# Long Answer Questions-II (PYQ)

## [6 Marks]

**Q.1. *A* card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.**

**Ans.**

Let $E_1$, $E_2$, $E_3$, $E_4$ and $A$ be event defined as

$E_1 =$ The lost card is a spade card.

$E_2 =$ The lost card is a non spade card.

and $A =$ Drawing three spade cards from the remaining cards.

Now, $P(E_1) = \frac{13}{52} = \frac{1}{4}, P(E_2) = \frac{39}{52} = \frac{3}{4}$

$$P\left(\frac{A}{E_1}\right) = \frac{^{12}C_3}{^{51}C_3} = \frac{220}{20825};$$
$$P\left(\frac{A}{E_2}\right) = \frac{^{13}C_3}{^{51}C_3} = \frac{286}{20825}$$

Here, required probability $= P\left(\frac{E_1}{A}\right)$

$$\therefore \quad P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{4}\times\frac{220}{20825}}{\frac{1}{4}\times\frac{220}{20825}+\frac{3}{4}\times\frac{286}{20825}}$$

$$= \frac{220}{220+3\times286} = \frac{220}{1078} = \frac{10}{49}$$

**Q.2. Bag *I* contains 3 red and 4 black balls and bag *II* contains 4 red and 5 black balls. Two balls are transferred at random from bag I to Bag *II* and then a ball is drawn from Bag *II*. The ball so drawn is found to be red in colour. Find the probability that the transferred balls were both black.**

**Ans.**

Let $E_1$, $E_2$, $E_3$ and $A$ be events such that

$E_1$ = Both transferred balls from bag I to bag II are red.

$E_2$ = Both transferred balls from bag I to bag II are black.

$E_3$ = Out of two transferred balls one is red and other is black.

$A$ = Drawing a red ball from bag II.

Here, $P\left(\frac{E_2}{A}\right)$ is required.

Now, $P(E_1) = \frac{{}^3C_2}{{}^7C_2} = \frac{3!}{2!1!} \times \frac{2! \times 5!}{7!} = \frac{1}{7}$

$P(E_2) = \frac{{}^4C_2}{{}^7C_2} = \frac{4!}{2!2!} \times \frac{2! \times 5!}{7!} = \frac{2}{7}$

$P(E_3) = \frac{{}^3C_1 \times {}^4C_1}{{}^7C_2} = \frac{3 \times 4}{7!} \times \frac{2!5!}{1} = \frac{4}{7}$

$P\left(\frac{A}{E_1}\right) = \frac{6}{11}, P\left(\frac{A}{E_2}\right) = \frac{4}{11}, P\left(\frac{A}{E_3}\right) = \frac{5}{11}$

$$\therefore\ P\left(\frac{E_2}{A}\right) = \frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{2}{7} \times \frac{4}{11}}{\frac{1}{7} \times \frac{6}{11} + \frac{2}{7} \times \frac{4}{11} + \frac{4}{7} \times \frac{5}{11}} = \frac{\frac{8}{77}}{\frac{6}{77} + \frac{8}{77} + \frac{20}{77}} = \frac{8}{77} \times \frac{77}{34} = \frac{4}{17}$$

**Q.3. Three bags contain balls as shown in the table below :**

| Bag | Number of white balls | Number of black balls | Number of red balls |
|---|---|---|---|
| **I** | 1 | 2 | 3 |
| **II** | 2 | 1 | 1 |
| **III** | 4 | 3 | 2 |

**A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from the III bag?**

**Ans.**

The distribution of balls in the three bags as per the question is shown below.

| Bag | Number of white balls | Number of black balls | Number of red balls | Total balls |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| **I** | 1 | 2 | 3 | 6 |
| **II** | 2 | 1 | 1 | 4 |
| **III** | 4 | 3 | 2 | 9 |

As bags are randomly chosen

$$P(\text{bag I}) = P(\text{bag II}) = P(\text{bag III}) = \frac{1}{3}$$

Let E be the event that one white and one red ball is drawn.

