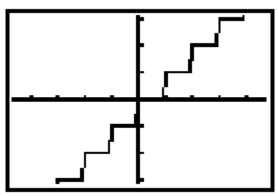
Mathematics
Class XII

Time: **3** hour Total Marks: 100

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. This graph does not represent a function. Make the required changes in this graph, and draw the graph, so that it represents a function.



2. Find the value of ‘α’ for which $\alpha(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

3. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point where the curve cuts the y-axis.

4. This 3 x 2 matrix gives information about the number of men and women workers in three factories I, II and III who lost their jobs in the last 2 months. What do you infer from the entry in the third row and second column of this matrix?

	Men workers	Women workers
Factory I	40	15
Factory II	35	40
Factory III	72	64

SECTION – B

5. Evaluate: $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$

6. Evaluate: $\int_{-1}^{1}\log\left(\frac{2-x}{2+x}\right)dx$

7. For what value of 'a' the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $a\hat{i}+6\hat{j}-8\hat{k}$ are collinear?

8. Write A^{-1} for $A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$.

9. Simplify $\cot^{-1}\frac{-1}{\sqrt{x^2-1}}$ for $x<-1$.

10. If $y=\tan^{-1}\frac{5x}{1-6x^2}$, $-\frac{1}{\sqrt{6}}x<-\frac{1}{\sqrt{6}}$, then prove that $\frac{dy}{dx}=\frac{2}{1+4x^2}+\frac{3}{1+9x^2}$.

11. Obtain the differential equation of the family of circles passing through the points (a,0) and (-a,0).

12. If $P(A)=\frac{2}{5}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{5}$, then find $P(\overline{A}/\overline{B})$.

SECTION – C

13. Differentiate $\frac{x^3\sqrt{5+x}}{(7-3x)^5\sqrt[3]{8+5x}}$, wrt x

OR

If y = acos(log x) + bsin(log x), prove that $x^2y''+xy'+y=0$,

14.

Let $f(x)=x+3$, $g(x)=x-3$; $x\in N$,

Show that (i) f is not an onto function (ii) gof is an onto function

15.

Find the distance between the parallel planes

$\vec{r}.\ 2\hat{i}-1\hat{j}+3\hat{k}\ =4$ and $\vec{r}.\ 6\hat{i}-3\hat{j}+9\hat{k}\ +13=0$

16.

A plane is at a distance of p units from the origin.

It makes an intercept of a,b,c with the x , y and z axis repectively.

Show that it satisfies the equation:

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$$

17. Four cards are drawn successively with replacement from a well shuffled deck of 52 cards. What is the probability that

(i) All the four cards are spades?

(ii) Only 3 cards are spades?

(iii) None is a spade?

18. Solve the equation:$(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$

19. Find the equation of a tangent to the curve given by $x=a\sin^3 t$, $y=b\cos^3 t$ at a point, where $t=\frac{\pi}{2}$.

20. Evaluate: $\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\sin 2x}dx$

21. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ then prove that $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}, n\in N$

OR

If ω is one of the cube roots of unity, evaluate the given determinant

$$\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

22. Show that the function f defined by $f(x) = |1 - x + |x||$, $x \in R$ is continuous.

OR

Show that a logarithmic function is continuous at every point in its domain.

23. Evaluate: $\int \frac{(3\sin\alpha - 2)\cos\alpha}{5 - \cos^2\alpha - 4\sin\alpha} d\alpha$

OR

Evaluate: $\int \frac{dx}{x^{\frac{1}{2}} + x^{\frac{1}{3}}}$

SECTION - D

24. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

OR

Find the points at which the function f given by $f(x) = (x - 2)^4(x+1)$

25. Obtain the inverse of the following matrix using elementary operations.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

26.

Calculate the area

(i) between the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and the x-axis between x = 0 to x = a

(ii) Triangle AOB is in the first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where OA = a and OB = b.

Find the area enclosed between the chord AB and the arc AB of the ellipse

(iii) Find the ratio of the two areas found.

OR

Find the smaller of the two areas in which the circle $x^2 + y^2 = 2a^2$ is divided by the parabola $y^2 = ax$, $a > 0$

27. Find the equation of a plane that is parallel to the x-axis and passes through the line common to two intersectiing planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right) - 1 = 0$ and $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right) = -4$

28. Two trainee carpenters A and B earn Rs. 150 and Rs. 200 per day respectively. A can make 6 frames and 4 stools per day while B can make 10 frames and 4 stools per day. How many days shall each work, if it is desired to produce atleast 60 frames and 32 stools at a minimum labour cost? Solve the problem graphically.

29. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine: (i) k (ii) $P(X < 3)$ (iii) $P(X > 6)$ (iv) $P(1 \leq X < 3)$

OR

In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses it or copies the answer. Let ½ be the probability that he knows the answer, ¼ be the probability that he guesses and ¼ be the probability that he copies it. Assuming that a student, who copies the answer, will be correct with the probability ¾, what is the probability that student knows the answer, given that he answered it correctly?

