coordinate equal to $x_0$ is written as $(x_0, 0, 0)$

(d) $x = x_0$ represents a plane parallel to the $YZ$-plane

**7.** The distance between the points $(1, 4, 5)$ and $(2, 2, 3)$ is

(a) 5 (b) 4 (c) 3 (d) 2

**8.** The perpendicular distance of the point $(6, 5, 8)$ from $Y$-axis is

(a) 5 units (b) 6 units

(c) 8 units (d) 10 units

**9.** If the origin is the centroid of a $\Delta ABC$ having vertices $A(a, 1, 3)$, $B(-2, b, -5)$ and $C(4, 7, c)$, then

(a) $a = -2$ (b) $b = 8$

(c) $c = -2$ (d) None of these

**10.** The centroid of the triangle, if the mid-point of the sides of triangles are $D(1, 2, -3)$, $E(3, 0, 1)$ and $F(-1, 1, -4)$, is **[NCERT Exemplar]**

(a) $(1, 2, -1)$ (b) $(1, 1, -2)$

(c) $(0, 1, -2)$ (d) None of these

## SHORT ANSWER Type I Questions

**11.** Describe the vertices and edges of the rectangular parallelopiped with vertex $(3, 5, 6)$ placed in the first octant with one vertex at origin and edges of parallelopiped lie along $X$, $Y$ and $Z$-axes. **[NCERT Exemplar]**

**12.** Find the locus of a point which moves such that the sum of its distance from points $A(0, 0, -\alpha)$ and $B(0, 0, \alpha)$ is constant.

**13.** Show that the points $(0, 4, 1)$, $(2, 3, -1)$, $(4, 5, 0)$ and $(2, 6, 2)$ are the vertices of a square.

**14.** Prove that the points $(5, 3, 2)$, $(3, 2, 5)$ and $(2, 5, 3)$ are the vertices of an equilateral triangle.

**15.** By using distance formula, show that the points $(2, -1, 0)$, $(-2, 3, -2)$ and $(0, 3, 0)$ are collinear.

**16.** Show that the points $(a, b, c)$, $(b, c, a)$ and $(c, a, b)$ are the vertices of an equilateral triangle.

## SHORT ANSWER Type II Questions

**17.** If a parallelopiped is formed by the planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes, then write the lengths of edges of the parallelopiped and length of the diagonal.

**18.** Show that the points (–1, – 6, 10), (1, – 3, 4), (– 5, – 1, 1) and (–7, – 4, 7) are the vertices of a rhombus.

**19.** Two vertices of a triangle are $A(3, 4, 2)$ and $B(1, 3, 2)$. The medians of the triangle intersect at (2, 4, 3). Find the remaining vertex $C$ of the triangle.

## CASE BASED Questions

**20.** Three students are standing in a park with sign boards "SAVE ENVIRONMENT", "Don't LITTER", "KEEP YOUR PLACE CLEAN". Their positions are marked by the points $A(0, 7, 10)$, $B(-1, 6, 6)$ and $C(-4, 9, 6)$. The three students are holding GREEN coloured ribbons together. Answer the following questions which are based on above it.

(i) The length of ribbon $AB$ is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $4\sqrt{3}$ (d) $5\sqrt{4}$

(ii) The length of ribbon $BC$ is

(a) $3\sqrt{2}$ (b) $2\sqrt{3}$ (c) $4\sqrt{3}$ (d) $6\sqrt{5}$

(iii) The length of ribbon $CA$ is

(a) 3 (b) 4 (c) 5 (d) 6

(iv) Which of the following is distance formula?

(a) $\sqrt{(x_2 - x_1)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2}$

(b) $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

(c) $\sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2 + (z_1 + z_2)^2}$

(d) None of the above

(v) Ribbons form the sides of a

(a) Isosceles triangle

(b) Equilateral teriangles

(c) Right angled triangle

(d) None of the above

**21.** Four students in traditional dresses represent four states of India, standing at the points represented by $O(0, 0, 0)$, $A(a, 0, 0)$, $B(0, b, 0)$ $C(0, 0, 0)$. If a girl representing BHARATMATA be placed in such a way that she is equidistant from the four students, then answer the following questions which are based on above it.

