

SAMPLE QUESTION PAPER

CLASS-XII (2016-17)

MATHEMATICS (041)

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION-A

Questions from 1 to 4 are of 1 mark each.

- 1. What is the principal value of $\tan^{-1}\left(\tan \frac{2\pi}{3}\right)$?
- 2. A and B are square matrices of order 3 each, $|A| = 2$ and $|B| = 3$. Find $|3AB|$
- 3. What is the distance of the point (p, q, r) from the x-axis?
- 4. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 3x^2 - 5$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $g(x) = \frac{x}{x^2+1}$. Find $g \circ f$

SECTION-B

Questions from 5 to 12 are of 2 marks each.

- 5. How many equivalence relations on the set $\{1,2,3\}$ containing (1,2) and (2,1) are there in all? Justify your answer.
- 6. Let l_i, m_i, n_i ; $i = 1, 2, 3$ be the direction cosines of three mutually perpendicular vectors in space. Show that $AA' = I_3$, where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.
- 7. If $e^y (x+1) = 1$, show that $\frac{dy}{dx} = -e^y$
- 8. Find the sum of the order and the degree of the following differential equations:

$$\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1+x) = 0$$

9. Find the Cartesian and Vector equations of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}$
10. Solve the following Linear Programming Problem graphically:
 Maximize $Z = 3x + 4y$
 subject to
 $x + y \leq 4, x \geq 0$ and $y \geq 0$
11. A couple has 2 children. Find the probability that both are boys, if it is known that (i) one of them is a boy (ii) the older child is a boy.
12. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which its area increases, when side is 10 cm long.

SECTION-C

Questions from 13 to 23 are of 4 marks each.

13. If $A + B + C = \pi$, then find the value of

$$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$$

OR

Using properties of determinant, prove that

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

14. It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ Rolle's theorem holds in $[1, 3]$ with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of 'a' and 'b'
15. Determine for what values of x, the function $f(x) = x^3 + \frac{1}{x^3}$ ($x \neq 0$) is strictly increasing or strictly decreasing

OR

Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$

16. Evaluate $\int_0^2 (x^2 + 3) dx$ as limit of sums.
17. Find the area of the region bounded by the y-axis, $y = \cos x$ and $y = \sin x$, $0 \leq x \leq \frac{\pi}{2}$
18. Can $y = ax + \frac{b}{a}$ be a solution of the following differential equation?

$$y = x \frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \dots\dots\dots (*)$$

 If no, find the solution of the D.E. (*).

OR

Check whether the following differential equation is homogeneous or not

$$x^2 \frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$$

Find the general solution of the differential equation using substitution $y=vx$.

19. If the vectors $\vec{p} = a\hat{i} + \hat{j} + \hat{k}$, $\vec{q} = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + c\hat{k}$ are coplanar, then for a, b, c $\neq 1$ show that

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

20. A plane meets the coordinate axes in A, B and C such that the centroid of ΔABC is the point (α, β, γ) . Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$
21. If a 20 year old girl drives her car at 25 km/h, she has to spend Rs 4/km on petrol. If she drives her car at 40 km/h, the petrol cost increases to Rs 5/km. She has Rs 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem. Write any one value reflected in the problem.
22. The random variable X has a probability distribution P(X) of the following form, where k is some number:
- $$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$
- (i) Find the value of k (ii) Find $P(X < 2)$ (iii) Find $P(X \leq 2)$ (iv) Find $P(X \geq 2)$
23. A bag contains $(2n + 1)$ coins. It is known that 'n' of these coins have a head on both its sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, find the value of 'n'.

SECTION-D

Questions from 24 to 29 are of 6 marks each

24. Using properties of integral, evaluate $\int_0^{\pi} \frac{x}{1 + \sin x} dx$

OR

Find: $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx$

25. Does the following trigonometric equation have any solutions? If Yes, obtain the solution(s):

$$\tan^{-1} \left(\frac{x+1}{x-1} \right) + \tan^{-1} \left(\frac{x-1}{x} \right) = -\tan^{-1} 7$$

OR

Determine whether the operation * define below on \mathbb{Q} is binary operation or not.

$$a * b = ab + 1$$

If yes, check the commutative and the associative properties. Also check the existence of identity element and the inverse of all elements in \mathbb{Q} .