$$P(E/\text{bag I}) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{3\times 2}{6\times 5} = \frac{1}{2}\ ; \qquad P(E/\text{bag II}) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{2\times 2}{4\times 3} = \frac{1}{3}$$

$$P(E/\text{bag III}) = \frac{{}^4C_1 \times {}^2C_1}{{}^9C_2} = \frac{4\times 2\times 2}{9\times 8} = \frac{2}{9}$$

Now, required probability

$$= P(\text{bag III}/\text{E}) = \frac{P(\text{bag III}).\,P(E/\text{bag III})}{P(\text{bag } I).\,P(E/\text{bag } I) + P(\text{bag II}).P(E/\text{bag II}) + P(\text{bag III}).P(E/\text{bag III})}$$

$$= \frac{\frac{1}{3}\times\frac{2}{9}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{9}} = \frac{\frac{1}{3}\times\frac{2}{9}}{\frac{1}{3}\left(\frac{1}{5}+\frac{1}{3}+\frac{2}{9}\right)}$$

$$= \frac{\frac{2}{9}}{\frac{9+15+10}{45}} = \frac{2}{9}\times\frac{45}{34} = \frac{5}{17}$$

**Q.4. There are three coins. One is two headed coin (having head on both faces), another is a biased coin that comes up head 75% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?**

**Ans.**

Let $E_1$, $E_2$, $E_3$ and $A$ be event defined as:

$E_1$= Selection of a two headed coin

$E_2$ = Selection of a biased coin.

$E_3$ = Selection of an unbiased coin

$A$ = Coin shows head after tossing.

Now, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$$P\left(\frac{A}{E_1}\right) = 1, \quad P\left(\frac{A}{E_2}\right) = \frac{75}{100} = \frac{3}{4}, \quad P\left(\frac{A}{E_3}\right) = \frac{1}{2}$$

Here, required probability = $P\left(\frac{E_1}{A}\right)$

By using Baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{3}\left(1+\frac{3}{4}+\frac{1}{2}\right)}$$

$$= \frac{1}{\frac{4+3+2}{4}} = \frac{4}{9}$$

**Q.5. Coloured balls are distributed in three bags as shown in the following table:**

| **Bag** | **Colour of the ball** | | |
|---|---|---|---|
| | **Black** | **White** | **Red** |
| **I** | **1** | **2** | **3** |
| **II** | **2** | **4** | **1** |
| **III** | **4** | **5** | **3** |

**A bag is selected at random and then two balls are randomly drawn from the selected bag. They happen to be black and red. What is the probability that they came from bag *I* ?**

**Ans.** Given distribution of the balls is shown in the table

| **Bag** | **Colour of the ball** | | |
|---|---|---|---|
| | **Black** | **White** | **Red** |
| I | 1 | 2 | 3 |
| II | 2 | 4 | 1 |
| III | 4 | 5 | 3 |

As bags are selected at random $P(\text{ bag } I) = \frac{1}{3} = P(\text{ bag II }) = P(\text{ bag III })$

Let $E$ be the event that 2 balls are 1 black and 1 red.

$P(E/\text{bag I}) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1}{5}$ ; $P(E/\text{bag II}) = \frac{{}^2C_1 \times {}^1C_1}{{}^7C_2} = \frac{2}{21}$

$P(E/\text{bag III}) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{2}{11}$

We have to determine

$$P(\text{bag I}/E) = \frac{P(\text{ bag } I).\ P(E\ /\ \text{ bag } I)}{\sum_{i\ I}^{\text{III}} P\ (\text{ bag } i).P(E/\text{ bag } i)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{2}{21} + \frac{1}{3} \times \frac{2}{11}} = \frac{\frac{1}{3} \times \frac{1}{5}}{\left(\frac{1}{5} + \frac{2}{21} + \frac{2}{11}\right)\frac{1}{3}}$$

$$= \frac{\frac{1}{5}}{\frac{1}{5} + \frac{2}{21} + \frac{2}{11}} = \frac{231}{551}$$

**Q.6. From a lot of 15 bulbs which include 5 defectives, a sample of 4 bulbs is drawn one by one with replacement. Find the probability distribution of number of defective bulbs. Hence, find the mean of the distribution.**

**Ans.** Let the number of defective bulbs be represented by a random variable *X*.

*X* may have values 0, 1, 2, 3, 4.

