Linear Inequalities

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. Two real numbers or two algebraic expressions related by the symbols <, s, z, > form an inequation. If the highest power of the variables used in the inequation is 1, then the inequation is called linear inequation.

(A) If - 3x + 17 <-13, then:

(a) x ∈ (10, ∞)

(b) x ∈ [10, ∞)

(c) x ∈ (-∞, 10]

(d) x ∈ [-10, 10)

(B) Given that x, y and b are real numbers and x<y, b < 0, then:

(a) $\frac{x}{b} < \frac{y}{b}$ (b) $\frac{x}{b} \leq \frac{y}{b}$

(c) $\frac{x}{b} > \frac{y}{b}$ (d) $\frac{x}{b} \geq \frac{y}{b}$

(C) If x 115, then:

(a) x ∈ (-4, 6)

(b) x ∈ [-4, 6]

(c) x ∈ (-∞,-4) U (6,-∞)

(d) x ∈ [-∞,-4) U [6, ∞)

(d)

If $\frac{|x-7|}{(x-7)} \geq 0$, then:

(a) x ∈ [7, ∞0)

(c) x ∈ (-∞, 7)

(b) x ∈ (7,∞)

(d) x ∈ (- ∞, 7]

(E) If x+31 10, then:

(a) x ∈ (-13, 7]

(b) x ∈ (13, 7]

(c) x ∈ (- ©, – 13] [7, oo)
(d) x ∈ - 0, - 13] [7,0)

Ans. (A) (a) x∈ (10,00)
Explanation: Given,
-3x + 17 < -13
Subtracting 17 from both sides,
-3x+17-17-13-17
⇒ -3x < -30
⇒ x 10 {since the division by negative number inverts the inequality sign}
⇒ x ∈ (10, ∞0)
(B)
(a) $\frac{x}{b} < \frac{y}{b}$
Explanation: Given that x, y and b are real numbers and x < y,b<0.
Consider, x < y
Divide both sides of the inequality by "b"

$\frac{x}{b} < \frac{y}{b}$ {since $b < 0$}

(C) (c) x E (-∞o, - 4) ∪ (6,∞)
Explanation: x - 1 >5
x-1<-5 and x-1>5
x<-4 and x> 6
Therefore, x∈ (-∞,-4) ∪ (6,∞)
(D) (b) x ∈ (7,00)
Explanation: Given,

$\frac{|x-7|}{(x-7)} \geq 0$

This is possible when x - 7≥ 0, and x-7= 0.
Here, x = 7 but x #7
Therefore, x> 7, i.e. x ∈ (7,∞).
(E) (d) x ∈ [-∞o, -13] [7,∞)

Explanation: Given,
$|x+3| \geq 10$
$\Rightarrow x+3-10$ or $x + 3 = 10$
$\Rightarrow$xs-13 or $x \geq 7$
$\Rightarrow x \in (-c, -13] \cup [7,6)$
200 to buy some

2. Amit's mother gave him packets of rice and maggie from the market. The cost of one packet of rice is 30 and that of one packet of maggie is 20. Let x denotes the number of packet of rice and y denotes the number of packets of maggie.

(A) Find the inequality that represents the given situation.
(B) If he buys 4 packets of rice and spends entire amount of Rs 200, then find the maximum number of packets of maggie that he can buy.
(C) Solve the following inequality for real x.
$4x + 3 < 5x + 7$

Ans. (A) Total amount = 200
Cost of one packet of rice = 30
And cost of one packet of maggie = * 20
Here, x and y denote the number of packets of rice and maggie respectively,
Total amount spent by Amit is $30x + 20y$.
;- Required inequality is $30x + 20y \leq 200$
(B) If he spends his entire amount, then
We have, $30x + 20y= 200$
Since, number of packet of rice = 4
:- At x = 4, equation (i) becomes
30 x 4 + 20y=200

120+20y=200

20y=200-120

20y=80

.. Maximum number of packets of maggie that he can buy is 4.

(C) Given that, $4x + 3 < 5x+7$

Now by subtracting 7 from both the sides, we get

$4x+3-7 <5x+7-7$

The above inequality becomes,

$4x-4 <5x$

Again, by subtracting 4x from both the sides,

$4x-4-4x < 5x-4x$

$x>-4$

:- The solutions of the given inequality are defined by all the real numbers greater than -4. The required solution set is $(-4,\infty)$.