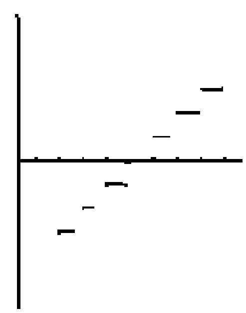
Arjun does not know that answer to one of the questions in the test. The evaluation process has negative marking. Which value would Arjun violate if he resorts of unfair means? How would an act like the above hamper his character development in the coming years?

Mathematics
Class XII
Solution

Time: **3** hour Total Marks: 100

SECTION A

1. If a vertical line intersects the graph of a relation in two or more points, then the relation is *not* a function.
Graph should have no vertical lines



2. Magnitude of α(î + ĵ + k)= 1, for it to be a unit vector.

$$\left|\alpha.(\hat{i}+\hat{j}+\hat{k})\right| = \sqrt{\alpha^2+\alpha^2+\alpha^2} = \sqrt{3\alpha^2} = \alpha\sqrt{3} = 1$$

$$\Rightarrow \alpha = \pm\frac{1}{\sqrt{3}}$$

3.

$$y = x^3 - x + 1$$

$$\frac{dy}{dx} = 3x^2 - 1$$

At a point where the curve cuts y-axis, x = 0

$$\left.\frac{dy}{dx}\right]_{x=0} = \left.3x^2 - 1\right]_{x=0} = -1$$

4. 64 women lost their jobs in factory III in the last two months.

SECTION B

5. $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$

$=\sin\left[\frac{\pi}{3}-\left(-\frac{\pi}{6}\right)\right]$

$=\sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right]$

$=\sin\frac{\pi}{2}=1$

6. $I=\int_{-1}^{1}\log\left(\frac{2-x}{2+x}\right)dx \quad ...(1)$

Let $f(x)=\log\left(\frac{2-x}{2+x}\right)$ and

$f(-x)=\log\left(\frac{2+x}{2-x}\right)=-\log\left(\frac{2-x}{2+x}\right)=-f(x).$

$\therefore$ $f(x)$ is an odd function.

Thus, $\int_{-1}^{1}\log\left(\frac{2-x}{2+x}\right)dx=0$

7.

Let $\vec{p}=2\hat{i}-3\hat{j}+4k$ and $\vec{q}=a\hat{i}+6\hat{j}-8k$

Two vectors $\vec{p}$ and $\vec{q}$ will be collinear if,

$$\vec{p}\times\vec{q}=0 \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & -3 & 4 \\ a & 6 & -8 \end{vmatrix}=0$$

$\Rightarrow \hat{i}(24-24)-\hat{j}(-16-4a)+k(12+3a)=0$

$\Rightarrow 0\hat{i}+(16+4a)\hat{j}+(12+3a)k=0\hat{i}+0\hat{j}+0k$

$\Rightarrow 16+4a=0$ and $12+3a=0$

$\Rightarrow a=-4$

8.

Given $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then

$|A| = 6 - 5 \neq 0$

and adj$A = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$, obtained from A by interchanging the main diagonal elements and multiplying by (-1) the non-diagonal elements.

$$\therefore A^{-1} = \frac{\text{adj}A}{|A|} = \frac{\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}}{1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

9.

Let $\sec^{-1} x = \theta.$

Then, $x = \sec\theta$ and for $x < -1, \frac{\pi}{2} < \theta < \pi$

$$\text{Given expression} = \cot^{-1}(-\cot\theta), \frac{\pi}{2} < \theta < \pi$$

$$= \cot^{-1}(\cot(\pi - \theta))$$

$$= \pi - \sec^{-1}\theta \text{ as } 0 < \pi - \theta < \frac{\pi}{2}$$

10.

$$y = \tan^{-1}\frac{5x}{1-6x^2}$$

$$\Rightarrow y = \tan^{-1}3x + \tan^{-1}2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{1+9x^2} + \frac{2}{1+4x^2}$$

11.

$$x^2+(y-b)^2=a^2+b^2 \quad \text{or} \quad x^2+y^2-2by=a^2 \quad ....(1)$$

$$2x+2y\frac{dy}{dx}-2b\frac{dy}{dx}=0 \Rightarrow 2b=\frac{2x+2y\frac{dy}{dx}}{\frac{dy}{dx}} \qquad ....(2)$$

Substituting in (1),

$$(x^2-y^2-a^2)\frac{dy}{dx}-2xy=0$$

12.

$$P(\bar{A}/\bar{B})=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$$

$$=\frac{1-P(A\cup B)}{1-P(B)}$$

$$=\frac{1-[P(A)+P(B)-P(A\cap B)]}{1-P(B)}$$

$$=\frac{7}{10}$$

SECTION – C

13.

