CHAPTER 13

RECTILINEAR FIGURE

Exercise 13.1

1. If two angles of a quadrilateral are 40° and 110° and the other two are in the ratio 3: 4, find these angles.

Solution:

We know that,

Sum of all four angles of a quadrilateral = 360°

Sum of two given angles = 40° + 110° = 150°

So, the sum of remaining two angles = 360° – 150° = 210°

Also given,

Ratio in these angles = 3: 4

Hence,

Third angle = $\frac{(210\times3)}{(3+4)}$

= $\frac{(210\times3)}{7}$

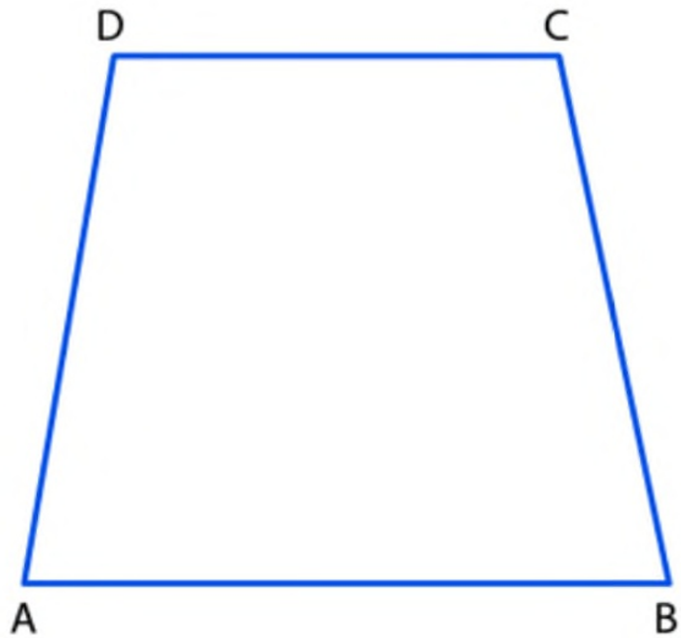
= 90°

And,

Fourth angle = $\frac{(210\times4)}{(3+4)}$

= $\frac{(210\times4)}{7}$

= 120°



2. If the angles of a quadrilateral, taken in order, are in the ratio 1: 2: 3: 4, Prove that it is a trapezium.

Solution:

Given,

In trapezium ABCD in which

$$\angle A : \angle B : \angle C : \angle D = 1 : 2 : 3 : 4$$

We know,

The sum of angles of the quad. ABCD = 360°

$$\angle A = \frac{(360° \times 1)}{10} = 36°$$

$$\angle B = \frac{(360° \times 2)}{10} = 72°$$

$$\angle C = \frac{(360° \times 3)}{10} = 108°$$

$$\angle D = \frac{(360° \times 4)}{10} = 144°$$

Now,

$$\angle A + \angle D = 36° + 114° = 180°$$

Since the sum of angles ∠A and ∠D is 180° and these are co-interior angles

Thus, AB || DC

Therefore, ABCD is a trapezium.

3. If an angle of a parallelogram is two–third of its adjacent angle, find the angles of the parallelogram.

Solution:

Here ABCD is a parallelogram

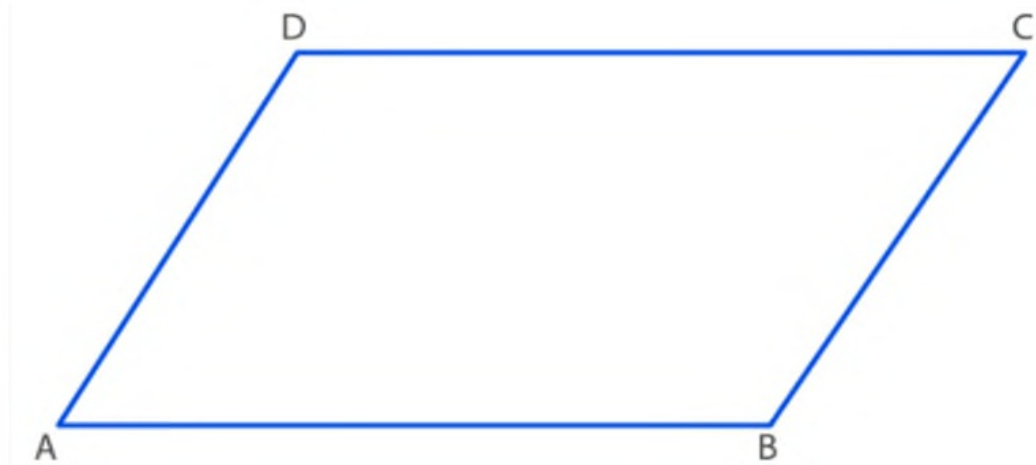
Let $\angle A = x°$

Then, $\angle A = \left(\frac{2x}{3}\right)^{o}$ (Given condition)

So,

$$\angle A + \angle B = 180°$$

(As the sum of adjacent angle in a parallelogram is 180°)



$$x^\circ + \frac{2}{3}x^\circ = 180^\circ$$

$$\Rightarrow \frac{3x + 2x}{3} = 180$$

$$\Rightarrow \frac{5x}{3} = 180$$

$$\Rightarrow 5x = 180 \times 3$$

$$\Rightarrow x = \frac{180 \times 3}{5}$$

$$\Rightarrow x = 36 \times 3$$

$$\Rightarrow x = 108$$

Hence, $\angle A = 108^\circ$

$$\angle B = \frac{2}{3} \times 108^\circ = 2 \times 36^\circ = 72^\circ$$

$\angle B = \angle D = 72^\circ$ (Opposite angle in a parallelogram is same)

Also,

$\angle A = \angle C = 108^\circ$ (Opposite angle in a parallelogram is same)

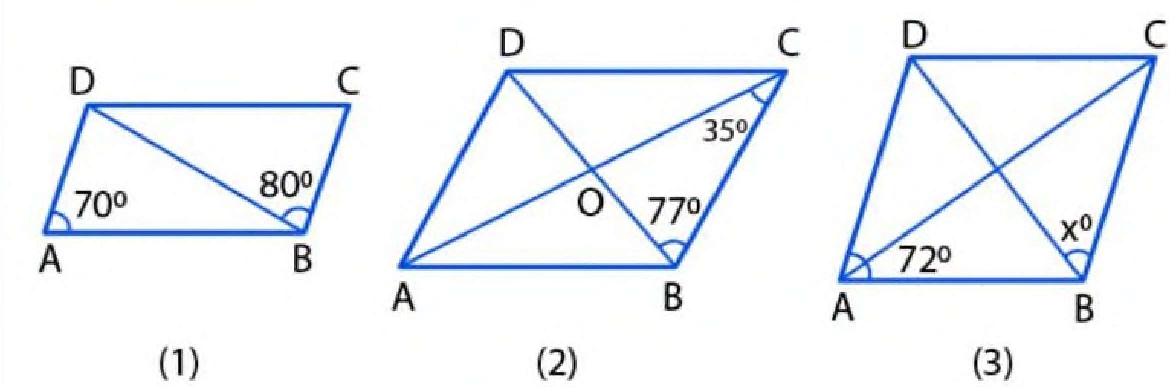
Therefore, angles of parallelogram are 108°, 72°, 108° and 72°.

4.

(a) In figure (1) given below, ABCD is a parallelogram in which DAB = 72°, DBC = 80°. Calculate angles CDB and ADB.

(b) In figure (2) given below, ABCD is a parallelogram. Find the angles of the AAOD.

(c) In figure (3) given below, ABCD is a rhombus. Find the value of x.



Solution:

(a) Since, ABCD is a || gm

We have, AB || CD

∠ADB = ∠DBC (Alternate angles)

∠ADB = 80° (Given, ∠DBC = 80°)

Now,

In ΔADB, we have

∠A + ∠ADB + ∠ABD = 180° (Angle sum property of a triangle)

70° + 80° + ∠ABD = 180°

150° + ∠ABD = 180°

$\angle ABD = 180° – 150° = 30°$

Now, $\angle CDB = \angle ABD$ (Since, AB || CD and alternate angles)

So,

$\angle CDB = 30°$

Hence, $\angle ADB = 80°$ and $\angle CDB = 30°$

(b) Given, $\angle BOC = 35°$ and $\angle CBO = 77°$

In ΔBOC, we have

$\angle BOC + \angle BCO + \angle CBO = 180°$ (Angle sum property of a triangle)

$\angle BOC = 180° – 112° = 68°$

Now, in || gm ABCD

We have,

$\angle AOD = \angle BOC$ (Vertically opposite angles)

Hence, $\angle AOD = 68°$

(c) ABCD is a rhombus

So, $\angle A + \angle B = 180°$ (Sum of adjacent angles of a rhombus is 180°)

$72° + \angle B = 180°$ (Given, $\angle A = 72°$)

$\angle B = 180° – 72° = 108°$

Hence,

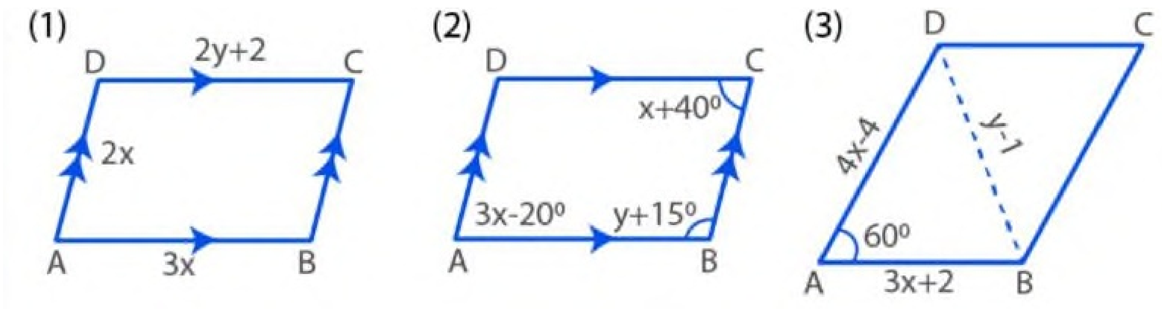
$x = \frac{1}{2} B = \frac{1}{2} \times 108° = 54°$

5.

(a) **In figure (1) given below, ABCD is a parallelogram with perimeter 40. Find the values of x and y.**

(b) **In figure (2) given below, ABCD is a parallelogram. Find the values of x and y.**

(c) **In figure (3) given below. ABCD is a rhombus. Find x and y.**



Solution:

(a) Since, ABCD is a parallelogram

So, AB = CD and BC = AD

$$\Rightarrow 3x = 2y + 2$$

$3x - 2y = 2$ …. (i)

Also, AB + BC + CD + DA = 40

$$\Rightarrow 3x + 2x + 2y + 2 + 2x = 40$$

$$7x + 2y = 40 - 2$$

$7x + 2y = 38$ …. (ii)

Now, adding (i) and (ii) we get

$3x - 2y = 2$

$7x + 2y = 38$

$10x = 40$

$$\Rightarrow x = \frac{40}{10} = 4$$

On substituting the value of x in (i), we get

$$3(4) - 2y = 2$$

$$12 - 2y = 2$$

$$2y = 12 - 2$$

$$\Rightarrow y = \frac{10}{2} = 5$$

Hence, $x = 4$ and $y = 5$

(b) In parallelogram ABCD, we have

∠A = ∠C (Opposite angles are same in || gm)

$$3x - 20° = x + 40°$$

$$3x - x = 40° + 20°$$

$$2x = 60°$$

$x = \frac{60°}{2} = 30°$ … (i)

Also, ∠A = ∠B = 180° (Sum of adjacent angles in || gm is equal to 180°)

$$3x - 20° + y - 15° = 180°$$

$$3x + y = 180° + 20° - 15°$$

$$3x + y = 180°$$

$3(30°) + y = 185°$ [Using (i)]

$$90° + y = 185°$$

$$y = 185° - 90° = 95°$$

Hence,

$$x = 30° \text{ and } 95°$$

(c) ABCD is a rhombus

So, AB = CD

$$3x + 2 = 4x - 4$$

$$3x - 4x = -4 - 2$$

$$-x = -6$$

$$x = 6$$

Now, in ΔABD we have

∠BAD = 60 and AB = AD

∠ADB = ∠ABD

So,

$\angle ADB = \frac{(180° - \angle BAD)}{2}$

$= \frac{(180° - 60°)}{2}$

$= \frac{120}{2} = 60°$

As ΔABD is an equilateral triangle, all the angles of the triangle are 60°

Hence, AB = BD

$$3x + 2 = y - 1$$

$3(6) + 2 = y - 1$ (Substituting the value of x)

$$18 + 2 = y - 1$$

$$20 = y - 1$$

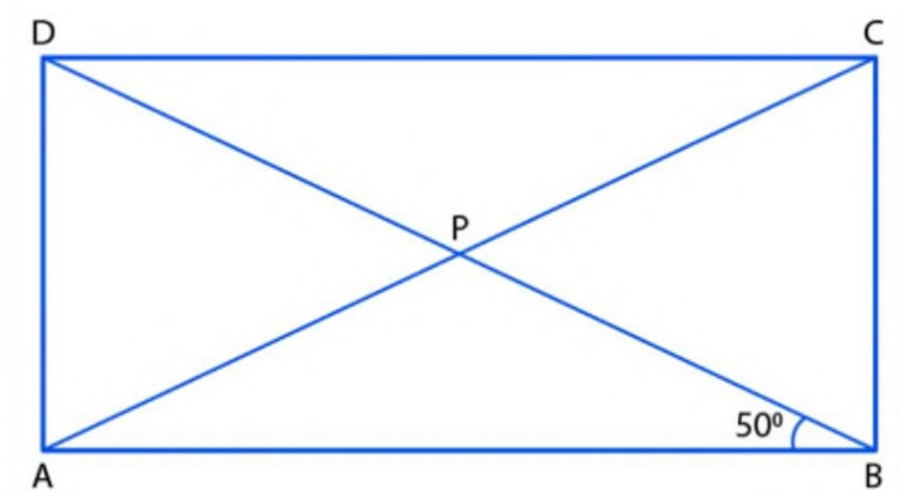
$$y = 20 + 1$$

$$y = 21$$

Thus, $x = 6$ and $y = 21$.

6. The diagonals AC and BD of a rectangle ABCD intersect each other at P. If ∠ABD = 50°, find ∠DPC.

Solution:



Given, ABCD is a rectangle

We know that the diagonals of rectangle are same and bisect each other

So, we have

AP = BP

∠PAB = ∠PBA (Equal sides have equal opposite angles)

∠PAB = 50° (Since, given ∠PBA = 50°)

Now, in ΔAPB,

∠APB + ∠ABP + ∠BAP = 180°

∠APB + 50° + 50° = 180°

∠APB = 180° – 100°

∠APB = 80°

Then,

∠DPB = ∠APB (Vertically opposite angles)

Hence,

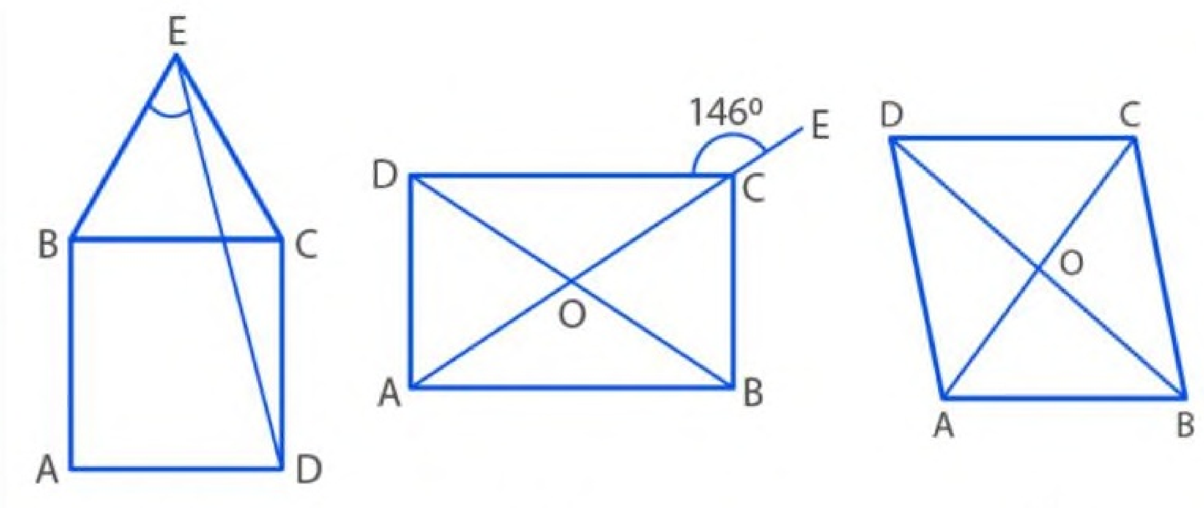
∠DPB = 80°

7.

(a) **In figure (1) given below, equilateral triangle EBC surmounts square ABCD. Find angle Bed represented by x.**

(b) **In figure, (2) given below, ABCD is a rectangle and diagonals intersect at O. AC is produced to E. If ∠ECD = 146°. Find the angles of the ΔAOB.**

(c) **If figure (3) given below, ABCD is rhombus and diagonals intersect at O. If ∠OAB: ∠OBA = 3: 2, find the angles of the ∠AOD.**



(1) (2) (3)

Solution:

(a) Since, EBC is an equilateral triangle, we have

EB = BC = EC …. (i)

Also, ABCD is a square

So, AB = BC = CD = AD …. (ii)

From (i) and (ii), we get

EB = EC = AB = BC = CD = AD …. (iii)

Now, in ΔECD

∠ECD = ∠BCD + ∠ECB

= 90° + 60°

= 150° …. (iv)

Also, EC = CD [From (iii)]

So, ∠DEC = ∠CDE … (v)

∠ECD + ∠DEC + ∠CDE = 180° [Angles sum property of a triangle]

150° + ∠DEC + ∠DEC = 180° [Using (iv) and (v)]

2∠DEC = 180° – 150° = 30°

∠DEC = $\frac{30°}{2}$

∠DEC = 15° …. (vi)

Now, ∠BEC = 60° [BEC is an equilateral triangle]

∠BEC + ∠DEC = 60°

$x° + 15° = 60°$ [From (vi)]

$$x = 60° - 15°$$

$$x = 45°$$

Hence, the value of x is 45°.

(b) Given, ABCD is a rectangle

$\angle ECD = 146°$

As ACE is a straight line, we have

$146° + \angle ACD = 180°$ [Linear pair]

$\angle ACD = 180° – 146° = 34°$ …. (i)

And, $\angle CAB = \angle ACD$ [Alternate angles] …. (ii)

From (i) and (ii), we have

$\angle CAB = 34° \Rightarrow \angle OAB = 34°$ …. (iii)

In ΔAOB

AO = OB [Diagonals of a rectangle are equal and bisect each other]

$\angle OAB = \angle OAB$ … (iv)

[Equal sides have equal angles opposite to them]

From (iii) and (iv),

$\angle OAB = 34°$ …. (v)

Now,

$\angle AOB + \angle OBA + \angle OAB = 180°$

$\angle AOB + 34° + 34° = 180°$ [Using (3) and (5)]

$\angle AOB + 68° = 180°$

$\angle AOB = 180° – 68° = 112°$

Hence, $\angle AOB = 112°$, $\angle OAB = 34°$ and $\angle OBA = 34°$

(c) Here, ABCD is a rhombus and diagonals intersect at O and ∠OAB: ∠OBA = 3: 2.

Let ∠OAB = $2x°$

Then,

∠OBA = $2x°$

We know that diagonals of rhombus intersect at right angle.

So, ∠OAB = 90°

Now, in ΔAOB

∠OAB + ∠OBA = 180°

$$90° + 3x° + 2x° = 180°$$

$$90° + 5x° = 180°$$

$$5x° = 180° - 90° = 90°$$

$$x° = \frac{90°}{5} = 18°$$

Hence,

∠OAB = $3x° = 3 \times 18° = 54°$

∠OBA = $2x° = 2 \times 18° = 36°$ and

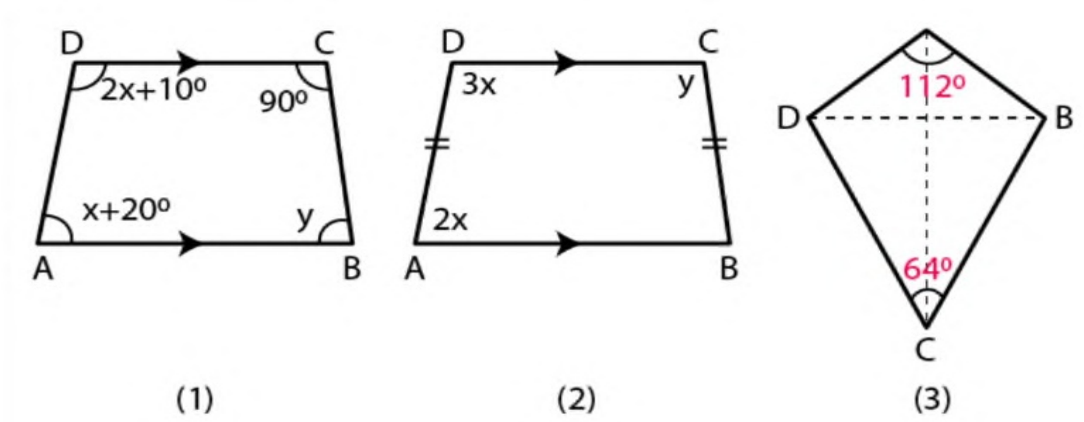
∠AOB = 90°

8.

