HOTS (Higher Order Thinking Skills)

Que 1. Find the sum of the first 15 multiples of 8.

Sol. The first 15 multiples of 8 are

8, 16, 24…120

Clearly, these numbers are in AP with first term a = 8 and common difference, d = 16 – 8 = 8

Thus, $S_{15} = \frac{15}{2}[2 \times 8 + (15 - 1) \times 8]$

$$= \frac{15}{2}[16 + 14 \times 8] = \frac{15}{2}[16 + 112] = \frac{15}{2} \times 128 = 15 \times 64 = 960$$

Que 2. Find the sum of all two digit natural numbers which when divided by 3 yield 1 as remainder.

Sol. Two digit natural number which when divided by 3 yield 1 as remainder are:

10, 13, 16, 19…… 97, which forms an AP.

with a = 10, d = 3, a_n = 97

a_n = 9 $\Rightarrow$ $a + (n - 1)\, d = 97$

or 10 + (n-1) 3 = 97 $\Rightarrow$ $(n - 1) = \frac{87}{3} = 29$ $\Rightarrow$ $n = 30$

Now, $S_{30} = \frac{30}{2}[2 \times 10 + 29 \times 3] = 15(20 + 87) = 15 \times 107 = 1605$

Que 3. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

Sol. Let the prizes be a + 60, a + 40, a +20, a, a – 20, a – 40, a – 60

Therefore, the sum of prizes is

a + 60 + a + 40 + a + 20 + a + a – 20 + a – 40 + a – 60 = 700

$\Rightarrow$ 7a = 700 $\Rightarrow$ $a = \frac{700}{7} = 100$

Thus, the value of seven prizes are

100 + 60, 100 + 40, 100 + 20, 100, 100 – 20, 100 – 40, 100 – 60

i.e., ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60, ₹ 40

Que 4. If the mth term of an AP is $\frac{1}{n}$ and nth term is $\frac{1}{m}$, then show that its (mn) th term is 1.

Sol. Let a and d be the first term and common difference respectively of the given AP. Then

$$\alpha_m = \alpha + (m - 1)\, d \quad \Rightarrow \quad \alpha + (m - 1)\, d = \frac{1}{n} \qquad ….(i)$$

And $\alpha_n = \alpha + (n - 1)\, d \quad \Rightarrow \quad \alpha + (n - 1)\, d = \frac{1}{m}$ ….(ii)

Subtracting (ii) from (i), we have

$$(m - n)\, d = \frac{1}{n} - \frac{1}{m} \quad \Rightarrow \quad (m - n)\, d = \frac{m-n}{mn}$$

$\therefore \quad d = \frac{1}{mn}$

Putting $d = \frac{1}{mn}$ in (i), we get

$$\alpha + (m-1)\,\frac{1}{mn} = \frac{1}{n} \qquad \Rightarrow \quad \alpha + \frac{m}{mn} - \frac{1}{mn} = \frac{1}{n}$$

$\Rightarrow \qquad \alpha - \frac{1}{mn} = 0 \qquad$ or $\quad \alpha = \frac{1}{mn}$

$\therefore$ mnth term = $\alpha + (mn-1)\,d = \frac{1}{mn} + (mn-1)\frac{1}{mn} = \frac{1+mn-1}{mn}$

$\therefore$ (mn) th term = 1

Que 5. If the sum of m terms of an AP is the same as the sum of its n terms, show that the sum of its (m + n) terms is zero.

Sol. Let a be the first term and d be the common difference of the given AP.
Then, $S_m = S_n$

$\Rightarrow \qquad \frac{m}{2}\{2\alpha + (m-1)\,d\} = \frac{n}{2}\{2\alpha + (n-1)\,d\}$

$\Rightarrow \qquad 2\alpha(m-n) + \{m\,(m-1)\ - n\,(n-1)\,\}\,d = 0$

$\Rightarrow \qquad 2\alpha\,(m-n) + \{(m^2 - n^2) - (m-n)\}\,d = 0$

$\Rightarrow \qquad (m-n)\,\{2\alpha + \ (m+n-1)\,d\} = 0$

$\Rightarrow \qquad 2\alpha + (m+n-1)\,d = 0 \qquad [\because\ m - n \neq 0]$
….(i)

Now, $S_{m+n} = \frac{m+n}{2}\{2a + (m+n-1)\,d\}$

$\Rightarrow \qquad S_{m+n} = \frac{m+n}{2} \times\ 0 = 0 \qquad$ [Using (i)]