Let $y = \dfrac{x^3\sqrt{5+x}}{(7-3x)^5\sqrt[3]{8+5x}}$

Taking log on both sides , we get

$\log y = 3\log x + \frac{1}{2}\log(5+x) - 5\log(7-3x) - \frac{1}{3}\log(8+5x)$

$\Rightarrow \frac{1}{y}.\frac{dy}{dx} = 3.\frac{1}{x} + \frac{1}{2}.\frac{1}{5+x}.1 - 5.\frac{1}{7-3x}(-3) - \frac{1}{3}.\frac{1}{8+5x}.5$

$\Rightarrow \frac{1}{y}.\frac{dy}{dx} = \frac{3}{x} + \frac{1}{2(5+x)} + \frac{15}{7-3x} - \frac{5}{3(8+5x)}$

$\Rightarrow \frac{dy}{dx} = y\left[\frac{3}{x} + \frac{1}{2(5+x)} + \frac{15}{7-3x} - \frac{5}{3(8+5x)}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{x^3\sqrt{5+x}}{(7-3x)^5\sqrt[3]{8+5x}}\left[\frac{3}{x} + \frac{1}{2(5+x)} + \frac{15}{7-3x} - \frac{5}{3(8+5x)}\right]$

OR

$y = a\cos(\log x) + b\sin(\log x)$

$\Rightarrow \frac{dy}{dx} = a\left[-\sin(\log x).\frac{1}{x}\right] + b.\left[\cos(\log x).\frac{1}{x}\right]$

$\Rightarrow x\frac{dy}{dx} = xy' = a\left[-\sin(\log x)\right] + b.\left[\cos(\log x)\right]$

Differentiating,

$xy'' + y' = a\left[-\cos(\log x).\frac{1}{x}\right] + b.\left[-\sin(\log x).\frac{1}{x}\right]$

$\Rightarrow x^2y'' + xy' = a\left[-\cos(\log x)\right] + b.\left[-\sin(\log x)\right] = -y$

$\Rightarrow \quad x^2y'' + xy' + y = 0$

14.

$f(x)=x+3,\ x\in N,$

$\text{Domain of } f=\{1,2,3,\ldots\}$

$\text{Range}=\{4,5,6,\ldots\}\neq\text{Codomain of } f\ =\{1,2,3,\ldots\}$

$\therefore$ f is not an onto function

f(x)= x+3

g(x) = $x-3$

$\therefore g[f(x)]=\left[x+3\right]-3$ =x

$\text{Domain of gof}=\{1,2,3,\ldots\}$

$\text{Range}=\{1,2,3,4,5,6,\ldots\}=\text{Codomain of gof}\ =\{1,2,3,\ldots\}$

$\therefore$ gof is an onto function

15.

Distance between the parallel planes

is given by $\dfrac{|d-k|}{|\vec{n}|}$

$$\vec{r}.\ 6\hat{i}-3\hat{j}+9\hat{k}\ +13=0$$

$$\Rightarrow \vec{r}.\ 2\hat{i}-\hat{j}+3\hat{k}\ =-\frac{13}{3}$$

$$\vec{r}.\ 2i-1\hat{j}+3\hat{k}\ =4 \text{ and } \vec{r}.\ 2\hat{i}-\hat{j}+3\hat{k}\ =-\frac{13}{3}$$

$\therefore$ the distance between the given parallel planes

$$\text{is}\frac{\left|4-\left(-\frac{13}{3}\right)\right|}{\sqrt{2+\ -1\ \ +3}}$$

$$=\frac{\left|4+\left(\frac{13}{3}\right)\right|}{\sqrt{4+1+9}}=\frac{\frac{25}{3}}{\sqrt{14}}=\frac{25}{3\sqrt{14}}$$

16.

The equation of the plane in the intercept form is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

$$\Rightarrow x\left(\frac{1}{a}\right)+y\left(\frac{1}{b}\right)+z\left(\frac{1}{c}\right)-1=0$$

Plane is at a distance of p units from the Origin

$$\frac{\left|0\left(\frac{1}{a}\right)+0\left(\frac{1}{b}\right)+0\left(\frac{1}{c}\right)-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}=p$$

$$\Rightarrow \frac{1}{p}=\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$$

17.

This is a case of bernoulli trials.

p = P(Success) = P(getting a spade in a single draw) = $\frac{13}{52}=\frac{1}{4}$

$q=P(\text{Failure})=1-p=1-\frac{1}{4}=\frac{3}{4}$

(i)All the four cards are spades = P(X = 4) = $^4C_4p^4q^0=\left(\frac{1}{4}\right)^4=\frac{1}{256}$

(ii)Only 3 cards are spades= P(X = 3) = $^4C_3p^3q^1=\frac{12}{256}=\frac{3}{64}$

(iii)None is a spade = P(X = 0) = $^4C_0p^0q^4=\left(\frac{3}{4}\right)^4=\frac{81}{256}$

18. Here, $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$

$$\Rightarrow (\tan^{-1}x+\cot^{-1}x)^2-2\tan^{-1}x.\cot^{-1}x=\frac{5\pi^2}{8}$$

$$\Rightarrow \left(\frac{\pi}{2}\right)^2-2\tan^{-1}x.\cot^{-1}x=\frac{5\pi^2}{8}$$

$$\Rightarrow 2\tan^{-1}x.\cot^{-1}x=\frac{\pi^2}{4}-\frac{5\pi^2}{8}$$

$$\Rightarrow 2\tan^{-1}x.\cot^{-1}x=\frac{(2-5)\pi^2}{8}=-\frac{3\pi^2}{8}$$

$$\Rightarrow 2\tan^{-1}x.\left(\frac{\pi}{2}-\tan^{-1}x\right)+\frac{3\pi^2}{8}=0$$