26. Find the value of x, y and z, if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$

OR

Verify: $A(\text{adj } A) = (\text{adj } A)A = |A| I$ for matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

27. Find $\frac{dy}{dx}$, if $y = e^{\sin^2 x} \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$
28. Find the shortest distance between the line $x - y + 1 = 0$ and the curve $y^2 = x$
29. Define skew lines. Using only vector approach, find the shortest distance between the following two skew lines:
- $$\vec{r} = (8 + 3\lambda) \hat{i} - (9 + 16\lambda) \hat{j} + (10 + 7\lambda) \hat{k}$$
- $$\vec{r} = 15 \hat{i} + 29 \hat{j} + 5 \hat{k} + \mu (3 \hat{i} + 8 \hat{j} - 5 \hat{k})$$

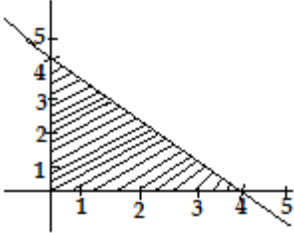
SAMPLE QUESTION PAPER

CLASS-XII (2016-17)

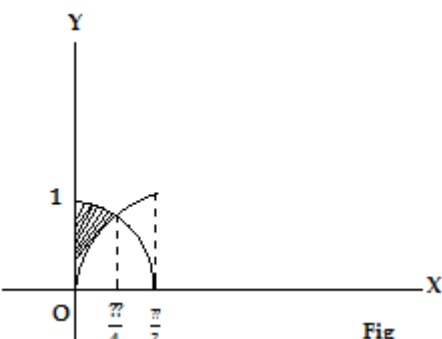
MATHEMATICS (041)

Marking Scheme

1.	$\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\left(-\tan\frac{\pi}{3}\right) = -\frac{\pi}{3}$	1
2.	$ 3AB = 3^3 A B = 27 \times 2 \times 3 = 162$	1
3.	Distance of the point (p, q, r) from the x-axis $=$ Distance of the point (p, q, r) from the point (p,0,0) $= \sqrt{q^2 + r^2}$	1
4.	$\text{gof}(x) = g\{f(x)\} = g(3x^2 - 5) = \frac{3x^2 - 5}{(3x^2 - 5)^2 + 1} = \frac{3x^2 - 5}{9x^4 - 30x^2 + 26}$	1
5.	Equivalence relations could be the following: $\{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ and (1) $\{(1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$ (1) So, only two equivalence relations.(Ans.)	2
6.	$AA' = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 \longrightarrow (1)$ because $l_i^2 + m_i^2 + n_i^2 = 1$, for each $i = 1, 2, 3 \longrightarrow 1/2$ $l_i l_j + m_i m_j + n_i n_j = 0$ ($i \neq j$) for each $i, j = 1, 2, 3 \longrightarrow 1/2$	2
7.	On differentiating $e^y (x + 1) = 1$ w.r.t. x, we get $e^y + (x + 1) e^y \frac{dy}{dx} = 0 \longrightarrow (1)$ $\Rightarrow e^y + \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -e^y \longrightarrow (1)$	2
8.	Here, $\left\{ \frac{d^2y}{dx^2} + (1 + x) \right\}^3 = -\frac{dy}{dx} \longrightarrow (1)$ Thus, order is 2 and degree is 3. So, the sum is 5 $\longrightarrow (1)$	2
9.	Here, $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z-8}{-6}$ is same as $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z-8}{6}$ Cartesian equation of the line is $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6} \longrightarrow (1)$ Vector equation of the line is $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k}) \longrightarrow (1)$	2