If *p* is the probability of getting defective bulb in a single draw then

$$\boldsymbol{p = \frac{5}{15} = \frac{1}{3}}$$

∴ $q$ = Probability of getting non-defective bulb = $1 - \frac{1}{3} = \frac{2}{3}$

Since, each trial in this problem is Bernaulli trials, therefore we can apply binomial distribution as

$P(X=r)={}^nC_r.p^r.q^{n-r}$, when $n=4$ and $r=0, 1, 2, 3, 4$

$P(X=0)={}^4C_0\left(\frac{1}{3}\right)^0.\left(\frac{2}{3}\right)^4=\frac{16}{81}$

$P(X=1)={}^4C_1\left(\frac{1}{3}\right)^1.\left(\frac{2}{3}\right)^3=4\times\frac{1}{3}\times\frac{8}{27}=\frac{32}{81}$

$P(X=2)={}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2=6\times\frac{1}{9}\times\frac{4}{9}=\frac{24}{81}$

$P(X=3)={}^4C_3\left(\frac{1}{3}\right)^3.\left(\frac{2}{3}\right)^1=4\times\frac{1}{27}\times\frac{2}{3}=\frac{8}{81}$

$P(X=4)={}^4C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0=\frac{1}{81}$

Now, probability distribution table is

| ***X*** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| ***P*(*X*)** | $\frac{16}{81}$ | $\frac{32}{81}$ | $\frac{24}{81}$ | $\frac{8}{81}$ | $\frac{1}{81}$ |

Now mean $E(X)=\sum p_ix_i$

$=0\times\frac{16}{81}+1\times\frac{32}{81}+2\times\frac{24}{81}+3\times\frac{8}{81}+4\times\frac{1}{81}$

Mean $=\frac{32}{81}+\frac{48}{81}+\frac{24}{81}+\frac{4}{81}=\frac{108}{81}=\frac{4}{3}$

**Q.7. From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs.**

**Ans.**

There are 3 defective bulbs & 7 non-defective bulbs.

Let *X* denote the random variable of “the number of defective bulbs” which can take values 0, 1, 2

Since bulbs are replaced

$\therefore \quad p = P(D) = \frac{3}{10}$ and $q = P(\bar{D}) = 1 - \frac{3}{10} = \frac{7}{10}$ [D stands for defective]

$\therefore$ Required probability distribution is

$$P(X=0) = \frac{{}^7C_2 \times {}^3C_0}{{}^{10}C_2} = \frac{7\times6}{10\times9} = \frac{7}{15}$$

$$P(X=1) = \frac{{}^7C_1 \times {}^3C_1}{{}^{10}C_2} = \frac{7\times3\times2}{10\times9} = \frac{7}{15}$$

$$P(X=2) = \frac{{}^7C_0 \times {}^3C_2}{{}^{10}C_2} = \frac{1\times3\times2}{10\times9} = \frac{1}{15}$$

The tabular form representation is

| ***X*** | 0 | 1 | 2 |
|---|---|---|---|
| ***P*(*X*)** | 7/15 | 7/15 | 1/15 |

**Q.8. There are two bags, bag *I* and bag *II*. bag *I* contains 4 white and 3 red balls while another bag *II* contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it is found to be white. Find the probability that it was drawn from Bag *I*.**

**Ans.**

Let $A$, $E_1$, $E_2$ be the events defined as follows:

$A$ : Ball drawn is white, $E_1$ : Bag I is chosen, $E_2$ : Bag II is chosen

Then we have to find $P(E_1 / A)$

Using Baye's theorem,

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{4}{7}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{3}{10}} = \frac{\frac{4}{7}}{\frac{40+21}{70}} = \frac{40}{61}$$

**Q.9. A bag contains 4 balls. Two balls are drawn at random and are found to be white. What is the probability that all balls are white?**

**Ans.**

Let us define the following events.

$E$: drawn balls are white; $A$ : 2 white balls in bag.

$B$: 3 white balls in bag; $C$: 4 white balls in bag.