$$\Rightarrow \pi\tan^{-1}x-2(\tan^{-1}x)^2+\frac{3\pi^2}{8}=0$$

$$\Rightarrow 16(\tan^{-1}x)^2-8\pi\tan^{-1}x-3\pi^2=0$$

$$\Rightarrow \tan^{-1}x=\frac{8\pi\pm\sqrt{64\pi^2+4\times16\times3\pi^2}}{2\times16}$$

$$\Rightarrow \tan^{-1}x=\frac{8\pi\pm\sqrt{256\pi^2}}{32}=\frac{8\pi\pm16\pi}{32}$$

$$\Rightarrow \tan^{-1}x=-\frac{\pi}{4}\text{ and }\frac{3\pi}{4}$$

$$\because -\frac{\pi}{2}<x<\frac{\pi}{2}\text{ so }\tan^{-1}x=-\frac{\pi}{4}$$

$$\Rightarrow x=\tan\left(-\frac{\pi}{4}\right)$$

$$\Rightarrow x=-1$$

19. Here, $x = a\sin^3 t$, $y = b\cos^3 t$

Differentiating (1) wrt *t*

$$\frac{dx}{dt} = 3a\sin^2 t \times \cos t \quad \text{and}$$

$$\frac{dy}{dt} = -3b\cos^2 t \times \sin t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3b\cos^2 t \times \sin t}{3a\sin^2 t \times \cos t} = -\frac{b}{a}\cot t$$

$\therefore$ Slope of the tangent at $t = \frac{\pi}{2}$

$$\left.\frac{dy}{dx}\right]_{\frac{\pi}{2}} = -\frac{b}{a}\cot\frac{\pi}{2} = 0$$

Hence, equation of tangent is given by

$$y - b\cos^3\frac{\pi}{2} = 0 \text{ or } y = 0$$

20.

Let $I=\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin 2x}dx$

$=\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\left[1-(\sin x-\cos x)^2\right]}dx$(1 mark)

Put $\sin x-\cos x=t$

$\Rightarrow(\cos x+\sin x)dx=dt$

For $x=\pi/4$, $t=0$ and

For $x=0$, $t=-1$

$\therefore\ I=\int_{-1}^{0}\frac{1}{9+16\left[1-(t)^2\right]}dt$

$=\int_{-1}^{0}\frac{1}{25-16(t)^2}dt$

$=\int_{-1}^{0}\frac{1}{5^2-(4t)^2}dt$

$\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

$=\frac{1}{40}\left[\log\left|\frac{5+4t}{5-4t}\right|\right]_{-1}^{0}$

$=\frac{1}{40}\log\left|\frac{1}{\frac{1}{9}}\right|$

$=\frac{1}{40}\log|9|$

21. We shall prove by principle of mathematical induction

Here, let $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

$P(n): A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$,

So, $A^1 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\Rightarrow P(1)$ is true.

Assuming result to be true for n = k i.e. P(k) to be true

$P(k):\ A^k = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$

We have to prove P (k +1)is true,

$P(k+1):\ A^{k+1} = A^1 A^k$

$$\Rightarrow\ A^{k+1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos k\theta - \sin\theta\sin k\theta & \cos\theta\sin k\theta + \sin\theta\cos k\theta \\ -\sin\theta\cos k\theta - \cos\theta\sin k\theta & -\sin\theta\sin k\theta + \cos\theta\cos k\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta + k\theta) & \sin(\theta + k\theta) \\ -\sin(\theta + k\theta) & \cos(\theta + k\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

$\Rightarrow$ P(k + 1)is true.

Thus by principle of mathematical induction

$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$ for all $n \in N$

OR

$$\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \Delta = \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix}$$

But, ω is one of the cube roots of unity

$\therefore 1+\omega+\omega^2 = 0$

$$\therefore \Delta = \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix}$$

Since I column of the determinant is zero therefore,
value of the given determinant Δ is zero.

22.

$f(x) = |1 - x + |x||$, $x \in R$

Let $g(x) = 1 - x + |x|, x \in R$

$1 - x$, being a polynomial function is continuous

$|x|$, being a modulus function is continuous

If a and b are two continuous functions, then a+b is a continuous function

$\therefore g(x) = 1 - x + |x|, x \in R$ is a continuous function

If it is continuous function, then $|g(x)|$ is continuous.

In the given problem,

$\therefore |g(x)| = |1 - x + |x||$..........is continuous.

but $f(x) = |1 - x + |x||$.......given

$\therefore f(x)$ is continuous.

OR

Domainof $\log x$ is $(0,\infty)$

$f(x)=\log x, x\in(0,\infty)$

Let $c\in(0,\infty)$

$$\lim_{x\to c} f(x)=\lim_{x\to c}\log x$$
$$=\lim_{h\to 0}\log(c+h), c>0$$
$$=\lim_{h\to 0}\log\left(c\left(1+\frac{h}{c}\right)\right), c>0$$
$$=\lim_{h\to 0}\log(c)+\lim_{h\to 0}\log\left(1+\frac{h}{c}\right)$$
$$=\log(c)+\lim_{h\to 0}\log\left(1+\frac{h}{c}\right)$$
$$=\log(c)+\lim_{h\to 0}\frac{\log\left(1+\frac{h}{c}\right)}{\frac{h}{c}}\times\frac{h}{c}$$
$$=\log(c)+\lim_{h\to 0}\frac{\log\left(1+\frac{h}{c}\right)}{\frac{h}{c}}\times\lim_{h\to 0}\frac{h}{c}$$