(a) In figure (1) given below, ABCD is a trapezium. Find the values of x and y.

(b) In figure (2) given below, ABCD is an isosceles trapezium. Find the values of x and y.

(c) **In figure (3) given below, ABCD is a kite and diagonals intersect at O. If ∠DAB = 112° and ∠DCB = 64°, find ∠ODC and ∠OBA.**



Solution:

(a) Given: ABCD is a trapezium

∠A = $x + 20°$, ∠B = y, ∠C = 92°, ∠D = $2x + 10°$

We have,

∠B + ∠C = 180° [Since AB || DC]

$$y + 92° = 180°$$

$$y = 180° - 92° = 88°$$

Also, ∠A + ∠D = 180°

$$x + 20° + 2x + 10° = 180°$$

$$3x + 30° = 180°$$

$$3x = 180° - 30° = 150°$$

$$x = \frac{150°}{3} = 50°$$

Hence, the value of $x = 50°$ and y = 88°.

(b) Given: ABCD is a isosceles trapezium BC = AD

$\angle$A = $2x$, $\angle$C = y, $\angle$D = $3x$

Since, ABCD is a trapezium and AB || DC

$$\Rightarrow \angle A + \angle D = 180°$$

$$2x + 3x = 180°$$

$$5x = 180°$$

$x = \frac{180°}{5} = 36°$ ….. (i)

Also, AB = BC and AB || DC

So, $\angle A + \angle C = 180°$

$$2x + y = 180°$$

$$2 \times 36° + y = 180°$$

$$72° + y = 180°$$

$$y = 180° - 72° = 108°$$

Hence, value of x = 72° and y = 108°.

(c) Given: ABCD is a kite and diagonal intersect at O.

$\angle$DAB = 112° and $\angle$DCB = 64°

As AC is the diagonal of kite ABCD, we have

$\angle$DCO = $\frac{64}{2} = 32°$

And, $\angle$DOC = 90° [Diagonal of kites bisect at right angles]

In ΔOCD, we have

$\angle$ODC = 180° – ($\angle$DCO + $\angle$DOC)

= 180° – (32° + 90°)

= 180° – 122°

= 58°

In ΔDAB, we have

∠OAB = $\frac{112}{2} = 56°$

∠AOB = 90° [Diagonal of kites bisect at right angles]

In ΔOAB, we have

∠OBA = 180° – (∠OAB + ∠AOB)

= 180° – (56° + 90°)

= 180° – 146°

= 34°

Hence, ∠ODC = 58° and ∠OBA = 34°.

9.

(i) Prove that each angle of a rectangle is 90°.

(ii) If the angle of a quadrilateral are equal, prove that it is a rectangle.

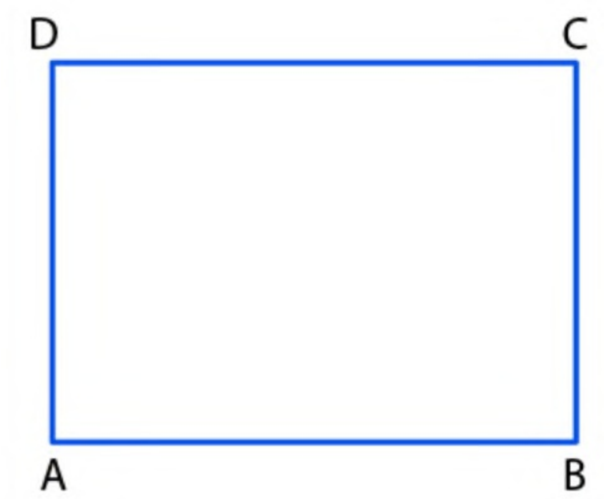
(iii) If the diagonals of a rhombus are equal, prove that it is a rectangle.

(iv) If the diagonals of a rhombus are equal, prove that it is a square.

(v) Prove that every diagonal of a rhombus bisects the angles at the vertices.

Solution:

(i) Given: ABCD is a rectangle



To prove: Each angles of rectangle = 90°

Proof:

In a rectangle opposite angles of a rectangle are equal

So, ∠A = ∠C and ∠B = ∠C

But, ∠A + ∠B + ∠C + ∠D = 360° [Sum of angles of a quadrilateral]

∠A + ∠B + ∠A + ∠B = 360°

2 (∠A + ∠B) = 360°

(∠A + ∠B) = 360°

∠A + ∠B = $\frac{360}{2}$

∠A + ∠B = 180°

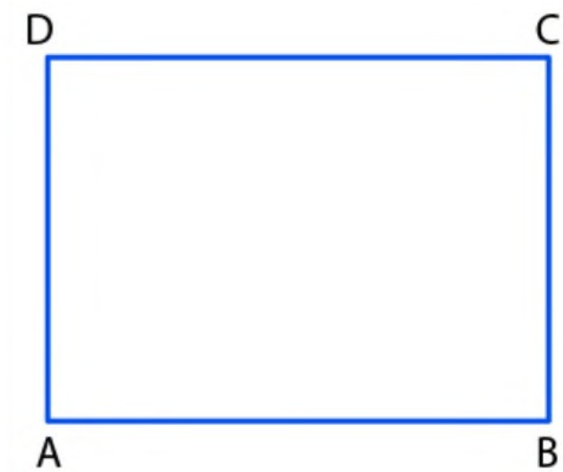
But, ∠A = ∠B [Angles of a rectangle]

So, ∠A = ∠B = 90°

Thus,

∠A = ∠B = ∠C = ∠D = 90°

Hence, each angle of a rectangle is 90°.



(ii) Given: In quadrilateral ABCD,

We have

$\angle A = \angle B = \angle C = \angle D$

To prove: ABCD is a rectangle

Proof:

$\angle A = \angle B = \angle C = \angle D$

$\Rightarrow \angle A = \angle C$ and $\angle B = \angle D$

But these are opposite angles of the quadrilateral

So, ABCD is a parallelogram

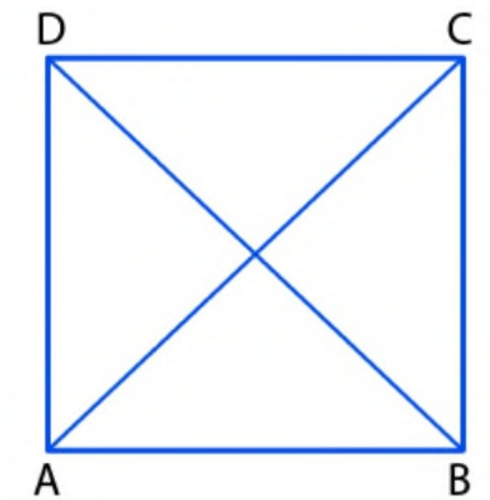
And, as $\angle A = \angle B = \angle C = \angle D$

Therefore, ABCD is a rectangle.

(iii) Given: ABCD is a rhombus in which AC = BD

To prove: ABCD is a square

Proof:



Join AC and BD.

Now, in ΔABC and ΔDCB we have

∠AB = ∠DC [Sides of a rhombus]

∠BC = ∠BC [Common]

∠AC = ∠BD [Given]

So, ΔABC ≅ΔDCB by S.S.S. axiom of congruency

Thus,

∠ABC = ∠DBC [By C.P.C.T]

But these are made by transversal BC on the same side of parallel line AB and CD.

So, ∠ABC = ∠DBC = 180°

∠ABC = 90°

Hence, ABCD is square.

(iv) Given: ABCD is rhombus.

To Prove: Diagonals AC and BD bisects ∠A, ∠C, ∠B and ∠D respectively

Proof:

In ΔAOD and ΔCOD, we have

AD = CD [Sides of a rhombus are all equal]

OD = OD [Common]

AO = OC [Diagonal of rhombus bisect each other]

So, ΔAOD≅ΔCOD by S.S.S. axiom of congruency

Thus,

∠AOD = ∠COD [By C.P.C.T]

So, ∠AOD = ∠COD = 180° [Linear pair]

∠AOD = 180°

∠AOD = 90°

And, ∠COD = 90°

Thus,

OD ⊥ AC = BD ⊥ AC

Also, ∠ADO = ∠CDO [By C.P.C.T]

So,

OD bisect ∠D

BD bisect∠D

Similarly, we can prove that BD bisect ∠B and AC bisect the ∠A and ∠C.

10. ABCD is parallelogram. If the diagonal AC bisects ∠A, then prove that:

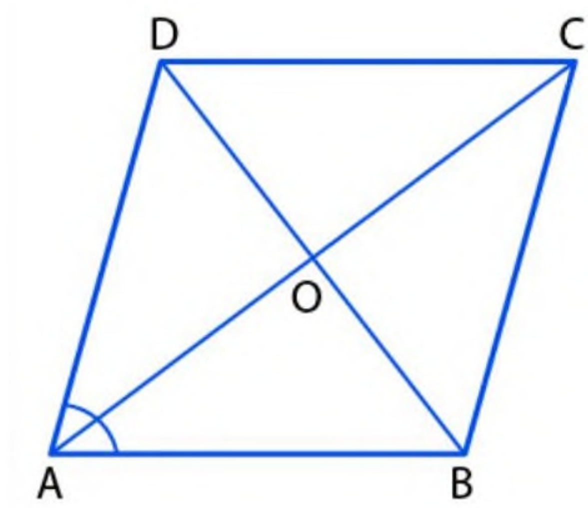
(i) AC bisect ∠C

(ii) ABCD is a rhombus

(iii) AC ⊥ BD.

Solution:

Given: In parallelogram ABCD in which diagonal AC bisect ∠A.



To prove:

(i) AC bisects ∠C

(ii) ABCD is a rhombus

(iii) AC ⊥ BD.

Proof:

(i) As AB || CD, we have [Opposite sides of a|| gm]

∠DCA = ∠CAB

Similarly, ∠DAC = ∠DCB

But, ∠CAB = ∠DCA [Since, AC bisects ∠A]

Hence,

∠DCA = ∠ACB and AC bisects ∠C.

(ii) As AC bisect ∠A and ∠C

And, ∠A = ∠C

Hence, ABCD is a rhombus.

(iii) Since, AC and BD are the diagonals of a rhombus and AC and BD bisect each other at right angles

Hence, AC ⊥ BD.

11.

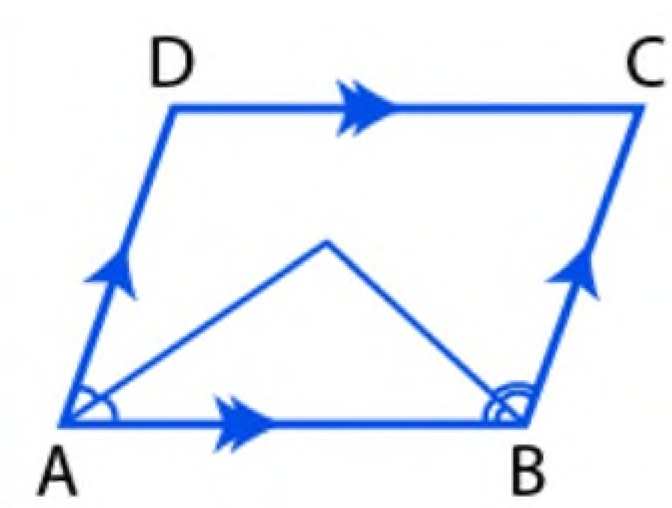
(i) Prove that bisectors of any two adjacent angles of a parallelogram are at right angles.

(ii) Prove the bisector of any two opposite angles of a parallelogram are parallel.

(iii) If the diagonals of a quadrilateral are equal and bisect each other at right angles, then prove that it is a square.

Solution:

(i) Given AM bisect angle A and BM bisects angle of || gm ABCD.



To probe: ∠AMB = 90°

Proof:

We have,

∠A + ∠B = 180° [AD || BC and AB is the transversal]

$$\Rightarrow \frac{1}{2}(\angle A + \angle B) = \frac{180}{2}$$

$$\frac{1}{2}\angle A + \frac{1}{2}\angle B = 90°$$

∠MAB + ∠MBA = 90° [Since, AM bisects ∠A and BM bisects ∠B]

Now, in ΔAMB

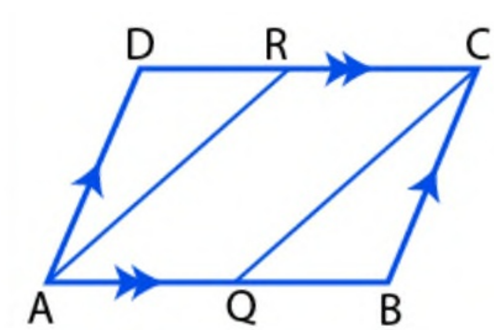
∠AMB + ∠MAB + ∠MBA = 180° [Angle sum property of a triangle]

∠AMB + 90° = 180°

∠AMB = 180° – 90° = 90°

Hence, bisectors of any two adjacent angles of a parallelogram are at right angles.

(ii) Given: A || gm ABCD in which bisector AR of ∠A meet DC in R and bisector CQ of ∠C meets AB in Q



To prove: AR || CQ

Proof:

In || gm ABCD, we have

∠A = ∠C [Opposite angles of || gm are equal]

$\frac{1}{2}\angle A = \frac{1}{2}\angle C$

∠DAR = ∠BCQ

[Since, AR is bisector of $\frac{1}{2}\angle A$ and CQ is the bisector of $\frac{1}{2}\angle C$]

Now, in ΔADR and ΔCBQ

∠DAR = ∠BCQ [Proved above]

AD = BC [Opposite sides of || gm ABCD are equal]

So, ΔADR ≅ΔCBQ, by A.S.A axiom of congruency

Then by C.P.C.T, we have

∠DAR = ∠BCQ

And,

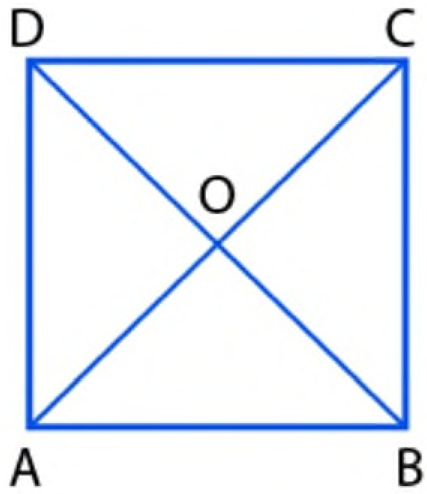
∠DAR = ∠RAQ [Alternate angles since, DC || AB]

Thus, ∠RAQ = ∠BCQ

But these are corresponding angles,

Hence, AR || CQ.

(iii) Given: In quadrilateral ABCD, diagonals AC and BD are equal and bisect each other at right angles



To prove: ABCD is a square

Proof:

In ΔAOB and ΔCOD, we have

AO = OC [Given]

BO = OD [Given]

∠AOB = ∠COD [Vertically opposite angles]

So, ΔAOB≅ΔCOD by S.A.S. axiom of congruency

By C.P.C.T, we have

AB = CD and ∠OAB = ∠OCD

But these are alternate angles

AB || CD

Thus, ABCD is a parallelogram

In a parallelogram, the diagonal bisect each other and are equal

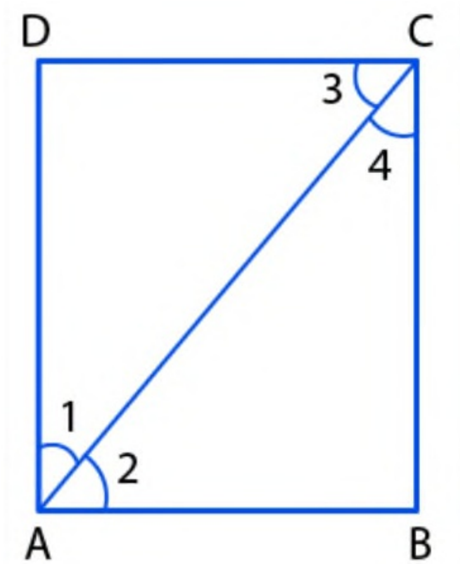
Hence, ABCD is a square.

12.

(i) **If ABCD is a rectangle in which the diagonal BD bisect ∠B, then show that ABCD is a square.**

(ii) **Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.**

Solution:



(i) ABCD is a rectangle and its diagonals AC bisects ∠A and ∠C.

To prove: ABCD is a square

Proof:

We know that the opposite sides of a rectangle are equal and each angle is 90°.

As AC bisects ∠A and ∠C

So, ∠1 = ∠2 and ∠3 = ∠4

But, ∠A = ∠C = 90°

∠2 = 45° and ∠4 = 45°

And, AB = BC [Opposite sides of equal angles]

But, AB = CD and BC = AD

So, AB = BC = CD = DA

Therefore, ABCD is a square.

(ii) In quadrilateral ABCD diagonals AC and BD are equal and bisect each other at right angles

To Prove: ABCD is a square

Proof:

In ΔAOB and ΔBOC, we have

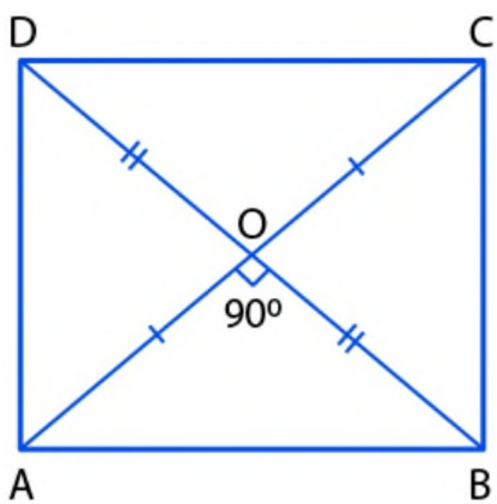
AO = CO [Diagonals bisect each other at right angles]

OB = OB [Common]

∠AOB = ∠COB [Each 90°]

So, ΔAOB≅ΔBOC by S.A.S. axiom of congruency

By C.P.C.T, we have



AB = BC …. (i)

Similarly, in ΔBOC and ΔCOD

OB = OD [Diagonals bisect each other at right angles]

OC = OC [Common]

∠BOC = ∠COD [Each 90°]

So, ΔBOC≅ΔCOD by S.A.S. axiom of congruency

By C.P.C.T, we have

BC = CD … (ii)

From (i) and (ii), we have

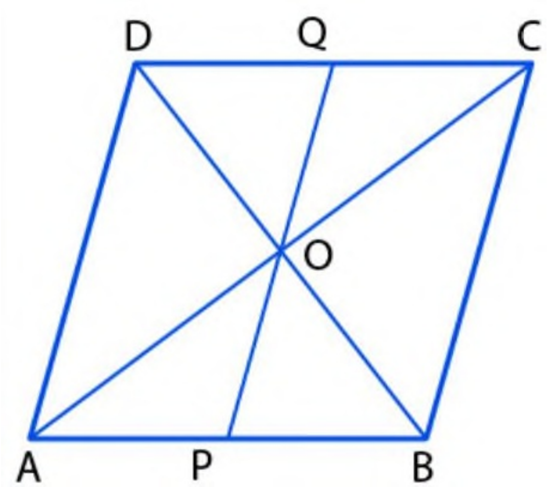
AB = BC = CD = DA

Hence, ABCD is a square.

13. P and Q are points on opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonal AC and BD. Show that PQ is bisected at O.

Solution:

Given: ABCD is a parallelogram, P and Q are the points on AB and DC. Diagonals AC and BD intersect each other at O.



To prove:

Diagonals of || gm ABCD bisect each other at O

So, AO = OC and BO = OD

Now, in ΔAOP and ΔCOQ we have

AO = OC [Proved]

∠OAP = ∠OCQ [Alternate angles]

∠AOP = ∠COQ [Vertically opposite angles]

So, ΔAOP≅ΔCOQ by S.A.S. axiom

Thus, by C.P.C.T, we have

OP = OQ

Hence, O bisects PQ.

14.

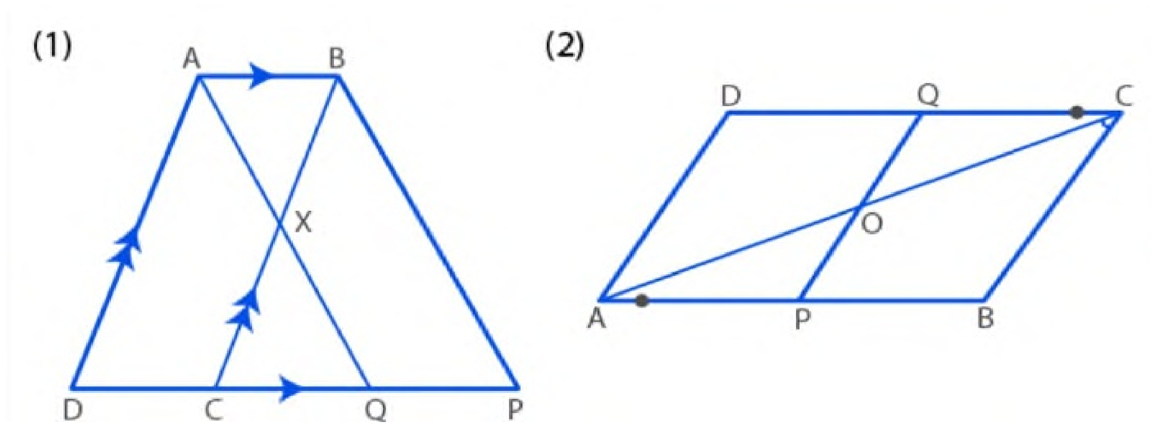
(a) In figure (1) given below, ABCD is a parallelogram and X is mid-point of BC. The line AX produced meets DC produced at Q. The parallelogram ABPQ is completed.