Then, $P(A) = P(B) = P(C) = \frac{1}{3}$

and $P\left(\frac{E}{A}\right) = \frac{^2C_2}{^4C_2} = \frac{1}{6}$, $P\left(\frac{E}{B}\right) = \frac{^3C_2}{^4C_2} = \frac{3}{6}$, $P\left(\frac{E}{C}\right) = \frac{^4C_2}{^4C_2} = 1$

By applying Baye's theorem

$$P\left(\frac{C}{E}\right) = \frac{P(C).P\left(\frac{E}{C}\right)}{P(A).P\left(\frac{E}{A}\right)+P(B).P\left(\frac{E}{B}\right)+P(C)P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{1}{3}\times1}{\left(\frac{1}{3}\times\frac{1}{6}\right)+\left(\frac{1}{3}\times\frac{3}{6}\right)+\left(\frac{1}{3}\times1\right)} = \frac{1}{\frac{1}{6}+\frac{3}{6}+1} = \frac{3}{5}$$

**Q.10. An urn contains 4 white and 3 red balls. Let *X* be the number of red balls in a random draw of three balls. Find the mean and variance of *X*.**

**Ans.**

Let $X$ be the number of red balls in a random draw of three balls.

As there are 3 red balls, possible values of $X$ are 0, 1, 2, 3.

$$P(0) = \frac{^3C_0\times{}^4C_3}{^7C_3} = \frac{4\times3\times2}{7\times6\times5} = \frac{4}{35}$$

$$P(1) = \frac{^3C_1\times{}^4C_2}{^7C_3} = \frac{3\times6\times6}{7\times6\times5} = \frac{18}{35}$$

$$P(2) = \frac{^3C_2\times{}^4C_1}{^7C_3} = \frac{3\times4\times6}{7\times6\times5} = \frac{12}{35}$$

$$P(3) = \frac{^3C_3\times{}^4C_0}{^7C_3} = \frac{1\times1\times6}{7\times6\times5} = \frac{1}{35}$$

For calculation of Mean & Variance

| ***X*** | ***P*(*X*)** | ***XP*(*X*)** | **$X^2P(X)$** |
|---|---|---|---|
| 0 | 4/35 | 0 | 0 |

| | | | |
|---|---|---|---|
| 1 | 18/35 | 18/35 | 18/35 |
| 2 | 12/35 | 24/35 | 48/35 |
| 3 | 1/35 | 3/35 | 9/35 |
| **Total** | **1** | **9/7** | **15/7** |

Mean = $\sum XP(X) = \frac{9}{7}$

Variance $= \sum X^2.P(X) - (\sum X.\ P\ (X))^2 = \frac{15}{7} - \frac{81}{49} = \frac{24}{49}$

**Q.11. A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is a number greater than 4. Find the probability that it is actually a number greater than 4.**

**Ans.**

Let $E_1$ be event getting number > 4

$E_2$ be event getting number ≤ 4

$P(E_1) = \frac{2}{6} = \frac{1}{3}$ $\qquad$ $P(E_2) = \frac{4}{6} = \frac{2}{3}$

Let $E$ be the event that man reports getting number > 4.

$P(E/E_1) = \frac{3}{5}$ $\qquad$ $P(E/E_2) = \frac{2}{5}$

By Baye's theorem

$$P(E_1/E) = \frac{P(E_1).P(E/E_1)}{P(E_1).P(E/E_1)+P(E_2).P(E/E_2)}$$

$$= \frac{\frac{1}{3}\times\frac{3}{5}}{\frac{1}{3}\times\frac{3}{5}+\frac{2}{5}\times\frac{2}{5}} = \frac{3}{3+4} = \frac{3}{7}$$

**Q.12. In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl.**

**Ans.**

Let $E_1$, $E_2$, $A$ be events such that

$E_1$ = student selected is girl; $E_2$ = student selected is Boy

$A$ = student selected is taller than 1.75 metres.

Here $P\left(\frac{E_1}{A}\right)$ is required.

