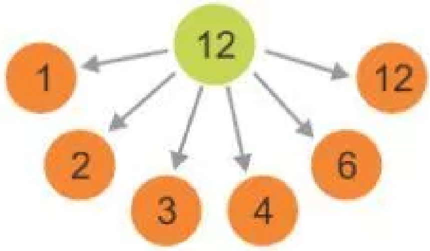
2. Number

Finding Factors And Multiples

FACTORS AND MULTIPLES

Factors: The numbers which completely divide the given number such that the remainder is zero. A number can have finite number of factors.
Example: Factors of the number 12 are: 1, 2, 3, 4, 6 and 12



Multiples: The numbers obtained by multiplying the given number by a natural number. A number can have infinite number of multiples.
Example:

Multiples of 6 are :	
6 × 2	12
6 × 3	18
6 × 4	24
....	....

Mr Sharma wanted to withdraw Rs. 1000 from his bank account to purchase books for his children. The cashier gave him 10 hundred-rupee notes, i.e., Rs. 10 × 100 = Rs. 1000
Mr Sharma got the required amount. But the cashier could also give the same amount in the following ways:

- 200 five-rupee notes
  $= \mathbf{200} \times ₹\ 5$
  $= ₹\ 1000$
- 100 ten-rupee notes
  $= \mathbf{100} \times ₹\ 10$
  $= ₹\ 1000$
- 50 twenty-rupee notes
  $= \mathbf{50} \times ₹\ 20$
  $= ₹\ 1000$
- 20 fifty-rupee notes
  $= \mathbf{20} \times ₹\ 50$
  $= ₹\ 1000$
- 2 five hundred-rupee notes
  $= \mathbf{2} \times ₹\ 500$
  $= ₹\ 1000$
- 1 thousand-rupee note
  $= \mathbf{1} \times ₹\ 1000$
  $= ₹\ 1000$

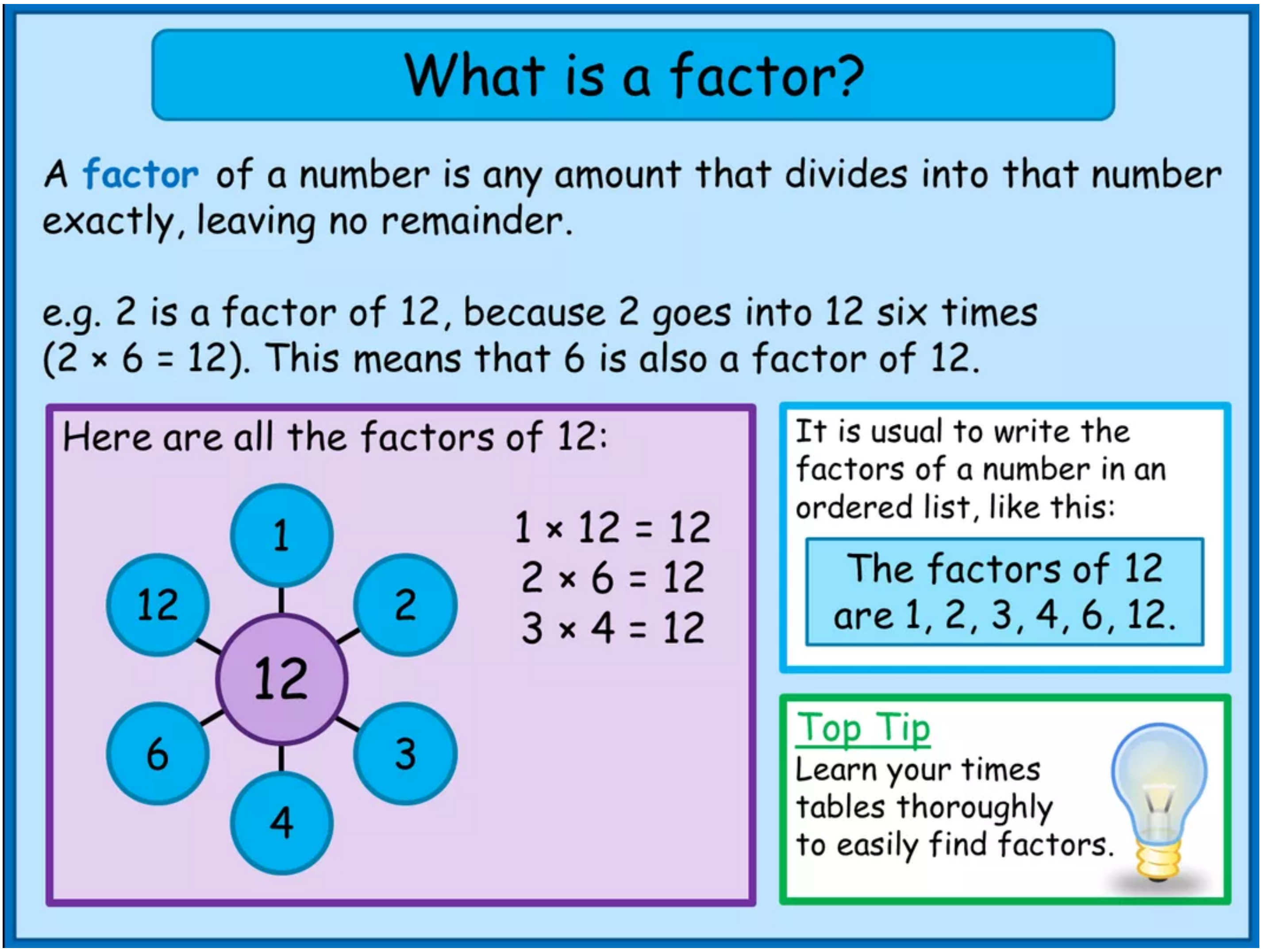
Here, we observe that in each case Mr Sharma got the same amount of Rs. 1000. These numbers 1, 2, 5, 10, 20, 50, 100, 200, 500, and 1000 are factors of 1000. Hence, 1000 is a multiple of these numbers.
Here we will discuss only the natural numbers, that is positive integers.
If a = b × c, we say b and c are factors of a and a is a multiple of c and b.

Factors

Factor: A number which divides a given number exactly (without leaving any remainder) is called a factor of the given number.

Example: Factors of 12

12 = 1 × 12
12 = 2 × 6
12 = 3 × 4
Here, 1, 2, 3, 4, 6, and 12 are factors of 12.



Properties of Factors

1. Every non-zero number is a factor of itself.
   Examples: 5 is a factor of 5. (5 ÷ 5 = 1)
   12 is a factor of 12. (12 ÷ 12 = 1)
2. 1 is a factor of every number.
   Examples: 1 is a factor of 5. (5 ÷ 1 = 5)
   1 is a factor of 12. (12 ÷ 1 = 12)
3. Every non-zero number is a factor of 0.
   Example: 5 and 12 are factors of 0 because
   0 ÷ 5 = 0 and 0 ÷ 12 = 0
4. The factors of a number are finite.

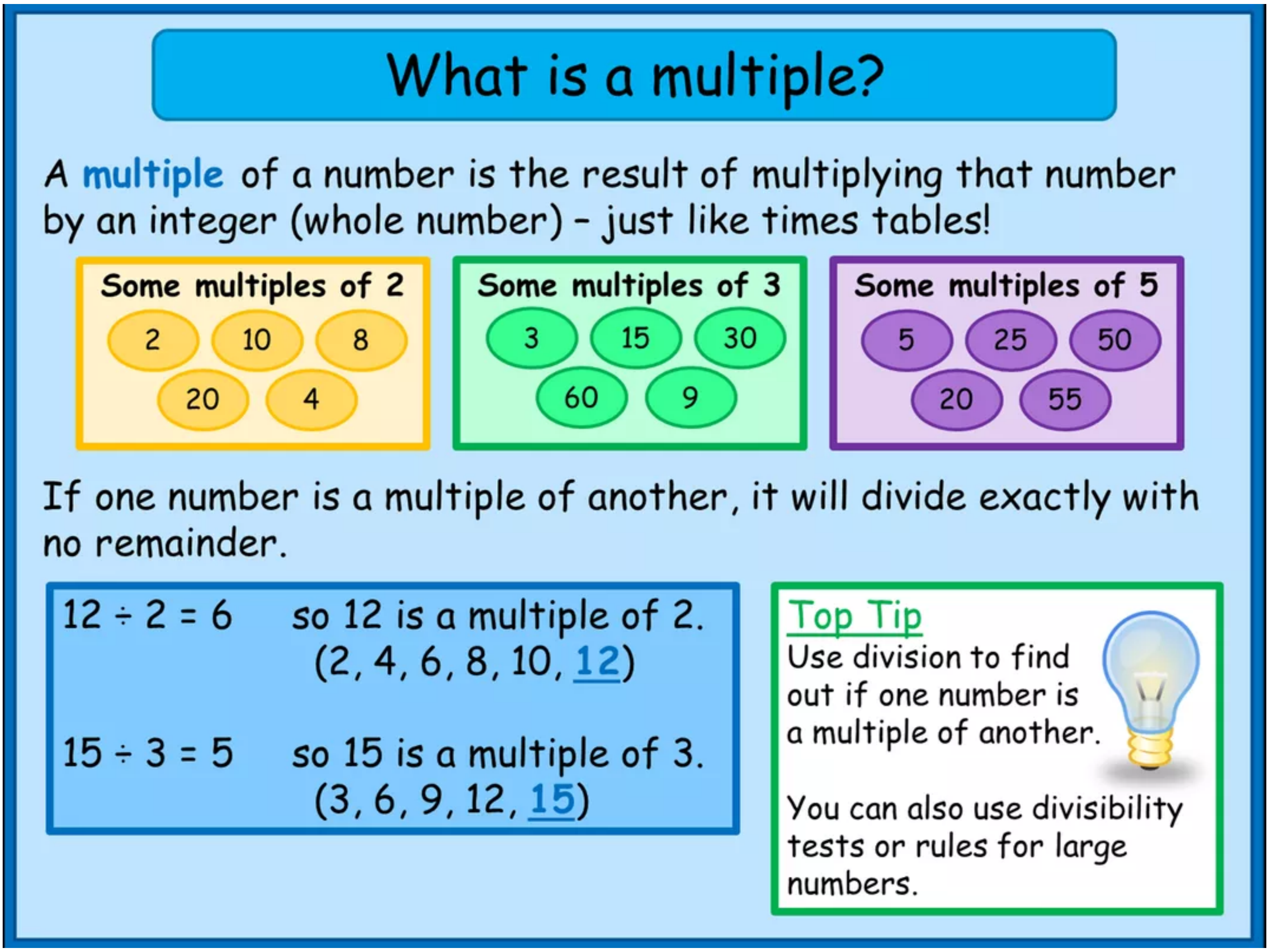
Multiples

Multiple: A multiple of any natural number is a number formed by multiplying it by another natural number.

Example: Multiples of 6 are 6 × 1 = 6; 6 × 2 = 12; 6 × 3 = 18; 6 × 4 = 24
Here, 6,12,18,24 are multiples of 6.

Example: Let us find the LCM and HCF of 24 and 36.
Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24
Factors of 36 = 1, 2, 3, 4, 6, 9, 12, 18, 36
Here, the highest common factor is 12.
∴ HCF = 12
Multiples of 24 = 24, 48, 72, 96,...
Multiples of 36 = 36, 72, 108,...

Here, the lowest common multiple is 72.
∴ LCM = 72



Properties of Multiples

1. Every number is a multiple of itself.
   Examples
   (a) 3 × 1 = 3; 3 is the multiple of 3
   (b) 7 × 1 = 7; 7 is the multiple of 7
2. Every number is the multiple of 1.
   Examples
   (a) 1 × 3 = 3; 3 is the multiple of 1
   (b) 1 × 7 = 7; 7 is the multiple of 1
3. The multiples of a number are infinite (unlimited).

Even numbers: A number which is a multiple of 2 is called an even number.

Example: 2, 4, 6, 8, 10,…

Odd numbers: A number which is not a multiple of 2 is called an odd number.

Example: 1, 3, 5, 7, 9, 11,…

Prime numbers: A number which is greater than 1, and has exactly two factors (1 and the number itself) is called a prime number.

Example: Factors of 2 = 1, 2

Factors of 3 = 1, 3
Factors of 5 = 1, 5
Factors of 7 = 1, 7

Factors of 11 = 1, 11
Here, 2, 3, 5, 7, 11 etc. are all prime numbers.

Composite numbers: A number, which is greater than 1 and has more than two factors is called a composite number.

Examples: Here,

Factors of 4 = 1, 2, 4
Factors of 6 = 1, 2, 3, 6
Factors of 8 = 1, 2, 4, 8
Factors of 9 = 1, 3, 9
Factors of 10 = 1, 2, 5

FINDING PRIME NUMBERS FROM 1 TO 100
We can find the prime numbers from 1 to 100 by following these steps (given by the Greek mathematician Eratosthenes).