(i) $x$-coordinate of girl representing BHARATMATA is

(a) $a$ (b) $\frac{a}{2}$ (c) $\frac{a}{3}$ (d) $\frac{a}{4}$

(ii) $y$-coordinate of girl representing BHARATMATA is

(a) $a$ (b) $\frac{b}{2}$ (c) $2b$ (d) $3b$

(iii) $z$-coordinate of girl representing BHARATMATA is

(a) $b$ (b) $c$ (c) $\frac{c}{2}$ (d) $2c$

(iv) Which concept is used for finding the coordinates of point?

(a) Distance formula (b) Section formula

(c) Mid-point formula (d) None of these

(v) Which of the following is coordinates of origin point?

(a) (0, 0, 0) (b) $(0, b, 0)$ (c) $(a, 0, 0)$ (d) $(0, 0, c)$

# HINTS & ANSWERS

**1.** (a) The coordinates of the origin $O$ are (0, 0, 0). The coordinates of any point on the $X$-axis will be as $(x, 0, 0)$ and the coordinates of any point in the $YZ$-plane will be as $(0, y, z)$.

**2.** (b) Since, $L$ is the foot of perpendicular from $P$ on the $XY$-plane, $z$-coordinate will be zero. Hence, coordinates of $L$ are (6, 7, 0).

**3.** (a) Since, $L$ is the foot of perpendicular from $P$ on the $X$-axis, $y$ and $z$-coordinates are zero. Hence, the coordinates of $L$ are (6, 0, 0).

**4.** (a) Locus of the point for which $y = 0$, $z = 0$ is $X$-axis, since on $X$-axis both $y = 0$ and $z = 0$.

**5.** (a) (i) The $X$-axis and $Y$-axis taken together determine a plane known as $XY$-plane.

(ii) The coordinates of a point in the $XY$-plane are of the form $(x, y, 0)$.

(iii) Coordinate planes divide the space into eight octants.

**6.** (a) The equation of the plane $z = 6$ represents a plane parallel to the $XY$-plane, having a $z$-intercept of 6 units.

**7.** (c) The distance between given points

$$= \sqrt{(2-1)^2 + (2-4)^2 + (3-5)^2} = \sqrt{1+4+4} = 3$$

**8.** (d) Perpendicular distance of the points (6, 5, 8) from $Y$-axis $= \sqrt{6^2 + 8^2} = 10$ units

**9.** (a) For centroid of $\Delta ABC$,

$$x = \frac{a-2+4}{3} = \frac{a+2}{3}$$

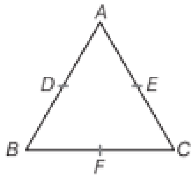
$$y = \frac{1+b+7}{3} = \frac{b+8}{3}$$

and $$z = \frac{3-5+c}{3} = \frac{c-2}{3}$$

But, given centroid is (0, 0, 0).

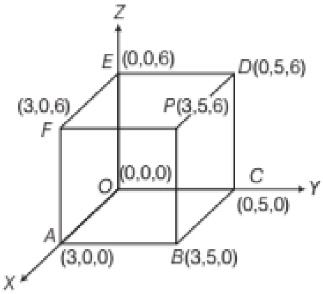
$$\therefore \quad \frac{a+2}{3}=0 \Rightarrow a=-2$$
$$\frac{b+8}{3}=0 \Rightarrow b=-8$$
$$\frac{c-2}{3}=0 \Rightarrow c=2$$

**10.** (b) Centroid of $\Delta ABC$ = Centroid of $\Delta DEF$




$$=\left(\frac{1+3-1}{3}, \frac{2+0+1}{3}, \frac{-3+1-4}{3}\right)=(1, 1, -2)$$

**11.**




**Ans.** (0, 0, 0), (9, 0, 0), (0, 5, 0), (0, 0, 6), (9, 5, 0), (9, 0, 6), (0, 5, 6); 6, 5, 9

**12.** Let $P(x, y, z)$ be the required point. According to question, $AP + BP = K$, where $K$ is any arbitrary constant

$$\Rightarrow \sqrt{(x-0)^2+(y-0)^2+(z+\alpha)^2} + \sqrt{(x-0)^2+(y-0)^2+(z-\alpha)^2} = K$$

**Ans.** $4K^2x^2+4K^2y^2+4z^2(K^2-4\alpha^2) + K^2(4\alpha^2-K^2)=0$

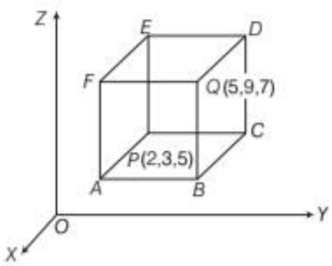
**13.** **Hint** Show that $AB = BC = CD = DA$ and $AC = BD$