Now, $P(E_1) = \frac{60}{100} = \frac{3}{5}$, $P(E_2) = \frac{40}{100} = \frac{2}{5}$

$P\left(\frac{A}{E_1}\right) = \frac{1}{100}$, $P\left(\frac{A}{E_2}\right) = \frac{4}{100}$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{3}{5}\times\frac{1}{100}}{\frac{3}{5}\times\frac{1}{100}+\frac{2}{5}\times\frac{4}{100}} = \frac{\frac{3}{500}}{\frac{3}{500}+\frac{8}{500}}$$

$$= \frac{3}{500}\times\frac{500}{11} = \frac{3}{11}$$

**Q.13. A factory has two machines *A* and *B*. Past record shows that machine *A* produced 60% of the items of output and machine *B* produced 40% of the items. Further, 2% of the items produced by machine *A* and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine *B*?**

**Ans.**

Let $E_1$, $E_2$ and $A$ be event such that

$E_1$ = Production of items by machine $A$

$E_2$ = Production of items by machine $B$

$A$ = Selection of defective items.

$$P(E_1) = \frac{60}{100} = \frac{3}{5}, P(E_2) = \frac{40}{100} = \frac{2}{5}$$

$$P\left(\frac{A}{E_1}\right) = \frac{2}{100} = \frac{1}{50}, P\left(\frac{A}{E_2}\right) = \frac{1}{100}$$

$P\left(\frac{E_2}{A}\right)$ is required

By Baye's theorem

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{2}{5}\times\frac{1}{100}}{\frac{3}{5}\times\frac{1}{50}+\frac{2}{5}\times\frac{1}{100}} = \frac{\frac{2}{500}}{\frac{3}{250}+\frac{2}{500}}$$
$$= \frac{2}{500}\times\frac{500}{6+2} = \frac{1}{4}$$

**Q.14. Two groups are competing for the position on the Board of Directors of a corporation. The probabilities that the first and the second group will win are 0. 6 and 0 .4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3, if the second group wins. Find the probability that the new product was introduced by the second group.**

**Ans.**

$P(G_I) = 0.6 \qquad P(G_{II}) = 0.4$

Let E is the event of introducing new product then

$P(E/G_I) = 0.7 \qquad P(E/G_{II}) = 0.3$

To find $P(G_{II}/E)$

Using Baye's theorem, we get

$$P(G_{II}/E) = \frac{P(G_{\text{II}}).P(E/G_{\text{II}})}{P(G_I).P(E/G_I)+P(G_{\text{II}}).P(E/G_{\text{II}})}$$

$$= \frac{0.4\times0.3}{0.6\times0.7+0.4\times0.3} = \frac{0.12}{0.42+0.12} = \frac{12}{54} = \frac{2}{9}$$

## Long Answer Questions-II (OIQ)

**[6 Marks]**

**Q.1. A bag I contains 5 red and 4 white balls and a Bag II contains 3 red and 3 white balls. Two balls are transferred from the Bag *I* to the Bag II and then one ball is drawn from the Bag II. If the ball drawn from the Bag II is red, then find the probability that one red and one white ball are transferred from the Bag I to the Bag II.**

**Ans.**


Let $E_1$, $E_2$, $E_3$ and $A$ be event such that

$E_1$ = Both transferred balls from bag I to bag II are red.

$E_2$ = Both transferred balls from bag I to bag II are white.

$E_3$ = Out of two transferred balls one in red and other is white.

$A$ = Drawing a red ball from bag II

$$P(E_1) = \frac{^5C_2}{^9C_2} = \frac{5\times4}{9\times8} = \frac{20}{72} = \frac{5}{18}$$

$$P(E_2) = \frac{^4C_2}{^9C_2} = \frac{4\times3}{9\times8} = \frac{12}{72} = \frac{3}{18}$$

$$P(E_3) = \frac{^5C_1\times\,^4C_1}{^9C_2} = \frac{5\times4\times2}{9\times8} = \frac{40}{72} = \frac{10}{18}$$

$$P\left(\frac{A}{E_1}\right) = \frac{5}{8};\quad P\left(\frac{A}{E_2}\right) = \frac{3}{8};\quad P\left(\frac{A}{E_3}\right) = \frac{4}{8}$$

We require $P\left(\frac{E_3}{A}\right)$.