Step 1: Prepare a list of numbers from 1 to 100. Step 2: As 1 is neither prime nor composite number, cross it out.
Step 3: Encircle '2' as a prime number and cross out all its other multiples.
Step 4: Encircle '3' as a prime number and cross out all its other multiples.
Step 5: Encircle '5' as a prime number and cross out all its other multiples.
Step 6: Continue this process till all the numbers are either encircled or crossed out.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

All the encircled numbers are **prime numbers** and the crossed out numbers (except 1) are **composite numbers**.
Numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97 are the prime numbers between 1 and 100.
This is called the **'Sieve of Eratosthenes'**.

Twin primes: Two prime numbers having a difference of 2 are known as twin primes.

Example: (3, 5), (5,7), (11,13), (17,19), etc are twin primes.

Co-primes: Two numbers are said to be co-primes if they have no common factor other than 1. In other words, two natural numbers are co-primes if their HCF is 1.

Example: (2, 3), (3, 4), (5, 6), (7, 8), and so on.

Example 1: Is 16380 a multiple of 28?

Solution: To check whether 16380 is a multiple of 28 or not, we have to divide 16380 by 28. If the remainder becomes zero, then it is a multiple of the number.

```
28)16380 (585
   140
   ---
    238
    224
    ---
     140
     140
     ---
       0
```

So, 16380 = 28 × 585, hence 16380 is a multiple of 28.

Example 2: Express 29 as the sum of three odd prime numbers.

Solution: 29 = 19 + 7 + 3
All 19, 7, and 3 are odd prime numbers.

DIVISIBILITY TESTS FOR 2, 3, 4, 5, 6, 7, 8, 9, 10, AND 11

If we want to know that a number is divisible by another number, we generally perform the actual division and see whether the remainder is zero or not. This process is time-consuming for division of large numbers. Therefore, to cut short our efforts, some divisibility tests of different numbers are given below.

Test of Divisibility by	Condition	Example
2	A number is divisible by 2, if its ones digit is 0, 2, 4, 6 or 8.	1372, 468, 500, 966 are divisible by 2, since their ones digit is 2, 8, 0 and 6 respectively.
3	A number is divisible by 3, if the sum of its digits is divisible by 3.	In 1881, the sum of digits is 1 + 8 + 8 + 1 = 18 which is divisible by 3. So 1881 is divisible by 3.
4	A number is divisible by 4, if the number formed by the last two digits is divisible by 4.	30776, 63784, 864 are all divisible by 4. Since last two digits of the numbers, i.e., 76, 84, and 64 are divisible by 4.
5	A number is divisible by 5, if its ones digit is either 5 or 0.	675, 4320, 145 all are divisible by 5 because their ones digit is 5 or 0.
6	A number is divisible by 6, if the number is divisible by 2 and 3.	In 5922, ones digit is 2, so it is divisible by 2. The sum of digits in 5922 is 5 + 9 + 2 + 2 = 18, which is divisible by 3. So, 5922 is divisible by 6.
7	A number is divisible by 7, if the difference between twice the last digit and the number formed by other digits is either 0 or a multiple of 7.	In number 2975, it is observed that the last digit in 2975 is 5. So, 297 – (2 x 5) = 287, which is a multiple of 7. Hence, 2975 is divisible by 7.
8	A number is divisible by 8, if the number formed by its last three digits is divisible by 8.	In 213456, the last three digits are 456 which is divisible by 8. So, the number 213456 is divisible by 8.
9	A number is divisible by 9, if the sum of its digits is divisible by 9.	In 538425, the sum of the digits are (5 + 3 + 8 + 4 + 2 + 5) = 27 which is divisible by 9. So, 538425 is divisible by 9.
10	A number is divisible by 10, if the digit at ones place of the number is 0.	The numbers 980, 63990 are all divisible by 10 because their ones digit is 0.
11	A number is divisible by 11, if the difference between the sum of digits at odd places and the sum of digits at even places is either 0 or a multiple of 11.	In number 27896, the sum of the digits at odd places are (2 + 8 + 6) = 16. The sum of the digits at even places are (7 + 9) = 16. Their difference is 16 – 16 = 0. So, the number 27896 is divisible by 11.

Example 3: Test whether 72148 is divisible by 8 or not?

Solution: Here, the number formed by the last three digits is 148, which is not divisible by 8.
So, 72148 is not divisible by 8.

Example 4: Test whether 8050314052 is divisible by 11 or not?

Solution: The sum of the digits at even places = 8 + 5 + 3 + 4 + 5 = 25
The sum of digits at the odd places = 0 + 0 + 1 + 0 + 2 = 3
Difference = 25 – 3 = 22 22 is divisible by 11.
So, the number 8050314052 is divisible by 11.

What Is Fundamental Theorem of Arithmetic

Fundamental Theorem of Arithmetic:

Statement: Every composite number can be decomposed as a product prime numbers in a unique way, except for the order in which the prime numbers occur.
For example:

(i) $30 = 2 \times 3 \times 5$, $30 = 3 \times 2 \times 5$, $30 = 2 \times 5 \times 3$ and so on.
(ii) $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$
or $432 = 3^3 \times 2^4$.
(iii) $12600 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$
$= 2^3 \times 3^2 \times 5^2 \times 7$

In general, a composite number is expressed as the product of its prime factors written in ascending order of their values.
Example: (i) $6615 = 3 \times 3 \times 3 \times 5 \times 7 \times 7$
$= 3^3 \times 5 \times 7^2$
(ii) $532400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 11 \times 11 \times 11$

Fundamental Theorem of Arithmetic Example Problems With Solutions

Example 1: Consider the number 6^n, where n is a natural number. Check whether there is any value of $n \in N$ for which 6^n is divisible by 7.

Sol. Since, $6 = 2 \times 3$; $6^n = 2^n \times 3^n$
⇒ The prime factorisation of given number 6^n
⇒ **6^n is not divisible by 7.**

Example 2: Consider the number 12^n, where n is a natural number. Check whether there is any value of $n \in N$ for which 12^n ends with the digit zero.

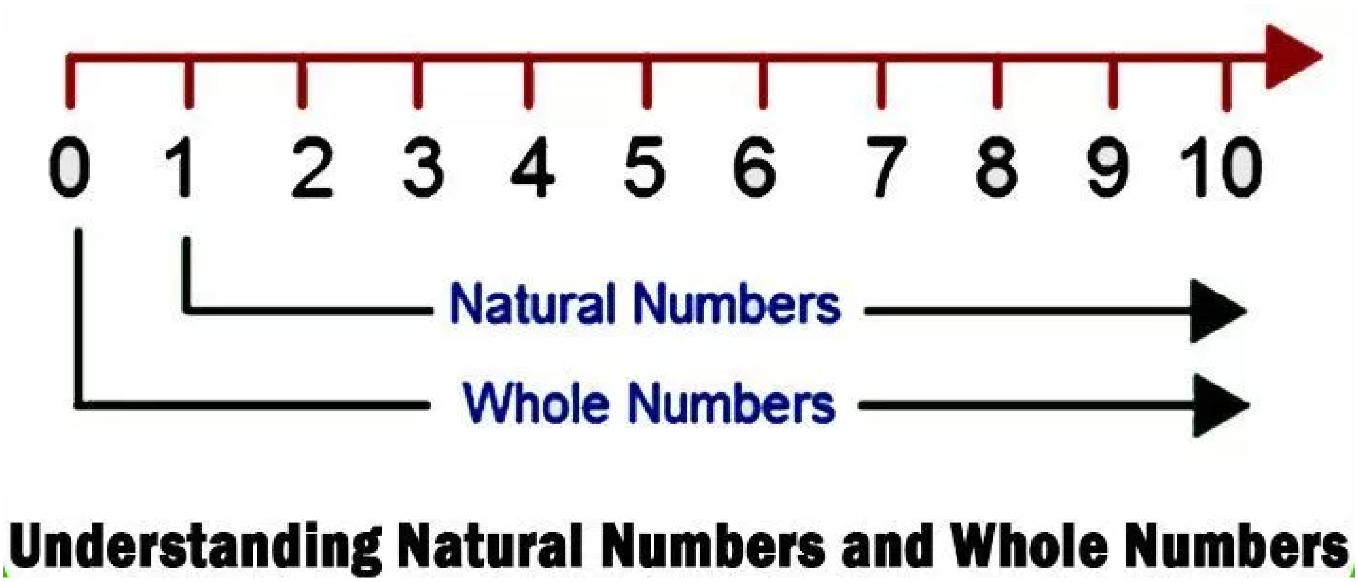
Sol. We know, if any number ends with the digit zero it is always divisible by 5.
If 12^n ends with the digit zero, it must be divisible by 5.
This is possible only if prime factorisation of 12^n contains the prime number 5.
Now, $12 = 2 \times 2 \times 3 = 2^2 \times 3$
⇒ $12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$
i.e., prime factorisation of 12^n does not contain the prime number 5.

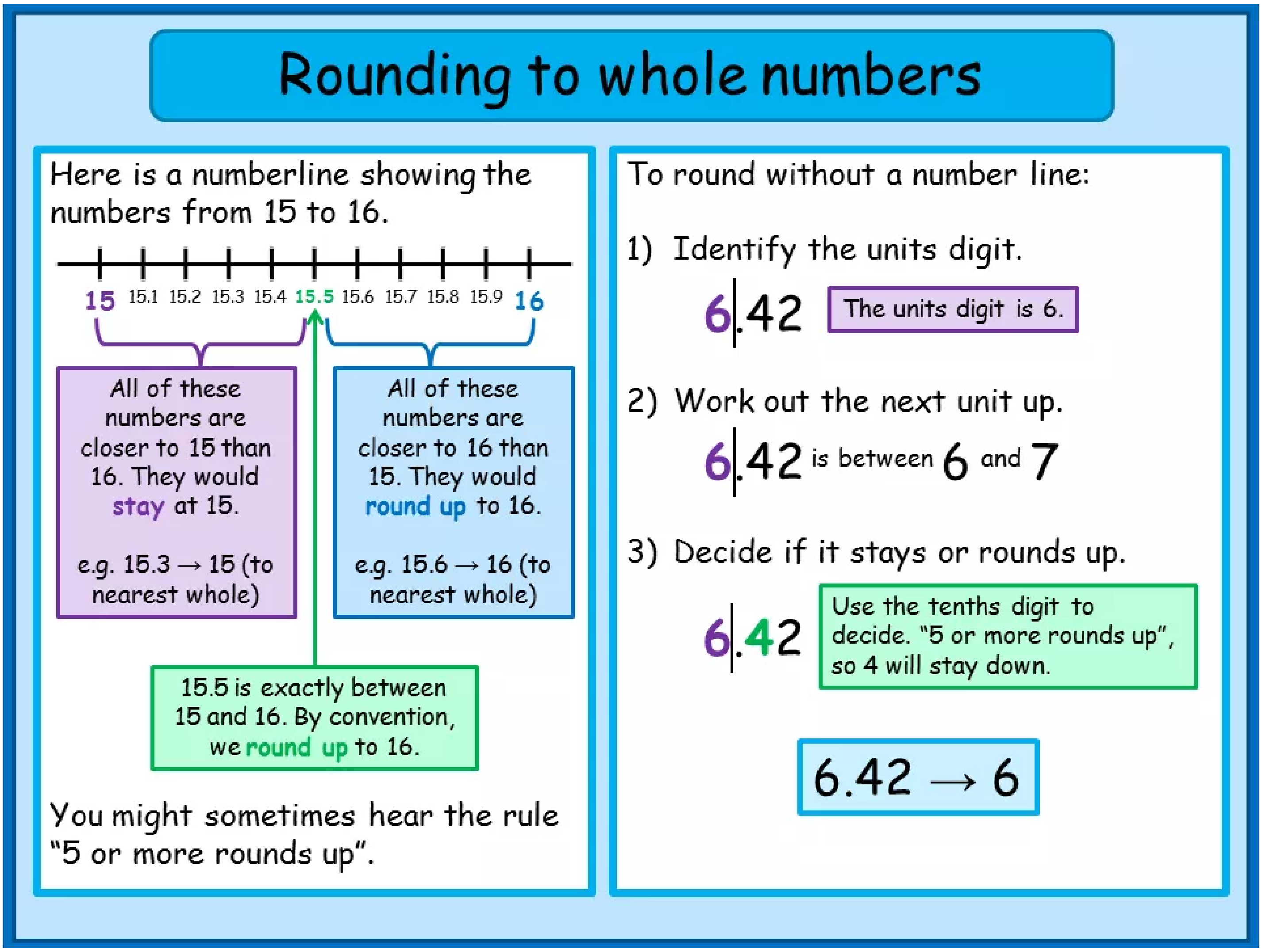
⇒ **There is no value of $n \in N$ for which 12^n ends with the digit zero.**

Whole Numbers And Its Properties

WHOLE NUMBERS

Now if we add zero (0) in the set of natural numbers, we get a new set of numbers called the **whole numbers**. Hence the set of whole numbers consists of zero and the set of natural numbers. It is denoted by W. i.e., W = {0, 1, 2, 3, . . .}. Smallest whole number is zero.