10.	<p>The feasible region is a triangle with vertices $O(0,0)$, $A(4,0)$ and $B(0,4)$</p> $\left. \begin{aligned} Z_0 &= 3 \times 0 + 4 \times 0 = 0 \\ Z_A &= 3 \times 4 + 4 \times 0 = 12 \\ Z_B &= 3 \times 0 + 4 \times 4 = 16 \end{aligned} \right\} (1)$ <p>Thus, maximum of Z is at $B(0,4)$ and the maximum value is 16 $\longrightarrow \frac{1}{2}$</p> 	2
11.	<p>Sample space = $\{ B_1B_2, B_1G_2, G_1B_2, G_1G_2 \}$, B_1 and G_1 are the older boy and girl respectively.</p> <p>Let E_1 = both the children are boys; E_2 = one of the children is a boy ; E_3 = the older child is a boy</p> <p>Then, (i) $P(E_1/E_2) = P\left(\frac{E_1 \cap E_2}{E_2}\right) = \frac{1/4}{3/4} = \frac{1}{3} \longrightarrow (1)$</p> <p>(ii) $P(E_1/E_3) = P\left(\frac{E_1 \cap E_3}{E_3}\right) = \frac{1/4}{2/4} = \frac{1}{2} \longrightarrow (1)$</p>	2
12.	<p>Here, $\text{Area}(A) = \frac{\sqrt{3}}{4} x^2$, where '$x$' is the side of the equilateral triangle $\longrightarrow \frac{1}{2}$</p> <p>So, $\frac{dA}{dt} = \frac{\sqrt{3}}{2} x \frac{dx}{dt} \longrightarrow (1)$</p> <p>$= \frac{\sqrt{3}}{2} (10) (2) = 10\sqrt{3} \text{ cm}^2/\text{sec} \longrightarrow \frac{1}{2}$</p>	2
13.	<p>As $A + B + C = \pi$,</p> $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} = \begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix} \longrightarrow (2)$ $= 0 \times \begin{vmatrix} 0 & \tan A \\ -\tan A & 0 \end{vmatrix} - \sin B \times \begin{vmatrix} -\sin B & \tan A \\ -\cos C & 0 \end{vmatrix} + \cos C \times \begin{vmatrix} -\sin B & 0 \\ -\cos C & -\tan A \end{vmatrix}$ $= 0 - \sin B \tan A \cos C + \cos C \sin B \tan A = 0 \text{ (Ans.)} \longrightarrow (2)$ <p style="text-align: center;">OR</p>	4

	<p>Let $\Delta = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$</p> <p>Applying $C_1 \rightarrow C_1 + C_3$, we get $\Delta = (a+b+c) \begin{vmatrix} 1 & a-b & a \\ 1 & b-c & b \\ 1 & c-a & c \end{vmatrix} \longrightarrow (1)$</p> <p>Applying $R_2 \rightarrow R_2 - R_1$, and $R_3 \rightarrow R_3 - R_1$, we get</p> $\Delta = (a+b+c) \begin{vmatrix} 1 & a-b & a \\ 0 & 2b-a-c & b-a \\ 0 & 2a+b+c & c-a \end{vmatrix} \longrightarrow (1)$ <p>Expanding Δ along first column, we have the result $\longrightarrow (2)$</p>	4
14.	<p>Since Rolle's theorem holds true, $f(1) = f(3)$</p> <p>i.e., $(1)^3 - 6(1)^2 + a(1) + b = (3)^3 - 6(3)^2 + a(3) + b$</p> <p>i.e., $a + b + 22 = 3a + b$</p> $\Rightarrow a = 11 \longrightarrow (2)$ <p>Also, $f'(x) = 3x^2 - 12x + a$ or $3x^2 - 12x + 11$</p> <p>As $f'(c) = 0$, we have</p> $3\left(2 + \frac{1}{\sqrt{3}}\right)^2 - 12\left(2 + \frac{1}{\sqrt{3}}\right) + 11 = 0$ <p>As it is independent of b, b is arbitrary. $\longrightarrow (2)$</p>	4
15.	<p>Here, $f'(x) = 3x^2 - 3x^{-4} = \frac{3(x^6 - 1)}{x^4} \longrightarrow (1)$</p> $= \frac{3(x^4 + x^2 + 1)}{x^4} (x+1)(x-1)$ <p>Critical points are -1 and $1 \longrightarrow (1)$</p> <p>$\Rightarrow f'(x) > 0$ if $x > 1$ or $x < -1$; and $f'(x) < 0$ if $-1 < x < 1$</p> $\left\{ \because \frac{3(x^4 + x^2 + 1)}{x^4} \text{ always +ive} \right\}$ <p>Hence, $f(x)$ is strictly increasing for $x > 1 \longrightarrow (1)$</p> <p>or $x < -1$; and strictly decreasing for</p> $(-1, 0) \cup (0, 1) [1] \longrightarrow (1)$ <p style="text-align: center;">OR</p> <p>Here, $\frac{dy}{dx} = 3x^2 - 11 \longrightarrow \frac{1}{2}$</p> <p>So, slope of the tangent is $3x^2 - 11$</p>	4