But $\lim_{x\to 0}\frac{\log(1+x)}{x}=1$

$$\therefore \lim_{h\to 0}\frac{\log\left(1+\frac{h}{c}\right)}{\frac{h}{c}}=1$$

$$\therefore \log(c)+\lim_{h\to 0}\frac{\log\left(1+\frac{h}{c}\right)}{\frac{h}{c}}\times\lim_{h\to 0}\frac{h}{c}$$
$$=\log(c)+1\times\lim_{h\to 0}\frac{h}{c}$$
$$=\log(c)+1\times 0$$
$$=\log c$$
$$=f(c)$$

$\Rightarrow$ logarithmic function is continuous at every point in its domain

23.

Let $I = \int \frac{(3\sin\alpha - 2)\cos\alpha}{5 - \cos^2\alpha - 4\sin\alpha} d\alpha$

Let $t = \sin\alpha \Rightarrow dt = \cos\alpha d\alpha$

$\Rightarrow I = \int \frac{(3t - 2)}{5 - (1 - t^2) - 4t} dt$

$\Rightarrow I = \int \frac{(3t - 2)}{t^2 - 4t + 4} dt = \int \frac{(3t - 2)}{(t - 2)^2} dt$

Using the method of Partial fractions

$\frac{(3t - 2)}{(t - 2)^2} = \frac{A}{(t - 2)} + \frac{B}{(t - 2)^2} \Rightarrow A = 3, B = 4$

$I = \int \left[\frac{3}{(t - 2)} + \frac{4}{(t - 2)^2} \right] dt$

$\Rightarrow I = \int \frac{3}{(t - 2)} dt + \int \frac{4}{(t - 2)^2} dt$

$= 3\log|\sin\alpha - 2| - \frac{4}{\sin\alpha - 2} + C$

OR

Let $I = \int \frac{dx}{x^{\frac{1}{2}} + x^{\frac{1}{3}}}$

$= \int \frac{dx}{x^{\frac{1}{3}}\left(1 + x^{\frac{1}{6}}\right)}$

Let $x^{\frac{1}{6}} = t \Rightarrow x = t^6 \Rightarrow dx = 6t^5 dt$

$\Rightarrow I = \int \frac{6t^5 dt}{t^2(1+t)} = \int \frac{6t^3 dt}{(1+t)}$

$= 6\int \frac{\left(t^3 + 1 - 1\right)dt}{(1+t)}$

$= 6\int \frac{\left(t^3 + 1\right)dt}{(1+t)} - 6\int \frac{dt}{(1+t)}$

$= 6\int \frac{(t+1)\left(t^2 - t + 1\right)dt}{(1+t)} - 6\int \frac{dt}{(1+t)}$

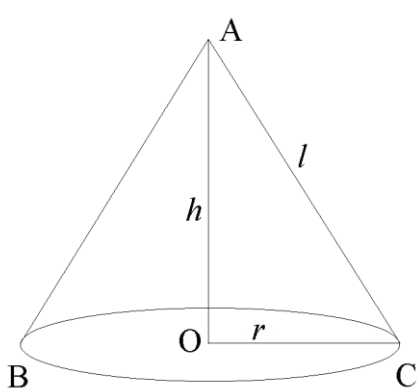
$= 6\int \left(t^2 - t + 1\right)dt - 6\int \frac{dt}{(1+t)}$

$= 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t\right] - 6\log|1+t|$

$= 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left|1 + x^{\frac{1}{6}}\right| + C$

SECTION D

24.



Here, Volume 'V' of the cone is $V = \frac{1}{3}\pi r^2 h \Rightarrow r^2 = \frac{3V}{\pi h}$ ...(1)

Surface area $S = \pi r l = \pi r\sqrt{h^2 + r^2}$ ...(2)

Where h = height of the cone

r = radius of the cone

l = Slant height of the cone

$S^2 = \pi^2 r^2 \left(h^2 + r^2\right)$ from equation (2)

Let, $s_1 = s^2$

Substituting the value of r^2 from equation (1), we have,

$$S_1 = \frac{3\pi V}{h}\left(h^2 + \frac{3V}{\pi h}\right) = 3\pi V h + \frac{9V^2}{h^2}$$

Differentiating S_1 with respect to h, we get

$$\frac{dS_1}{dh} = 3\pi V + 9V^2\left(\frac{-2}{h^3}\right)$$

$$\frac{dS_1}{dh} = 0 \text{ for maxima/minima}$$

$$3\pi V + 9V^2\left(\frac{-2}{h^3}\right) = 0$$

$$\Rightarrow 3\pi V = 9V^2\left(\frac{2}{h^3}\right)$$

$$\Rightarrow h^3 = \frac{6V}{\pi}$$

$$\frac{d^2S_1}{dh^2} = \frac{54V^2}{h^4}$$

$$\frac{d^2S_1}{dh^2} > 0 \text{ at } h^3 = \frac{6V}{\pi^2}$$

Therefore curved surface area is minimum at $\frac{\pi h^3}{6} = V$

Thus, $\frac{\pi h^3}{6} = \frac{1}{3}\pi r^2 h \Rightarrow h^2 = 2r^2$

$$\Rightarrow h = \sqrt{2}r$$

Hence for least curved surface the altitude is $\sqrt{2}$ times radius.