Prove that:

(i) The triangle ABX and QCX are congruent:

(ii) DC = CQ = QP

(b) **In figure (2) given below, points P and Q have been taken on opposite sides AB and CD respectively of a parallelogram ABCD such that AP = CQ. Show that AC and PQ bisect each other**.



Solution:

(a) Given: ABCD is parallelogram and X is midpoint of BC. The line AX produced meets DC produced at Q and ABPQ is a || gm.

To Prove:

(i) ΔABX ≅ΔQCX
(ii) DC = CQ = QP

Proof:

In ΔABX andΔQCX, we have

BX = XC [X is the midpoint of BC]

∠AXB = ∠CXQ [Vertically opposite angles]

∠XCQ = ∠XBA [Alternate angle, since AB || CQ]

So, ΔABX ≅ΔQCX by A.S.A. axiom of congruence

Now, by C.P.C.T

CQ = AB

But,

AB = DC and AB = QP [As ABCD and ABPQ are || gm]

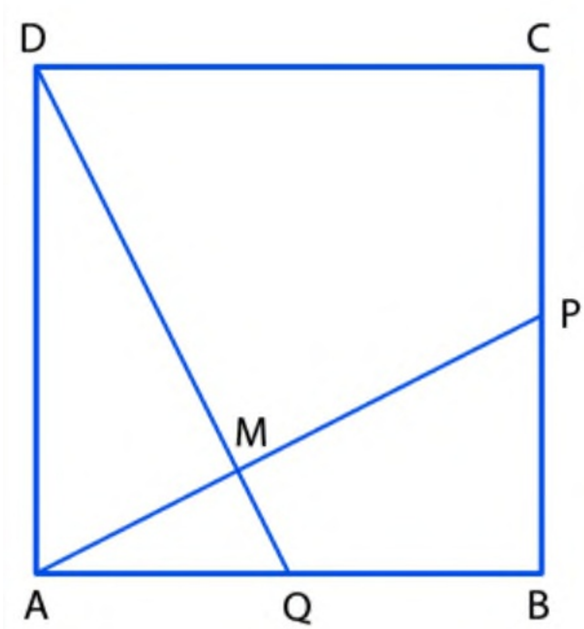
Hence,

DC = CQ = QP

15. ABCD is a square. A is joined to a point P on BC and D is joined to a point Q on AB. If AP = DQ. Prove that AP and DQ are perpendicular to each other.

Solution:

Given: ABCD is a square. P is any point on BC and Q is any point on AB and these point are taken such that AP = DQ.



To prove: AP ⊥ DQ

Proof:

In ΔABPandΔADQ, we have

AP = DQ	[Given]
AD = AB	[Sides of square ABCD]
∠DAQ = ∠ABP	[Each 90°]

So, ΔABP≅ΔADQ by R.H.S. axiom of congruence

Now, by C.P.C.T

∠BAP = ∠ADQ

But, ∠BAD = 90°

∠BAD = ∠BAP + ∠PAD … (i)

90° = ∠BAP + ∠PAD

∠BAP + ∠PAD = 90°

∠BAP + ∠ADQ = 90°

Now, in ΔADB we have

(∠MAD + ∠ADM) + ∠AMD = 180° [Angles sum property of a triangle]

90° + ∠AMD = 180° [From (i)]

∠AMD = 180° – 90° = 90°

So, DM ⊥ AP

⇒ DQ ⊥ AP

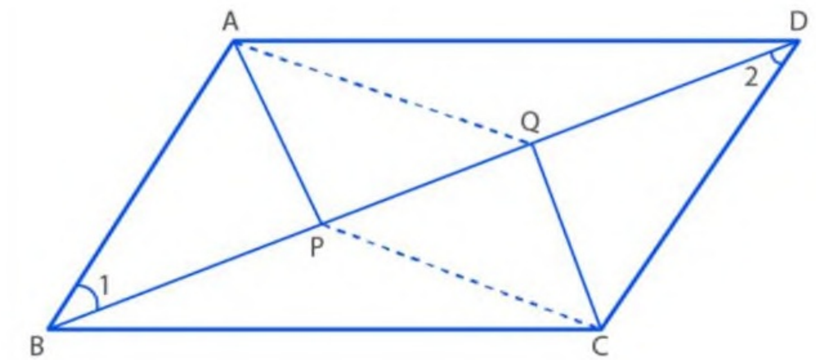
Hence, AP ⊥ DQ

16. If P and Q are points of trisection of the diagonal BD of a parallelogram ABCD, prove that CQ || AP.

Solution:

Given : ABCD is a ||gm is which BP = PQ = QD

To prove : CQ || AP



Proof :

In ||gm ABCD, we have

AB = CD [Opposite sides of a || gm are equal]

And BD is the transversal

So, $\angle 1 = \angle 2$ [Alternate interior angles] ….(i)

Now, in ΔABP and ΔDCQ

AB = CD [Opposite sides of a || gm are equal]

$\angle 1 = \angle 2$ [From (i)]

BP = QD [Given]

So, $\Delta ABP \cong \Delta DCQ$ by S.A.S. axiom of congruency

Then by C.P.C.T, we have

AP = QC

Also, $\angle APB = \angle DQC$ [By C.P.C.T]

-$\angle APB = -\angle DQC$ [Multiplying both sides by -1]

$180^{\circ} - \angle APB = 180^{\circ} - \angle DQC$ [Adding 180° both sides]

$$\angle APQ = \angle CQP$$

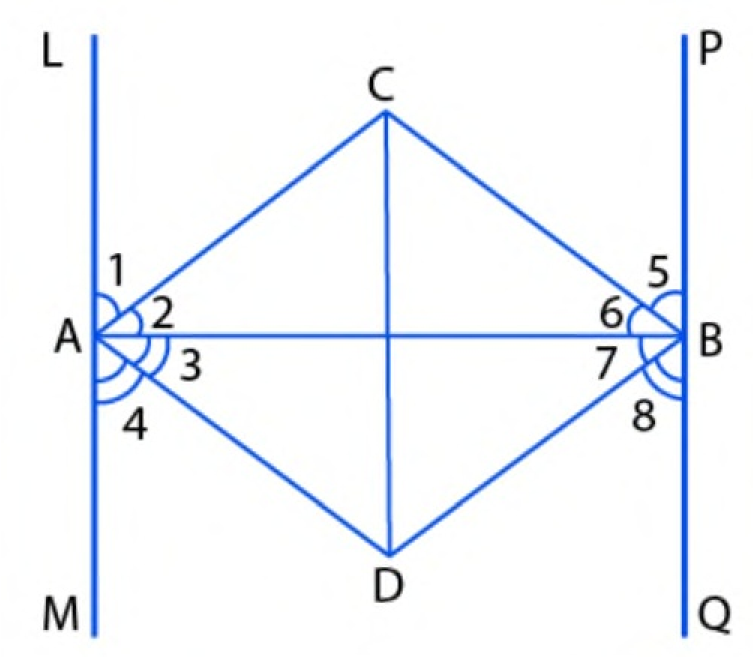
But, these are alternate angles

Hence, AP || QC ⇒ $CQ \parallel AP$.

17. A transversal cuts two parallel lines at A and B. The two interior angles at A are bisected and so are the two interior angles at B; the four bisectors form a quadrilateral ABCD. Prove that

(i) ABCD is a rectangle.

(ii) CD is parallel to the original parallel lines.



Solution:

Given : LM || PQ and AB is the tranversal line cutting $\angle M$ at A and PQ at B

AC, AD, BC and BD is the bisector of $\angle LAB, \angle BAM, \angle PAB$ and $\angle ABQ$ respectively.

AC and BC intersect at C and AD and BD intersect at D.

A quadrilateral ABCD is formed.

To prove : (i) ABCD is a rectangle

(ii) CD || LM and PQ

Proof:

(i) $\angle LAB + \angle BAM = 180^{\circ}$ [LAM is a straight line]

$$\frac{1}{2}(\angle LAB + \angle BAM) = 90^{\circ}$$

$$\frac{1}{2}\angle LAB + \frac{1}{2}\angle BAM = 90^{\circ}$$

$\angle 2 + \angle 3 = 90^{\circ}$ [Since, AC and AD is bisector of $\angle LAB \& \angle BAM$ respectively]

$$\angle CAD = 90^{\circ}$$

$$\angle A = 90^{\circ}$$

(2) Similarly, $\angle PBA + \angle QBA = 180^{\circ}$ [PBQ is a straight line]

$$\frac{1}{2}(\angle PBA + \angle QBA) = 90^{\circ}$$

$\frac{1}{2}\angle PBA + \frac{1}{2}\angle QBA = 90^{\circ}$

$\angle 6 + \angle 7 = 90^{\circ}$ [Since, BC and BD is bisector of $\angle PAB$ & $\angle QBA$ respectively.]

$$\angle CBD = 90^{\circ}$$

$$\angle B = 90^{\circ}$$

(3) $\angle LAB + \angle ABP = 180^{\circ}$ [Sum of co-interior angles is 180° and given LM || PQ]

$$\frac{1}{2}\angle LAB + \frac{1}{2}\angle ABP = 90^\circ$$

$$\angle 2 + \angle 6 = 90^\circ$$

[Since, AC and BC is bisector of $\angle LAB$ &$\angle PBA$ respectively]

(4) In ΔACB,

$\angle 2 + \angle 6 + \angle C = 180^\circ$ [Angles sum property of a triangle]

$$(\angle 2 + \angle 6) + \angle C = 180^\circ$$

$90^\circ + \angle C = 180^\circ$ [Using (3)]

$$\angle C = 180^\circ - 90^\circ$$

$$\angle C = 90^\circ$$

(5) $\angle MAB + \angle ABQ = 180^\circ$ [Sum of co-interior angles is 180° and given LM || PQ]

$$\frac{1}{2}\angle MAB + \frac{1}{2}\angle ABQ = 90^\circ$$

$\angle 3 + \angle 7 = 90^\circ$ [Since, AD and BD is bisector of $\angle MAB$ & $\angle ABQ$ respectively]

(6) In ΔADB,

$\angle 3 + \angle 7 + \angle D = 180^\circ$ [Angles sum property of a triangle]

$$(\angle 3 + \angle 7) + \angle C = 180^\circ$$

$90^\circ + \angle D = 180^\circ$ [Using (5)]

$$\angle D = 180^\circ - 90^\circ$$

$$\angle D = 90^\circ$$

(7) $\angle LAB + \angle BAM = 180^{\circ}$

$\angle BAM = \angle ABP$ [From(1) and (2)]

$$\frac{1}{2}\angle BAM = \frac{1}{2}\angle ABP$$

$\angle 3 = \angle 6$ [Since, AD and BC is bisector of $\angle BAM$ & $\angle ABP$ respectively]

Similarly, $\angle 2 = \angle 7$

(8) in $\Delta ABC\ and\ \Delta ABD,$

$\angle 2 = \angle 7$ [From (7)]

AB = AB [common]

$\angle 6 = \angle 3$ [from (7)]

So, $\Delta ABC \cong \Delta ABD$ by A.S.A. axiom of congruency

Then, by C.P.C.T. we have

AC = DB

Also, CB = AD

(9) $\angle A = \angle B = \angle C = \angle D = 90^{\circ}$ [From(1),(2), (3) and (4)]

AC = DB [Proved in (8)]

CB = AD [Proved in (8)]

Hence, ABCD is a rectangle.

(10) Since, ABCD is a rectangle [From (9)]

OA = OD [Diagonals of rectangle bisect each other]

(11) In $\Delta AOD,$ we have

OA = OD [From (10)]

$\angle 9 = \angle 3$ [Angles opposite to equal sides are equal]

(12) $\angle 3 = \angle 4$ [AD bisects $\angle MAB$]

(13) $\angle 9 = \angle 4$ [From (11) and (12)]

But these are alternate angles.

OD || LM ⇒ $CD \parallel LM$

Similarly, we can prove that

$$\angle 10 = \angle 8$$

But these are alternate angles,

So, OD = PQ ⇒ $CD||PQ$

(14) CD || LM [Proved in (13)]

CD || PQ [Proved in (13)]

18. In a parallelogram ABCD, the bisector of $\angle A$ meets DC in E and AB = 2 AD. Prove that :

(i) BE bisects $\angle B$

(ii) $\angle AEB$ is a right angle

Solution:

Given: ABCD is a ||gm in which bisectors of angles A and B meets in E and AB = 2AD

To prove : (i) BE bisects $\angle B$

(ii) $\angle AEB = 90^\circ$

Proof:

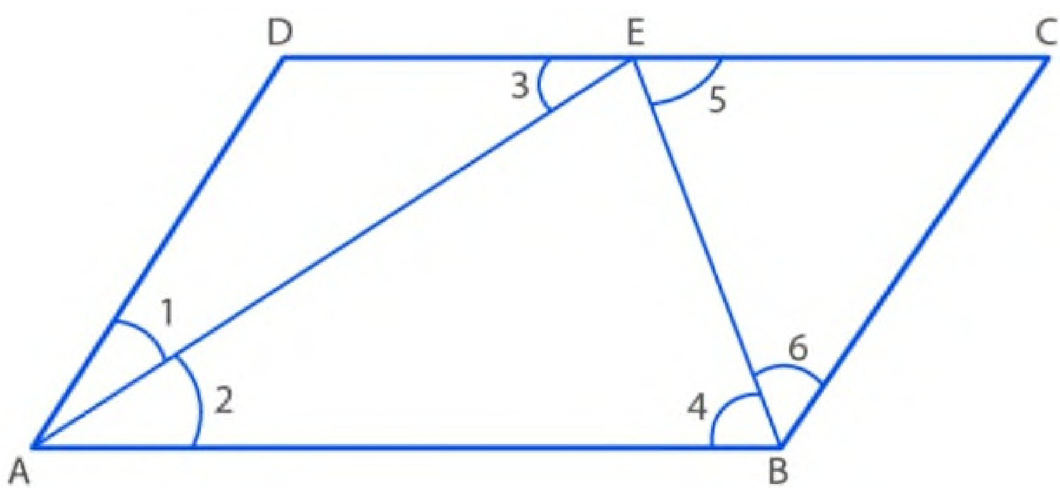
(1) In || gm ABCD

$\angle 1 = \angle 2$ [AD bisects angles $\angle A$]

(2) AB || DC and AE is the transversal

$\angle 2 = \angle 3$ [Alternate angles]

(3) $\angle 1 = \angle 2$ [From (1) and (2)]



(4) in ΔADE, we have

$\angle 1 = \angle 3$ [Proved in (3)]

DE = AD [Sides opposite to equal angles are equal]

$\Rightarrow$ AD = DE

(5) AB = 2AD [Given]

$\frac{AB}{2}$ = AD

$\frac{AB}{2}$ = DE [Using (4)]

$\frac{DC}{2}$ = DE [AB = DC, Opposite sides of a|| gm are equal]

So, E is the mid-point of D.

⇒ DE = EC

(6) AD = BC [Opposite sides of a|| gm are equal]

(7) DE = BC [From (4) and (6)]

(8) EC = BC [From (5) and (7)]

(9) In $\Delta BCE,$ we have

EC = BC [Proved in (8)]

$\angle 6 = \angle 5$ [Angles opposite to equal sides are equal]

(10) AB || DC and BE is the transval

$\angle 4 = \angle 5$ [Alternate angles]

(11) $\angle 4 = \angle 6$ [From (9) and (10)]

So, BE is bisector of $\angle B$

(12) $\angle A + \angle B = 180^\circ$ [Sum of co-interior angles is equal to 180°, AD || BC]

$$\frac{1}{2}\angle A + \frac{1}{2}\angle B = \frac{180^\circ}{2}$$

$\angle 2 + \angle 4 = 90^\circ$ [AE is bisector of $\angle A$ and BE is bisector of $\angle B$]

(13) In ΔAPB,

$$\angle AEB + \angle 2 + \angle 4 = 180^\circ$$

$$\angle AEB + 90^\circ = 180^\circ$$

Hence, $\angle AEB = 90^\circ$

19. ABCD is a parallelogram, bisectors of angles A and B meet at E which lie on DC. Prove that AB.

Solution:

Given : ABCD is a parallelogram in which bisector of $\angle A$ and $\angle B$ meets DC in E

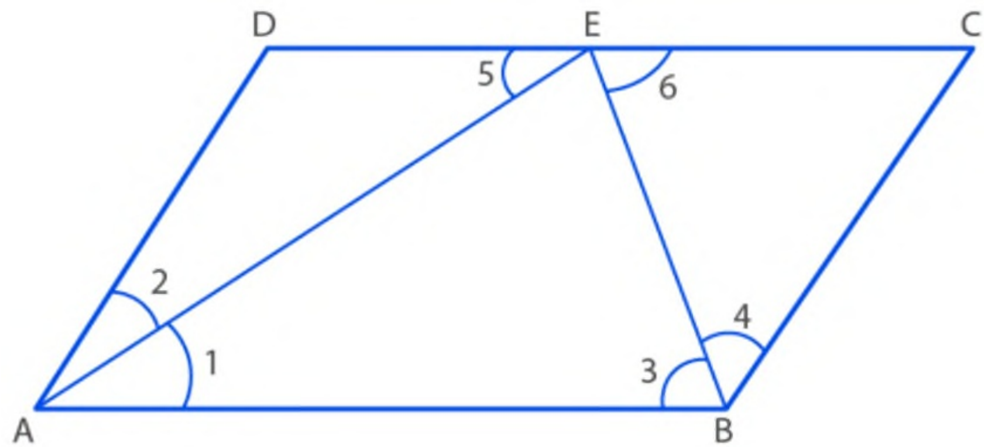
To prove : AB = 2AD

Proof :

In Parallelogram ABCD, we have

AB || DC

$\angle 1 = \angle 5$ [Alternate angles, AE is transversal]



$\angle 1 = \angle 2$ [AE is bisector of $\angle A$, given]

Thus,

$\angle 2 = \angle 5$ …(i)

Now, in ΔAED

DE = AD [Sides opposite to equal angles are equal]

$\angle 3 = \angle 6$ [Alternate angles]

$\angle 3 = \angle 4$ [Since, BE is bisector of $\angle B$ (given)]

Thus, $\angle 4 = \angle 6$ ….(ii)

In ΔBCE, we have

BC = EC [Sides opposite to equal angles are equal]

AD = BC [Opposite sides of || gm are equal]

AD = DE = EC [From (i) and (ii)]

AB = DC [Opposite sidesof a || gm are equal]

$\mathrm{AB} = \mathrm{DE} + \mathrm{EC}$

$= \mathrm{AD} + \mathrm{AD}$

Hence,

AB = 2AD

20. ABCD is a square and the diagonals intersect at O. If P is a point on AB such that AO = AP, prove that $3\angle POB = \angle AOP$.

Solution:

Given : ABCD is a square and the diagonals intersect at O. P is the point on AB such that AO = AP

To prove : $3\angle POB = \angle AOP$

Proof :

(1) In square, ABCD, AC is a diagonal

So, $\angle CAB = 45^\circ$

$$\angle OAP = 45^\circ$$

(2) In ΔAOP,

$\angle OAP = 45^\circ$ [From(1)]

AO = AP [Sides oppsite to equal angles are equal]

Now,

$\angle AOP + \angle APO + \angle OAP = 180^\circ$ [Angles sum property of a triangle]

$$\angle AOP + \angle AOP + 45^\circ = 180^\circ$$

$2\angle AOP = 180^\circ$ - 45°

$$\angle AOP = \frac{135^\circ}{2}$$

(3) $\angle AOB = 90^\circ$ [Diagonals of a square bisect at right angles]

So, $\angle AOP + \angle POB = 90^\circ$

$\frac{135^\circ}{2} + \angle POB = 90^\circ$ [From (2)]

$\angle POB = 90^\circ - \frac{135^\circ}{2}$

$= \frac{(180^\circ - 135^\circ)}{2}$

$= \frac{45^\circ}{2}$

$3\angle POB = \frac{135^\circ}{2}$ [Multiplying both sides by 3]

Hence,

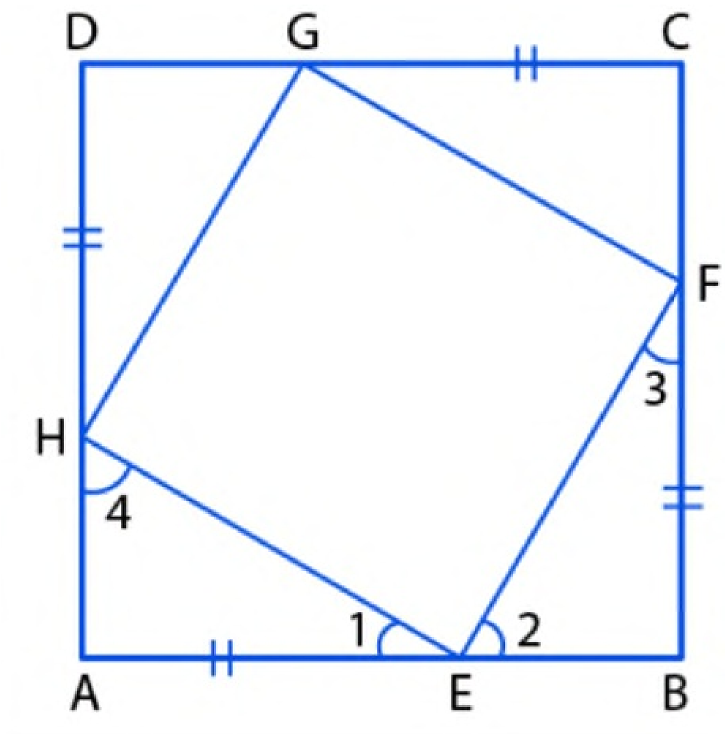
$\angle AOP = 3\angle POB$ [From (2) and (3)]

21. ABCD is a square. E, F, G and H are points on the sides AB, BC, CD and DA respectively such that AE = BF = CG = DH. Prove that EFGH is a square.