	<p>Slope of the given tangent line is 1.</p> <p>Thus, $3x^2 - 11 = 1 \longrightarrow (1)$</p> <p>that gives $x = \pm 2$</p> <p>When $x = 2$, $y = 2 - 11 = -9$</p> <p>When $x = -2$, $y = -2 - 11 = -13$</p> <p>Out of the two points $(2, -9)$ and $(-2, -13) \longrightarrow (2)$</p> <p>only the point $(2, -9)$ lies on the curve</p> <p>Thus, the required point is $(2, -9) \longrightarrow \frac{1}{2}$</p>	
16.	<p>Here, $f(x) = x^2 + 3$, $a = 0$, $b = 2$ and $nh = b - a = 2 \longrightarrow (1)$</p> $\int_0^2 (x^2 + 1) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a + \overline{n-1}h)] \longrightarrow (1)$ $= \lim_{h \rightarrow 0} h [3 + 1^2h^2 + 3 + 2^2h^2 + 3 + \dots + (n-1)^2h^2 + 3]$ $= \lim_{h \rightarrow 0} h [3n + h^2 \{1^2 + 2^2 + 3^2 + \dots + (n-1)^2\}]$ $= \lim_{h \rightarrow 0} [3nh + h^3 \{ \frac{(n-1)n(2n-1)}{6} \}]$ $= \lim_{h \rightarrow 0} [3nh + \{ \frac{(nh-h)nh(2nh-h)}{6} \}] \longrightarrow (1)$ $= \lim_{h \rightarrow 0} [3 \times 2 + \{ \frac{(2-h)2(4-h)}{6} \}]$ $= 6 + \frac{16}{6}, \text{ i.e., } \frac{26}{3} \longrightarrow (1)$	4
17.	<p>The rough sketch of the bounded region is shown on the right. $\longrightarrow (1)$</p> <p>Required area $= \int_0^{\pi/4} \cos x dx - \int_0^{\pi/4} \sin x dx \longrightarrow (1)$</p> $= (\sin x + \cos x) \Big _0^{\pi/4} \longrightarrow (1)$ $= \sin \frac{\pi}{4} + \cos \frac{\pi}{4} - \sin 0 - \cos 0$ $= \frac{2}{\sqrt{2}} - 1, \text{ i.e., } (\sqrt{2} - 1) \text{ sq units } \longrightarrow (1)$  <p style="text-align: center;">Fig</p>	4
18.	<p>$y = ax + \frac{b}{a} \dots (1)$</p> <p>gives $\frac{dy}{dx} = a \longrightarrow \left(1 \frac{1}{2}\right)$</p> <p>Substituting this value of 'a' in (1), we get</p>	4