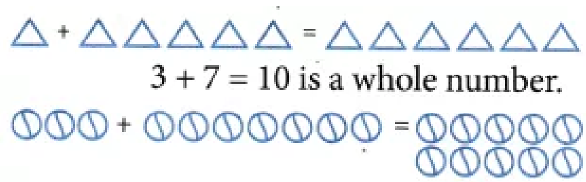
Properties of whole numbers

All the properties of numbers satisfied by natural numbers are also satisfied by whole numbers. Now we shall learn some fundamental properties of numbers satisfied by whole numbers.

Properties of Addition

(a) Closure Property: The sum of two whole numbers is always a whole number. Let a and b be two whole numbers, then a + b = c is also a whole number.
This property is called the closure property of addition

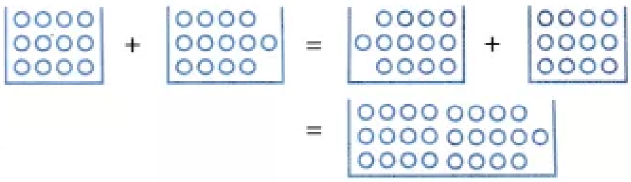
Example: 1 + 5 = 6 is a whole number.



(b) Commutative Property: The sum of two whole numbers remains the same if the order of numbers is changed. Let a and b be two whole numbers, then
a + b = b + a
This property is called the commutative property of addition.

Examples: $12 + 13 = 13 + 12$

$(25) = (25)$



$0 + 8 = 8 + 0$

$(8) = (8)$

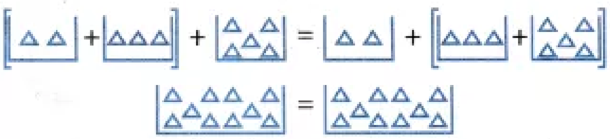
the sum remains same

(c) Associative Property: The sum of three whole numbers remains the same even if the grouping is changed. Let a, b, and c be three whole numbers, then
$(a + b) + c = a + (b + c)$
This property is called the associative property of addition.

Examples: $(2+ 3) + 5 = 2 + (3 +5)$

$5 + 5 = 2 + 8$

$10 = 10$



$12 + (13 + 7) = (12 + 13) + 7$

$12 + 20 = 25 + 7$

$32 = 32$

(d) Identity Element: If zero is added to any whole number, the sum remains the number itself. As we can see that $0+a=a=a+0$ where a is a whole number.

Examples: $0 + 3 = 3 = 3 + 0$

$0 + 312 = 312 = 312 + 0$

$0 + 27 = 27 = 27 + 0$

Therefore, the number zero is called the additive identity, as it does not change the value of the number when addition is performed on the number.

Examples
17 – 5 = 12 is a whole number.
5 – 17 = – 12 is not a whole number.

(b) Commutative Property: If a and b are two whole numbers, then a – b ≠ b – a. It shows that subtraction of two whole numbers is not commutative. Hence, commutative property does not hold good for subtraction of whole numbers, i.e.,
a – b ≠ b – a.

Example: 3 – 4 = – 1 and 4 – 3 = 1
∴ 3 – 4 ≠ 4 – 3

(c) Associative Property: If a, b, and c are whole numbers, then (a – b) – c ≠ a – (b – c). It shows that subtraction of whole numbers is not associative. Hence, associative property does not hold good for subtraction of whole numbers.

Example: (40 – 25) – 10 = 15 – 10 = 5
40 – (25 – 10) =40- 15 = 25
∴ (40 – 25) – 10 ≠ 40 – (25 – 10)

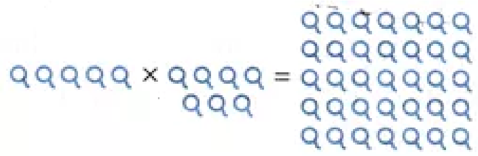
(d) Property of Zero: If we subtract zero from any whole number, the result remains the number itself.

Example: 7 – 0 = 7 5 – 0 = 5

Properties of Multiplication

(a) Closure Property: If a and b are two whole numbers, then a × b = c will always be a whole number. Hence, closure property holds good for multiplication of whole numbers.

Example: 5 × 7 = 35 (a whole number)



6 × 1 = 6 (a whole number)

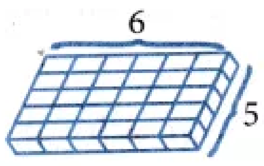
(b) Commutative Property: If a and b are two whole numbers, then the product of two whole numbers remains unchanged if the order of the numbers is interchanged, i.e.,
a × b = b × a.

Example: 6 × 5 = 5 × 6 30 = 30

i. e., 6 rows of 5 or 5 rows of 6 give the same results.

Properties of Subtraction

(a) Closure Property: The difference of two whole numbers will not always be a whole number. Let a and b be two whole numbers, then a – b will be a whole number if a > b or a = b. If a < b, then the result will not be a whole number. Hence, closure property does not hold good for subtraction of whole numbers.

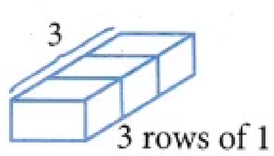


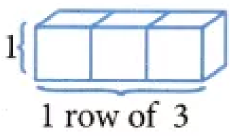
so, 6 × 5 = 30 = 5 × 6

(c) Associative Property: If a, b, and c are whole numbers, then the product of three whole numbers remains unchanged even if they are multiplied in any order. Hence, associative property does hold good for multiplication of whole numbers, i.e.,
(a × b) × c = a × (b × c)

Example:
(4 × 5) × 8 = 4 × (5 × 8)
20 × 8 = 4 × 40
160 = 160

(d) Multiplicative Identity: If any whole number is multiplied by 1, the product remains the number itself. Let a whole number be a, then
a × 1 = a = 1 × a.





3 × 1 = 3 = 1 × 3

Examples
75 × 1 = 75 = 1 × 75
3 × 1 = 3 = 1 × 3
Hence, 1 is called the multiplicative identity.

(e) Multiplicative Property of Zero: Any whole number multiplied by zero gives the product as zero.If a is any whole number, then 0 × a = a × 0 = 0.

Example: 3 × 0 = 0 × 3 = 0

Properties of Division

(a) Closure Property: If a and b are whole numbers, then a ÷ b is not always a whole number. Hence, closure property does not hold good for division of whole numbers.

Example: 7 ÷ 5 = $\frac{7}{5}$ is not a whole number.
7 ÷ 7 = 1 is a whole number.

(b) Commutative Property: If a and b are whole numbers, then a ÷ b ≠ b ÷ a. Hence, commutative property does not hold good for division of whole numbers.
Example: 18 ÷ 3 = 6 is a whole number.
3 ÷ 18 = $\frac{3}{18}$ = $\frac{1}{6}$ is not a whole number.
∴ 3 ÷ 18 ≠ 18 ÷ 3

(c) Associative Property: If a, b, and c are whole numbers then (a ÷ b) ÷ c ≠ a ÷ (b ÷ c). Hence, associative property does not hold good for division of whole numbers.

Example: (15 ÷ 3) ÷ 5 = 5 ÷ 5 = 1
15 ÷ (3 ÷ 5) = 15 ÷ 3/5 = 15 × 5/3
= 25
∴ (15 ÷ 3) ÷ 5 ≠ 15 ÷ (3 ÷ 5)

(d) Property of Zero: If a is a whole number then 0 ÷ a = 0 but a ÷ 0 is undefined.
Example: 6 ÷ 0 is undefined.

Note:

- Product of zero and a whole number gives zero.
  $a \times 0 = 0$
- Zero divided by any whole number gives zero.
  $0 \div a = 0$
  $a \div 0 =$ undefined
- Any number divided by 1 is the number itself.
  $a \div 1 = a$

DISTRIBUTIVE PROPERTY

You are distributing something as you separate or break it into parts.
Example: Raj distributes 4 boxes of sweets. Each box comprises 6 chocolates and 10 candies. How many sweets are there in these 4 boxes?
∴ Chocolates in 1 box = 6
Choclolates in 4 boxes = 4 × 6 = 24
Candies in 1 box =10
Candies in 4 boxes = 4 × 10 = 40
Total number of sweets in 4 boxes
= 4 × 6 + 4 × 10 = 4 × (6 + 10)
= 4 × 16 = 64

Hence, we conclude the following:

(a) Multiplication distributes over addition, i.e., $a(b + c) = ab + ac$, where a, b, c are whole numbers.

Example: 10 × (6 + 5) = 10 × 6 + 10 × 5
10 × 11 = 60 + 50
110 = 110
This property is called the distributive property of multiplication over addition.

(b) Similarly, multiplication distributes over subtraction, i.e., $a \times (b - c) = ab - ac$ where a, b, c are whole numbers and $b > c$.

Example: 10 × (6 – 5) = 10 × 6 – 10 × 5
10 × 1 = 60 – 50
10 = 10
This property is called the distributive property of multiplication over subtraction.

Example 1: Determine the following by suitable arrangement.
2 × 17 × 5

Solution: 2 × 17 × 5 = (2 × 5) × 17
= 10 × 17 = 170

Example 2: Solve the following using distributive property.
97 × 101

Solution: 97 × 101 = 97 × (100 + 1)
= 9700 + 97 = 9797

Example 3: Tina gets 78 marks in Mathematics in the half-yearly Examination and 92 marks in the final Examination. Reenagets 92 marks in the half- yearly Examination and 78 marks in the final Examination in Mathematics. Who has got the higher total marks?

Solution: Tina gets the following marks = 78 + 92 = 170 Total marks
Reena gets the following marks = 92 + 78 = 170 Total marks
So, both of them got equal marks.

Example 4: A fruit seller placed 12 bananas, 10 oranges, and 6 apples in a fruit basket. Tarun buys 3 fruit baskets for a function. What is the total number of fruits in these 3 baskets?

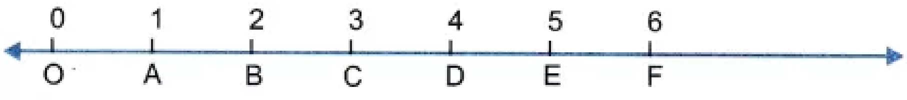
Solution: Number of bananas in 3 baskets = 12 × 3 = 36 bananas
Number of oranges in 3 baskets = 10 × 3 = 30 oranges

Number of apples in 3 baskets = 6×3 = 18 apples
Total number of fruits = 36 + 30+ 18 = 84

Alternative Method
Total number of fruits in 3 baskets
= 3 × [12 +(10 + 6)]
= 3 × [12 + 16]
= 3 × 28 = 84

Representation Of Whole Numbers On A Number Line

We can represent whole numbers-on a straight line. To represent a set of whole numbers on a number line, let's first draw a straight line and mark a point O on it. After that, mark points A, B, C, D, E, F on the line at equal distance, on the right side of point O.

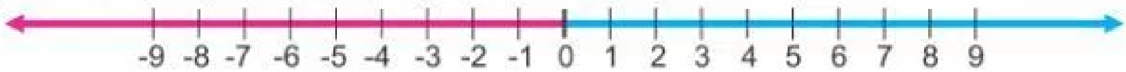


Now, OA = AB = BC = CD and so on
Let OA = 1 unit
OB = OA + AB = 1 + 1 = 2 units
OC = OB + BC = 2 + 1 = 3 units
OD = OC + CD = 3 + 1 = 4 units and so on.
Let the point O correspond to the whole number 0, then points A, B, C, D, E, ….. correspond to the whole numbers 1, 2, 3, 4, 5,…. In this way every whole number can be represented on the number line.

What is an Integer and give some Examples

Introduction

Integers consist of whole numbers and negative numbers.
Z = {...., -3, -2, -1, 0, 1, 2, 3,....}



Property	Operations on Integers			
Name	Addition	Subtraction	Multiplication	Division*
Closure	a + b ∈ Z	a - b ∈ Z	a × b ∈ Z	a ÷ b ∉ Z
Commutative	a + b = b + a	a - b ≠ b - a	a × b = b × a	a ÷ b ≠ b ÷ a
Associative	(a + b) + c = a + (b + c)	(a - b) - c ≠ a - (b - c)	(a × b) × c = a × (b × c)	(a ÷ b) ÷ c ≠ a ÷ (b ÷ c)
Distributive	a × (b + c) = ab + ac	a × (b - c) = ab - ac	Not applicable	Not applicable
where a, b, c ∈ Z				*b is a non-zero integer

While studying the properties of whole numbers, we found that the closure property does not hold good for the subtraction of natural numbers as well as whole numbers. This is because of the following:
12 – 10 = 2 is a whole number as well as a natural number.
12-12 = 0 is a whole number but not a natural number.
12 – 18 = – 6 which is neither a whole number nor a natural number.