Solution:

Given : ABCD is a square in which E, F, G and H are points on AB, BC, CD and DA

such that AE = BF = CG = DH



EF, FG, GH and HE are joined

To Prove : EFGH is a square

Proof :

Since, AE = BF = CG = DH

So, EB = FC = GD = HA

Now, in ΔAEH and $\Delta\ BFE$

AE = BF [Given]

AH = EB [Proved]

$\angle A = \angle B$ [Each 90°]

So, $\Delta AEH \cong\ \Delta BFE$ by S.A.S. axiom of congruency

Then, by C.P.C.T. we have

EH = EF

And $\angle 4 = \angle 2$

But $\angle 1 + \angle 4 = 90^{\circ}$

$$\angle 1 + \angle 2 = 90^{\circ}$$

Thus, $\angle HEF = 90^{\circ}$

Hence, EFGH is a square.

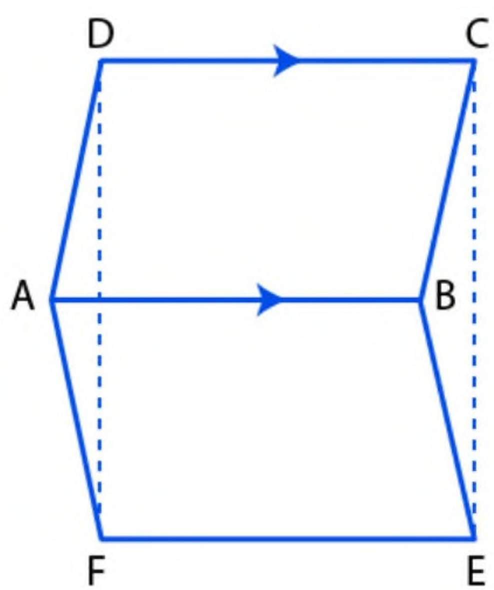
22. (a) In the figure (1) given below, ABCD and ABEF are parallelograms, Prove that

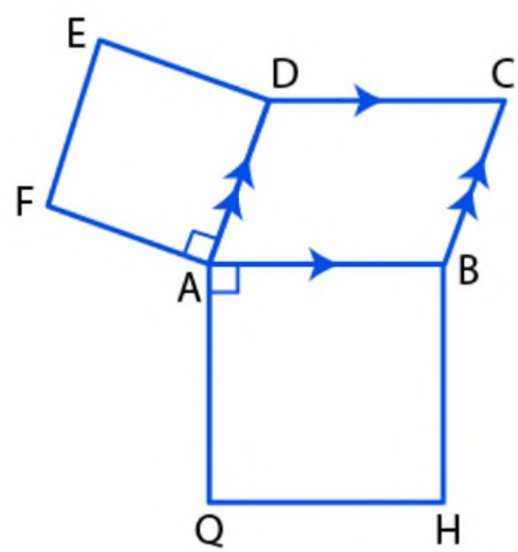
(i) CDEF is a parallelogram

(ii) FD = EC

(iii) $\Delta AFD = \Delta BEC$

(b) In the figure (2) given below, ABCD is a parallelogram, ADEF and AGHB are two squares. Prove that FG = AC.





Solution:

Given : ABCD and ABEF are || gms

To prove : (i) CDEF is a parallelogram

(ii) FD = EC

(iii) $\Delta AFD = \Delta BEC$

Proof :

(1) DC || AB and DC = AB [ABCD is a || gm]

(2)FE ||AB and FE = AB [ABEF is a || gm]

(3) DC || FE and DC = FE [From (1) and (2)]

Thus, CDEF is a || gm

(4) CDEF is a || gm

So, FD = EC

(5) In ΔAFD and $\Delta BEC,$ we have

AD = BC [Opposite sides of || gm ABCD are equal]

AF = BE [Opposite sides of || gm ABEF are equal]

FD = BE [From (4)]

Hence, $\Delta AFD \cong \Delta BEC$ by S.S.S, axiom of congruency

(b) Given : ABCD is a || gm, ADEF and AGHB are two squares

To Prove : FG = AC

Proof :

(1) $\angle FAG + 90^\circ + 90^\circ + \angle BAD = 360^\circ$ [At a point total angle is 360°]

$$\angle FAG = 360^\circ - 90^\circ - 90^\circ - \angle BAD$$

$$\angle FAG = 180^\circ - \angle BAD$$

(2) $\angle B + \angle BAD = 180^\circ$ [Adjacent angle in || gm is equal to 180°]

$\angle B = 180^\circ - \angle BAD$

(3) $\angle FAG = \angle B$ [From (1) and (2)]

(4) In ΔAFG and ΔABC, we have

AF = BC [FADE and ABCD both are squares on the same base]

Similarly, AG = AB

$\angle FAG = \angle B$ [From (3)]

So, $\Delta\ AFG \cong \Delta ABC\ by\ S.A.S.$ axiom of congruency

Hence, by C.P.C.T

FG = AC

23. ABCD is a rhombus in which $\angle A = 60^\circ$. Find the ratio AC : BD.

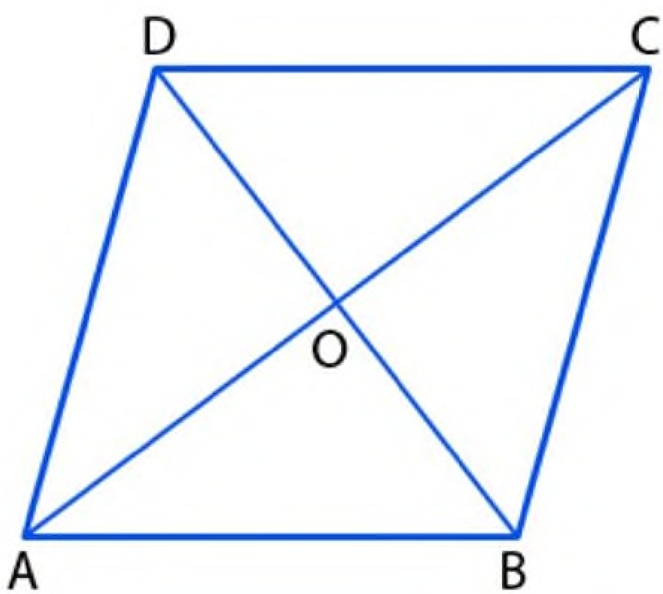
Solution:

Let each side of the rhombus ABCD be a

$$\angle A = 60^\circ$$

So, ABD is an equilateral triangle

$$\Rightarrow BD = AB = a$$



We know that, the diagonals of a rhombus bisect each other at right angles

So, in right triangle AOB, we have

$AO^2 + OB^2 = AB^2$ [By Pythagoras Theorem

$$AO^2 = AB^2 - OB^2$$

$= a^2 - \left(\frac{1}{2}a\right)^2$

$= a^2 - \frac{a^2}{4}$

$= \frac{3a^2}{4}$

AO = $\sqrt{\frac{3a^2}{4}} = \frac{\sqrt{3a}}{2}$

But, AC = 2 AO = $2 \times \frac{3a}{2}$ = 3a

Hence,

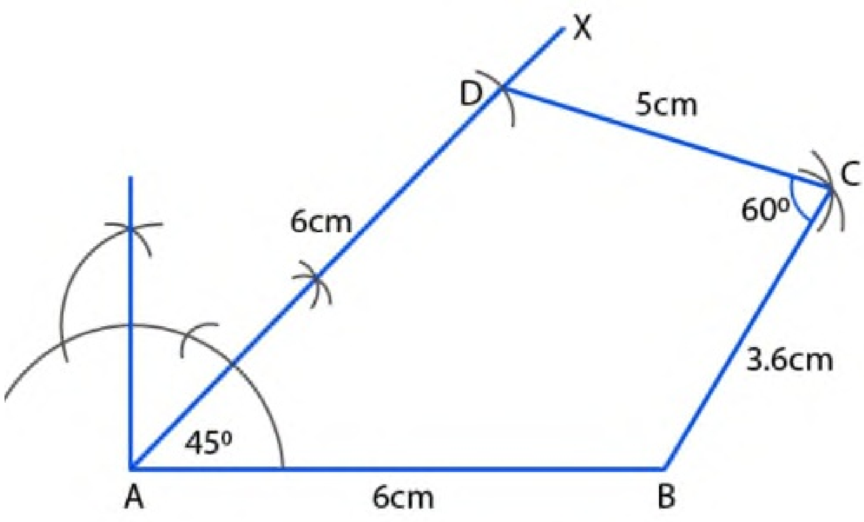
AC : BD = $\sqrt{3a}$: a = $\sqrt{3}$: 1

Exercise 13.2

Question 1

Using ruler and compasses only , construct the quadrilateral ABCD in which ∠BAD = 45° , AD = AB = 6cm, BC = 3.6cm, CD = 5cm . measure ∠BCD.

Solution:



Steps of construction:

(i) Draw a line segment AB = 6cm

(ii) At A, draw a ray AX making an angle of 45° and cut of AD = 6cm

(iii)with centre B and radius. 3.6 and with centre D and radius 5cm, draw two arcs intersecting each other at C.

(iv) join BC and DC.

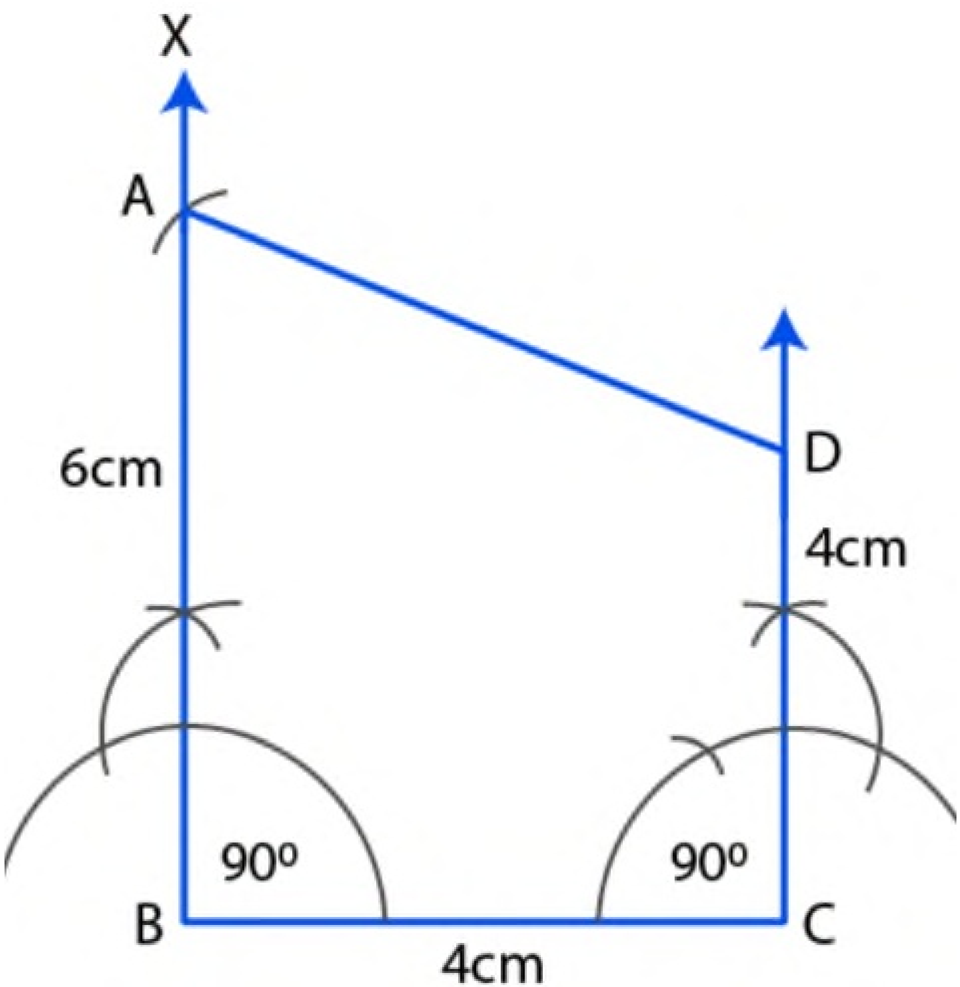
Thus, ABCD is the required quadrilateral.

On measuring ∠BCD, it is 60°

Question 2.

Draw a quadrilateral ABCD with AB = 6cm, BC = 4Cm , CD = 4cm and ∠BC = ∠BCD = 90°

Solution :



Steps of construction:

(i) Draw a line segment BC = 4cm

(ii) At B and C draw rays BX and CY making an angle of 90 each

(iii) From BX, cut off BA = 6cm and from CY, cut off CD = 4cm

(iv) join AD.

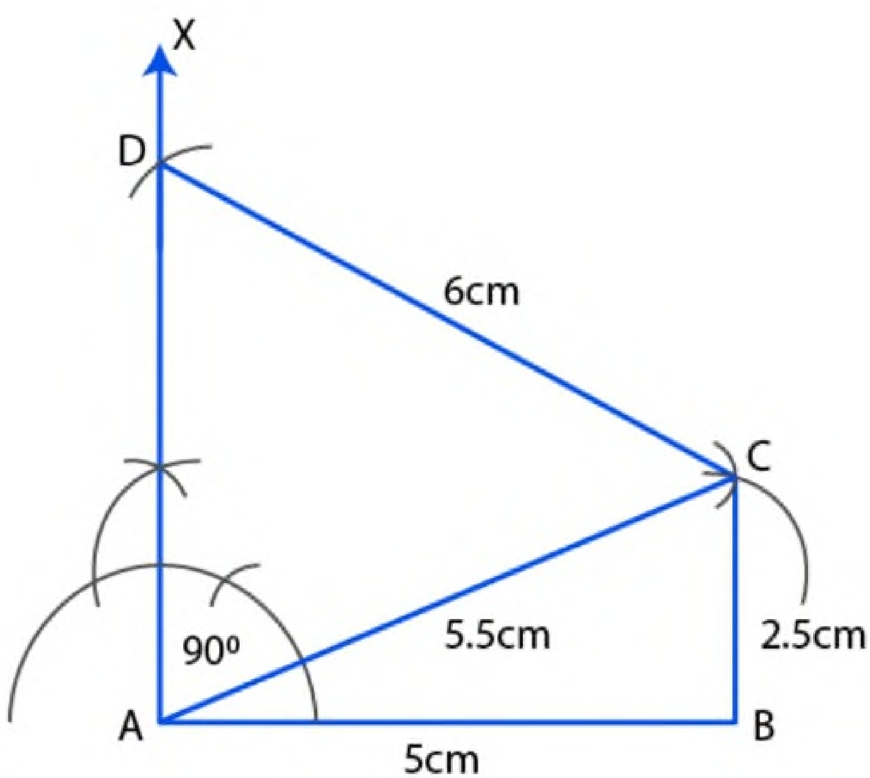
Thus, ABCD is the required quadrilateral.

Question 3.

Using ruler and compasses only, construct the quadrilateral ABCD given that AB = 5cm , BC = 2.5 cm, CD = 6cm , ∠BAD = 90°

And the diagonal AC = 5.5 cm.

Solution:



Steps of construction:

(i) Draw a line segment AB = 5cm

(ii) with centre A and radius 5.5cm and with centre B and radius 2.5 cm draw arcs which intersect each other at C.

(iii) join AC and BC.

(iv) At A draw a ray AX making an angle of 90.

(v) with centre C and radius 6cm, draw an arc intersecting AX at D

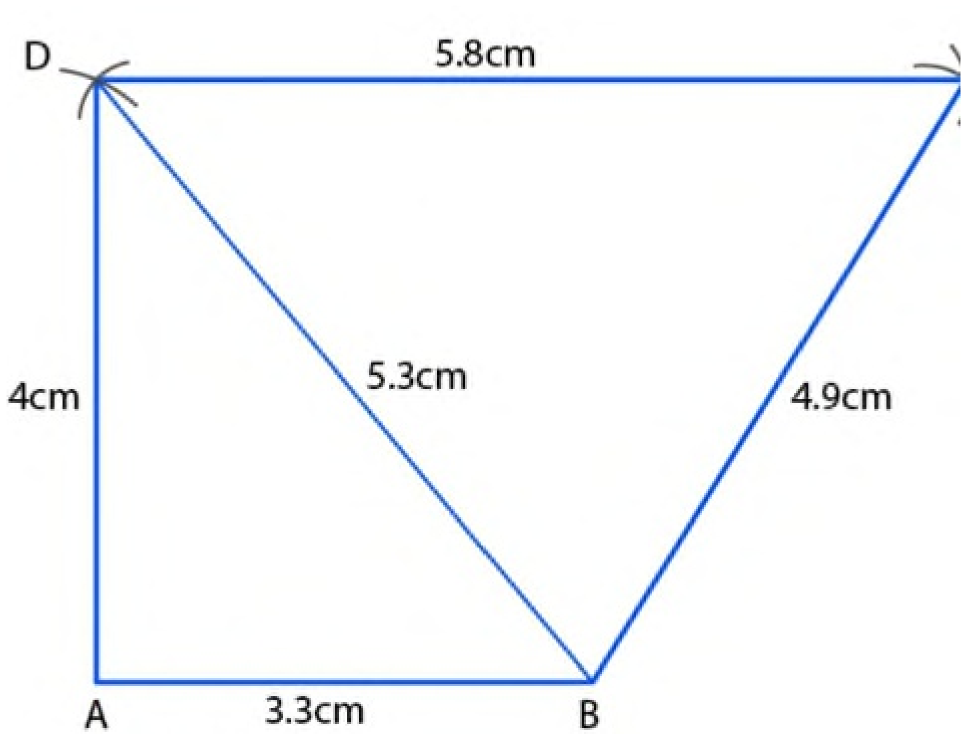
(vi) join CD.

Thus, ABCD is the required quadrilateral .

Question 4.

Construct a quadrilateral ABCD in which AB = 3.3 cm, BC = 4.9 cm , CD = 5.8 cm, DA = 4 and BD = 5.3cm.

Solution:



c

steps of construction:

(i) Draw a line segment AB = 3.3cm

(ii) with centre A and radius 4cm, and with centre B and radius 5.3cm, draw arcs intersecting each other at D.

(iii) join AD and BD.

(iv) with centre B and radius 4.9 cm and with centre D and radius 5.8cm, draw arcs intersecting each other at C.

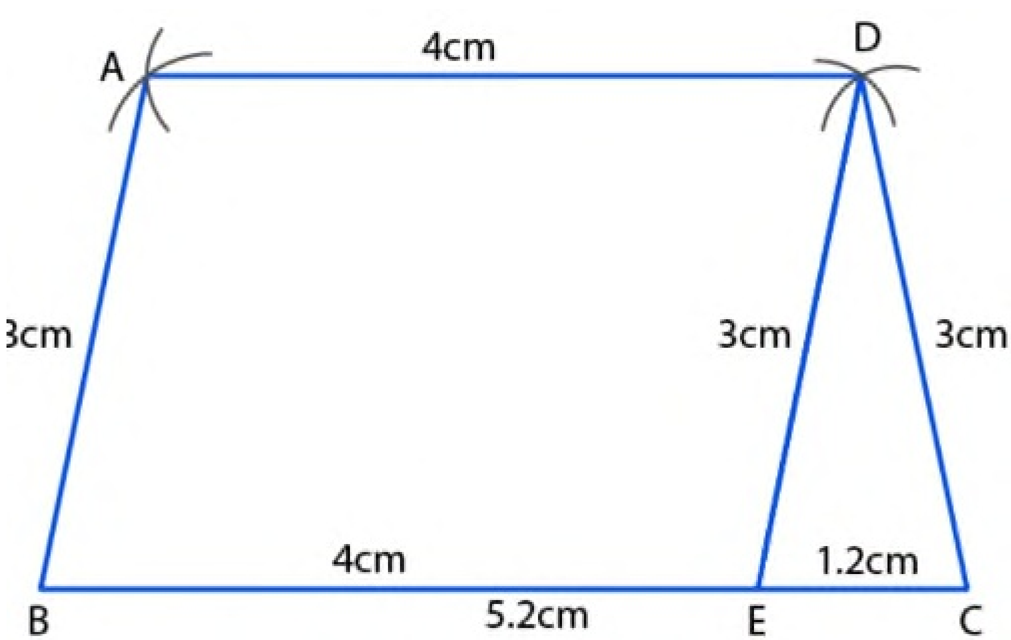
(v) join BC and DC.