	$y = x \frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \longrightarrow \left(1 \frac{1}{2}\right)$ <p>Thus, $y = ax + \frac{b}{a}$ is a solution of the following differential equation $y = x \frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \longrightarrow 1$</p> <p style="text-align: center;">OR</p> <p>Given differential equation can be written as</p> $\frac{dy}{dx} = \frac{1+xy + \cos\left(\frac{y}{x}\right)}{x^2} = \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{x^2}\right] \dots\dots(1)$ <p>Let $F(x,y) = \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{x^2}\right]$.</p> <p>Then $F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} + \left[\frac{1 + \cos\left(\frac{\lambda y}{\lambda x}\right)}{(\lambda x)^2}\right]$</p> $= \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{\lambda^2 x^2}\right] \neq F(x,y)$ <p>Hence, the given D.E. is not a homogeneous equation. $\longrightarrow (1)$</p> <p>Putting $y = vx$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$ in (1), we get</p> $v + x \frac{dv}{dx} = v + \frac{1 + \cos v}{x^2}$ $\Rightarrow \frac{dv}{1 + \cos v} = \frac{1}{x^3} dx$ $\Rightarrow \sec^2\left(\frac{v}{2}\right) dv = \frac{2}{x^3} dx \longrightarrow (1)$ <p>Integrating both sides, we get</p> $2 \tan \frac{v}{2} = -\frac{1}{x^2} + C \longrightarrow 1 \frac{1}{2}$ $\text{or } 2 \tan \frac{y}{2x} = -\frac{1}{x^2} + C \longrightarrow \frac{1}{2}$	4
19.	<p>Since the vector \vec{p}, \vec{q} and \vec{r} are coplanar</p> $\therefore [\vec{p}, \vec{q}, \vec{r}] = 0$ $[\vec{p} \quad \vec{q} \quad \vec{r}] = 0 \longrightarrow (1)$ <p>i.e., $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$</p> $\begin{matrix} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{matrix} \longrightarrow (1)$ <p>or $\begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0$</p>	4

	$\Rightarrow a(b-1)(c-1) - 1(1-a)(c-1) - 1(1-a)(b-1) = 0$ i.e., $a(1-b)(1-c) + (1-a)(1-c) + (1-a)(1-b) = 0 \longrightarrow (1)$ Dividing both the sides by $(1-a)(1-b)(1-c)$, we get $\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$ i.e., $-\left(1 - \frac{1}{1-a}\right) + \frac{1}{1-b} + \frac{1}{1-c} = 0$ i.e., $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1 \longrightarrow (1)$									
20.	We know that the equation of the plane having intercepts a, b and c on the three coordinate axes is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \longrightarrow (1)$ Here, the coordinates of A, B and C are (a,0,0), (0,b,0) and (0,0,c) respectively. The centroid of ΔABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) \longrightarrow (1)$ Equating $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ to (α, β, γ) , we get $a = 3\alpha, b = 3\beta$ and $c = 3\gamma \longrightarrow (1)$ Thus, the equation of the plane is $\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$ or $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3 \longrightarrow (1)$	4								
21.	Let the distance covered with speed of 25 km/h = x km and the distance covered with speed of 40 km/h = y km $\left(\frac{1}{2}\right)$ Total distance covered = z km The L.P.P. of the above problem, therefore, is $\longrightarrow (1)$ Maximize $z = x + y$ subject to constraints $\left. \begin{array}{l} 4x + 5y \leq 200 \\ \frac{x}{25} + \frac{y}{40} \leq 1 \\ x \geq 0, y \geq 0 \end{array} \right\} \begin{array}{l} \longrightarrow (1) \\ \longrightarrow (1) \\ \longrightarrow (1) \end{array}$ Any one value $\longrightarrow \left(\frac{1}{2}\right)$	4								
22.	Here, <table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>k</td><td>2k</td><td>3k</td></tr></table> (i) Since $P(0) + P(1) + P(2) = 1$, we have	X	0	1	2	P(X)	k	2k	3k	4
X	0	1	2							
P(X)	k	2k	3k							