OR

$f(x) = (x-2)^4(x+1)^3$

$f'(x) = 3(x-2)^4(x+1)^2 + 4(x+1)^3(x-2)^3$

$= (x-2)^3(x+1)^2\left[3(x-2)+4(x+1)\right]$

$= (x-2)^3(x+1)^2\left[3x-6+4x+4\right]$

$= (x-2)^3(x+1)^2\left[7x-2\right]$

$f'(x) = 0 \Rightarrow (x-2)^3(x+1)^2\left[7x-2\right] \Rightarrow x = -1, \frac{2}{7}, 2$

Let us examine the behaviour of $f'(x)$, slightly to the left and right of each of these three values of x

(i) $x = -1$:

When $x<-1$; $f'(x) > 0$

When $x>-1$; $f'(x) > 0$

$\Rightarrow x = -1$ is neither a point of local maxima nor minima

$\Rightarrow$ It may be a point of inflexion

(ii) $x = \frac{2}{7}$

When $x<\frac{2}{7}$; $f'(x) > 0$

When $x>\frac{2}{7}$; $f'(x) < 0$

$\Rightarrow x = \frac{2}{7}$ is a point of local maxima

$f\left(\frac{2}{7}\right) = \left(\frac{2}{7}-2\right)^4\left(\frac{2}{7}+1\right)^3 = \left(\frac{-12}{7}\right)^4\left(\frac{9}{7}\right)^3 = \frac{2^8 \times 3^{10}}{7^7}$

$\Rightarrow$ The local maximum value is $\frac{2^8 \times 3^{10}}{7^7}$

(iii) $x = 2$

When $x < 2$; $f'(x) < 0$

When $x > 2$; $f'(x) > 0$

$\Rightarrow x = 2$ is a point of local minima

$f(2) = (2-2)^4(2+1)^3 = 0$

$\Rightarrow$ The local minimum value is 0

25. **Since A = IA**

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$$

Applying $R_3 \to \frac{1}{2} R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_3$

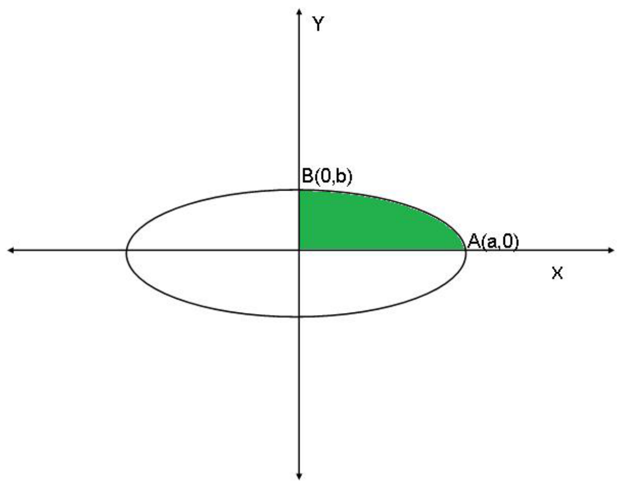
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_3$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\-4 & 3 & -1\\\frac{5}{2} & -\frac{3}{2} & \frac{1}{2}\end{bmatrix}A$$

Hence $A^{-1}=\begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\-4 & 3 & -1\\\frac{5}{2} & -\frac{3}{2} & \frac{1}{2}\end{bmatrix}$

26.



(i) between the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and the x-axis between $x = 0$ to $x = a$

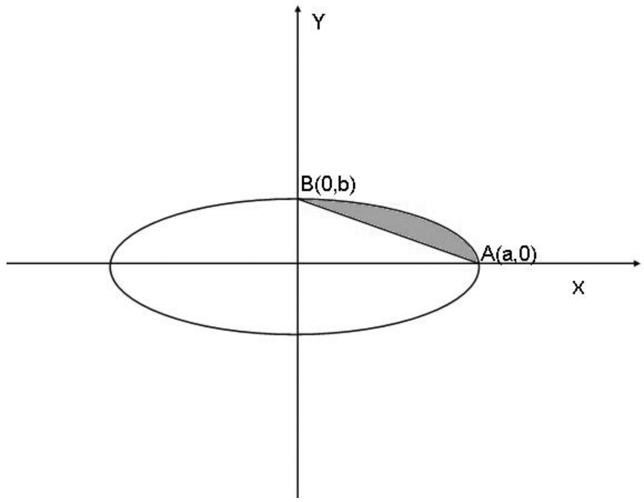
$$\int_0^a b\sqrt{1-\frac{x^2}{a^2}}dx=\frac{b}{a}\int_0^a\sqrt{a^2-x^2}dx$$

$$=\frac{b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$=\frac{b}{2a}\left[\left(0+a^2\sin^{-1}(1)\right)-\left(0+a^2\sin^{-1}(0)\right)\right]$$

$$=\frac{b}{2a}\left[a^2\times\frac{\pi}{2}\right]$$

$$\therefore\int_0^a b\sqrt{1-\frac{x^2}{a^2}}dx=\frac{1}{4}\pi ab$$



(ii) Area of triangle AOB is in the first quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where OA = a and OB = b.