Thus, ABCD is the required quadrilateral.

Question 5.

Construct a trapezium ABCD in which AD || BC, AB = CD = 3 Cm , BC = 5.2cm and AD = 4cm.

Solution:



Steps of construction:

(i) Draw a line segment BC = 5.2 cm

(ii) From BC, cut of BE = AD = 4cm

(iii) with centre E and C , and radius 3cm, draw arcs intersecting each other at D.

(iv) Join ED and CD.

(v) With centre D and radius 4cm and with centre B and radius 3cm, draw arcs intersecting each other at A.

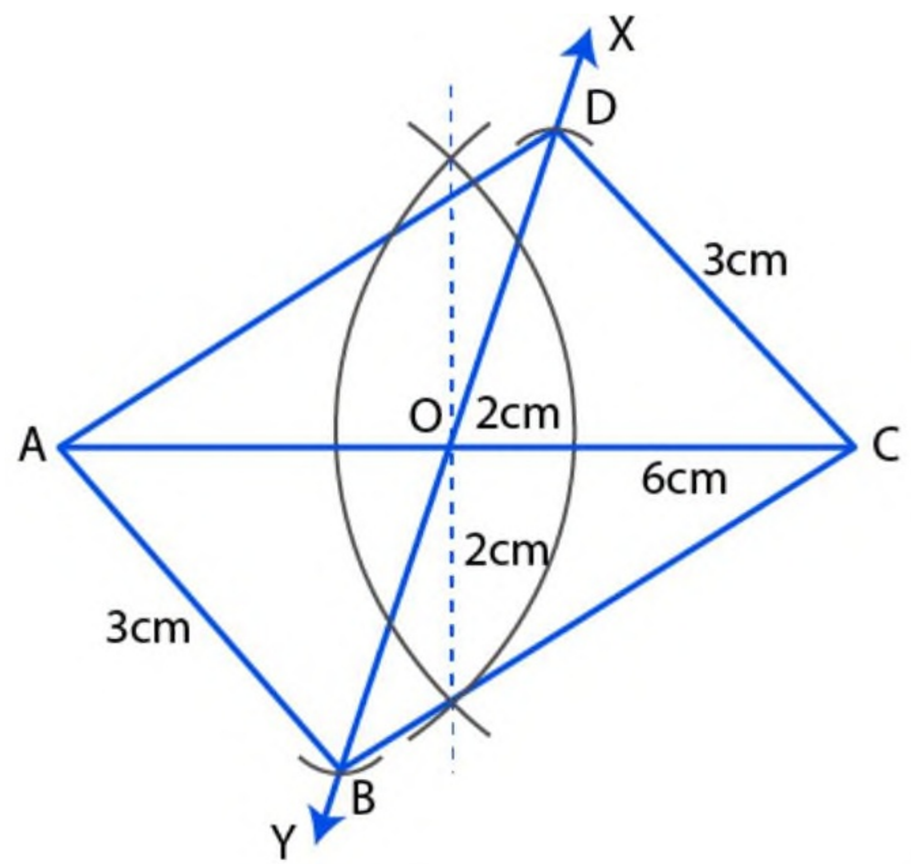
(vi) Join BA and DA

Thus, ABCD is the required trapezium.

Question 6.

Construct a trapezium ABCD in which AD ∥ BC , B = 60 , AB = 5cm. BC = 6.2cm and CD = 4.8cm.

Solution:



Steps of construction:

(i) Draw a line segment BC = 6.2cm

(ii) At B, draw a ray BX making an angle of 60 and cut off AB = 5cm

(iii) from A , draw a line AY parallel to BC.

(iv) with centre C and radius 4.8cm, draw are which intersects AY at D and D' .

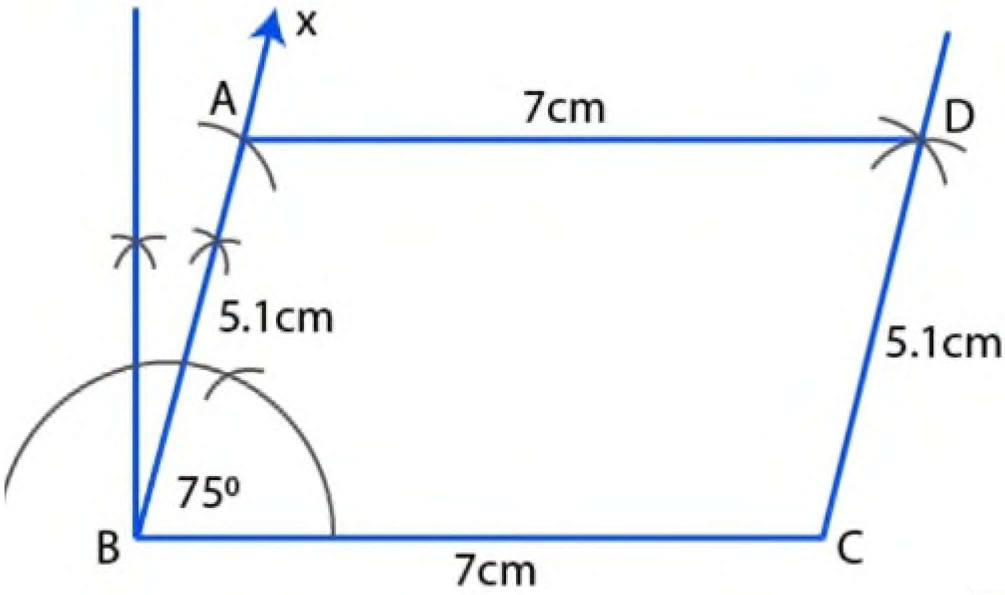
(v) join CD and CD'

Thus, ABCD and ABCD' are the required two trapeziums.

Question 7.

Using ruler and compasses only, construct a parallelogram ABCD with AB = 5.1 cm, BC = 7cm and ABC = 75.

Solution:



Steps of construction:

(i) Draw a line segment BC = 7cm

(ii) A to B, draw a ray Bx making an angle of 75 and cut off AB = 5.1cm

(iii) with centre A and radius 7cm with centre c and radius 5.1cm,

Draw arcs intersecting each other at D.

(iv) join AD and CD.

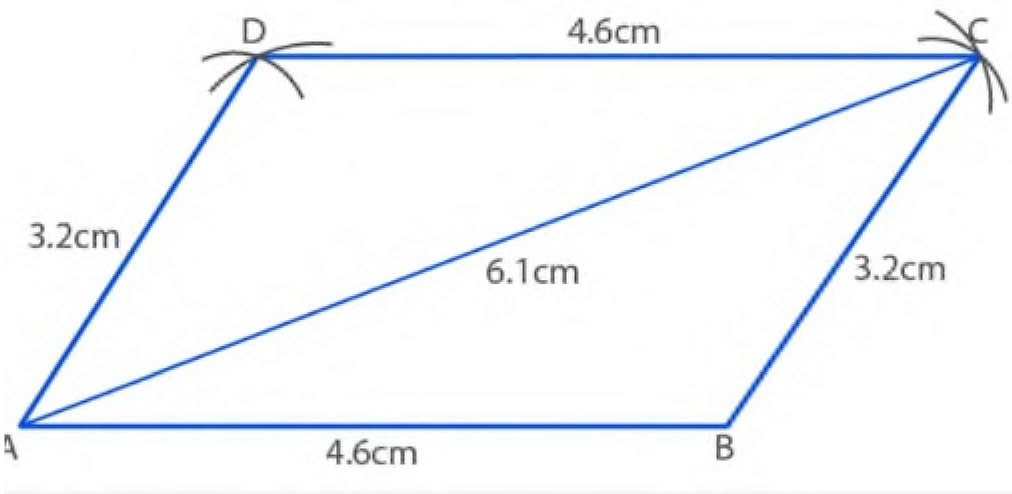
Thus, ABCD is the required parallelogram.

Question 8.

Using ruler and compasses only, construct a parallelogram

ABCD in which AB = 4.6cm BC = 3.2 and AC = 6.1cm.

Solution:



Steps of construction:

(i) Draw a line segment AB = 4.6cm

(ii) with centre A and radius 6.1cm and with centre B and radius 3.2cm, draw arcs intersecting each other at C.

(iii) Join AC and BC.

(iv) Again , with centre A and radius 3.2cm and with centre C and radius 4.6cm, draw arcs intersecting each other at C.

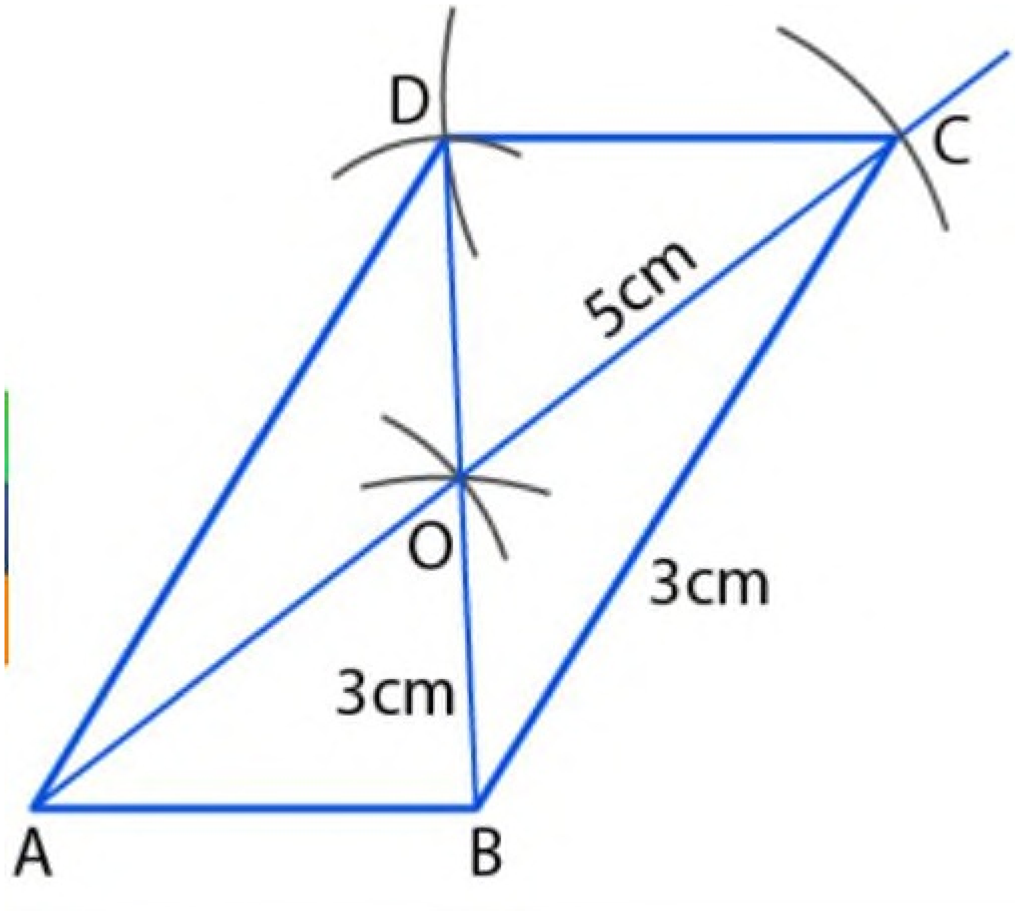
(v) Join AD and CD.

Thus, ABCD is the required parallelogram.

Question 9.

Using ruler and compasses, construct a parallelogram ABCD give that AB = 4cm, AC = 10cm , BD = 6cm. Measure BC.

Solution:



Steps of construction:

(i) construct triangle OAB such that

OA = $\frac{1}{2} \times AC = \frac{1}{2} \times 10cm = 5cm$

OB = $\frac{1}{2} \times BD = \frac{1}{2} \times 6cm = 3cm$

As, diagonals of ∥ gm bisect each other and AB = 4cm

(ii) Produce AO to C such that OA = OC = 5cm

(iii) Produce BO to D such that OB = OD = 3cm

(iv) join AD, BC and CD

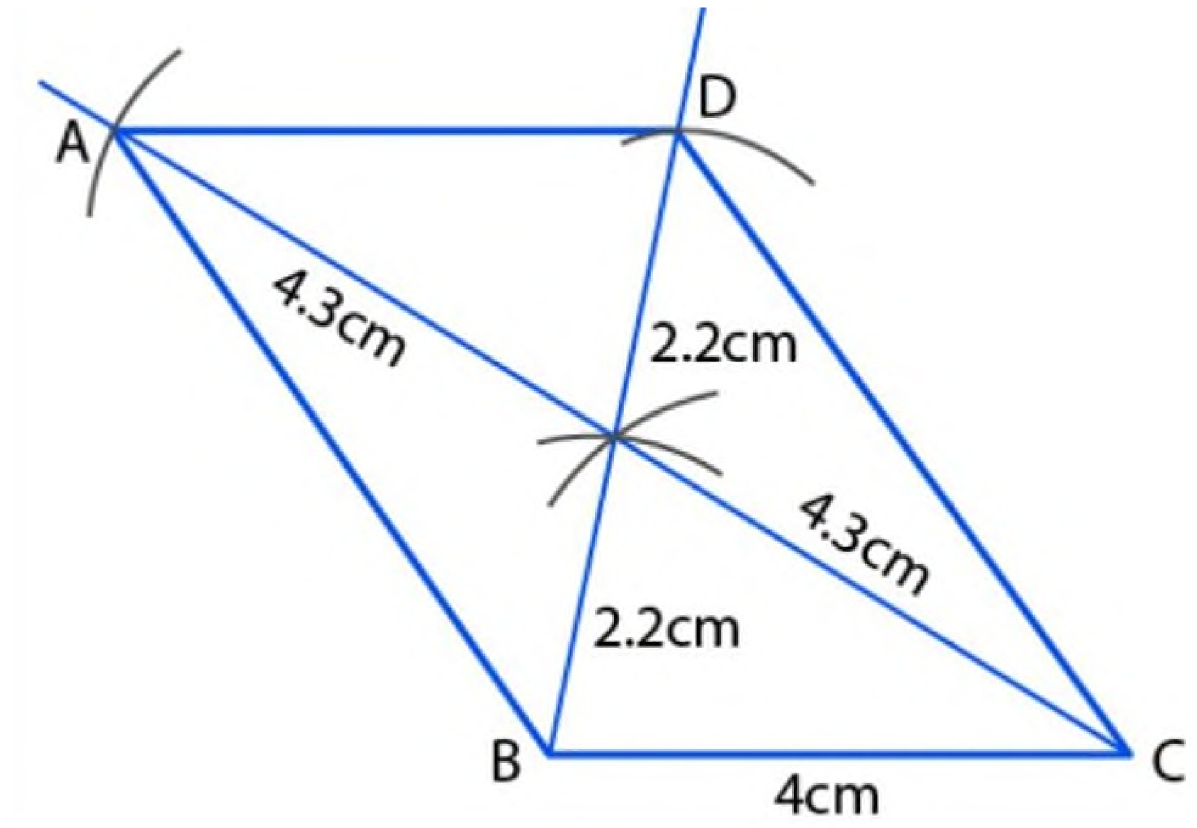
Thus , ABCD is the required parallelogram

(v) measure BC which is equal to 7.2cm

Question 10.

Using ruler and compasses only, construct a parallelogram ABCD such that BC = 4cm , diagonal AC = 8.6 cm and diagonal BD = 4.4cm. measure the side AB.

Solution:



Steps of construction;

(i) Construct triangle OBC such that

OB = $\frac{1}{2} \times BD = \frac{1}{2} \times 4.4cm = 2.2cm$

OC = $\frac{1}{2} \times AC = \frac{1}{2} \times 8.6cm = 4.3$

Since, diagonals of ∥ gm bisect each other and BC = 4cm

(ii) produce BO to D such that BO = OD = 2.2cm

(iii) Produce CO to A such that CO = OA = 4.3cm

(iv) Join AB, AD and CD

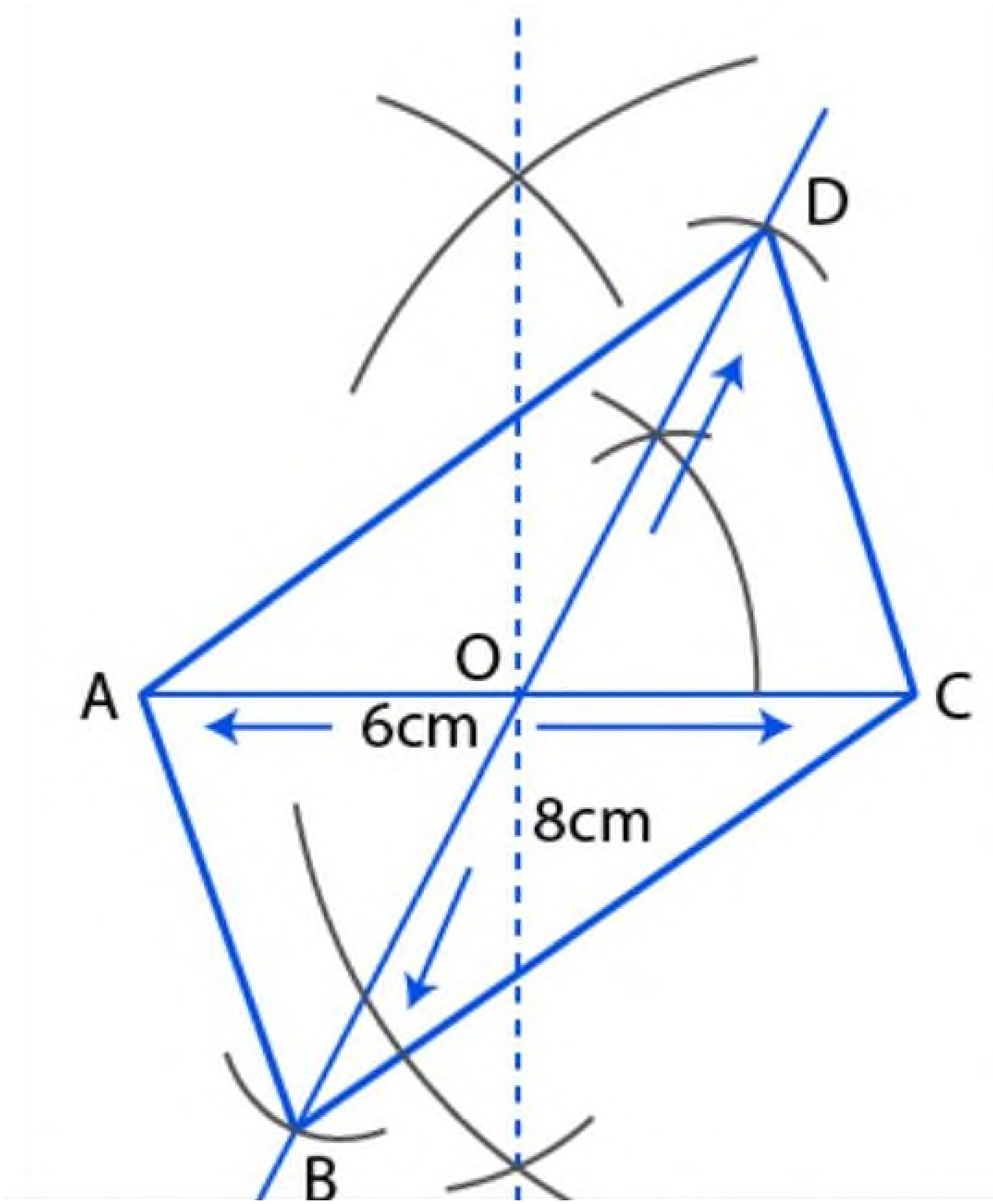
Thus, ABCD is the required parallelogram.

(v) Measure the side AB, AB = 5.6cm

Question 11.

Use ruler and compasses to construct a parallelogram with diagonals 6cm and 8 cm in length having given the acute angle between them is 60. Measure one of the longer sides.

Solution:



Steps of construction:

(i) Draw AC = 6cm

(ii) Find the mid – point O of AC . [since , diagonals of ∥ gm bisect each other]

(iii) Draw line POQ such that POC = 60° and OB = OD = $\frac{1}{2}\ BD = \frac{1}{2}x$ 8cm = 4cm

So, from OP cut OD = 4cm and from OQ cut OB = 4cm

(iv) Join AB, BC, CD and DA.

Thus, ABCD is the required parallelogram.