	$k + 2k + 3k = 1$ $\text{i.e., } 6k = 1, \text{ or } k = \frac{1}{6} \longrightarrow (1)$ $\text{(ii) } P(X < 2) = P(0) + P(1) = k + 2k = 3k = \frac{1}{2}; \longrightarrow (1)$ $\text{(iii) } P(X \leq 2) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k = 1 \longrightarrow (1)$ $\text{(iv) } P(X \geq 2) = P(2) = 3k = \frac{1}{2} \longrightarrow (1)$	
23.	<p>Let the events be described as follows:</p> <p>E_1 : a coin having head on both sides is selected.</p> <p>E_2 : a fair coin is selected.</p> <p>A : head comes up in tossing a selected coin</p> $P(E_1) = \frac{n}{2n+1}; P(E_2) = \frac{n+1}{2n+1}; P(A/E_1) = 1; P(A/E_2) = \frac{1}{2} \longrightarrow (2)$ <p>It is given that $P(A) = \frac{31}{42}$. So,</p> $P(E_1) P(A/E_1) + P(E_2) P(A/E_2) = \frac{31}{42}$ $\Rightarrow \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2} = \frac{31}{42} \longrightarrow (1)$ $\Rightarrow \frac{1}{2n+1} \left[n + \frac{n+1}{2} \right] = \frac{31}{42}$ $\Rightarrow 42(3n+1) = 62(2n+1)$ $\Rightarrow 2n = 20, \text{ or } n = 10 \longrightarrow (1)$	4
24.	$I = \int_0^{\pi} \frac{x}{1+\sin x} dx = \int_0^{\pi} \frac{\pi-x}{1+\sin(\pi-x)} dx \quad (1)$ $= \pi \int_0^{\pi} \frac{1}{1+\sin x} dx - \int_0^{\pi} \frac{x}{1+\sin x} dx$ $\Rightarrow 2I = \pi \int_0^{\pi} \frac{1}{1+\sin x} dx \quad (1)$ $\Rightarrow \frac{\pi}{2} \int_0^{\pi} \frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)} dx$ $\Rightarrow \frac{\pi}{2} \int_0^{\pi} \frac{1}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)} dx$ $\Rightarrow \frac{\pi}{4} \int_0^{\pi} \sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right) dx \quad (1)$ $\Rightarrow I = \frac{\pi}{4} \left[-2\tan\left[\left(\frac{\pi}{4}-\frac{x}{2}\right)\right] \right]_0^{\pi} \quad (2)$ $\Rightarrow I = \frac{\pi}{4} [2 - (-2)] = \pi \quad (1)$ <p style="text-align: center;">OR</p>	6

	<p>Let $I = \int \frac{\sin x}{\sin^3 x + \cos^3 x} dx = \int \frac{\tan x \sec^2 x}{\tan^3 x + 1} dx$ (½)</p> <p>On substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get (1)</p> <p>$I = \int \frac{t}{t^3 + 1} dt = \int \frac{t}{(t+1)(t^2 - t + 1)} dt$ (½)</p> <p>$= -\frac{1}{3} \int \frac{1}{t+1} dt + \frac{1}{3} \int \frac{t+1}{t^2 - t + 1} dt$</p> <p>$= -\frac{1}{3} \log t+1 + \frac{1}{6} \int \frac{(2t-1)+3}{t^2 - t + 1} dt$ (1)</p> <p>$= -\frac{1}{3} \log t+1 + \frac{1}{6} \int \frac{2t-1}{t^2 - t + 1} dt + \frac{1}{2} \int \frac{1}{t^2 - t + 1} dt$</p> <p>$= -\frac{1}{3} \log t+1 + \frac{1}{6} \log t^2 - t + 1 + \frac{1}{2} \int \frac{1}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dt$</p> <p>$= -\frac{1}{3} \log t+1 + \frac{1}{6} \log t^2 - t + 1 + \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)$ (2)</p> <p>$= -\frac{1}{3} \log \tan x + 1 + \frac{1}{6} \log \tan^2 x - \tan x + 1 + \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + c$ (1)</p>	6
25.	<p>$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = -\tan^{-1} 7$</p> <p>$\Rightarrow \tan^{-1}\left[\frac{\left(\frac{x+1}{x-1}\right) + \left(\frac{x-1}{x}\right)}{1 - \left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)}\right] = -\tan^{-1} 7$, if $\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right) < 1$(*) (2)</p> <p>$\Rightarrow \tan^{-1}\left[\frac{x(x+1) + (x-1)^2}{(x-1)x - (x+1)(x-1)}\right] = -\tan^{-1} 7$</p> <p>$\Rightarrow \frac{(x^2+x) + (x^2+1-2x)}{(x^2-x) - (x^2-1)} = \tan[-\tan^{-1} 7]$</p> <p>$\Rightarrow \frac{2x^2 - x + 1}{-x + 1} = -7$ (1)</p> <p>$\Rightarrow 2x^2 - 8x + 8 = 0$</p> <p>$\Rightarrow (x - 2)^2 = 0$</p> <p>$\Rightarrow x = 2$ (1)</p> <p>Let us now verify whether $x = 2$ satisfies the condition (*)</p> <p>For $x = 2$,</p> <p>$\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right) = 3 \times \frac{1}{2} = \frac{3}{2}$ which is not less than 1</p> <p>Hence, this value does not satisfy the condition (*) (1)</p> <p>i.e., there is no solution to the given trigonometric equation. (1)</p> <p style="text-align: center;">OR</p> <p>Given * on Q, defined by $a*b = ab+1$</p> <p>Let, $a \in Q, b \in Q$ then</p> <p>$ab \in Q$</p>	6