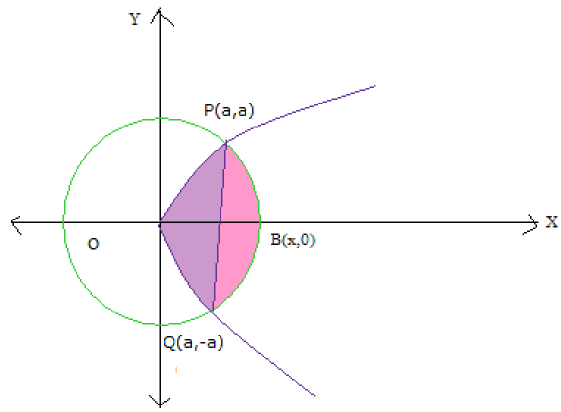
=the area enclosed between the chord AB and the arc AB of the ellipse .

= Area of Ellipse (In quadrant I)- Area of ΔAOB$=\int_0^a b\sqrt{1-\frac{x^2}{a^2}}dx-\frac{1}{2}ab=\frac{1}{4}\pi ab-\frac{1}{2}ab$

$=\frac{(\pi-2)ab}{4}$

(iii) Ratio $=\frac{\frac{1}{4}\pi ab}{\frac{(\pi-2)}{4}ab}=\frac{\pi}{\pi-2}$

OR



The circle is $x^2 + y^2 = 2a^2 \Rightarrow C\ (0, \sqrt{2}a)$

The parabola is $y^2 = ax, a > 0 \Rightarrow y^2 = 4\frac{1}{4}ax, a > 0$

Their point of intersection is given by :$x^2 + ax = 2a^2$

$\Rightarrow x^2 + ax - 2a^2 = 0$

$\Rightarrow \left(x + 2a\right)\left(x - a\right) - 0$

$\Rightarrow x = a, -2a$

$\Rightarrow x = a$

$\Rightarrow y^2 = a^2 \Rightarrow y = \pm a$

$\Rightarrow$ shade region is the smaller of the two areas

in which the circle is divided by the parabola

$$A=2\left[\int_0^a \sqrt{ax}\,dx+\int_a^{\sqrt{2}a}\left(2a^2-x^2\right)^{1/2}dx\right]$$

$$=2\sqrt{a}\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^a+2\left[\frac{x}{2}\left(2a^2-x^2\right)^{1/2}+\frac{2a^2}{2}\sin^{-1}\frac{x}{\sqrt{2}a}\right]_a^{\sqrt{2}a}$$

$$=\frac{4}{3}\sqrt{a}\left[a^{3/2}\right]+\left[x\left(2a^2-x^2\right)^{1/2}+2a^2\sin^{-1}\frac{x}{\sqrt{2}a}\right]_a^{\sqrt{2}a}$$

$$=\frac{4}{3}\sqrt{a}\left[a^{3/2}\right]+\left[2a^2\sin^{-1}\frac{\sqrt{2}a}{\sqrt{2}a}-a\left(2a^2-a^2\right)^{1/2}-2a^2\sin^{-1}\frac{a}{\sqrt{2}a}\right]$$

$$=\frac{4}{3}\sqrt{a}\left[a^{3/2}\right]+\left[2a^2\sin^{-1}1-a\left(a^2\right)^{1/2}-2a^2\sin^{-1}\frac{1}{\sqrt{2}}\right]$$

$$=\frac{4}{3}\sqrt{a}\left[a^{3/2}\right]+\left[2a^2\frac{\pi}{2}-a^2-2a^2\frac{\pi}{4}\right]$$

$$=\frac{4}{3}\sqrt{a}\left[a^{3/2}\right]+2a^2\frac{\pi}{4}-a^2$$

$$=\frac{4}{3}\sqrt{a}\left[a^{3/2}\right]+a^2\frac{\pi}{2}-a^2\text{sq. units}$$

27.

$\vec{r}.\ (\hat{i} + \hat{j} + \hat{k}) - 1 = 0$ and $\vec{r}.\ (2\hat{i} + 3\hat{j} - \hat{k}) = -4$

$\Rightarrow x + y + z - 1 = 0$ and $2x + 3y - z + 4 = 0$

The required plane passes through the line common to two intersectiing planes

$\Rightarrow x + y + z - 1 + k(2x + 3y - z + 4) = 0$

$\Rightarrow x(1 + 2k) + y(1 + 3k) + z(1 - k) + (-1 + 4k) = 0 ......(1)$

The required plane is parallel to the X-axis whose d.cs are 1,0,0

$\therefore 1.(1 + 2k) + 0.(1 + 3k) + 0.(1 - k) = 0$

$\Rightarrow (1 + 2k) = 0 \Rightarrow k = -\frac{1}{2}$

Substituting in (1), we get

$x(1 + 2\left(-\frac{1}{2}\right)) + y(1 + 3\left(-\frac{1}{2}\right)) + z(1 - \left(-\frac{1}{2}\right)) + (-1 + 4\left(-\frac{1}{2}\right)) = 0$

$\Rightarrow x(0) + y(\frac{-1}{2}) + z(\frac{3}{2}) - 3 = 0$

$\Rightarrow -y + 3z - 6 = 0 \Rightarrow y - 3z + 6 = 0$

28. Let the two carpenters work for x days and y days respectively.

Our problem is to minimize the objective function.