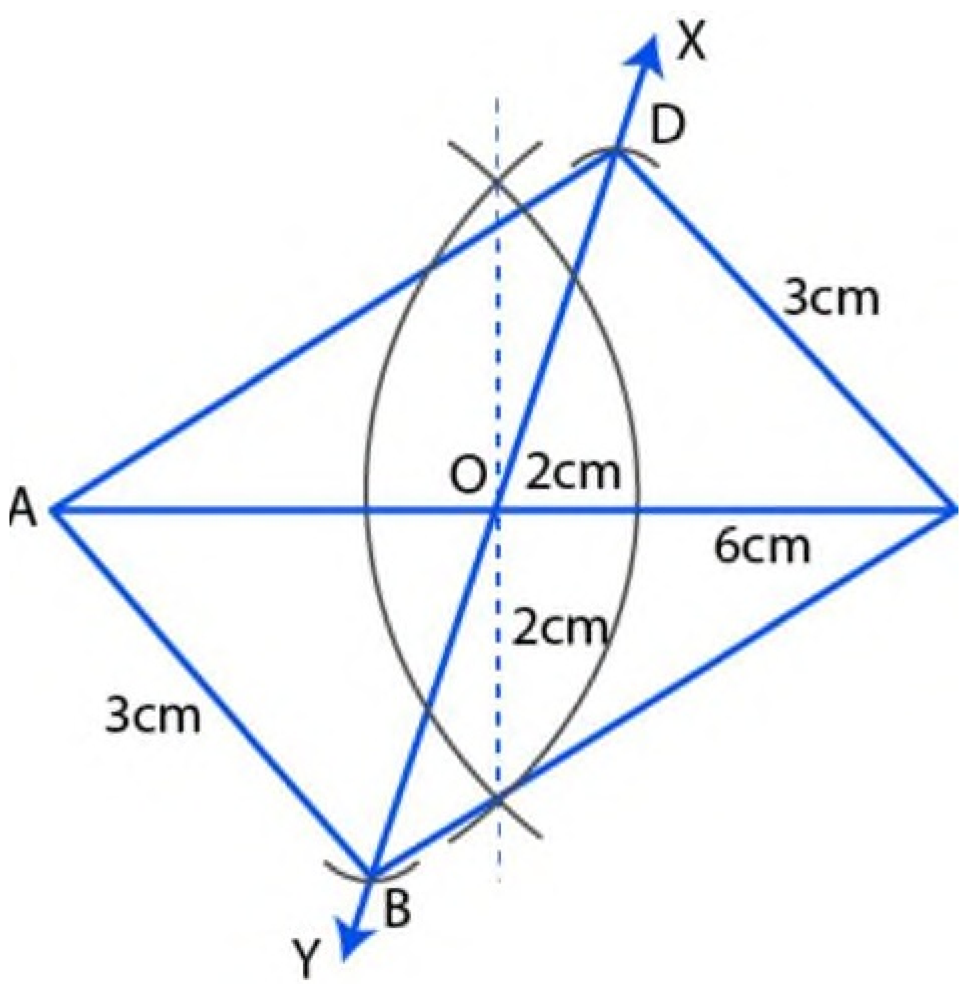
(v) Measure the length of side AD = 6.1cm

Question 12.

Using ruler and compasses only, draw a parallelogram whose diagonals are 4cm and 6cm long and contain an angle of 75°

Measure and write down the length of one of the shorter sides of the parallelogram.

Solution:



c

steps of construction:

(i) Dra a line segment AC = 6cm

(ii) Bisect AC at O.

(iii) At O , Draw a ray XY making an angle of 75° at O.

(iv) From OX and OY , cut off OD = OB = $\frac{4}{2}$ = 2cm

(v) Join AB , BC, CD and DA

Thus, ABCD is the required parallelogram

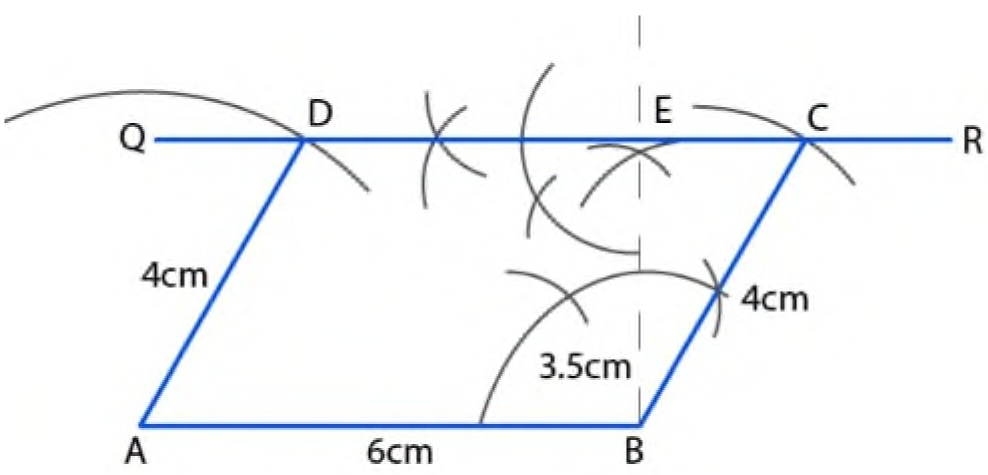
On measuring one of the shorter sides, we get

AB = CD = 3cm

Question 13.

Using ruler and compasses only, construct a parallelogram ABCD with AB = 6cm, altitude = 3.5cm and side BC = 4cm. Measure the acute angles of the parallelogram.

Solution:



Steps of construction:

(i) Draw AB = 6cm

(ii) At B, Draw BP ⊥ AB

(iii) From BP, cut BE = 3.5 cm = height of ∥ gm

(iv) through E draw QR parallel to AB

(v) With B as centre and radius BC = 4cm draw an arc which cuts QR at C.

(vi) Since, opposite sides of ∥ gm are equal

So, AD = BC = 4cm

(vii) with A as centre and radius = 4cm draw an arc which cuts QR at D.

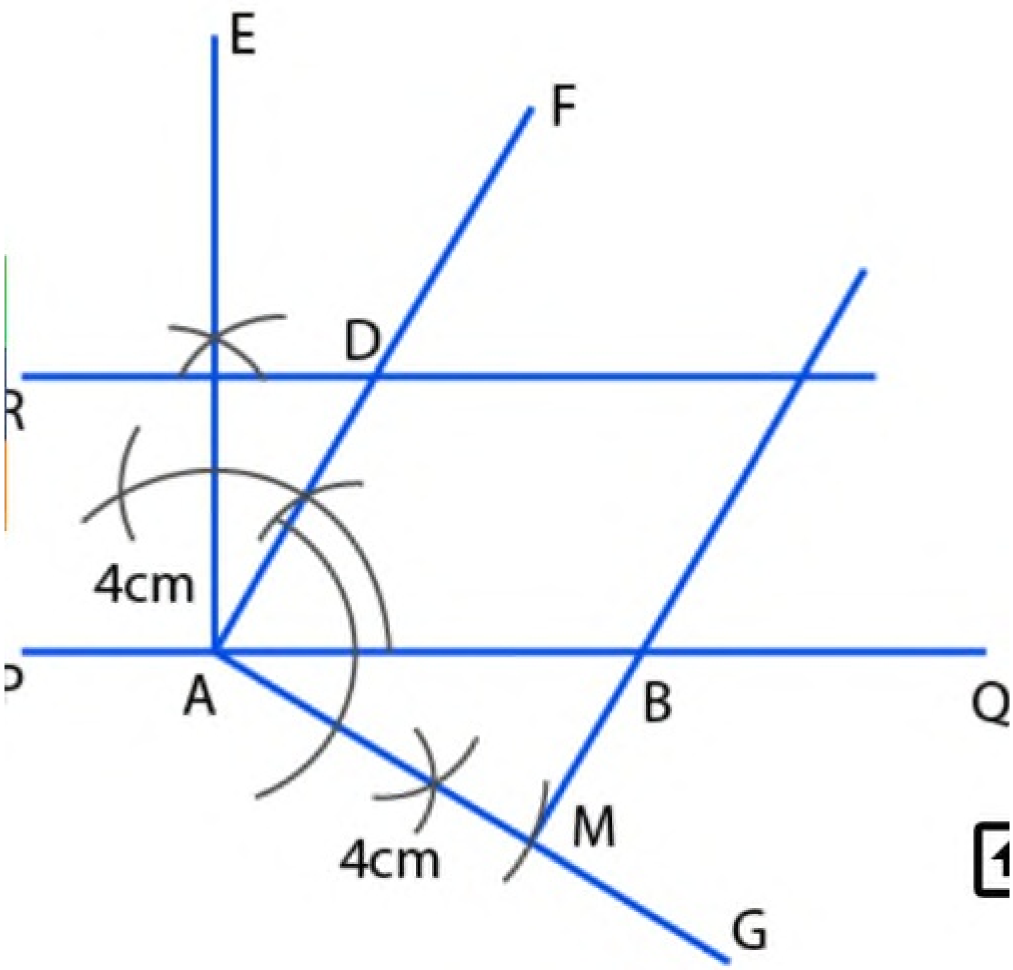
Thus, ABCD is the required parallelogram.

(viii) to measure the acute angle of parallelogram which is equal to 61°

Question 14.

The perpendicular distances between the pairs of opposite sides of a parallelogram ABCD are3cm and 4cm and one of its angles measures 60°. Using ruler and compasses only, construct ABCD.

Solution:



Steps of construction:

(i) Draw a straight – line PQ, take a point A on it.

(ii) At A, construct ∠QAF = 60°

(iii) At A draw AE ⊥ PQ from AE cut AN = 3cm

(iv) Through N draw a straight line to PQ to meet AF at D.

(v) At D, Draw AG ⊥ AD, from AG cut of AM = 4cm

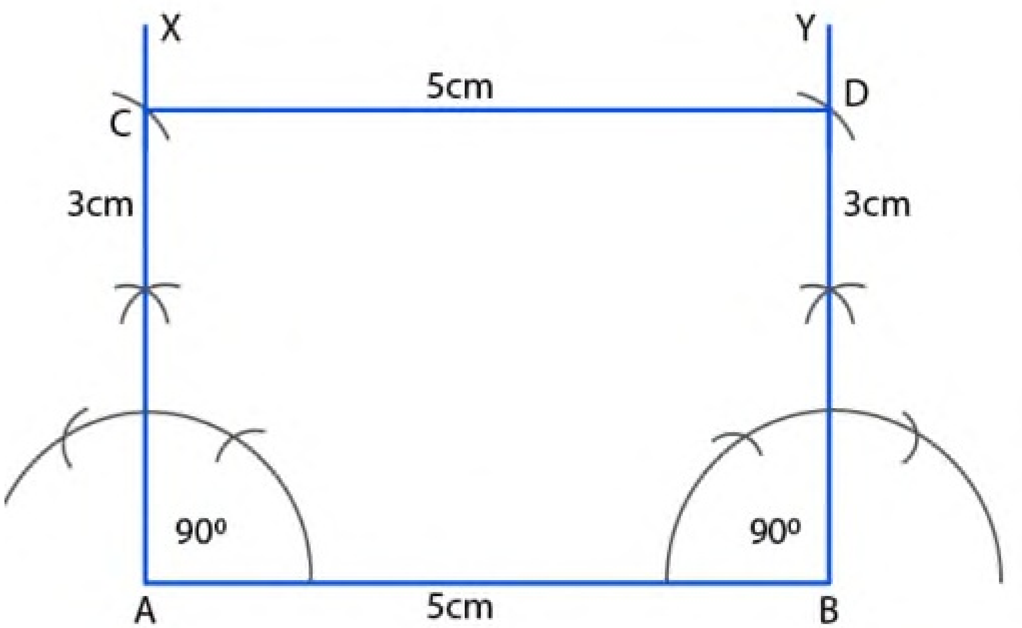
(vi) Thought M, Draw at straight line parallel toad to meet AQ in B and ND in C.

Then , ABCD is the required parallelogram

Question 15

Using ruler and compasses, construct a rectangle ABCD with AB = 5cm and AD = 3cm.

Solution:



Steps of construction:

(i) Draw a straight – line AB = 5cm

(ii) At A and B construct ∠XAB and ∠YBA = 90°

(iii) From A and B cut off AC and BD = 3cm each

(iv) join CD

Thus, ABCD is the required rectangle.

Question 16.

Using ruler and compasses only, construct a rectangle each of whose diagonals measures 6cm and the diagonals intersect at an angle of 45.

Solution:

Steps of construction:

(i) Draw a line segment AC = 6cm

(ii) Bisect AC at O.

(iii) At O draw a ray XY making an angle of 45° at o .

(iv) From xy, cut off OB = OD $\frac{6}{2}$ = 3cm each

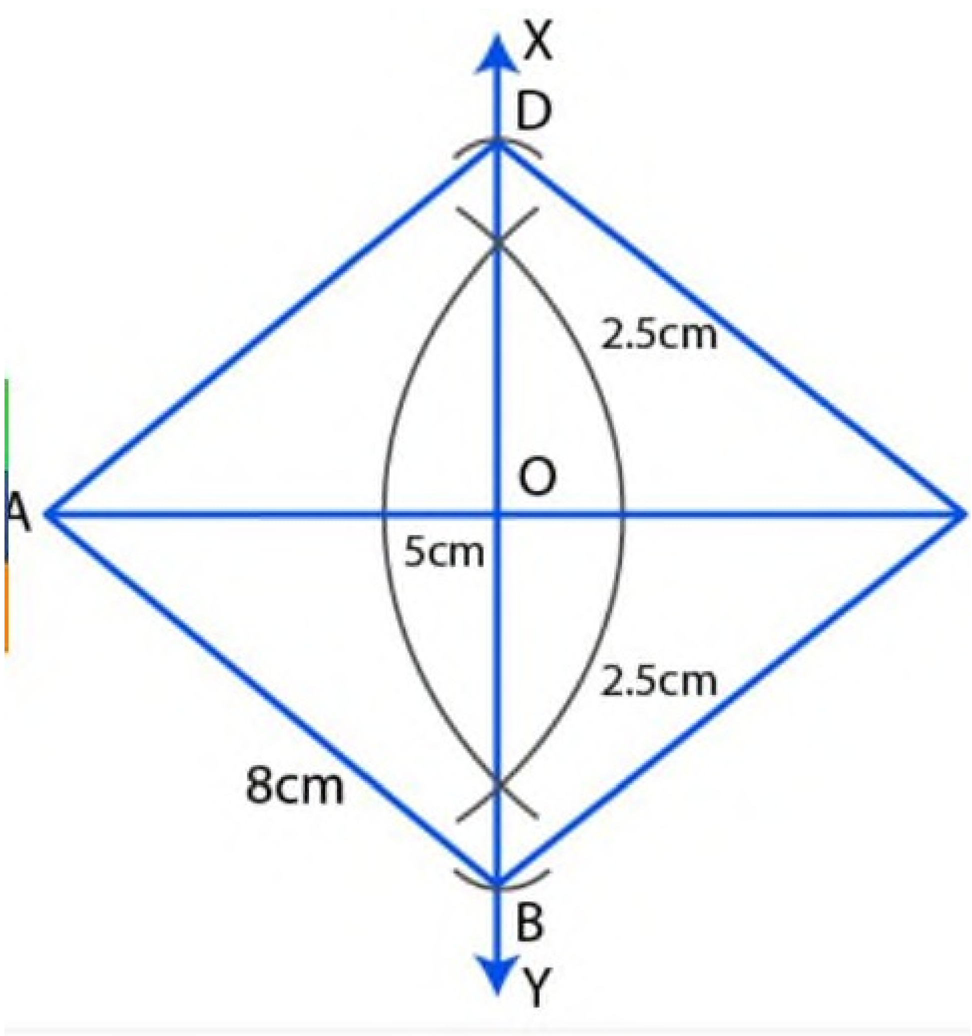
(v) join AB, BC CD and DA.

Thus, ABCD is the required rectangle.

Question 17.

Using ruler and compasses only, construct a square having a diagonal of length 5cm. Measure its sides correct to the nearest millimetre.

Solution:



c

steps of construction:

(i) Draw a line segment AC = 5cm

(ii) Draw its percentage bisector XY bisecting it at O

(iii) From XY, cut off

OB = OD = $\frac{5}{2}$ = 2.5cm

(iv) join AB, BC, CD and DA.

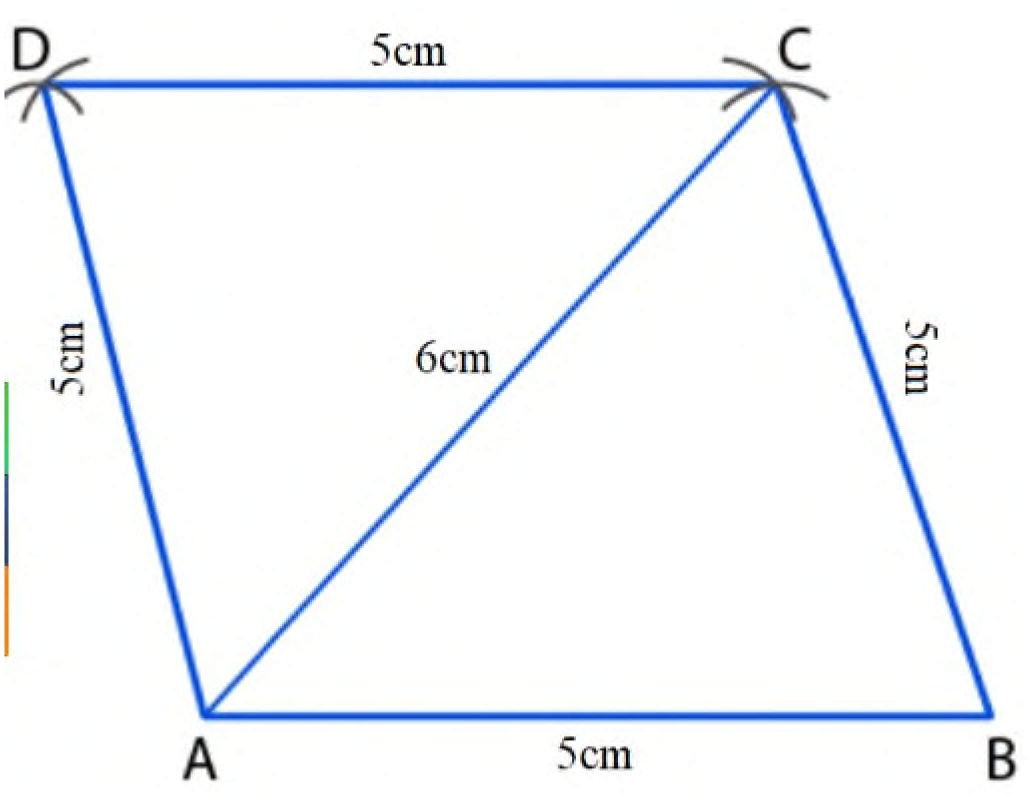
Thus, ABCD is the required square

On measuring its sides, each = 3.6cm (approximately)

Question 18.

Using ruler and compasses only construct A rhombus ABCD given that AB 5cm, AC = 6cm measure BAD.

Solution:



Steps of construction:

(i) Draw a line segment AB = 5cm

(ii) with centre A and radius 6cm, with centre B and radius 5cm draw arcs intersecting each other at C.

(iii) Join AC and BC

(iv) with centre A and C and radius 5cm, draw arc intersecting each other 5cm, draw arcs intersecting each other at D

(v) Join AD and CD.

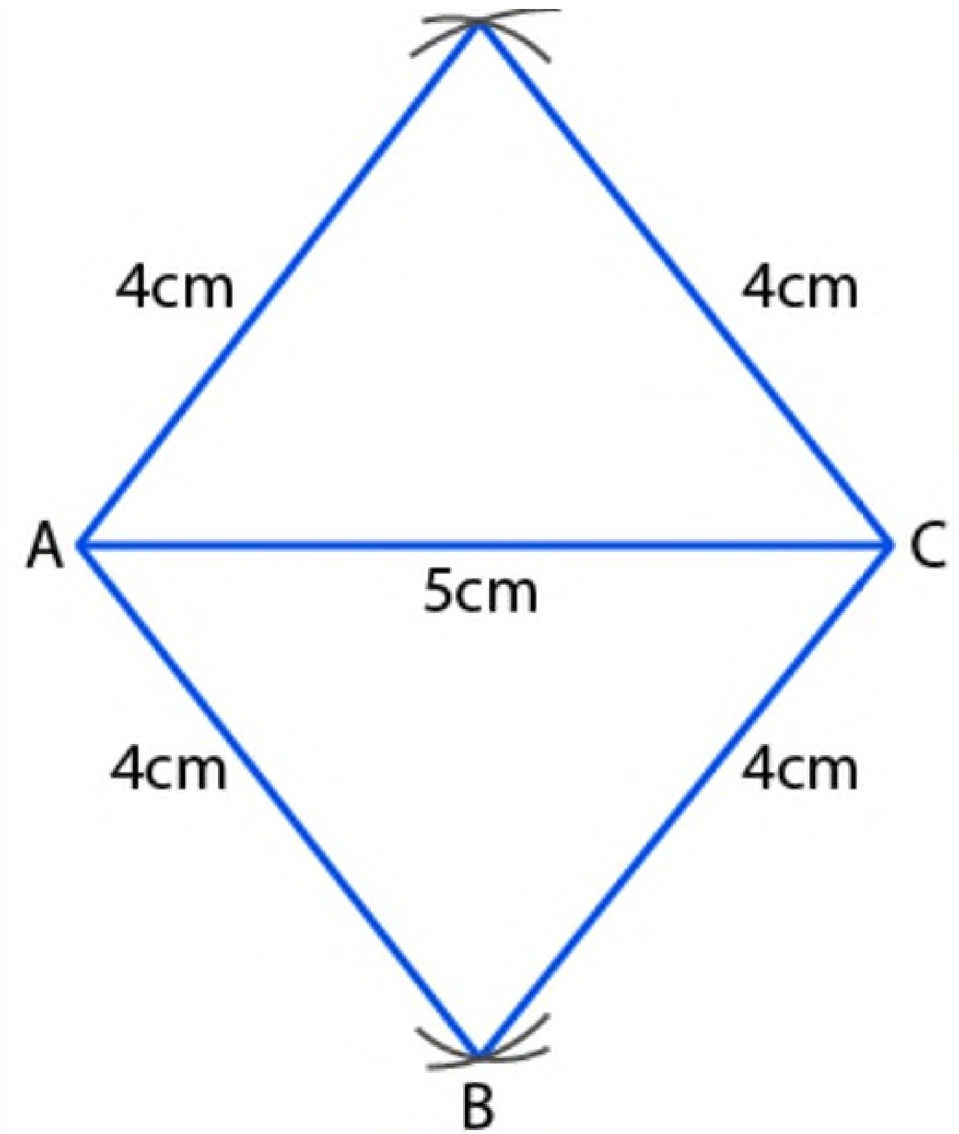
Thus, ABCD is a rhombus

On measuring , ∠BAD = 106°.