Let us study an example

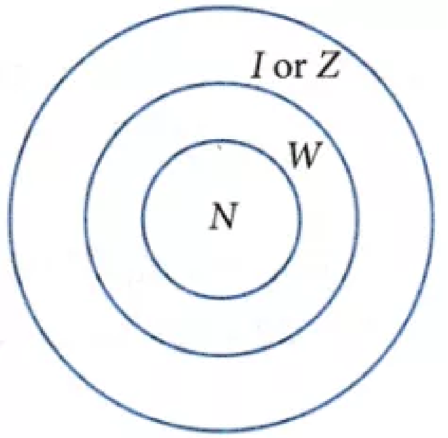
Rohit bought 3 basketballs from a wholesale sports goods shop for Rs 200 each. He sold the first basketball for Rs 220, the second basketball for Rs 200, and the third basketball for Rs 195. On the first basketball, he gained Rs. 20, i.e., Rs. 220 – Rs 200 = + Rs 20; on the second basketball he did not gain anything, i.e., Rs 200 – Rs 200 = Rs 0; on the third basketball, he lost Rs 5, i.e., Rs 195 – Rs 200 = – Rs 5.
Similarly, in our daily life we come across so many situations such as:
(i) 2°C rise in temperature or 2°C fall in temperature.
(ii) Rs 200 profit or Rs 200 loss
(iii) 4 km above the sea level or 4 km below the sea level etc.
In the above cases, the idea of oppositeness has been introduced. To avoid confusion or mistakes.

Opposite of Natural numbers

In mathematics, Rs. 200 profit means +200 and Rs. 200 loss means -200, +2°C means 2°C rise in temperature and -2°C means 2°C fall in temperature, +4 km means 4 km above the sea level and – 4 km means 4 km below the sea level. Similarly, in natural numbers, the opposite of 1 is -1, opposite of 2 is -2, opposite of 3 is -3, and so on.
We, therefore, need to extend the whole number system to include such 'negative numbers' which are opposite of natural numbers. In order to have opposites of 1, 2, 3,… we introduce -1, -2, -3,…. All the numbers with '+ve' sign are called positive numbers and all the numbers with '-ve' sign are called negative numbers. Zero is neither positive nor negative.
This new collection of numbers are called integers which include all positive numbers, negative numbers, and zero. Numbers 1, 2, 3,… are called positive integers and -1, -2, -3,… are called negative integers.

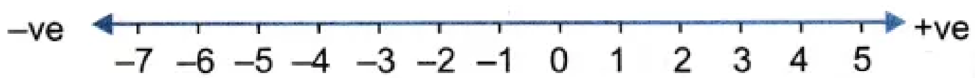
Note:
Set of negative numbers, positive numbers, and zero together are called **integers**.

Natural numbers (N) = 1, 2, 3,4,…
Whole numbers (W) = 0,1,2, 3,…
Integers (Z or I) = -3, -2, -1, 0,1, 2, 3 ,…



Ordering of Integers

We know that a whole number is greater than any whole number to its left on a number line. Same is true for the integers also. The number line given below shows whole numbers and negative numbers with zero in the middle.



All the positive integers (which are greater than zero) lie on the right side of zero and all the negative integers (which are less than zero) lie on the left side of zero at equal distance from each other. A number placed to the right of another is greater than it.

Examples:

7 > 3 as 7 is to the right of 3.
0 > -1 as 0 is to the right of-1.

Note:

1 is the smallest positive integer but -1 is the largest negative integer, e., -1 is always greater than -2, -4,…

From the examples, we note that:

(a) Since 0 is to the right of every negative integer, so 0 is greater than every negative integer.
(b) Since 0 is to the left of every positive integer, so 0 is less than every positive integer.
(c) Every positive integer is to the right of every negative integer. Hence positive integers are greater than negative integers.
(d) The greater the number is the lesser to its opposite.

Absolute value of integers

The absolute value of an integer is its numerical value regardless of its sign. It indicates its size or magnitude. So, absolute value is either zero or positive. It is never negative. The absolute value of 6 is written as | 6 | = 6 and absolute value of -5 = | -5 | = 5.
Note that | 0 | = 0; |-117 | = 117; but – | 117 | = -117.

Example 1: Write the opposite of each of the following:

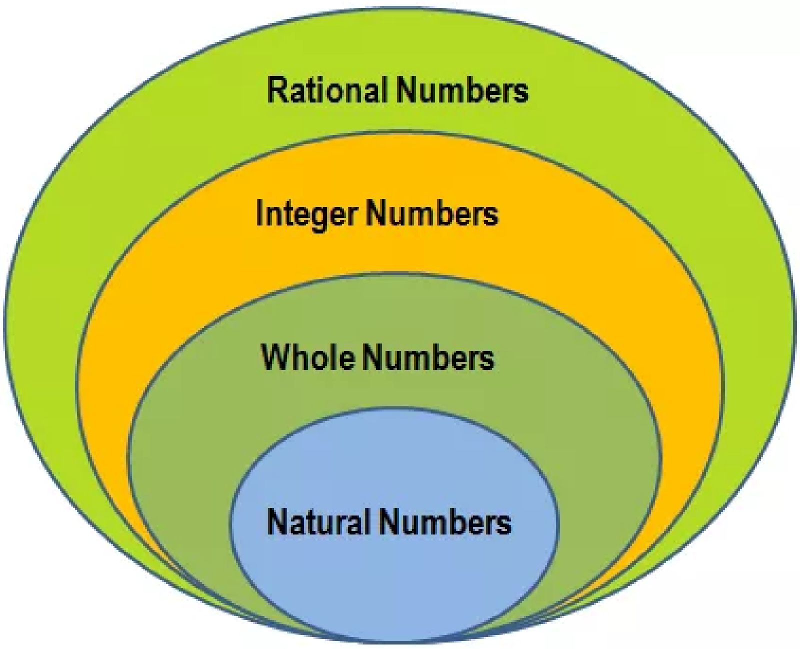
(a) 6 km north
(b) 14 km above the sea level
(c) Depositing money in the bank Solution
(a) 6 km south
(b) 14 km below the sea level
(c) Withdrawing money from the bank

Example 2: Find the absolute value of | -51 | and -I 13 |.

Solution: Absolute value of (-51) is | -51 | = 51 Absolute value of -| 13 | = -13

What is a Rational Number?

A rational number is a number which can be put in the form $\frac{p}{q}$, where p and q are both integers and q ≠ 0.
p is called numerator (N^r) and q is called denominator (D^r).

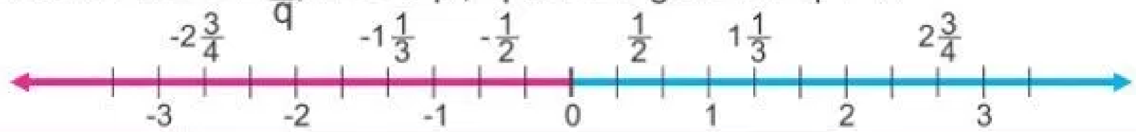


- A rational number is either a terminating or non-terminating but recurring (repeating) decimal.
- A rational number may be positive, negative or zero.
  Examples:

$3, 4, \frac{7}{3}, \frac{5}{2}, -\frac{3}{7}, 2.7, 3.923, 1.42\overline{7}, 1.2343434$, etc.

- The sum, difference and the product of two rational numbers is always a rational number.
- The quotient of a division of one rational number by a non-zero rational number is a rational number. Rational numbers satisfy the closure property under addition, subtraction, multiplication and division.

A number of the form $\frac{p}{q}$, where 'p', 'q' are integers and $q \neq 0$.



Property	Operations on Rational Numbers			
Name	Addition	Subtraction	Multiplication	Division*
Closure	$a + b \in Q$	$a - b \in Q$	$a \times b \in Q$	$a \div b \in Q$
Commutative	$a + b = b + a$	$a - b \neq b - a$	$a \times b = b \times a$	$a \div b \neq b \div a$
Associative	$(a + b) + c = a + (b + c)$	$(a - b) - c \neq a - (b - c)$	$(a \times b) \times c = a \times (b \times c)$	$(a \div b) \div c \neq a \div (b \div c)$
Distributive	$a \times (b + c) = ab + ac$	$a \times (b - c) = ab - ac$	Not applicable	Not applicable
Where a, b, c $\in$ Q (set of rational numbers), *b is a non-zero rational number				

Results

Since every number is divisible by 1, we can say that :

1. Every natural number is a rational number, but every rational number need not be a natural number.
   For example, $3 = \frac{3}{1}$, $5 = \frac{5}{1}$, $9 = \frac{9}{1}$ and so on.
   but, $\frac{7}{9}, \frac{11}{13}, \frac{5}{7}$ are rational numbers but not natural numbers.
2. Zero is a rational number because $\left(0 = \frac{0}{1} = \frac{0}{2} = \ldots\right)$.
3. Every integer is a rational number, but every rational number may not be an integer.
   For example $\frac{-2}{1}, \frac{-5}{1}, \frac{0}{1}, \frac{3}{1}, \frac{5}{1}$, etc. are all rationals, but rationals like $\frac{3}{2}, \frac{-5}{2}$ etc. are not integers.
4. Rational numbers can be positive and negative.

   Eg : $\frac{2}{3}, \frac{-7}{-8}, \frac{8}{11}, \frac{-9}{-3}$ etc. are positive rational numbers

   & $\frac{-2}{3}, \frac{7}{-8}, \frac{-8}{11}, \frac{11}{-20}$ etc. are negative rational numbers.
5. Every positive rational number is greater than zero.
6. Every negative rational number is less than zero.
7. Every positive rational number is greater than every negative rational number.
8. Every negative rational number is smaller than every positive rational number.

Equivalent Rational Numbers

∵ Rational no. can be written with different N^r and D^r.

Eg :

$\frac{-5}{7} = \frac{-5 \times 2}{7 \times 2} = \frac{-10}{14}$ $\quad \therefore \frac{-5}{7}$ is same as $\frac{-10}{14}$

$\frac{-5}{7} = \frac{-5 \times 3}{7 \times 3} = \frac{-15}{21}$ $\quad \frac{-5}{7}$ is same as $\frac{-15}{21}$

$\frac{-5}{7} = \frac{-5 \times -1}{7 \times -1} = \frac{5}{-7}$ $\quad \frac{-5}{7}$ is same as $\frac{5}{-7}$

Such rational number that are equal to each other are said to be equivalent to each other.

Example: Write $\frac{2}{5}$ in an equivalent form so that the numerator is equal to −56.
Solution:

Multiplying both the numerator and denominator of $\frac{2}{5}$ by −28, we have

$$\frac{2 \times (-28)}{5 \times (-28)} = \frac{-56}{-140}.$$

Lowest Form of a Rational Number

A rational number is said to be in lowest form if the numerator and the denominator have no common factor other than 1.

Example: Write the following rational numbers in the lowest form :

(i) $\frac{-36}{180}$ (ii) $\frac{-64}{256}$

Solution:

(i) Here, HCF of 36 and 180 is 36, therefore, we divide the numerator and denominator of $\frac{-36}{180}$ by 36, we have

$$\frac{-36 \div 36}{180 \div 36} = \frac{-1}{5}$$

So, the lowest form of $\frac{-36}{180}$ is $\frac{-1}{5}$

(ii) Here, HCF of 64 and 256 is 64.

Dividing the numerator and denominator of $\frac{-64}{256}$ by 64, we have

$$\frac{-64 \div 64}{256 \div 64} = \frac{-1}{4}$$

So, the lowest form of $\frac{-64}{256}$ is $\frac{-1}{4}$.

Standard Form of a Rational Number

A rational number $\frac{p}{q}$ is said to be in its standard form if
(i) its denominator 'q' is positive
(ii) the numerator and denominator have no common factor other than 1.
For example : $\frac{3}{2}, \frac{-5}{2}, \frac{1}{7}$, etc.

Example: Express the rational number $\frac{14}{-21}$ in standard form.
Solution:
The given rational number is $\frac{14}{-21}$.
1. Its denominator is negative. Multiply both the numerator and denominator by −1 to change it to positive, i.e.,

$$\frac{14}{-21} = \frac{14 \times (-1)}{(-21) \times (-1)} = \frac{-14}{21}$$

2. The greatest common divisor of 14 and 21 is 7. Dividing both numerator and denominator by 7, we have

$$\frac{-14}{21} = \frac{(-14) \div 7}{21 \div 7} = \frac{-2}{3}$$

which is the required answer.

Equality of Rational Numbers

Method-1: If two or more rational numbers have the same standard form, we say that the given rational numbers are equal.

Example: Are the rational numbers $\frac{8}{-12}$ and $\frac{-50}{75}$ equal?
Solution: We first express these given rational numbers in the standard form.
The first rational number is $\frac{8}{-12}$.
(i) Multiplying both the numerator and denominator by −1.

$$\text{We have, } \frac{8}{-12} = \frac{8 \times (-1)}{(-12) \times (-1)} = \frac{-8}{12}$$

(ii) Dividing both the numerator and denominator by the greatest common divisor of 8 and 12, which is 4.

$$\text{We have, } \frac{8}{-12} = \frac{(-8) \div 4}{12 \div 4} = \left[\frac{-2}{3}\right]$$

Again, the second rational number is $\frac{-50}{75}$.
(i) The denominator is positive.
(ii) Dividing both numerator and denominator by the greatest common divisor of 50 and 75, which is 25.

$$\text{We have, } \frac{-50}{75} = \frac{(-50) \div 25}{75 \div 25} = \left[\frac{-2}{3}\right]$$

Clearly, both the rational numbers have the same standard form.

