CBSE Test Paper 05
CH-09 Sequences and Series

1. If a, 4, b are in A.P.; a, 2, b are in G.P.; then a, 1, b are in
   a. A.P.
   b. none of these
   c. G.P.
   d. H.P.
2. If a, b, c, d , e are in G. P., then $\frac{e}{c}$ equals
   a. $\frac{b}{a}$
   b. $\frac{c}{b}$
   c. $\frac{d}{b}$
   d. $\frac{d}{c}$
3. The A.M. between two positive numbers a and b is twice the G.M. between them. The ratio of the numbers is
   a. none of these
   b. $(\sqrt{3}+1):(\sqrt{3}-1)$
   c. $(2+\sqrt{3}):(2-\sqrt{3})$
   d. $(2+3):(\sqrt{2}-3)$
4. The first, second and last terms of an A.P. are a, b and 2 a. The number of terms in the A.P. is
   a. $\frac{b}{b-a}$
   b. $\frac{a}{b-a}$
   c. $\frac{a}{b+a}$
   d. $\frac{b}{b+a}$
5. The nth term of the sequence 5 + 55 + 555 + ... is
   a. none of these
   b. $\frac{5}{9}(10^n-1)$
   c. $5\times10^{n-1}$
   d. $5\times11^{n-1}$
6. Fill in the blanks:

   The sum of the following series of n terms: $2^3 + 4^3 + 6^3 + 8^3 + ...$ is ________.

7. Fill in the blanks:

   7^{th} term of an arithmetic sequence whose first term is 2 and the common difference of zero is ________.

8. How many numbers of two digits are divisible by 7?

9. Find the sum of n terms of an A.P. whose k^{th} term is 5k + 1.

10. Find the 10^{th} term of GP: 5, 25, 125 ...

    Also, find its n^{th} term.

11. The sum of n terms of two A. P are in the ratio (3n+8) : (7n+15). Find the ratio of their 12^{th} terms.

12. Is 667 a term of an AP 11, 18, 25,...?

13. If the sum of first pterms of an A.P. is equal to the sum of the first q terms, then find the sum of the first (p + q) terms.

14. If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A. M. between a and b. Then find the value of n.

15. If the sum of p terms of an A.P. is q and p, show that the sum of p + q terms is - (p + q).

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Solution

1. (d) H.P.

   Explanation:

   As a,4,b are in AP so, ...(i)

   Also a,2,b are in GP so, ab = 4...................(ii)

   from (i) and (ii)

   a+b=2ab

   $\frac{1}{a}+\frac{1}{b}=2\ \frac{2}{\frac{1}{a}+\frac{1}{b}}=1$

   hence a, 1 , b are in HP

2. (c) $\frac{d}{b}$

   Explanation:

   If a, b, c, d, e are in G.P we have $\frac{b}{a}=\frac{c}{b}=\frac{d}{c}=\frac{e}{d}\therefore\frac{c}{b}=\frac{e}{d}\Rightarrow\frac{e}{c}=\frac{d}{b}$

3. (c) $\left(2+\sqrt{3}\right):\left(2-\sqrt{3}\right)$

   Explanation:

   Given a and b are two positive numbers

   Also given A.M=2.G.M

   $\frac{a+b}{2}=2\sqrt{ab}$

   $\Rightarrow\frac{a+b}{2\sqrt{ab}}=\frac{2}{1}$

   Applying componendo dividendo

   $\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{2+1}{2-1}$

   $\Rightarrow\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2}=\frac{3}{1}$

   $\Rightarrow\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{\sqrt{3}}{1}$

Applying componendo dividendo again we get

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\Rightarrow \frac{a}{b} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)^2} = \frac{4+2\sqrt{3}}{4+2\sqrt{3}} = \frac{2(2+\sqrt{3})}{2(2-\sqrt{3})} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

4. (a) $\frac{b}{b-a}$

Explanation:

Given $a_1 = a, a_2 = b$ and $a_n = 2a$

Hence d=b-a

Now $a_n = a + (n-1)d \Rightarrow 2a = a + (n-1)(b-a)$

$\Rightarrow a = (n-1)(b-a)$

$\Rightarrow \frac{a}{b-a} + 1 = n \Rightarrow n = \frac{b}{b-a}$

5. (b) $\frac{5}{9}(10^n - 1)$

Explanation:

5+55+555+.........

$= \frac{5}{9}\{9 + 99 + 999 + \ldots\ldots\ldots\ldots\}$

$= \frac{5}{9}\left\{[10-1] + \left[10^2-1\right] + \left[10^3-1\right] + \ldots\ldots\ldots\ldots\ldots\ldots + [10^n - 1] + \ldots\ldots\ldots\right\}$

Hence by inspection we get nth term is $\frac{5}{9}[10^n - 1]$

6. $8\left[\frac{n(n+1)}{2}\right]^2$

7. 2

8. First two digit number divisible by 7 is 14 and last two digit number divisible by 7 is 98. So, we have to determine the number of terms in the sequence 14,21,28,..., 98. Let there be n terms in this sequence. Then,