Question 19.

Using ruler and , construct rhombus ABCD with sides of length 4cm and diagonal AC of length 5cm . measure ∠ABC.

Solution: D



Steps of construction:

(i) Draw a line segment AC = 5cm

(ii) with centre A and c and radius 4cm, draw arcs intersecting each other above and below AC at D and B.

(iii) join AB, BC , CD and DA.

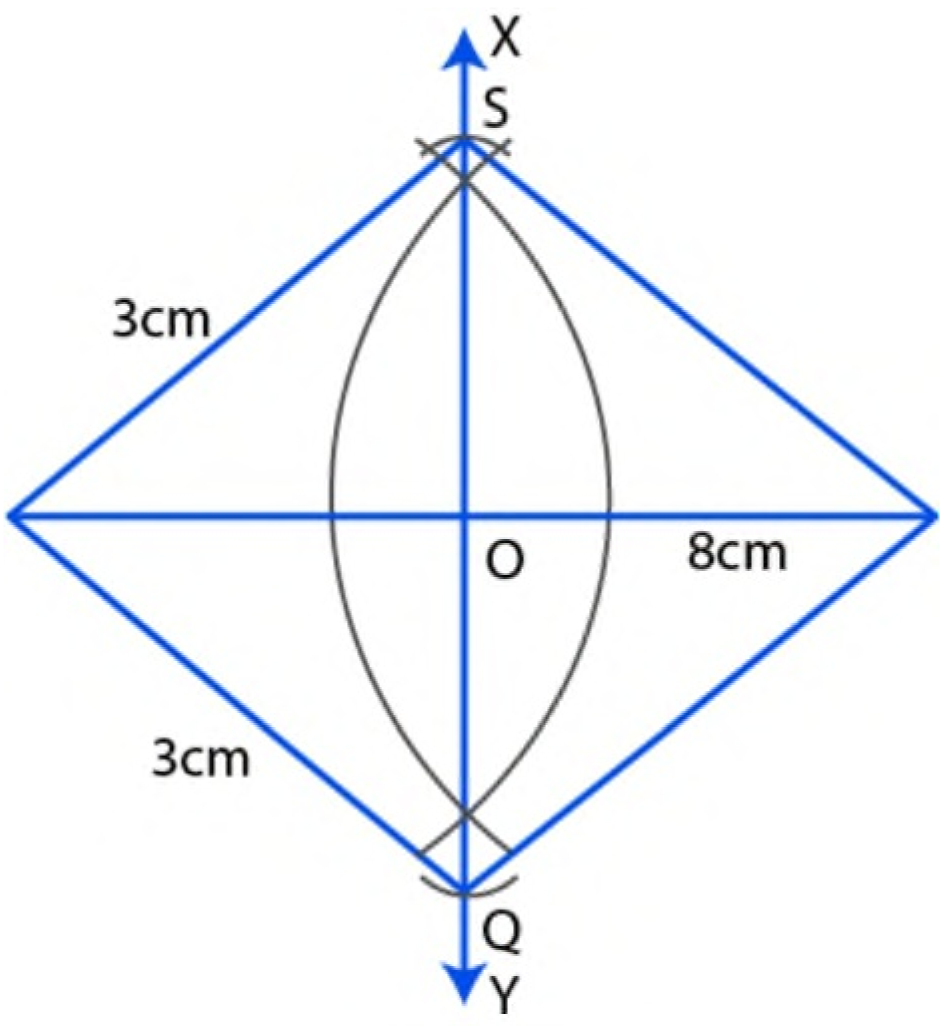
Thus, ABCD is the required rhombus.

Question 20.

Construct a rhombus PQRS whose diagonals PR and QS are 8 cm and 6cm respective

Solution:

P



Steps of construction:

(i) Draw a line segment PR = 8cm

(ii) Draw its perpendicular bisector XY intersecting it at O.

(iii) From XY, cut off OQ = OS = $\frac{6}{2} = 3cm\ each$

(iv) join PQ, PR ,RS and SP

Thus, PQRS and SP

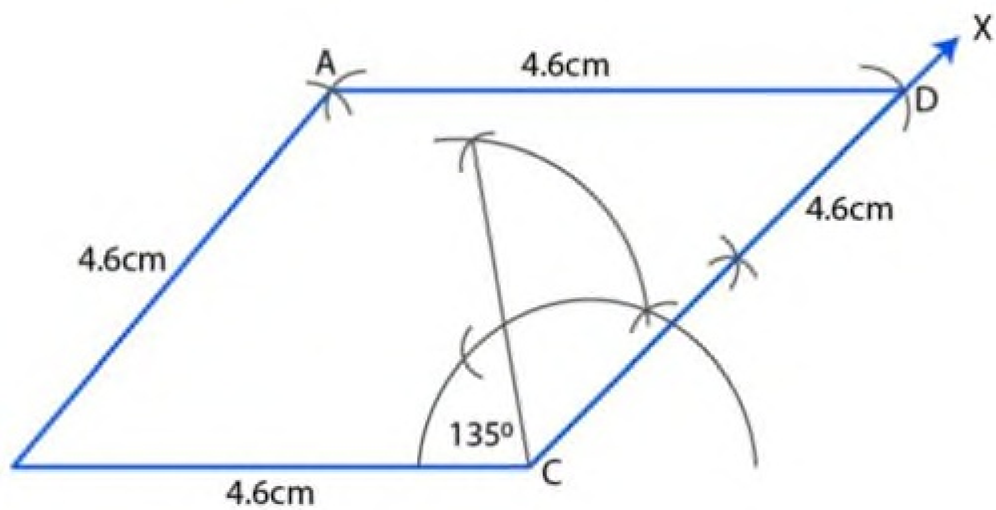
Thus, PQRS is the required rhombus.

Question 21.

Construct a rhombus ABCD of side 4.6 cm and ∠BCD = 135°.

By using ruler and compasses only.

Solution:



Steps of construction:

(i) Draw a line segment BC = 4.6cm

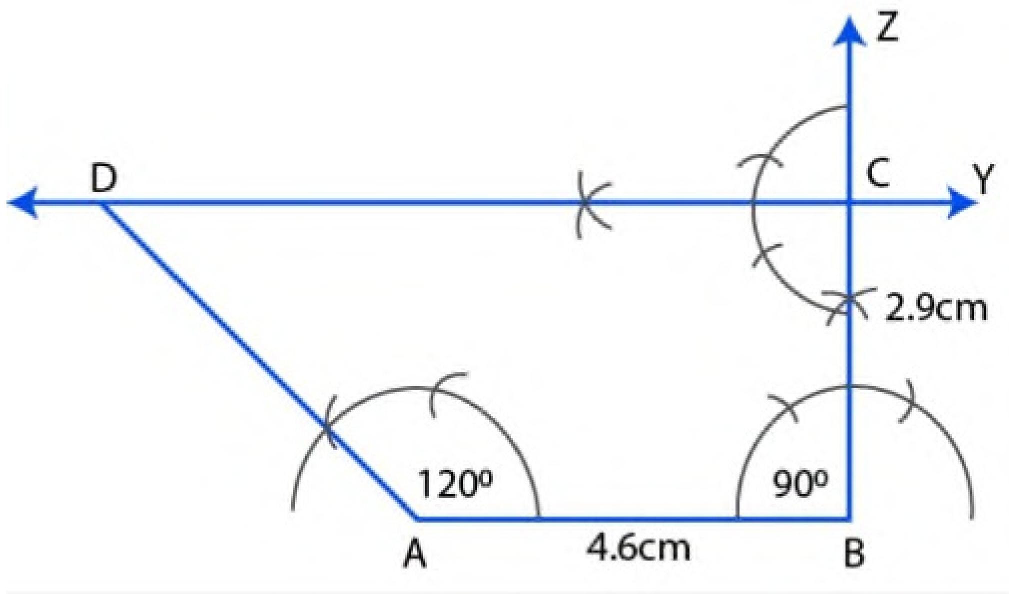
(ii) At C, Draw a ray CX making an angle of 135° and cut off CD = 4.6cm

(iii) with centres B and D, and radius 4.6cm draw arcs intersecting each other at A.

Thus, ABCD is the required rhombus.

Question 22.

Construct a trapezium in which AB || CD, AB = CD, AB = 4.6cm, ∠ABC = 90° ∠DAB = 120° and the distance between parallel sides is 2.9 cm.



Steps of construction:

(i) Draw a line segment AB = 4.6cm

(ii) At B, Draw a ray BZ making an angle of 90 and cut off BC = 2.9cm (distance between AB and CD)

(iii) At C, Draw a parallel line XY to AB.

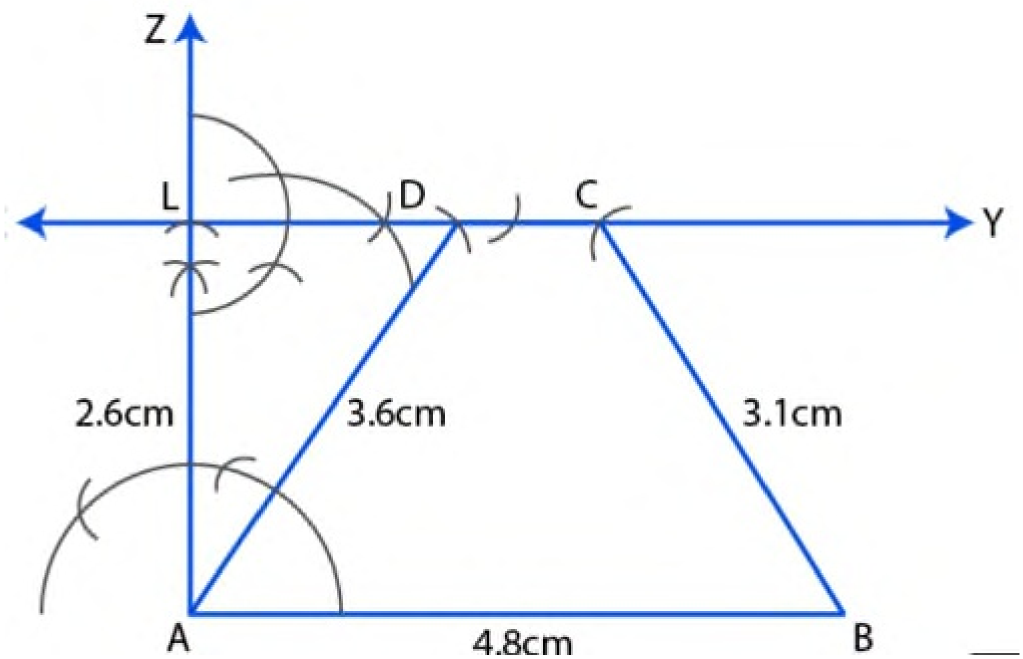
(iv) At A, Draw a ray making an angle of 120 meeting XY at D.

Thus. ABCD is the required trapezium.

Question 23.

Construct a trapezium ABCD when one of parallel sides AB = 4.8 cm, height = 2.6cm, BC = 3.1 cm and AD = 3.6cm

Solution:



Step construction:

(i) Draw a line segment AB = 4.8cm

(ii) At A, draw a ray AZ making an angle of 90 cut off AL = 2.6cm

(iii) At L, draw a line XY parallel to AB .

(iv) with centre A and radius 3.6cm and with centre B and radius3.1cm, draw arcs intersecting XY at D and C respectively.

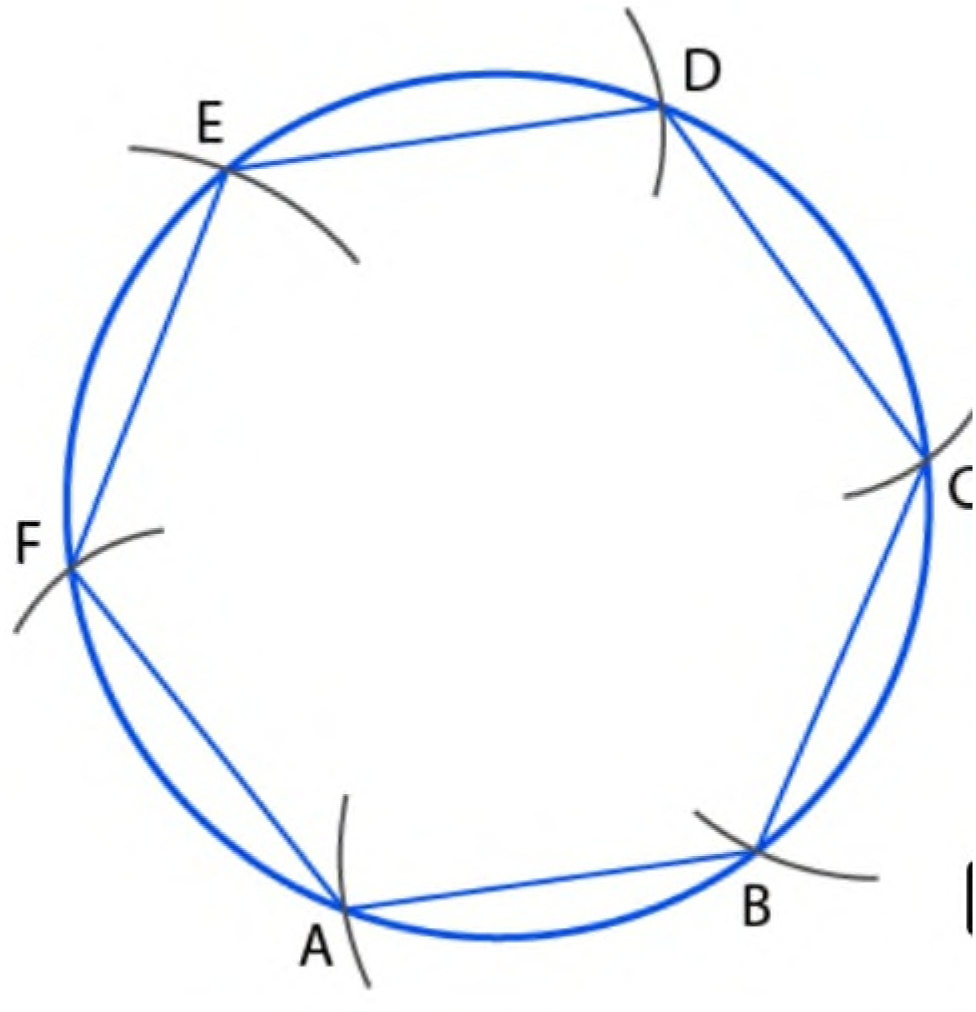
(v) join AD and BC

Thus, ABCD is the required trapezium.

Question 24.

Construct a regular hexagon of side 2.5cm.

Solution:



Steps of construction:

(i) With O as centre and radius = 2.5cm, draw a circle

(ii) take any point A on the circumference of circle.

(iii) with A as centre and radius = 2.5cm, draw an arc which cuts the circumference of circle at B.

(iv) with B as centre and radius= 2.5cm, draw and arc which cuts the circumference of circle at C.

(v) with C as centre and radius = 2.5cm , draw an arc which cuts the circumference of circle at D.

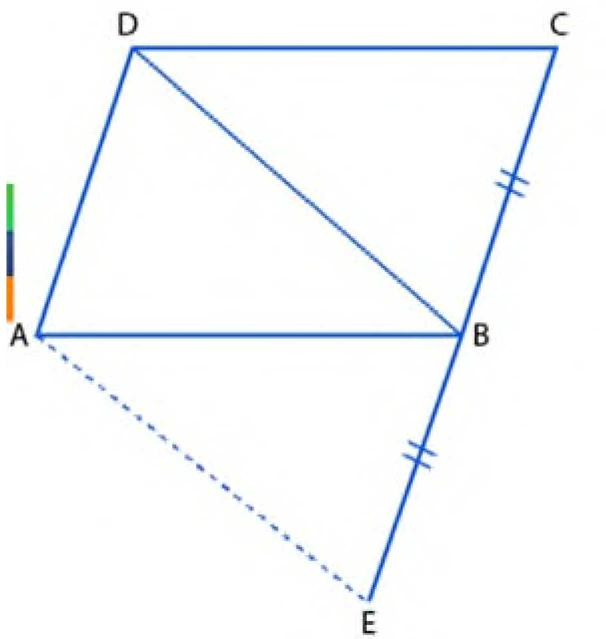
(vi) with D as centre and radius = 2.5cm, draw an arc which cuts the circumference of circle at E.

(vii) with E as centre and radius = 2.5cm. draw an arc which cuts the circumference of circle at F.

(viii) join AB, BC , CD , DE , EF and FA.

<u>**Chapter test**</u>

1. in the given figure, ABCD is a parallelogram . CB is produced to E such that BE = BC. Prove that AEBD is a parallelogram.



Solution

Given ABCD is a ∥ gm in which CB is produced to E such that BE = BC

BD and AE are joined

To prove : AEBD is a parallelogram

Proof:

In Δ AEB and Δ BDC

EB = BC [given]

∠ABE = ∠DCB [corresponding angles]

AB = DC [opposite sides of ∥ gm]

Thus, ΔAEB ≅ Δ BDC by S.A.S axiom

So, by C.P.C.T

But , AD = CB = BE [given]

As the opposite sides are equal and ∠AEB = ∠DBC

But these are corresponding angles

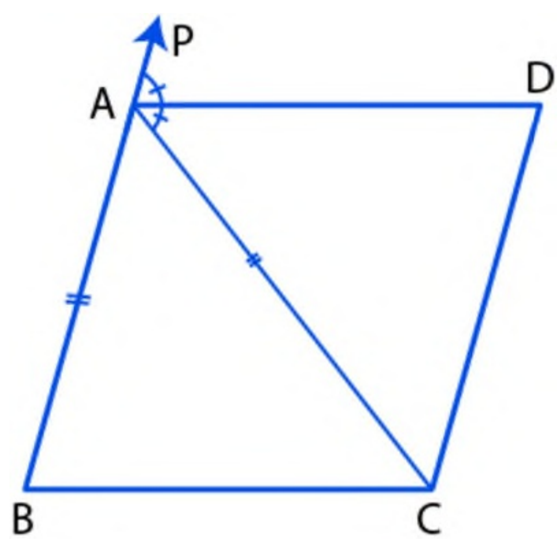
Hence, AEBD is a parallelogram .

2. in the given figure , ABC is an isosceles triangle in which AB = AC. AD bisects exterior angle PAC and CD ∥ BA . show that (i) ∠DAC = ∠BCA (ii) ABCD is parallelogram.

Solution

Given: In isosceles triangle ABC, AB = AC . AD is the bisector of ext. ∠PAC and CD ∥ BA

To prove: (i) ∠DAC = ∠BCA



(ii) ABCD is a ∥ gm

Proof:

In Δ ABC

AB = AC [given]

∠C = ∠B [angle opposite to equal sides]

Since , ext. ∠PAC = ∠B + ∠C

= ∠C + ∠C

= 2∠C

= 2∠BCA

So, ∠DAC = 2 ∠BCA

∠ DAC = ∠ BCA

But these are alternate angles

Thus, AD ∥ BC

But, AB ∥ AC

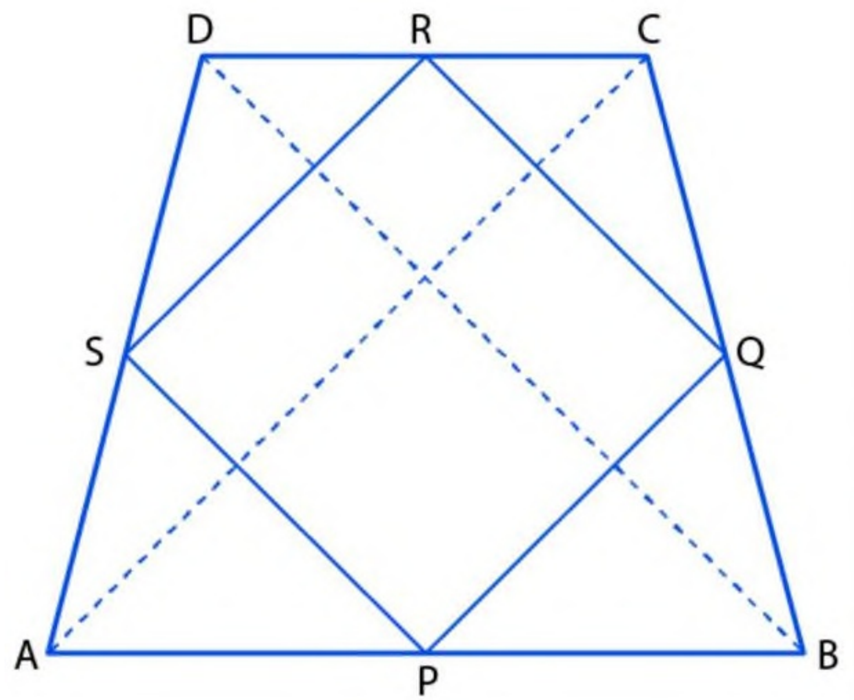
Hence ABCD is a ∥ gm .