	<p>and $(ab+1) \in \mathbb{Q}$</p> <p>$\Rightarrow a*b = ab+1$ is defined on \mathbb{Q}</p> <p>$\therefore *$ is a binary operation on \mathbb{Q} (1)</p> <p>Commutative: $a*b = ab+1$</p> <p>$b*a = ba+1$</p> <p>$= ab+1 \quad (\because ba = ab \text{ in } \mathbb{Q})$</p> <p>$\Rightarrow a*b = b*a$</p> <p>So $*$ is commutative on \mathbb{Q} (1)</p> <p>Associative: $(a*b)*c = (ab+1)*c = (ab+1)c+1$</p> <p>$= abc+c+1$</p> <p>$a*(b*c) = a*(bc+1)$</p> <p>$= a(bc+1)+1$</p> <p>$= abc+a+1$</p> <p>$\therefore (a*b)*c \neq a*(b*c)$</p> <p>So $*$ is not associative on \mathbb{Q} (1)</p> <p>Identity Element : Let $e \in \mathbb{Q}$ be the identity element, then for every $a \in \mathbb{Q}$</p> <p>$a*e = a$ and $e*a = a$</p> <p>$ae+1 = a$ and $ea+1 = a$</p> <p>$\Rightarrow e = \frac{a-1}{a}$ and $e = \frac{a-1}{a}$ (1)</p> <p>e is not unique as it depend on 'a', hence identity element does not exist for $*$ (1)</p> <p>Inverse: since there is no identity element hence, there is no inverse. (1)</p>	6
26.	<p>The relation $A' = A^{-1}$ gives $A'A = A^{-1}A = I$ (1)</p> <p>Thus, $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\left(1 \frac{1}{2}\right)$</p> <p>$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$</p> <p>$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (2)</p> <p>$\Rightarrow 2x^2 = 1; 6y^2 = 1 \text{ and } 3z^2 = 1$</p> <p>$\Rightarrow x = \pm \frac{1}{\sqrt{2}}; y = \pm \frac{1}{\sqrt{6}}; z = \pm \frac{1}{\sqrt{3}}$ $\left(1 \frac{1}{2}\right)$</p> <p style="text-align: center;">OR</p>	6