**14.** **Hint** Show that $AB = BC = CA$

**15.** To prove, collinear, $AB + BC = AC$

**16.** Let the given points are $A$, $B$ and $C$ and then show that $AB = BC = CA$

**17.** Let $P \equiv (2, 3, 5)$ and $Q \equiv (5, 9, 7)$




Now,

$PE$ = Distance between parallel planes $ABCP$ and $FQDE$

$PA$ = Distance between parallel planes $ABQF$ and $PCDE$

$PC$ = Distance between parallel planes $APEF$ and $BCDQ$

and length of diagonal = distance between $P$ and $Q$.

**Ans.** 3 units, 6 units, 2 units and 7 units.

**18.** **Hint** Show that $AB = BC = CD = DA$ and $AC \neq BD$.

**19.** Let third vertex of $\Delta ABC$ be $C(x, y, z)$.

We know that intersection of medians of triangle is known as centroid of a triangle.

$$\therefore \left(\frac{3+1+x}{3}, \frac{4+3+y}{3}, \frac{2+2+z}{3}\right)=(2, 4, 3)$$ **Ans.** $C(2, 5, 5)$

**20.** (i) (b) Given that, $A(0, 7, 10)$, $B(-1, 6, 6)$ and $C(-4, 9, 6)$.

$$\therefore AB=\sqrt{(0+1)^2+(7-6)^2+(10-6)^2}=\sqrt{1+1+16}=\sqrt{18}=3\sqrt{2} \text{ units}$$

(ii) (a) $BC=\sqrt{(-1+4)^2+(6-9)^2+(6-6)^2}=\sqrt{3^2+3^2+0}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$ units

(iii) (d) $CA=\sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}=\sqrt{16+4+16}=\sqrt{36}=6$ units

(iv) (b) Distance between the points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$

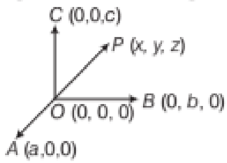
$$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

(v) (c) $\therefore AB=3\sqrt{2}$, $BC=3\sqrt{2}$ and $CA=6$

Here, $AB^2+BC^2=CA^2$

Thus, $\Delta ABC$ is right angled triangle at $B$

**21.** (i) (b) Let $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$ and $O(0, 0, 0)$ be four points equidistant from the point $P(x, y, z)$.




Then, $PA = PB = PC = OP$

Now, $OP = PA \Rightarrow OP^2 = PA^2$

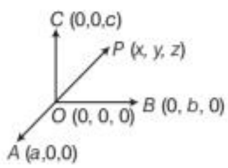
$$\Rightarrow x^2+y^2+z^2=(x-a)^2+(y-0)^2+(z-0)^2$$

$$[\because \text{distance}=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}]$$

$$\Rightarrow x^2+y^2+z^2=x^2+a^2-2ax+y^2+z^2$$

$$\Rightarrow 0=-2ax+a^2 \Rightarrow x=\frac{a}{2}$$

(ii) (b) Let $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$ and $O(0, 0, 0)$ be four points equidistant from the point $P(x, y, z)$.



Then, $PA = PB = PC = OP$

Now, $OP = PA \Rightarrow OP^2 = PA^2$

$\Rightarrow x^2 + y^2 + z^2 = x^2 + (y - b)^2 + (z - 0)^2$

$\Rightarrow x^2 + y^2 + z^2 = x^2 + y^2 + b^2 - 2by + z^2$

$\Rightarrow \qquad b^2 = 2by \Rightarrow y = \frac{b}{2}$

(iii) (c) Let $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$ and $O(0, 0, 0)$ be four points equidistant from the point $P(x, y, z)$.

Then, $PA = PB = PC = OP$

Now, $OP = PA \Rightarrow OP^2 = PA^2$

$\Rightarrow x^2 + y^2 + z^2 = x^2 + y^2 + (z - c)^2$

$\Rightarrow x^2 + y^2 + z^2 = x^2 + y^2 + z^2 + c^2 - 2zc$

$\Rightarrow \qquad 0 = -2zc + c^2$

$\Rightarrow \qquad z = \frac{c}{2}$

(iv) (a) Distance formula

(v) (a) Coordinates of origin is (0,0, 0)