Therefore, $\frac{8}{-12} = \frac{-50}{75}$

Method-2: In this method, to test the equality of two rational numbers, say $\frac{a}{b}$ and $\frac{c}{d}$, we use cross multiplication in the following way : $\frac{a}{b} = \frac{c}{d}$

Then a × d = b × c

If a × d = b × c, we say that the two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equal.

Example: Check the equality of the rational numbers $\frac{-7}{21}$ and $\frac{3}{-9}$.

Solution:

The given rational numbers are

$\frac{-7}{21}$ and $\frac{3}{-9}$.

By cross multiplication, we get

$$(-7) \times (-9) = 21 \times 3$$

i.e., $63 = 63$.

Clearly, both sides are same. Thus, we can say that $\frac{-7}{21} = \frac{3}{-9}$.

Comparison of Rational Numbers

Comparing fraction. We compare two unequal fractions, each is written as another equal fraction so that both have the same denominators. Then the fraction with greater numerator is greater.

Example : To compare $\frac{7}{6}$ and $\frac{5}{8}$, find the L.C.M. of 6 and 8 (it is 24) and

$$\frac{7}{6} = \frac{7 \times 4}{6 \times 4} = \frac{28}{24}$$

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

As $\frac{28}{24} > \frac{15}{24}$ (as $28 > 15$)

$\Rightarrow \frac{7}{6} > \frac{5}{8}$

Quicker method of comparison of $\frac{a}{b}$ and $\frac{c}{d}$ is that $\frac{a}{b} > \frac{c}{d}$ if $ad > bc$.

$\frac{7}{6} > \frac{5}{8}$ as $(7 \times 8 > 6 \times 5)$

To compare two negative rational numbers, we compare them ignoring their negative signs and then reverse the order.

For example,

$\frac{-9}{13}$ and $\frac{-5}{3}$,

we first compare $\frac{9}{13}$ and $\frac{5}{3}$.

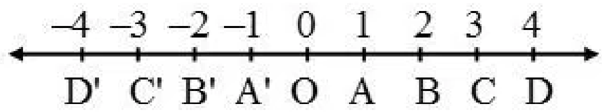
$\frac{9}{13} < \frac{5}{3} \qquad (\because 9 \times 3 < 13 \times 5 \Rightarrow 27 < 65)$

and conclude that $\frac{-9}{13} > \frac{-5}{3}$.

Note :
Every positive rational number is greater than negative rational number.

Representation of Rational Numbers on Number Line

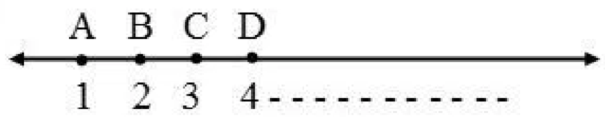
We know that the natural numbers, whole numbers and integers can be represented on a number line. For representing an integer on a number line, we draw a line and choose a point O on it to represent '0'. We can represent this point 'O' by any other alphabet also. Then we mark points on the number line at equal distances on both sides of O. Let A, B, C, D be the points on the right hand side and A′, B′, C′, D′ be the points on the left of O as shown in the figure.



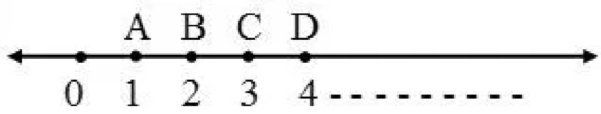
The points on the left side of O, i.e., A′, B′, C′, D′, etc. represent negative integers −1, −2, −3, −4 whereas, points on the right side of O, i.e., A, B, C, D represent positive integers 1, 2, 3, 4 etc. Clearly,

the points A and A′ representing the integers 1 and −1 respectively are on opposite sides of O, but at equal distance from O. Same is true for B and B′ ; C and C′ and other points on the number line.

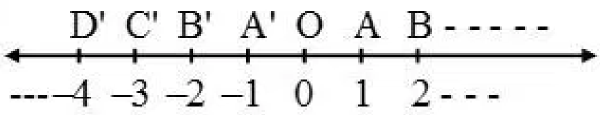
(1) Natural Numbers



(2) Whole Numbers



(3) Integers



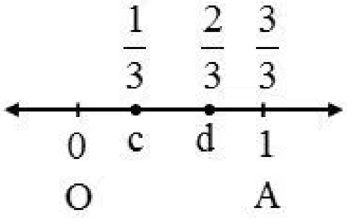
Negative numbers are in left side of zero (0) & positive numbers are in right side.
∵ negative numbers are less than positive numbers
∴ If we move on number line from right to left we are getting smaller numbers.
Also OA = distance of 1 from 0
OD′ = distance of −4 from 0
D′A = distance between −4 and 1. etc.

(4) Rational Numbers

(a) If $N^r < D^r$:

We divide line segment OA (i.e. distance between 0 & 1) in equal parts as denominator (Dr).

Eg : $\frac{1}{3}$ and $\frac{2}{3}$ =



$\because$ D^r is 3, so we divide OA in three equal parts by points c and d.

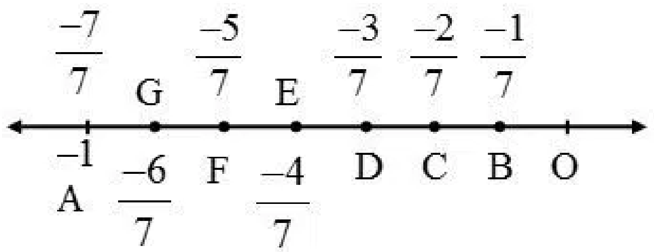
$\therefore$ $c = \frac{1}{3}$, $d = \frac{2}{3}$ and $A = \frac{3}{3} = 1$

Eg : $\frac{-1}{7}$ and $\frac{-4}{7}$

$\because$ D^r is 7 $\therefore$ we divide OA in 7 equal parts by points B, C, D, E, F, G. So, these points represent

$\frac{-1}{7}, \frac{-2}{7}, \frac{-3}{7}, \frac{-4}{7}, \frac{-5}{7}, \frac{-6}{7}, \frac{-7}{7}$ respectively.

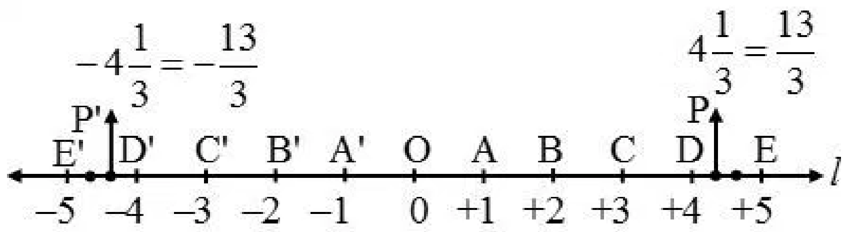
$\therefore$ $\frac{-1}{7}$ by B and $\frac{-4}{7}$ by E.



(b) If $N^r > D^r$:

Example: Represent $\frac{13}{3}$ and $-\frac{13}{3}$ on number line.

Solution:



Draw a line l and mark zero on it

$$\frac{13}{3} = 4\frac{1}{3} = 4 + \frac{1}{3} \quad \text{and} \quad \frac{-13}{3} = -\left(4 + \frac{1}{3}\right)$$

Therefore, from O mark OA, AB, BC, CD and DE to the right of O such that OA = AB = BC = CD = DE = 1 unit.
Clearly,
Point A,B,C,D,E represents the Rational numbers 1, 2, 3, 4, 5 respectively.

Since we have to consider 4 complete units and a part of the fifth unit, therefore divide the fifth unit DE into 3 equal parts. Take 1 part out of these 3 parts. Then point P is the representation of number $\frac{13}{3}$ on the number line. Similarly, take 4 full unit lengths to the left of 0 and divide the fifth unit D'E' into 3 equal parts. Take 1 part out of these three equal parts. Thus, P' represents the rational number $-\frac{13}{3}$.

Rational Number Example Problems With Solutions

Example 1: Is zero a rational number? can you write it in the form , where p and q are integers and q ≠ 0?

Solution:

Yes, zero is a rational number.

It can be written as $\frac{0}{1} = \frac{0}{2} = \frac{0}{3}$ etc.

where denominator q ≠ 0, it can be negative also.

Example 2: Find five rational numbers between 3/5 and 4/5.

Solution:

A rational number between r and s is $\frac{r+s}{2}$.

A rational number between

$\frac{3}{5}$ and $\frac{4}{5} = \frac{1}{2}\left(\frac{3}{5}+\frac{4}{5}\right) = \frac{7}{10}$.

And a rational number between

$\frac{3}{5}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{3}{5}+\frac{7}{10}\right) = \frac{13}{20}$

Similarly; $\frac{5}{8}, \frac{27}{40}, \frac{31}{40}$ are between $\frac{3}{5}$ and $\frac{4}{5}$.

So, five rational number between

$\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{5}{8}, \frac{13}{20}, \frac{7}{10}, \frac{31}{40}, \frac{27}{40}$

Example 3: Find six rational numbers between 3 and 4.

Solution: We can solve this problem in two ways.

Method 1:

A rational number between r and s is $\frac{r+s}{2}$.

Therefore, a rational number between 3 and

$4 = \frac{1}{2}(3+4) = \frac{7}{2}$

A rational number between 3 and

$\frac{7}{2} = \frac{1}{2}\ \frac{6+7}{2} = \frac{13}{4}$.

We can accordingly proceed in this manner

to find three more rational numbers between 3 and 4.

Hence, six rational numbers between 3 and 4

are $\frac{15}{8}, \frac{13}{4}, \frac{27}{8}, \frac{7}{2}, \frac{29}{8}, \frac{15}{4}$.

Method 2 :

Since, we want six numbers,

we write 3 and 4 as rational numbers with denominator 6 + 1,

i.e., $3 = \frac{21}{7}$ and $4 = \frac{28}{7}$.

Then we can check that $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$,

and $\frac{27}{7}$ are all between 3 and 4.

Hence, the six numbers between 3 and 4

are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$, and $\frac{27}{7}$

Example 4: Find two rational & two irrational numbers between 4 and 5.

Solution:

Rational numbers $\frac{4+5}{2} = 4.5$ **Ans.**

& $\frac{4.5+4}{2} = \frac{8.5}{2} = 4.25$ **Ans.**

Irrational numbers 4.12316908………

4.562381032…….

What Is Irrational Number

1. A number is irrational if and only if its decimal representation is non-terminating and non-repeating. e.g.$\sqrt{2}, \sqrt{3}, \Pi$…………… etc.

Definition

An **irrational number** can NOT be written in the form $\frac{n}{d}$, where n and d are integers and d is nonzero.

Examples

$\pi \qquad \sqrt{3} \qquad \sqrt{5}$

Examples of Irrational Numbers

$\sqrt{20}$

4.47213594...

Pi π

3.1415926535897932
384626433832795...
(and more)

$\frac{\sqrt{3}}{2}$

0.8660254...

2. Rational number and irrational number taken together form the set of real numbers.
3. If a and b are two real numbers, then either
   (i) a > b or (ii) a = b or (iii) a < b
4. Negative of an irrational number is an irrational number.
5. The sum of a rational number with an irrational number is always irrational.
6. The product of a non-zero rational number with an irrational number is always an irrational number.
7. The sum of two irrational numbers is not always an irrational number.
8. The product of two irrational numbers is not always an irrational number.
9. In division for all rationals of the form $\frac{p}{q}$($q \neq 0$), p & q are integers, two things can happen either the remainder becomes zero or never becomes zero.

Type (1) Example: $\frac{7}{8}$ = 0.875

```
8 ) 70 ( 0.875
    64
    ---
     60
     56
    ---
      40
      40
    ---
      ×
```

This decimal expansion 0.875 is called **terminating**.
∴ If remainder is zero then decimal expansion ends (terminates) after finite number of steps. These decimal expansion of such numbers terminating.

Type (2) Example: $\frac{1}{3}$ = 0.333……… = $0.\overline{3}$

$$
\begin{array}{l}
3\overline{)10}(0.33\ldots\ldots \\
\ \ \underline{9} \\
\ \ 10 \\
\ \ \underline{\ 9} \\
\ \ \ 1\ldots\ldots
\end{array}
$$

or $\frac{1}{7}$ = 0.142857142857….. = $0.\overline{142857}$

$$
\begin{array}{l}
7\overline{)10}(0.14285\ldots. \\
\ \ \underline{7} \\
\ \ 30 \\
\ \ \underline{28} \\
\ \ \ 20 \\
\ \ \ \underline{14} \\
\ \ \ \ 60 \\
\ \ \ \ \underline{56} \\
\ \ \ \ \ 40 \\
\ \ \ \ \ \underline{35} \\
\ \ \ \ \ \ 50 \\
\ \ \ \ \ \ \underline{49} \\
\ \ \ \ \ \ \ 1\ldots.
\end{array}
$$

In both examples remainder is never becomes zero so the decimal expansion is never ends after some or infinite steps of division. These type of decimal expansions are called **non terminating.**

In above examples, after Ist step & 6 steps of division (respectively) we get remainder equal to dividend so decimal expansion is repeating (recurring).
So these are called **non terminating recurring decimal expansions**.
Both the above types (1 & 2) are rational numbers.