	<p>Here, $A = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} = 1(0+0) + 1(9+2) + 2(0-0) = 11$ (1)</p> <p>$\Rightarrow A I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$(1) (1/2)</p> <p>$\text{adj } A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$ (2)</p> <p>Now, $A(\text{adj } A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ (1)</p> <p>and $(\text{adj } A)A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ (1)</p> <p>Thus, it is verified that $A(\text{adj } A) = (\text{adj } A)A = A I$ (1/2)</p>	6
27.	<p>Putting $x = \cos 2\theta$ in $\left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$, we get (1)</p> $2 \tan^{-1} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$ <p>i.e., $2 \tan^{-1} \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} = 2 \tan^{-1}(\tan \theta) = 2\theta = \cos^{-1} x$ (2)</p> <p>Hence, $y = e^{\sin^2 x} \cos^{-1} x$</p> <p>$\Rightarrow \log y = \sin^2 x + \log (\cos^{-1} x)$</p> <p>$\Rightarrow \frac{1}{y} \times \frac{dy}{dx} = 2 \sin x \cos x + \frac{1}{\cos^{-1} x} \times \frac{-1}{\sqrt{1-x^2}} = \sin 2x - \frac{1}{\cos^{-1} x \sqrt{1-x^2}}$ (2)</p> <p>$\Rightarrow \frac{dy}{dx} = e^{\sin^2 x} \cos^{-1} x \left[\sin 2x - \frac{1}{\cos^{-1} x \sqrt{1-x^2}} \right]$ (1)</p>	6
28.	<p>Let (t^2, t) be any point on the curve $y^2 = x$. Its distance (S) from the line $x - y + 1 = 0$ is given by $\frac{1}{\sqrt{2}}$</p> $S = \left \frac{t^2 - t - 1}{\sqrt{1+1}} \right \quad \frac{1}{\sqrt{2}}$ $= \frac{t^2 - t + 1}{\sqrt{2}} \quad \left\{ \because t^2 - t + 1 = \left(t - \frac{1}{2}\right)^2 + \frac{3}{4} > 0 \right\} \quad (1)$ $\Rightarrow \frac{dS}{dt} = \frac{1}{\sqrt{2}} (2t-1) \quad (1)$ <p>and $\frac{d^2S}{dt^2} = \sqrt{2} > 0$ (1)</p> <p>Now, $\frac{dS}{dt} = 0 \Rightarrow \frac{1}{\sqrt{2}} (2t-1) = 0$,i.e., $t = \frac{1}{2}$ (1)</p> <p>Thus, S is minimum at $t = \frac{1}{2}$</p>	6

So, the required shortest distance is $\frac{(\frac{1}{2})^2 - (\frac{1}{2}) + 1}{\sqrt{2}} = \frac{3}{4\sqrt{2}}$, or $\frac{3\sqrt{2}}{8}$ (1)

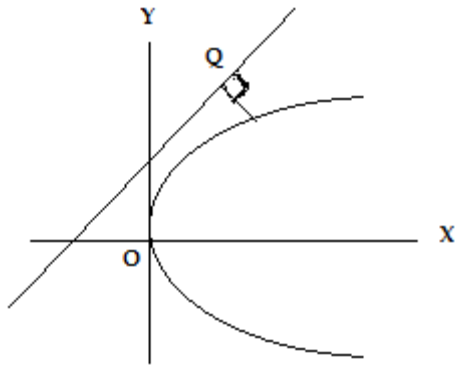


Fig. 1

29. 1) the line which are neither intersecting nor parallel. (1)

2) The given equations are

$$\vec{r} = 8\hat{i} - 9\hat{j} + 10\hat{k} + \mu(3\hat{i} - 16\hat{j} + 7\hat{k}) \dots\dots\dots(1) \quad (\frac{1}{2})$$

$$\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}) \dots\dots\dots(2)$$

$$\text{Here, } \vec{a}_1 = 8\hat{i} - 9\hat{j} + 10\hat{k}; \quad \vec{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k}$$

$$\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}; \quad \vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$$

$$\text{Now, } \vec{a}_2 - \vec{a}_1 = (15 - 8)\hat{i} + (29 + 9)\hat{j} + (5 - 10)\hat{k} = 7\hat{i} + 38\hat{j} - 5\hat{k} \quad (\frac{1}{2})$$

and

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k} \quad (1)$$

$$\Rightarrow (\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (24\hat{i} + 36\hat{j} + 72\hat{k}) \cdot (7\hat{i} + 38\hat{j} - 5\hat{k}) = 1176 \quad (1)$$

$$\text{Shortest distance} = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| \quad (1)$$

$$= \left| \frac{1176}{\sqrt{24^2 + 36^2 + 72^2}} \right| = \frac{1176}{\sqrt{7056}} = \frac{1176}{84} = \frac{98}{7} \quad (1)$$