3. prove that the quadrilateral obtained by joining the mid-points of an isosceles trapezium is a rhombus.

Solution

Given : ABCD is an isosceles trapezium in which AB ∥ DC and AD = BC

P,Q,R and S are the mid – points of the sides AB, BC, CD and DA respectively PQ, QR, RS and SP are joined.



To prove : PQRS is a rhombus

Construction : join AC and BD

Proof :

Since, ABCD is an isosceles trapezium

Its diagonals are equal

AC = BD

Now, in Δ ABC

P and Q are the mid- points of AB and BC

So, PQ ∥ AC and PQ = $\frac{1}{2}$ AC ......(i)

Similarly, in Δ ADC

S and R mid – point of CD and AD

So, SR ∥ AC and SR = $\frac{1}{2}$ AC....(ii)

From (i) and (ii) we have

PQ ∥ SR and PQ = SR

Thus, PQRS is a parallelogram.

Now, in Δ APS and ΔBPQ

AP = BP [p is the mid – point]

AS = BQ [half of equal sides]

∠ A = ∠B [as ABCD is an isosceles triangle]

So, Δ APS ≅ Δ BPQ by SAS axiom of congruency

Thus by C.P.C.T we have

PS = PQ

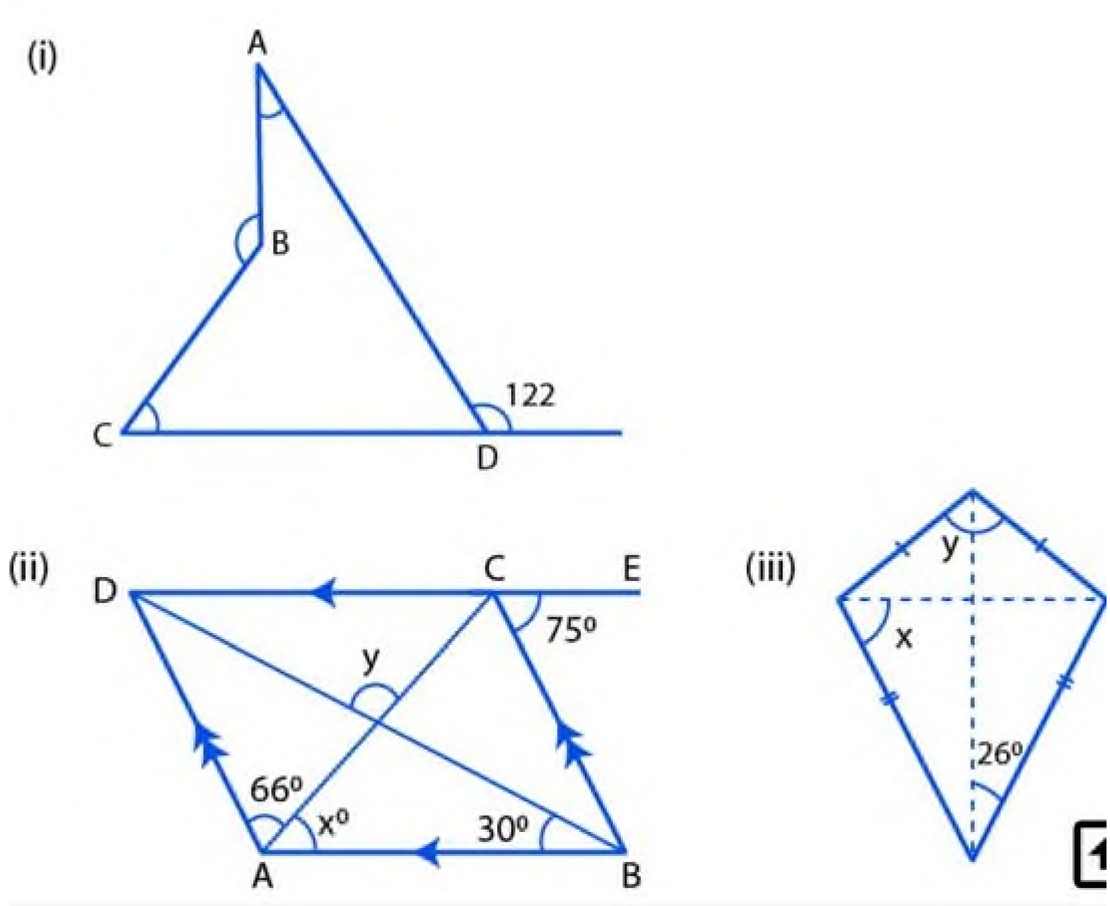
But there are the adjacent sides of a parallelogram

So, sides of PQRS are equal

Hence, PQRS is a rhombus

Hence proved

4. Find the size of each lettered angle in the following figures.



Solution

(i) as CDE is a straight line

$\angle ADE + \angle ADC = 180^o$

$122^o + \angle ADC = 180^o$

$\angle ADC = 180^o - 122^o = 58^o$.....(i)

$\angle$ ABC $= 360^o - 140^o = 220^o$.....(ii)

[at any point the angle is 360^o]

Now, in quadrilateral ABCD we have

$\angle ADC + \angle BCD + \angle BDA + \angle ABC = 360^o$

$58^o + 53^o + x + 220^o = 360^o$ [using (i) and (ii)]

$331^o + x = 360^o$

$X = 360^o - 331^o$

$X = 29^o$

(ii) As DE ∥ AB[given]

$\angle ECB = \angle CBA$ [alternate angles]

$75^o = \angle CBA$

$= \angle CBA = 75^o$

Since , AD ∥ BC we have

$(x + 66^o) + 75^o = 180^o$

$X + 141^o = 180^o$

$X = 180^o - 141^o$

$X = 39^o$.....(i)

Now in Δ AMB

$X + 30^o + \angle AMB = 180^o$ [Angles sum property of a triangle]

$39^o + 30^o + \angle AMB = 180^o$ [from (i)]

$69^o + \angle AMB = 180^o$

$\angle AMB = 180^o - 69^o = 111^o$....(ii)

Since, $\angle AMB = y$ [vertically opposite angles]

$= y = 111^o$

Hence $x = 39^o$ and $y = 111^o$

(iii) In ΔABD

AB = AD [given]

$\angle ABD = \angle ADB$ [angles opposite to equal sides are equal]

$\angle ABD = 42^o$ [since, given $\angle ADB = 42^o$]

And ,

$\angle ABD + \angle ADB + \angle BAD = 180^o$ [angles sum property of a triangle]

$42^o + 42^o + y = 180^o$

$84^o + y = 180^o$

$Y = 180^o - 84^o$

$Y = 96^o$

$\angle BCD = 2 \times 26° = 52°$

In ΔBCD,

As BC = CD [given]

$\angle CBD = \angle CDB = X$ [angle opposite to equal sides are equal]

$\angle CBD + \angle CDB + \angle BCD = 180^o$

$X + x + 52^o = 180^o$

$2x + 52^o = 180^o$

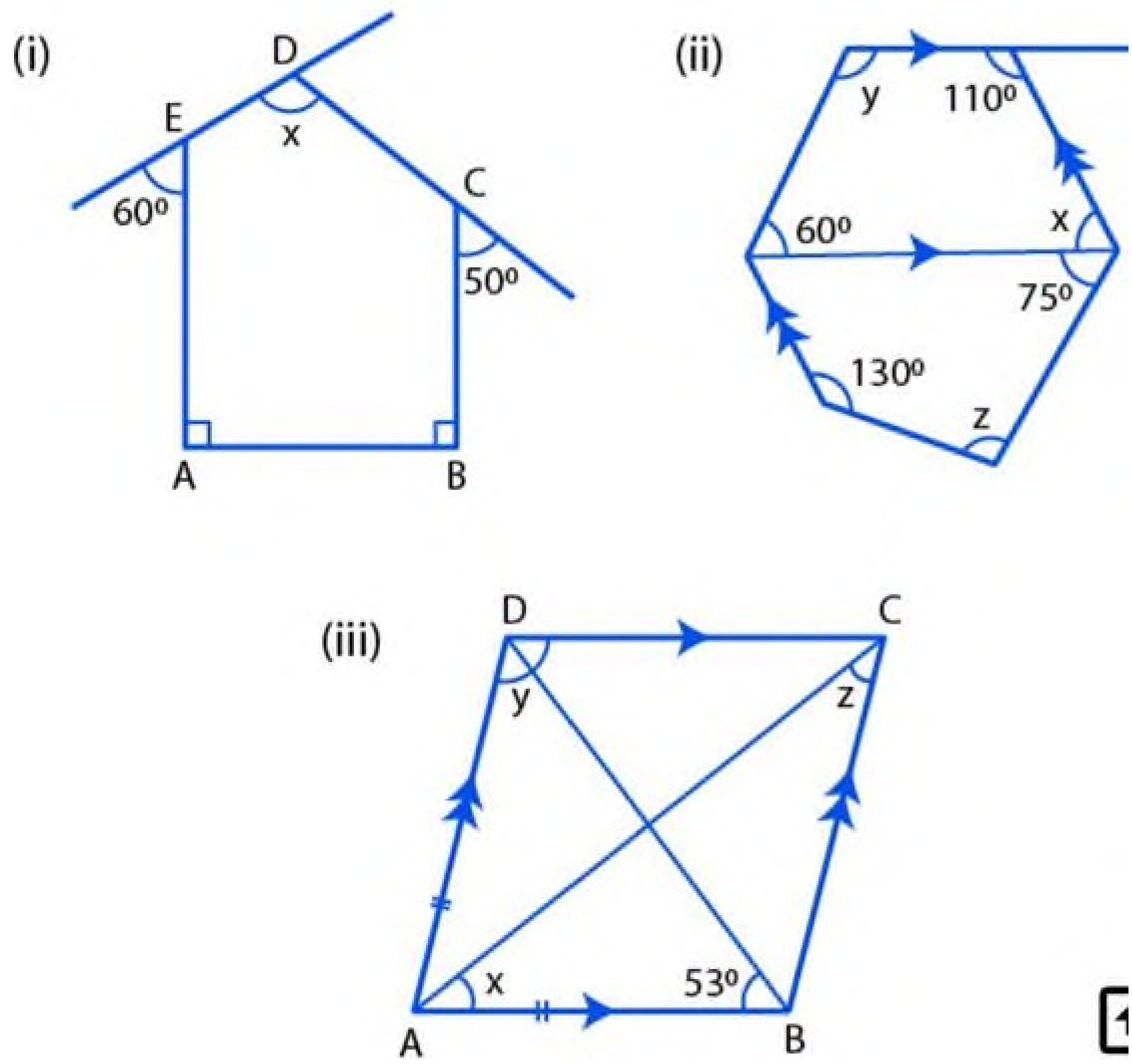
$2x = 180^o - 52^o$

$X = \frac{120°}{2}$

$X = 64^o$

Hence , $x = 64^o$ and $y = 90^o$

5. find the size of each lettered angle in the following figures:



Solution

(i) here , AB ∥ CD and BC ∥ AD

So, ABCD is a ∥ gm

Y = 2 × ∠ABD

Y = 2 × 53° = 106°....(1)

Also , $y + \angle DAB = 180^o$

$\angle DAB = 180^o - 106^o$

$= 74^o$

Thus , $x = \frac{1}{2}\angle DAB$ [as AC bisects $\angle DAB$]

$X = \frac{1}{2} \times 74^o = 37^o$

And $\angle DAB = x = 37^o$.....(ii)

Also , $\angle DAB = z$ ....(iii) [Alternative angles]

From (ii) and (iii)

$Z = 37^o$

Hence $x = 37^o$, $y = 106^o$ and $z = 37^o$

(ii) As ED is a straight line, we have

$60^o + \angle AED = 180^o$ [linear pair]

$\angle AED = 180^o - 60^o$

$\angle AED = 120^o$....(i)

Also , as CD is a straight line

$50^o + \angle BCD = 180^o$ [linear pair]

$\angle BCD = 180^o - 50^o$

$\angle BCD = 130^o$.....(ii)

In pentagon ABCDE, we have

$\angle A + \angle B + \angle AED + \angle BCD + \angle x = 540^o$[sum of interior angles in pentagon is 540^o]

$90^o + 90^o + 120^o + 130^o + x = 540^o$

$430^o + x = 540^o$

$X = 540^o – 430^o$

$X = 110^o$

Hence , value of $x = 110^o$

(iii) in given figure, AD ∥ BC [given]

$60^o + y = 180^o$ and $x + 110^o = 180^o$

$Y = 180^o – 60^o$ and $x = 180^o - 110^o$

$Y = 120^o$ and $x = 70^o$

Since , CD ∥ AF [given]

$\angle FAD + 75^o + Z + 130^o = 360^o$

$70^o + 75^o + z + 130^o = 360^o$

$275^o + z = 360^o$

$Z = 360^o – 275^o = 85^o$

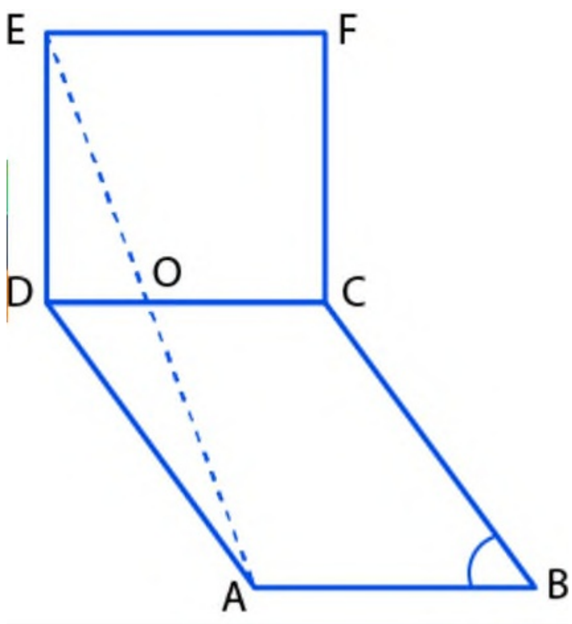
Hence ,

$X = 70^o$, $y = 120^o$ and $z = 85^o$

6. in the adjoining figure, ABCD is a rhombus and DCFE is a square. If ∠ABC = 56º , find (i) ∠DAG (ii) ∠ FEG (iii) ∠GAC (iv) ∠ AGC.

Solution

Here ABCD and DCEF is a rhombus and square respectively .



So AB = BC = DC = AD...(i)

Also , DC = EF = FC =EF....(ii)

From(i) and (ii) , we have

AB = BC = DC = AD = EF = FC = EF .....(iii)

∠ABC = 56º[Given]

∠ADC = 56º [opposite angle in rhombus are equal]

So, ∠EDA = ∠EDC + ∠ADC = 90º + 56º = 146º

In Δ ADE ,

DE = AD [from(iii)]

∠DEA = ∠DAE [equal sides have equal opposite angles]

$\angle DEA = \angle DAG = \frac{180^\circ - \angle EDA}{2}$

$= \frac{180^\circ - 146^\circ}{2}$

$= \frac{34°}{2} = 17°$

$\angle DAG = 17°$

Also

$\angle DEG = 17°$

$\angle FEB = \angle E - \angle DEG$

$= 90° – 17°$

$= 73°$

In rhombus ABCD

$\angle DAB = 180° - 56° = 124°$

$\angle DAC = \frac{124°}{2}$ [since , AC diagonals bisect the $\angle A$]

$\angle DAC = 62°$

$\angle GAC = \angle DAC - \angle DAG$

$= 62° – 17°$

$= 45°$

In Δ EDG,

$\angle D + \angle DEG + \angle DGE = 180°$ [angles sum property of a triangle]

$90° + 17° + \angle DGE = 180°$

$\angle DGE = 180° – 107° = 73°$ .....(iv)

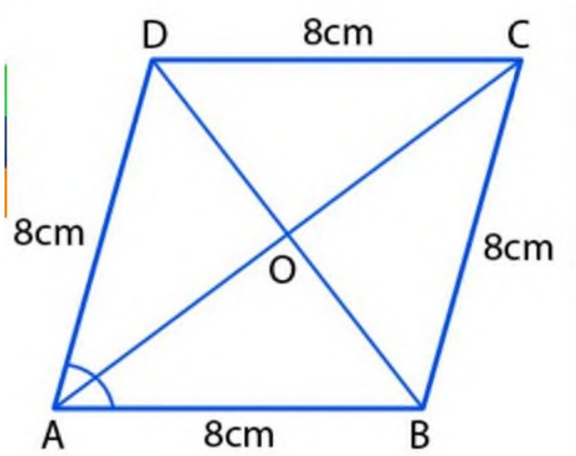
Thus, $\angle AGC = \angle DGE$.....(v) [vertically opposite angles]

Hence from (iv) and (v) we have

$\angle AGC = 73°$

7. if one angle of a rhombus is 60° and the length of a side is 8 cm, find the lengths of its diagonals.

Solution



Each side of rhombus ABCD is 8 cm

So, AB = BC = CD = DA = 8 cm

Let $\angle A = 60^{\circ}$

So , ΔABD is an equilateral triangle

Then ,

AB = BD = AD = 8cm

As we know the diagonals of a rhombus bisect each other at right angle

AO = OC, BO = OD = 4cm and $\angle AOB = 90^{\circ}$

Now, in right ΔAOB

By Pythagoras theorem

$AB^2 = AO^2 + OB^2$

$8^2 + AO^2 + 4^2$

$64 = AO^2 + 16$

$AO^2 = 64 - 16 = 48$

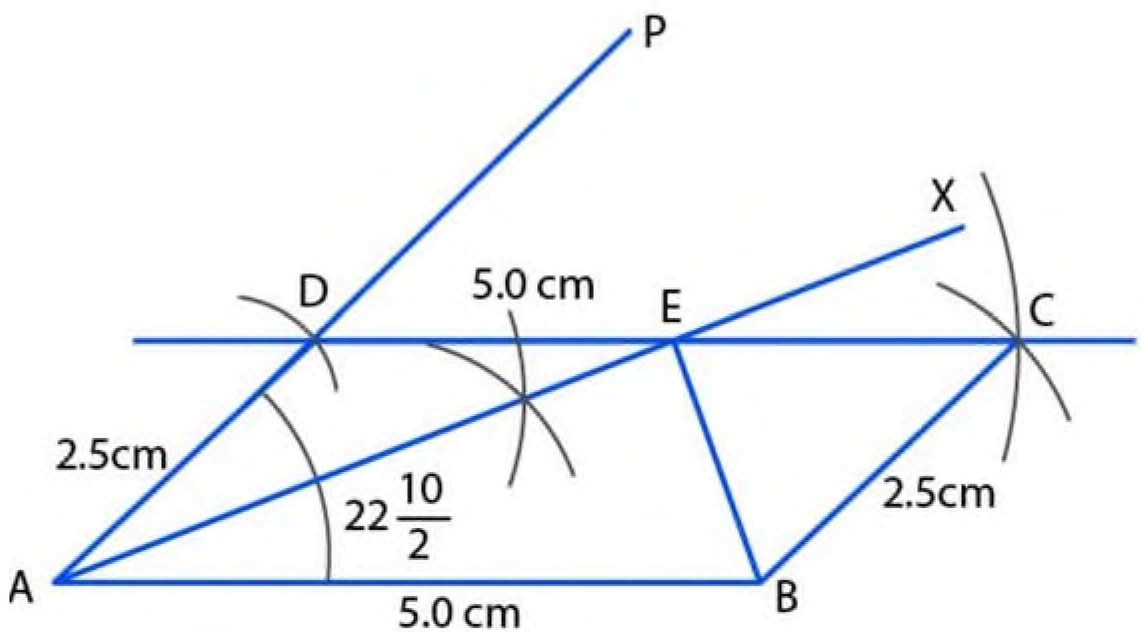
$AO = \sqrt{48} = 3\sqrt{3}CM$

But , AC = 2AO

Hence ,$AC = 2 \times 4\sqrt{3} = 8\sqrt{3}$ cm .

8. using ruler and compasses only , construct a parallelogram ABCD with AB = 5 cm , AD = 2.5 cm and ∠BAD = 45°. If the bisector of ∠BAD meets DC at E, prove that ∠AEB is a right angle.

Solution



Steps of construction :

(i) Draw AB = 5.0 cm

(ii) draw BAP = 45° on side AB

(iii) take A as centre and radius 2.5 cm cut the line AP at D

(iv) take D as a centre and radius 5.0 cm draw an arc

(v) take B as a centre and radius equal to 2.5 cm cut the arc of step (iv) at c .

(vi) join BC and CD

(vii) ABCD is the required parallelogram

(viii) draw the bisector of ∠BAD, which cuts the DC at E

(ix) join EB

(x) measure ∠AEB which is equal to 90^o.