98 = nth term $\Rightarrow$ 98 = 14 + (n -1) $\times$ 7

$\Rightarrow$ 7n = 91 $\Rightarrow$ n = 13

9. Given: a_k = 5k + 1

Putting k = 1 and k=n , we get

a = 5$\times$ 1 + 1 = 6 and a_n = 5_n + 1

$\therefore S_n = \frac{n}{2}(a+l)$

$\Rightarrow S_n = \frac{n}{2}(6+5n+1) = \frac{n}{2}(5n+7)$

10. We have, 5 + 25 + 125 + ... is GP.

    Here, a = 5 and r = $\frac{25}{5}$ = 5

    We know that, $T_n = ar^{n-1} = 5\,(5)^{n-1} = 5^n$

    and $T_{10} = 5\,(5)^{10-1} = 5^{10}$

11. Let a_1, a_2 and d_1, d_2 are the first term and common difference of two A. P. S respectively.

    A T Q $\frac{\frac{n}{2}\,[2a_1+(n-1)d_1]}{\frac{n}{2}\,[2a_2+(n-1)d_2]} = \frac{3n+8}{7n+15}$

    $\frac{\text{12 th term of Ist A.P}}{\text{12th term of 2ndA.P}} = \frac{a_1+11d_1}{a_2+11d_2}$

    put n = 23 in eq (i)

    $\frac{2a_1+22d_1}{2a_2+22d_2} = \frac{3\times23+8}{7\times23+15}$

    $\frac{a_1+11d_1}{a_2+11d_2} = \frac{7}{16}$

12. Given AP is 11, 18, 25,...

    Here, a = 11, d = 18 - 11 = 7 and a_n = 667

    $\because$ a + (n - 1)d = a_n

    $\therefore$ 11 + (n - 1)7 = 667

    $\Rightarrow$ 11 + 7n - 7 = 667

    $\Rightarrow$ 7n + 4 = 667

    $\Rightarrow$ 7n = 667 - 4

    $\Rightarrow$ 7n = 663

    $\therefore$ n = $\frac{663}{7}$, which is not a whole number. Hence, it is not the term of the given AP.

13. Let a be the first term and d be the common difference of given A.P.

    $\therefore S_p = \frac{p}{2}[2a+(p-1)d]$ and $S_q = \frac{q}{2}[2a+(q-1)d]$

    According to question, $S_p = S_q$

$\Rightarrow \frac{p}{2}[2a+(p-1)d] = \frac{q}{2}[2a+(q-1)d]$

$\Rightarrow$ 2ap + p^2d - pd = 2aq + q^2d - qd

$\Rightarrow$ 2ap - 2aq = q^2d - p^2d + pd - qd

$\Rightarrow$ 2a(p - q) = [-(p^2 - q^2)d + (p - q)d]

$\Rightarrow$ 2a(p - q) = [-(p - q) (p + q)d + (p - q)d]

$\Rightarrow$ 2a(p - q) = (p - q) [1 - p - q]d

$\Rightarrow a = \frac{(1-p-q)d}{2}$

Now $S_{p+q} = \frac{p+q}{2}\left[\frac{2(1-p-q)d}{2} + (p+q-1)d\right]$

$= \frac{p+q}{2}[d - pd - qd + pd + qd - d]$

$\Rightarrow S_{p+q} = \frac{p+q}{2} \times 0 = 0$

14. $\frac{a^n+b^n}{a^{n-1}+b^{n-1}} = \frac{a+b}{2}$

$2a^n + 2b^n = (a + b)(a^{n-1} + b^{n-1})$

$2a^n + 2b^n = a^n + ab^{n-1} + b.a^{n-1} + b^n$

$a^n + b^n = a.b^{n-1} + b.d^{n-1}$

$a^n - b.a^{n-1} = a.b^{n-1} - b^n$

$a^{n-1}(a - b) = b^{n-1}(a - b)$

$\left(\frac{a}{b}\right)^{n-1} = \frac{a-b}{a-b}$

$\left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^0 \left(\because \left(\frac{a}{b}\right)^0 = 1\right)$

n - 1 = 0

n = 1

15. Let a be the first term and d the common difference of the given A.P.

$\therefore S_p = \frac{p}{2}$ [2a + (p - 1)d] = q

$\Rightarrow$ 2a + (p - 1)d $= \frac{2q}{p}$ ....(i)

And $S_q = \frac{q}{2}$ [2a + (q - 1)d] = p

$\Rightarrow$ 2a + (q - 1)d $= \frac{2p}{q}$ ....(ii)

Subtracting eq. (ii) from eq. (i) we get

(p - q)d = $\frac{2q}{p} - \frac{2p}{q} \Rightarrow$ (p - q)d $= \frac{2(q^2 - p^2)}{pq}$

$\Rightarrow$ (p - q)d $= \frac{-2}{pq}$ $(p^2 - q^2)$

$\Rightarrow$ (p - q)d $= \frac{-2}{pq}$ (p + q)(p - q) $\Rightarrow d = \frac{-2}{pq}$ (p + q)

Substituting the value of d in eq. (i) we get

2a + (p - 1) $\left[\frac{-2(p+q)}{pq}\right] = \frac{2q}{p}$

$\Rightarrow 2a = \frac{2q}{p} + \frac{2(p-1)(p+q)}{pq}$

$\Rightarrow a = \frac{q}{p} + \frac{(p-1)(p+q)}{pq}$

$a = \frac{q^2+p^2+pq-p-q}{pq}$

Now $S_{p+q} = \frac{p+q}{2}$ [2a + (p + q - 1)d

$= \frac{p+q}{2}\left[\frac{2q^2+2p^2+2pq-2q-2q}{pq} + \frac{(p+q-1)[-2(p+q)]}{pq}\right]$

$= \frac{p+q}{2}\left[\frac{2q^2+2p^2+2pq-2p-2q-2p^2-2pq+2p-2pq-2q^2+2q}{pq}\right]$

$= \frac{p+q}{2}\left[\frac{-2pq}{pq}\right]$ = -(p + q) hence proved.