Types (3) Example: The decimal expansion 0.327172398……is not ends any where, also there is no arrangement of digits (not repeating) so these are called **non terminating not recurring**. These numbers are called **irrational numbers**.

Example:

0.1279312793	rational	terminating
0.1279312793….	rational	non terminating
or $0.\overline{12793}$		recurring
0.32777	rational	terminating
or $0.32\overline{7}$	rational	non terminating
0.32777…….		& recurring
0.5361279	rational	terminating
0.3712854043….	irrational	non terminating non recurring
0.10100100010000	rational	terminating
0.10100100010000….	irrational	non terminating non recurring.

Irrational Number Example Problems With Solutions

Example 1: Insert a rational and an irrational number between 2 and 3.

Sol. If a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b. Also, if a,b are rational numbers, then $\frac{a+b}{2}$ is a rational number between them.
∴ A rational number between 2 and 3 is
$\frac{2+3}{2}$ = 2.5
An irrational number between 2 and 3 is
= $\sqrt{2 \times 3}$ = $\sqrt{6}$

Example 2: Find two irrational numbers between 2 and 2.5.

Sol. If a and b are two distinct positive rational numbers such that ab is not a perfect square of a rational number, then is an irrational number lying between a and b.
∴ Irrational number between 2 and 2.5 is
= $\sqrt{2 \times 2.5}$ = $\sqrt{5}$
Similarly, irrational number between 2 and $\sqrt{5}$ is $\sqrt{2 \times \sqrt{5}}$
So, required numbers are $\sqrt{5}$ and $\sqrt{2 \times \sqrt{5}}$

Example 3: Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Sol. We know that, if a and b are two distinct positive irrational numbers, then $\sqrt{ab}$ is an irrational number lying between a and b.
∴ Irrational number between $\sqrt{2}$ and $\sqrt{3}$ is = $\sqrt{\sqrt{2} \times \sqrt{3}}$ = $6^{1/4}$
Irrational number between $\sqrt{2}$ and $6^{1/4}$ is $\sqrt{\sqrt{2} \times 6^{\frac{1}{4}}}$ = $2^{1/4} \times 6^{1/8}$.
Hence required irrational number are $6^{1/4}$ and
$2^{1/4} \times 6^{1/8}$.

Example 4: Find two irrational numbers between 0.12 and 0.13.

Sol. Let a = 0.12 and b = 0.13. Clearly, a and b are rational numbers such that a < b.
We observe that the number a and b have a 1 in the first place of decimal. But in the second place of decimal a has a 2 and b has 3. So, we consider the numbers
c = 0.1201001000100001 ……
and, d = 0.12101001000100001…….
Clearly, c and d are irrational numbers such that a < c < d < b.

Example 5: Prove that is $\sqrt{2}$ irrational number

Sol. Let us assume, to the contrary, that $\sqrt{2}$ is rational. So, we can find integers r and s (≠0) such that $\sqrt{2} = \frac{r}{s}$. Suppose r and s not having a common factor other than 1. Then, we divide by the common factor to get $\sqrt{2} = \frac{a}{b}$ where a and b are coprime.
So, $b\sqrt{2}$ = a.
Squaring on both sides and rearranging, we get $2b^2 = a^2$. Therefore, 2 divides a^2. Now, by Theorem it following that 2 divides a.
So, we can write a = 2c for some integer c.
Substituting for a, we get $2b^2 = 4c^2$, that is,
$b^2 = 2c^2$.
This means that 2 divides b^2, and so 2 divides b (again using Theorem with p = 2).
Therefore, a and b have at least 2 as a common factor.
But this contradicts the fact that a and b have no common factors other than 1.
This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.
So, we conclude that $\sqrt{2}$ is irrational.

Example 6: Prove that is $\sqrt{3}$ irrational number.
Sol. Let us assume, to contrary, that is rational. That is, we can find integers a and b (≠0) such that $\sqrt{2} = \frac{a}{b}$. Suppose a and b not having a common factor other than 1, then we can divide by the common factor, and assume that a and b are coprime.
So, $b\sqrt{3}$ = a.
Squaring on both sides, and rearranging, we get $3b^2 = a^2$.
Therefore, a^2 is divisible by 3, and by Theorem, it follows that a is also divisible by 3.
So, we can write a = 3c for some integer c.
Substituting for a, we get $3b^2 = 9c^2$, that is,
$b^2 = 3c^2$.
This means that b^2 is divisible by 3, and so b is also divisible by 3 (using Theorem with p = 3).
Therefore, a and b have at least 3 as a common factor.
But this contradicts the fact that a and b are coprime.
This contradicts the fact that a and b are coprime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational.
So, we conclude that $\sqrt{3}$ is irrational.

Example 7: Prove that $7-\sqrt{3}$ is irrational

Sol.

Method I :

Let $7-\sqrt{3}$ is rational number
∴ $7-\sqrt{3} = \frac{p}{q}$ (p, q are integers, q ≠ 0)
∴ $7 - \frac{p}{q} = \sqrt{3}$
⇒ $\sqrt{3} = \frac{7q-p}{q}$
Here p, q are integers
∴ $\frac{7q-p}{q}$ is also integer
∴ LHS = $\sqrt{3}$ is also integer but this $\sqrt{3}$ is contradiction that is irrational so our assumption is wrong that $7-\sqrt{3}$ is rational
∴ $7-\sqrt{3}$ is irrational proved.

Method II :
Let $7-\sqrt{3}$ is rational
we know sum or difference of two rationals is also rational
∴ $7-7-\sqrt{3}$
= $\sqrt{3}$ = rational
but this is contradiction that $\sqrt{3}$ is irrational
∴ $7-\sqrt{3}$ is irrational proved.

Example 8: Prove that $\frac{\sqrt{5}}{3}$ is irrational.

Sol. Let $\frac{\sqrt{5}}{3}$ is rational
∴ $3\left(\frac{\sqrt{5}}{3}\right)$ = $\sqrt{5}$ is rational
(∵ Q product of two rationals is also rational)
but this is contradiction that $\sqrt{5}$ is irrational
∴ $\frac{\sqrt{5}}{3}$ is irrational proved.

Example 9: Prove that $2\sqrt{7}$ is irrational.

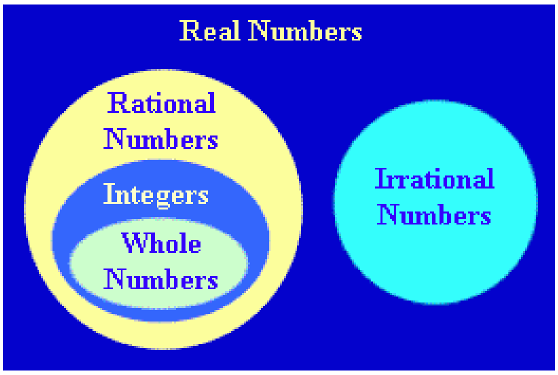
Sol. Let is rational
∴ $2\sqrt{7}\times\left(\frac{1}{2}\right)$ = $\sqrt{7}$
(∵ Q division of two rational no. is also rational)
∴ $\sqrt{7}$is rational
but this is contradiction that is irrational
∴ $2\sqrt{7}$ is irrational

Example 10: Find 3 irrational numbers between 3 & 5.

Solution: ∵ 3 and 5 both are rational
The irrational are 3.127190385..............
3.212325272930.........
3.969129852937...........

Rational and Irrational Numbers

Both rational and irrational numbers are real numbers.



This Venn Diagram shows the relationships between sets of numbers. Notice that rational and irrational numbers are contained in the large blue rectangle representing the set of Real Numbers.

1. A **rational number** is a number that can be expressed as a fraction or ratio. The numerator and the denominator of the fraction are both integers.
2. When the fraction is divided out, it becomes a terminating or repeating decimal. (The repeating decimal portion may be one number or a billion numbers.)
3. Rational numbers can be ordered on a number line.

Examples of rational numbers are:

6 or $\frac{6}{1}$	can also be written as	6.0
-2 or $\frac{-2}{1}$	can also be written as	-2.0
$\frac{1}{2}$	can also be written as	0.5
$\frac{-5}{4}$	can also be written as	-1.25
$\frac{2}{3}$	can also be written as	0.666666666... $0.\overline{6}$
$\frac{21}{55}$	can also be written as	0.38181818... $0.3\overline{18}$
$\frac{53}{83}$	can also be written as	0.62855421687... the decimals will repeat after 41 digits
Be careful when using your calculator to determine if a decimal number is irrational. The calculator may not be displaying enough digits to show you the repeating decimals, as was seen in the last example above.		

Hint: When given a rational number in decimal form and asked to write it as a fraction, it is often helpful to "say" the decimal out loud using the place values to help form the fraction.

2	.	3	4	5	6
o	a	t	h	t	ten-
n	n	e	u	h	t
e	d	n	n	o	h
s		t	d	u	o
		h	r	s	u
		s	e	a	s
			d	n	a
			t	d	n
			h	t	d
			s	h	t
				s	h
					s

Examples: Write each rational number as a fraction:

Rational number in decimal form	Rational number in fractional form
1. 0.3	$\frac{3}{10}$
2. 0.007	$\frac{7}{1000}$
3. -5.9	$-5\frac{9}{10} = -\frac{59}{10}$

Hint: When checking to see which fraction is larger, change the fractions to decimals by dividing and compare their decimal values.

Examples:

	Which of the given numbers is greater?	Using full calculator display to compare the numbers.
1.	$\frac{2}{3}, \frac{1}{4}$	.6666666667 > .25
2.	$-\frac{7}{3}, -\frac{11}{3}$	-2.333333333 > -3.666666667

An **irrational number** cannot be expressed as a fraction.

1. Irrational numbers cannot be represented as terminating or repeating decimals.
2. Irrational numbers are non-terminating, non-repeating decimals.

3. **Examples of irrational numbers are:**

$$\pi = 3.141592654\ldots..$$
$$\sqrt{2} = 1.414213562\ldots..$$
$$\text{and } 0.12122122212\ldots$$

Note: Many students think that π is the terminating decimal, 3.14, but it is not. Yes, certain math problems ask you to use π as 3.14, but that problem is rounding the value of to make your calculations easier. π is actually a non-ending decimal and is an irrational number.

Imaginary Unit and Standard Complex Form

The Imaginary Unit is defined as
i =√-1

The reason for the name "imaginary" numbers is that when these numbers were first proposed several hundred years ago, people could not "imagine" such a number.

It is said that the term "imaginary" was coined by René Descartes in the seventeenth century and was meant to be a derogatory reference since, obviously, such numbers did not exist. Today, we find the imaginary unit being used in mathematics and science. Electrical engineers use the imaginary unit (which they represent as j) in the study of electricity.

Imaginary numbers occur when a quadratic equation has no roots in the set of real numbers.

$$x^2 + 1 = 0$$
$$x^2 = -1$$
$$\sqrt{x^2} = \sqrt{-1}$$
$$x = \pm\sqrt{-1}$$
$$x = +\sqrt{-1} \text{ or } x = -\sqrt{-1}$$
$$* \quad i = \sqrt{-1} \text{ or } -i = -\sqrt{-1}$$

A pure imaginary number can be written in bi form where b is a real number and i is √-1

A complex number is any number that can be written in the standard form a + bi, where a and b are real numbers and i is the imaginary unit. .

A complex number is a real number a, or a pure imaginary number bi, or the sum of both.

Note these examples of complex numbers written in standard a + bi form: 2 + 3i, -5 + bi .

Complex Number: standard a + b*i* form	a	b*i*
$7 + 2i$	7	$2i$
$1 - 5i$	1	$-5i$
$8i$	0	$8i$
$\frac{-2+3i}{5} \Rightarrow \frac{-2}{5} + \frac{3i}{5}$	$\frac{-2}{5}$	$\frac{3i}{5}$

Complex Numbers

"**Complex number** is the combination of real and imaginary numbers"

Basic concepts of complex number

Definition: A number of the form x + iy where x, y ϵ R and i = √-1 is called a complex number and 'i' is called iota.