Now, by Baye's theorem

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3).P\left(\frac{A}{E_3}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{10}{18}\times\frac{4}{8}}{\frac{5}{18}\times\frac{5}{8}+\frac{3}{18}\times\frac{3}{8}+\frac{10}{18}\times\frac{4}{8}}$$

$$= \frac{\frac{40}{144}}{\frac{25}{144}+\frac{9}{144}+\frac{40}{144}} = \frac{40}{144}\times\frac{144}{74} = \frac{20}{37}$$

**Q.2. In a village there are three mohallas *A*, *B* and *C*. In *A*, 60% persons believe in honesty, while in *B*, 70% and in *C*, 80%. A person is selected at random from village and found, he is honest. Find the probability that he belongs to mohalla *B*.**

**Ans.**

Let the event be defined as

$E_1$ = Selection of mohalla *A*

$E_2$ = Selection of mohalla *B*

$E_3$ = Selection of mohalla *C*

*A* = Selection of honest person

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{3}$$

$$P\left(\frac{A}{E_1}\right) = \frac{60}{100} = \frac{3}{5};$$

$$P\left(\frac{A}{E_2}\right) = \frac{70}{100} = \frac{7}{10}$$

$$P\left(\frac{A}{E_3}\right) = \frac{80}{100} = \frac{4}{5};$$

$$P\left(\frac{E_2}{A}\right) = \text{ required}$$

$$P\left(\frac{E_2}{A}\right)=\frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$=\frac{\frac{1}{3}\cdot\frac{7}{10}}{\frac{1}{3}\cdot\frac{3}{5}+\frac{1}{3}\cdot\frac{7}{10}+\frac{1}{3}\cdot\frac{4}{5}}=\frac{\frac{7}{30}}{\frac{3}{15}+\frac{7}{30}+\frac{4}{15}}$$

$$=\frac{7}{6+7+8}=\frac{7}{21}=\frac{1}{3}$$

**Q.3. A person wants to construct a hospital in a village for welfare. The probabilities are 0.40 that some bad elements oppose this work, 0.80 that the hospital will be completed if there is not any oppose of any bad elements and 0.30 that the hospital will be completed if bad elements oppose. Determine the probability that the construction of hospital will be completed.**

**Ans.**

Let the event be defined as

$A$ = Construction of hospital will be completed

$E_1$ = There will be oppose of bad elements

$E_2$ = There will be no oppose of any bad element

$$P(E_1)=0.40=\frac{4}{10},P(E_2)=1-0.40=0.60=\frac{6}{10}$$

$$P\left(\frac{A}{E_1}\right)=\frac{30}{100}=\frac{3}{10};$$

$$P\left(\frac{A}{E_2}\right)=\frac{80}{100}=\frac{8}{10}$$

$P(A)$ = required

$$P(A)=P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)$$

$$=\frac{4}{10}\cdot\frac{3}{10}+\frac{6}{10}\cdot\frac{8}{10}=\frac{12}{100}+\frac{48}{100}=\frac{60}{100}=\frac{3}{5}$$

**Q.4. In answering a question on a *MCQ* test with 4 choices per question, a student knows the answer, guesses or copies the answer. Let $\frac{1}{2}$ be the probability that he knows the answer, $\frac{1}{4}$ be the probability that he guesses and $\frac{1}{4}$ that he copies it. Assuming that a student, who copies the answer, will be correct has the probability $\frac{3}{4}$, what is the probability that the student knows the answer, given that he answered it correctly?**

**Ans.**

Let $A$ be the event that he knows the answer, $B$ be the event that he guesses, $C$ be the event that he copies and $X$ be the event that he answered correctly.

Then, $P(A) = \frac{1}{2}, P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{4}$

Also, $P\left(\frac{X}{A}\right) = 1$, $P\left(\frac{X}{B}\right) = \frac{1}{4}$ and $P\left(\frac{X}{C}\right) = \frac{3}{4}$

Thus, Required probability = $P\left(\frac{A}{X}\right)$

$$P\left(\frac{A}{X}\right) = \frac{P\left(\frac{X}{A}\right) \times P(A)}{P\left(\frac{X}{A}\right) \times P(A) + P\left(\frac{X}{B}\right) \times P(B) + P\left(\frac{X}{C}\right) \times P(C)}$$

$$= \frac{\frac{1}{2} \times 1}{\frac{1}{2} \times 1 + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4}} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{16} + \frac{3}{16}} = \frac{2}{3}$$