$C = 150x + 200y$

Subject to the constraints

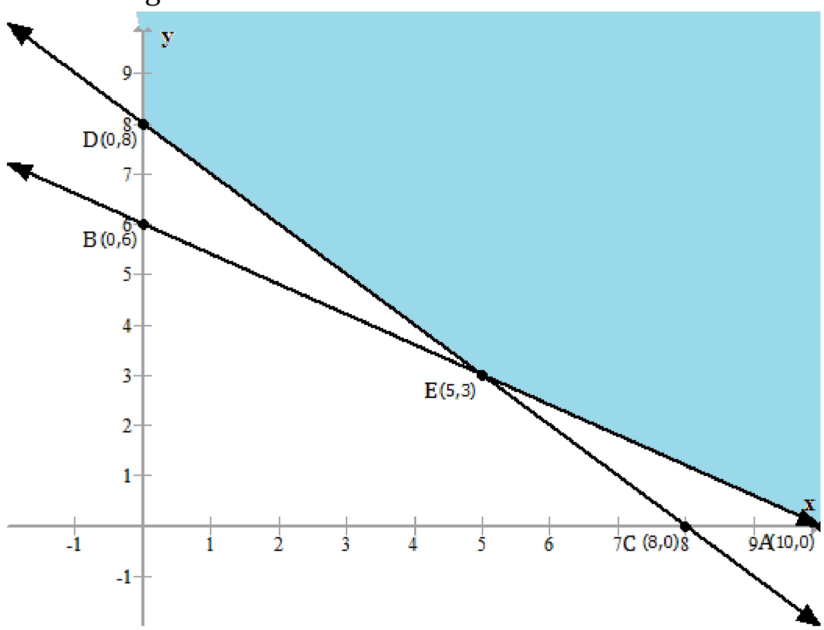
$6x + 10y \geq 60 \Leftrightarrow 3x + 5y \geq 30$

$4x + 4y \geq 32 \Leftrightarrow x + y \geq 8$

And

$x \geq 0, y \geq 0$

Feasible region is shaded.



This region is unbounded.

Corner points	Objective function values $C = 150x + 200y$
A(10, 0)	1500
E(5, 3)	1350
D(0, 8)	1600

The labour cost is the least, when carpenter A works for 5 days and carpenter B works for 3 days.

29.

$$(i)\sum_{i=0}^{7} P(X_i) = 1$$

$$\Rightarrow \left[0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k\right] = 1$$

$$\Rightarrow 10k^2 + 9k = 1 \Rightarrow 10k^2 + 9k - 1 = 0 \Rightarrow 10k^2 + 10k - k - 1 = 0$$

$$\Rightarrow 10k(k+1) - (k+1) = 0$$

$$\Rightarrow (k+1)(10k-1) = 0$$

$$\Rightarrow k = -1, k = \frac{1}{10}$$

k, also being a probability cannot be negative

$$\Rightarrow k = \frac{1}{10}$$

$$(ii)P(X<3) = P(0) + P(1) + P(2) = 0 + k + 2k = 3k = \frac{3}{10}$$

$$(ii)P(X>6) = P(7) + P(8) = 2k^2 + 7k^2 + k = 2\left(\frac{1}{10}\right)^2 + 7\left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right) = \frac{19}{100}$$

$$(iii)P(1 \le X < 3) = P(1) + P(2) = k + 2k = 3k = \frac{3}{10}$$

OR

Let E_1, E_2, E_3 and A be the events defined as follows:

Let E_1 be the event that the student knows the answer.

Let E_2 be the event that the student guesses the answer.

Let E_3 be the event that the student copies the answer.

Let A be the event that the answer is correct.

$P(E_1)=\frac{1}{2}; P(E_2)=\frac{1}{4}; P(E_3)=\frac{1}{4};$

Probability that he answers correctly given that he knew the answer is 1.

That is, $P(A|E_1)=1$

If E_2 has already occurred, then the student guesses. Since there are four choices out of which only one is correct, therefore, the probability that he answers correctly given that he has made a guess is $\frac{1}{4}$.

That is $P(A|E_2)=\frac{1}{4}$.

It is given that, $P(A|E_3)=\frac{3}{4}$

By Bayes Theorem, we have,

Required Probability=$P\left(E_1 \mid A\right)$

$$= \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$= \frac{\frac{1}{2}\times 1}{\frac{1}{2}\times 1+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{16}+\frac{3}{16}}$$

$$= \frac{\frac{1}{2}}{\frac{8}{16}+\frac{1}{16}+\frac{3}{16}}$$

$$= \frac{\frac{1}{2}}{\frac{12}{16}}$$

$$= \frac{1}{2}\times\frac{16}{12}$$

$$= \frac{1}{2}\times\frac{4}{3}$$

$$= \frac{2}{3}$$

Thus, the probability that student knows the answer, given that he answered it correctly is $\frac{2}{3}$.

Arjun is dishonest, as he copies from the other student(s).

Copying once may become habit forming as he may continue resort to dishonest means in the coming years.