A complex number is usually denoted by z and the set of complex number is denoted by C.
i.e., C = {x + iy : x ϵ R, y ϵ R, i = √-1}
For example, 5 + 3i, −1 + i, 0 + 4i, 4 + 0i etc. are complex numbers.
(i) Euler was the first mathematician to introduce the symbol i (iota) for the square root of − 1 with property $i^2 = -1$. He also called this symbol as the imaginary unit.
(ii) For any positive real number a, we have

$$\sqrt{-a} = \sqrt{-1 \times a} = \sqrt{-1}\,\sqrt{a} = i\sqrt{a}$$

(iii) The property $\sqrt{a}\sqrt{b} = -\sqrt{ab}$ is valid only if at least one of a and b is non-negative. If a and b are both negative then $\sqrt{a}\sqrt{b} = -\sqrt{|a|\,|b|}$.
(2) Integral powers of iota (i) : Since hence we have i = √-1, $i^2 = -1$, $i^3 = -i$ and $i^4 = 1$. To find the value of i^n (n>4) first divide n by 4. Let q be the quotient and r be the remainder.
i.e., n = 4q + r where 0≤r≤3
$i^n = i^{4q+r} = (i^4)^q.(i)^r = (1)^q.(i)^r$
In general we have the following results $i^{4n} = 1$, $i^{4n+1} = i$, $i^{4n+2} = -1$, $i^{4n+3} = -i$, where n is any integer.

Real and Imaginary parts of a complex number

If x and y are two real numbers, then a number of the form is called a complex number. Here 'x' is called the real part of z and 'y' is known as the imaginary part of z. The real part of z is denoted by Re(z) and the imaginary part by Im(z).
If z = 3 − 4i, then Re(z) = 3 and Im(z) = − 4.

A complex number z is purely real if its imaginary part is zero i.e., Im(z) = 0 and purely imaginary if its real part is zero i.e., Re(z) = 0.

Algebraic operations with complex numbers

Let two complex numbers be $z_1 = a + ib$ and $z_2 = c + id$.

Addition $(z_1 + z_2)$: $(a+ib)+(c+id)=(a+c)+i(b+d)$

Subtraction $(z_1 - z_2)$: $(a+ib)-(c+id)=(a-c)+i(b-d)$

Multiplication $(z_1 . z_2)$: $(a+ib)(c+id)=(ac-bd)+i(ad+bc)$

Division (z_1 / z_2) : $\dfrac{a+ib}{c+id}$

(where at least one of c and d is non-zero)

$$\frac{a+ib}{c+id}=\frac{(a+ib)}{(c+id)}.\frac{(c-id)}{(c-id)} \quad \text{(Rationalization)}$$

$$\frac{a+ib}{c+id}=\frac{(ac+bd)}{c^2+d^2}+\frac{i(bc-ad)}{c^2+d^2}.$$

Properties of algebraic operations on complex numbers

Let z_1, z_2 and z_3 are any three complex numbers then their algebraic operations satisfy following properties :

(i) Addition of complex numbers satisfies the commutative and associative properties

i.e., $z_1 + z_2 = z_2 + z_1$ and $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

(ii) Multiplication of complex numbers satisfies the commutative and associative properties.

i.e., $z_1z_2 = z_2z_1$ and $(z_1z_2)z_3 = z_1(z_2z_3)$

(iii) Multiplication of complex numbers is distributive over addition

i.e., $z_1(z_2 + z_3) = z_1z_2 + z_1z_3$ and $(z_2 + z_3)z_1 = z_2z_1 + z_3z_1$

Equality of two complex numbers

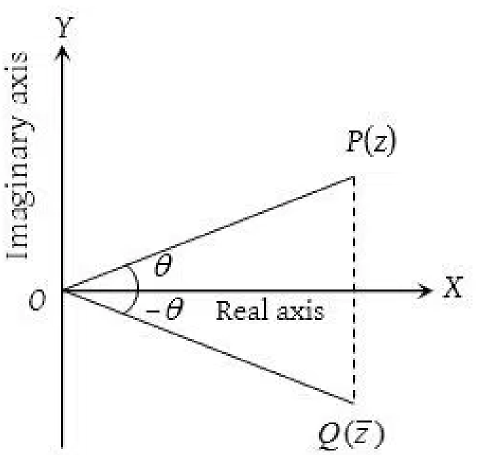
Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are said to be equal if and only if their real and imaginary parts are separately equal.

i.e., $z_1 = z_2 \Leftrightarrow x_1 + iy_1 = x_2 + iy_2 \Leftrightarrow x_1 = x_2$ and $y_1 = y_2$.

Complex numbers do not possess the property of order i.e., (a+ib) < (or) > (c+id) is not defined. For example, the statement (9+6i) > (3+2i) makes no sense.

Conjugate of a complex number

(1) Conjugate complex number: If there exists a complex number $z = a+ib$, $(a,b) \in R$, then its conjugate is defined as $\bar{z} = a - ib$.



Hence, we have $\text{Re}(z)=\dfrac{z+\bar{z}}{2}$ and $\text{Im}(z)=\dfrac{z-\bar{z}}{2i}$.

Geometrically, the conjugate of z is the reflection or point image of z in the real axis.

(2) Properties of conjugate: If z, z_1 and z_2 are existing complex numbers, then we have the following results:

(i) $\overline{(\bar{z})} = z$

(ii) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

(iii) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$

(iv) $\overline{z_1 z_2} = \bar{z}_1\, \bar{z}_2$, In general $\overline{z_1 . z_2 . z_3 ..... z_n} = \bar{z}_1 . \bar{z}_2 . \bar{z}_3 ..... \bar{z}_n$

(v) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$, $z_2 \neq 0$

(vi) $(\bar{z})^n = \overline{(z^n)}$

(vii) $z + \bar{z} = 2\text{Re}(z) = 2\text{Re}(\bar{z}) =$ purely real

(viii) $z - \bar{z} = 2i\text{Im}(z) =$ purely imaginary

(ix) $z\bar{z} = |z|^2 =$ purely real

(x) $z_1 \bar{z}_2 + \bar{z}_1 z_2 = 2\text{Re}(z_1 \bar{z}_2) = 2\text{Re}(\bar{z}_1 z_2)$

(3) Reciprocal of a complex number: For an existing non-zero complex number z = a+ib, the reciprocal is given by

$$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{z . \bar{z}} = \frac{\bar{z}}{|z|^2}$$

Modulus of a complex number

Modulus of a complex number z = a+ib is defined by a positive real number given by $|z| = \sqrt{a^2 + b^2}$ where a, b real numbers. Geometrically |z| represents the distance of point P from the origin, i.e. |z| = OP.
If the corresponding complex number is known as unimodular complex number. Clearly z lies on a circle of unit radius having centre (0, 0).

Properties of modulus

(i) $|z| \geq 0 \Rightarrow |z| = 0$ if $z = 0$ and $|z| > 0$ if $z \neq 0$.

(ii) $-|z| \leq \text{Re}(z) \leq |z|$ and $-|z| \leq \text{Im}(z) \leq |z|$

(iii) $|z| = |\bar{z}| = |-z| = |-\bar{z}| = |zi|$

(iv) $z\bar{z} = |z|^2 = |\bar{z}|^2$

(v) $|z_1 z_2| = |z_1||z_2|$.

In general $|z_1 z_2 z_3 \ldots\ldots z_n| = |z_1||z_2||z_3|\ldots.|z_n|$

(vi) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, (z_2 \neq 0)$

(vii) $|z^n| = |z|^n, n \in N$

(viii) $|z_1 \pm z_2|^2 = (z_1 \pm z_2)(\bar{z}_1 \pm \bar{z}_2) = |z_1|^2 + |z_2|^2 \pm (z_1\bar{z}_2 + \bar{z}_1 z_2)$

or $|z_1|^2 + |z_2|^2 \pm 2\,\text{Re}(z_1\bar{z}_2)$

(ix) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 \Rightarrow \frac{z_1}{z_2}$ is purely imaginary or

$\text{Re}\left(\frac{z_1}{z_2}\right) = 0$

(x) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left\{|z_1|^2 + |z_2|^2\right\}$

(Law of parallelogram)

Square root of a complex number

Let z = a+ib be a complex number,

$$\text{Then } \sqrt{a+ib} = \pm\left[\sqrt{\frac{|z| + a}{2}} + i\sqrt{\frac{|z| - a}{2}}\right], \text{ for } b > 0$$

$$= \pm\left[\sqrt{\frac{|z| + a}{2}} - i\sqrt{\frac{|z| - a}{2}}\right], \text{ for } b < 0.$$

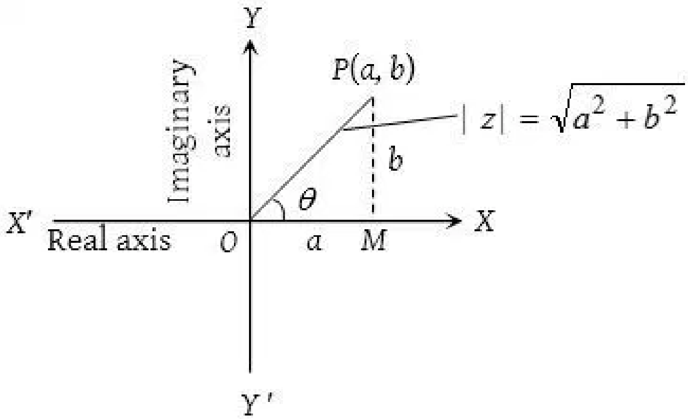
To find the square root of a–ib replace i by –i in the above results.

Various representations of a complex number

A complex number can be represented in the following form:

(1) Geometrical representation (Cartesian representation):

The complex number z = a+ib = (a, b) is represented by a point P whose coordinates are referred to rectangular axes *XOX′*and *YOY′* which are called real and imaginary axis respectively. This plane is called argand plane or argand diagram or complex plane or Gaussian plane.



Distance of any complex number from the origin is called the modulus of complex number and is denoted by |z|, i.e., $|z| = \sqrt{a^2 + b^2}$.
Angle of any complex number with positive direction of x–axis is called amplitude or argument of z.

$$i.e., \; amp\,(z) = arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$$

(2) Trigonometrical (Polar) representation:

In Δ OPM, let OP = r, then a = r cos *θ* and b = r sin *θ* Hence z can be expressed as z = r(cos *θ* + i sin *θ*) where r = |z| and *θ* = principal value of argument of z.
For general values of the argument
z = r[(cos(2*nπ* + *θ)* + i sin (2*nπ* + *θ)*]
Sometimes (cos *θ* + i sin *θ*) is written in short as *cisθ*.

(3) Vector representation:

If P is the point (a, b) on the argand plane corresponding to the complex number z = a+ib.

$$\text{Then } \overrightarrow{OP} = a\hat{i} + b\hat{j}, \; \therefore |\overrightarrow{OP}| = \sqrt{a^2 + b^2} = |z| \text{ and}$$

$$arg\,(z) = \text{direction of the vector } \overrightarrow{OP} = \tan^{-1}\left(\frac{b}{a}\right)$$

(4) Eulerian representation (Exponential form):

Since we have $e^{i?}$ = (cos *θ* + i sin *θ*) and thus z can be expressed as z = $re^{i?}$, where |z|=r and arg (z)

Adding and Subtracting Complex Numbers

To Add or Subtract Complex Numbers

- Change all imaginary numbers to *bi* form.
- Add (or subtract) the real parts of the complex numbers.
- Add (or subtract) the imaginary parts of the complex numbers.
- Write the answer in the form $a + bi$.

Add like terms

Find the sum of the real components.

Find the sum of the imaginary components (the components with the i after them).

1. Add: $(7 + 5i) + (8 - 3i)$

$$(7+5i)+(8-3i) = (7+8)+(5-3)i$$
$$= 15+2i$$

2. Add: $(2 + 3i) + (-8 - 6i)$

$$(2+3i)+(-8-6i) = (2+-8)+(3+-6)i$$
$$= -6-3i$$

3. Express the sum of $\left(3+\sqrt{-64}\right)$ and $\left(10-\sqrt{-25}\right)$ in the form $a+bi$.

$$\left(3+\sqrt{-64}\right)+\left(10-\sqrt{-25}\right) = (3+8i)+(10-5i)$$
$$= (3+10)+(8-5)i$$
$$= 13+3i$$

4. Add $1+7i\sqrt{2}$ and $-3-6i\sqrt{2}$.

$$\left(1+7i\sqrt{2}\right)+\left(-3-6i\sqrt{2}\right) = (1+-3)+(7+-6)i\sqrt{2}$$
$$= -2+1i\sqrt{2}$$

Subtract like terms

Find the difference of the real components.

Find the difference of the imaginary components (the components with the i after them).

Subtracting Rule:

(m+ni) – (p+qi) = (m-p) + (n-q)i

1. Subtract: (5 + 8*i*) - (2 + 2*i*) $(5+8i)-(2+2i)=(5-2)+(8-2)i$ $=3+6i$
2. Subtract: (4 + 10*i*) - (-12 + 20*i*) $(4+10i)-(-12+20i)=(4--12)+(10-20)i$ $=16-10i$
3. Subtract $\left(8-\sqrt{-100}\right)$ from $\left(9+\sqrt{-36}\right)$. $\left(9+\sqrt{-36}\right)-\left(8-\sqrt{-100}\right)=(9+6i)-(8-10i)$ $=(9-8)+(6--10)i$ $=1+16i$
4. Subtract $\left(7+3i\sqrt{5}\right)$ from $\left(-9+4i\sqrt{5}\right)$. $\left(-9+4i\sqrt{5}\right)-\left(7+3i\sqrt{5}\right)=(-9-7)+\left(4\sqrt{5}-3\sqrt{5}\right)i$ $=-16+i\sqrt{5}$

Multiplying and Dividing Complex Numbers

Multiplication:

Multiplying two complex numbers is accomplished in a manner similar to multiplying two binomials. You can use the FOIL process of multiplication, distributive multiplication, or your personal favorite means of multiplication.

Distributive Multiplication:
$(2+3i)\cdot(4+5i)=2(4+5i)+3i(4+5i)$ $=8+10i+12i+15i^2$ $=8+22i+15(-1)$ $=8+22i-15$ $=-7+\mathbf{22}i$ **Answer**   Be sure to replace i^2 with (-1) and proceed with the simplification. Answer should be in **a** + **b***i* form.

$(a+bi)(c+di) = a(c+di) + bi(c+di)$

$= ac + adi + bci + bdi^2$

$= ac + adi + bci + bd(-1)$

$= ac + adi + bci - bd$

$= (ac - bd) + (adi + bci)$

$= (ac - bd) + (ad + bc)i$ answer in $a+bi$ form

The conjugate of a complex number a + bi is the complex number a − bi.
For example, the conjugate of 4 + 2i is 4 − 2i.
(Notice that only the sign of the bi term is changed.)

The product of a complex number and its conjugate is a real number, and is always positive.

$(a + bi)(a - bi) = a^2 + abi - abi - b^2i^2$

$= a^2 - b^2(-1)$ (the middle terms drop out)

$= \mathbf{a^2 + b^2}$ Answer

This is a real number (no i's) and since both values are squared, the answer is positive.

Division:

Complex Numbers

(Division of Complex Numbers)

To divide complex numbers, we write the division as a fraction, then multiply the top and the bottom of the fraction by the conjugate of the denominator.

Example F. Simplify $\frac{3 - 2i}{4 + 3i}$

Multiply the conjugate of the denominator (4 – 3i) to the top and the bottom.

$$\frac{(3 - 2i)}{(4 + 3i)} * \frac{(4 - 3i)}{(4 - 3i)} = \frac{12 - 8i - 9i + \overset{-6}{6i^2}}{4^2 + 3^2} = \frac{6 - 17i}{25} = \frac{6}{25} - \frac{17i}{25}$$

Using the quadratic formula, we can solve all 2nd degree equations and obtain their complex number solutions.

Example G. Solve $2x^2 - 2x + 3 = 0$ and simplify the answers.

To find $b^2 - 4ac$ first: $a = 2$, $b = -2$, $c = 3$, so $b^2 - 4ac = -20$.

Hence $x = \frac{2 \pm \sqrt{-20}}{4} = \frac{2 \pm 2\sqrt{-5}}{4}$

When dividing two complex numbers,
1. write the problem in fractional form,
2. rationalize the denominator by multiplying the numerator and the denominator by the conjugate of the denominator.

(Remember that a complex number times its conjugate will give a real number.
This process will remove the i from the denominator.)

Example: $(4+2i) \div (3-i)$

Dividing using the conjugate:

$$\frac{(4+2i)}{(3-i)} = \frac{(4+2i)}{(3-i)} \cdot \frac{(3+i)}{(3+i)}$$

$$= \frac{12+4i+6i+2i^2}{9+3i-3i-i^2} = \frac{12+10i+2(-1)}{9-(-1)}$$

$$= \frac{10+10i}{10} = \frac{1+i}{1} = 1+i$$

Answer

Absolute Value of Complex Numbers

Geometrically, the absolute value of a complex number is the number's distance from the origin in the complex plane.

The absolute value of a complex number

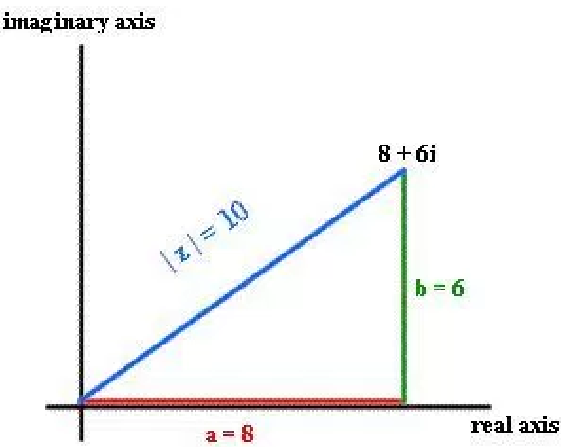
$z = a + bi$ is written as $|z|$.

It is a nonnegative real number defined as:

$$|z| = \sqrt{a^2 + b^2}$$

In the diagram at the left, the complex number 8 + 6i is plotted in the complex plane on an Argand diagram (where the vertical axis is the imaginary axis). For this problem, the distance from the point 8 + 6i to the origin is 10 units. Distance is a positive measure.

Notice the Pythagorean Theorem at work in this problem.



A complex number can be represented by a point, or by a vector from the origin to the point. When thinking of a complex number as a vector, the absolute value of the complex number is simply the length of the vector, called the magnitude.

The formula for finding the absolute value of a complex number,

$$|a+bi| = |z| = \sqrt{a^2+b^2}$$

can be derived from the Pythagorean theorem,

$$c^2 = a^2 + b^2 \text{ (see example 2 below).}$$

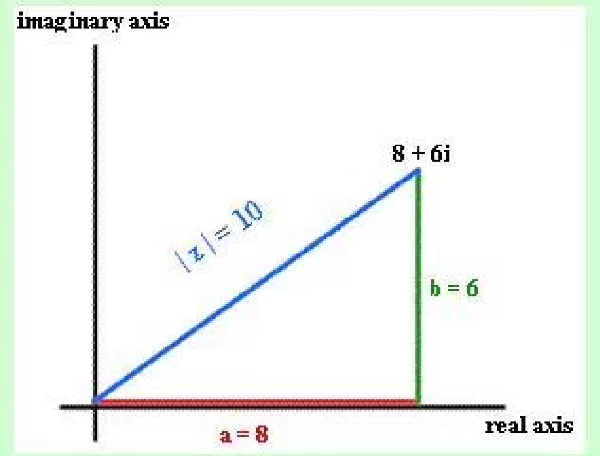
In the Pythagorean Theorem, c is the hypotenuse and when represented in the coordinate plane, is always positive. This same idea holds true for the distance from the origin in the complex plane. Using the absolute value in the formula will always yield a positive result.

To find the absolute value of a complex number a + bi:
1. Be sure your number is expressed in a + bi form
2. Pick out the coefficients for a and b
3. Substitute into the formula

$$|z| = \sqrt{a^2+b^2}$$

Example 1:

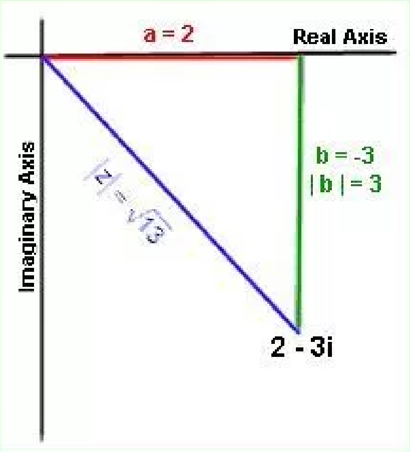
Plot z = 8 + 6i on the complex plane, connect the graph of z to the origin (see graph below), then find | z | by appropriate use of the definition of the absolute value of a complex number.



$$z = 8+6i$$
$$|z| = \sqrt{a^2+b^2}$$
$$|z| = \sqrt{8^2+6^2}$$
$$|z| = \sqrt{64+36}$$
$$|z| = \sqrt{100}$$
$$|z| = 10$$

Example 2:

Find the | z | by appropriate use of the Pythagorean Theorem when z = 2 – 3i.

	You can find the distance | z | by using the Pythagorean theorem. Consider the graph of **2 - 3*i*** shown at the left. The horizontal side of the triangle has length | a |, the vertical side has length | b |, and the hypotenuse has length | z |. By applying the Pythagorean Theorem, you have, $\|z\|^2 = a^2 + b^2$. **Notice:** you can drop the absolute value symbols for a and b since $\|a\|^2 = a^2$ and $\|b\|^2 = b^2$. You must keep the absolute value symbol for z to insure that the final answer will be positive. Solving this equation for | z |, you have just derived the formula for the absolute value of a complex number: $\|z\| = \sqrt{a^2 + b^2}$ $\|z\| = \sqrt{2^2 + (-3)^2}$ $\|z\| = \sqrt{4 + 9}$ $\|z\| = \sqrt{13}$

Example 3:

If z = – 8 – 15i, find | z |.

$$z = -8 - 15i$$

$$|z| = \sqrt{a^2 + b^2}$$

$$|z| = \sqrt{(-8)^2 + (-15)^2}$$

$$|z| = \sqrt{64 + 225}$$

$$|z| = \sqrt{289}$$

$$|z| = 17$$

Representing Complex Numbers Graphically (+ & -)

Due to their unique nature, complex numbers cannot be represented on a normal set of coordinate axes.

In 1806, J. R. Argand developed a method for displaying complex numbers graphically as a point in a coordinate plane. His method, called the Argand diagram, establishes a relationship between the x-axis (real axis) with real numbers and the y-axis (imaginary axis) with imaginary numbers.

In the Argand diagram, a complex number a + bi is the point (a,b) or the vector from the origin to the point (a,b).

Graph the complex numbers:

1. 3 + 4i (3,4)

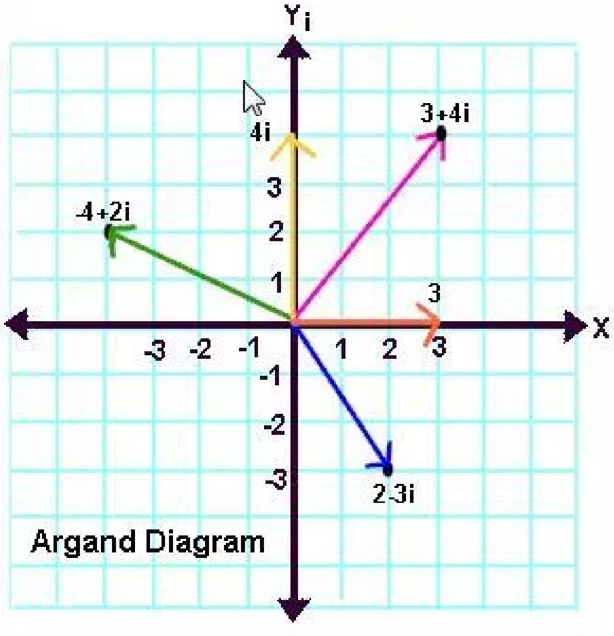
2. 2 – 3i (2,-3)

3. -4 + 2i (-4,2)

4. 3 (which is really 3 + 0i) (3,0)

5. 4i (which is really 0 + 4i) (0,4)

The complex number is represented by the point, or by the vector from the origin to the point.



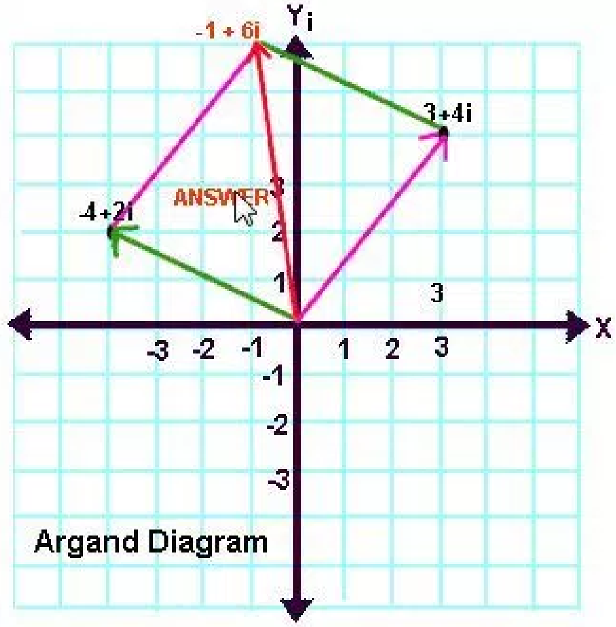
Add 3 + 4i and -4 + 2i graphically.

Graph the two complex numbers 3 + 4i and -4 + 2i as vectors.

Create a parallelogram using these two vectors as adjacent sides.

The answer to the addition is the vector forming the diagonal of the parallelogram (read from the origin).

This new vector is called the resultant vector.



Subtract 3 + 4i from -2 + 2i

Subtraction is the process of adding the additive inverse.
(-2 + 2i) − (3 + 4i)
= (-2 + 2i) + (-3 − 4i)
= (-5 − 2i)

Graph the two complex numbers as vectors.

Graph the additive inverse of the number being subtracted.

Create a parallelogram using the first number and the additive inverse. The answer is the vector forming the diagonal of the parallelogram.