Class XII Session 2024-25
Subject - Mathematics
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ’s and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A = $[a_{ij}]$ is a scalar matrix of order n $\times$ n such that a_{ij} = k for all i, then trace of A is equal to **[1]**

a) $\frac{n}{k}$ b) n - k

c) nk d) n + k

2. Let A and B be two invertible matrices of order 3 $\times$ 3. If det (ABA') = 8 and det (AB^{-1}) = 8, then det $(BA^{-1}B')$ is equal to **[1]**

a) 16 b) $\frac{1}{4}$

c) 1 d) $\frac{1}{16}$

3. If A is a 3$\times$3 matrix and |A| = -2, then value of |A (adj A)| is **[1]**

a) -2 b) 8

c) 2 d) -8

4. If y = log (sin e^x), then $\frac{dy}{dx}$ is: **[1]**

a) cot e^x b) e^x cot e^x

c) cosec e^x d) e^x cosec e^x

5. The direction ratios of a line parallel to z-axis are: **[1]**

a) $\langle 0, 0, 0 \rangle$ b) $\langle 1, 1, 0 \rangle$

c) $\langle 1, 1, 1 \rangle$ d) $\langle 0, 0, 1 \rangle$

6. What is integrating factor of $\frac{dy}{dx}$ + y sec x = tan x? **[1]**

a) sec x
b) $e^{\sec x}$
c) sec x + tan x
d) log (sec x + tan x)

7. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let Z = px + qy, where p, q > 0. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is **[1]**

a) q = 3p
b) q = 2p
c) p = q
d) p = 2q

8. Which of the following is the principal value branch of $\operatorname{cosec}^{-1}x$? **[1]**

a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
c) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$
d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

9. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} dx = ?$ **[1]**

a) (e - 1)
b) $\left(\frac{1}{e} + 1\right)$
c) (e + 1)
d) $\left(\frac{1}{e} - 1\right)$

10. The order of the single matrix obtained from $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}_{3\times2} \left\{ \begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix}_{2\times3} - \begin{bmatrix} 0 & 1 & 23 \\ 1 & 0 & 21 \end{bmatrix}_{2\times3} \right\}$ is **[1]**

a) 2×3
b) 3×3
c) 3×2
d) 2×2

11. The solution set of the inequality 3x + 5y < 4 is **[1]**

a) an open half-plane not containing the origin.
b) an open half-plane containing the origin.
c) a closed half plane containing the origin.
d) the whole XY-plane not containing the line 3x + 5y = 4.

12. If the vectors $4\hat{i} + 11\hat{j} + m\hat{k}$, $7\hat{i} + 2\hat{j} + 6\hat{k}$ and $\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar, then m = **[1]**

a) 38
b) -10
c) 0
d) None of these

13. $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$ = ? **[1]**

a) $(a^2 - b^2 - c^2 - d^2)$
b) $(a^2 + b^2 + c^2 - d^2)$
c) $(a^2 - b^2 + c^2 - d^2)$
d) $(a^2 + b^2 + c^2 + d^2)$

14. If $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P(B/A) + P(A/B)$ equals **[1]**

a) $\frac{1}{3}$
b) $\frac{1}{4}$
c) $\frac{5}{12}$
d) $\frac{7}{12}$

15. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is **[1]**

a) $e^x + e^y = \frac{x^3}{3} + c$
b) $e^x - e^y = \frac{x^3}{3} + c$
c) $y = e^{x-y} - x^2 e^{-y} + c$
d) $e^y - e^x = \frac{x^3}{3} + C$

16. The area of a triangle with vertices A, B, C is given by **[1]**

a) $|\overrightarrow{AB} \times \overrightarrow{AC}|$

b) $\frac{1}{8}|\overrightarrow{AC} \times \overrightarrow{AB}|$

c) $\frac{1}{4}|\overrightarrow{AC} \times \overrightarrow{AB}|$

d) $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

17. If f(x) = $\begin{cases} \mathrm{kx} + 5, \text{ when x} \le 2 \\ \mathrm{x} + 1, \text{ when x} > 2 \end{cases}$ is continuous at x = 2 then k = ? **[1]**

a) -2

b) -1

c) 2

d) -3

18. The lines l_1 and l_2 intersect. The shortest distance between them is **[1]**

a) infinity

b) negative

c) positive

d) zero

19. **Assertion (A):** The rate of change of area of a circle with respect to its radius r when r = 6 cm is 12π cm^2/cm. **[1]**

Reason (R): Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** A relation R = {(a, b): |a - bl <3} defined on the set A = {1, 2, 3, 4} is reflexive. **[1]**

Reason (R): A relation R on the set A is said to be reflexive if for (a, b) ∈ R and (b, c) ∈ R, we have (a, c) ∈ R.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the value of $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$. **[2]**

OR

Write the interval for the principal value of function and draw its graph: $\cos^{-1}$ x.

22. A particle moves along the curve 6y = x^3 + 2. Find the points on the curve at which y-coordinates is changing 2 times as fast as x - coordinates. **[2]**

23. Show that $f(x) = \cos(2x + \frac{\pi}{4})$ is an increasing function on $(\frac{3\pi}{8}, \frac{7\pi}{8})$ **[2]**

OR

Show that the function given by f (x) = sin x is neither increasing nor decreasing in (0, π)

24. Integrate the function: $\frac{1}{9x^2+6x+5}$ **[2]**

25. Find the matrix X for which: $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$ **[2]**

Section C

26. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$ **[3]**

27. A doctor claims that 60% of the patients he examines are allergic to some type of weed. What is the probability that exactly 3 of his next 4 patients are allergic to weeds? **[3]**

28. Evaluate the integral: $\int \sec^{-1}\sqrt{x}dx$ **[3]**

OR

Prove that: $\int_0^{\pi} \frac{x}{1-\cos\alpha\sin x}dx = \frac{\pi(\pi-\alpha)}{\sin\alpha}$

29. Solve the initial value problem: $\frac{dy}{dx}$ + y tan x = 2x + x^2 tan x, y(0) = 1 **[3]**

OR

Solve differential equation: $\frac{dy}{dx} - y\tan x = e^x$

30. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram. **[3]**

OR

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d} = 15$.

31. Find all points of discontinuity of f where f is defined as follows,f(x) = $\begin{cases} |x| + 3, x \leq -3 \\ -2x, -3 < x < 3 \\ 6x + 2, x \geq 3 \end{cases}$ **[3]**

Section D

32. Find the area of the region $\left\{(x, y) : 0 \leqslant y \leqslant (x^2 + 1), 0 \leqslant y \leqslant (x + 1), 0 \leqslant x \leqslant 2\right\}$ **[5]**

33. Let f : W → W be defined as f(n) = n - 1, if n is odd and f(n) = n + 1, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers. **[5]**

OR

Let A = {1, 2, 3} and R = {(a, b):a, b ∈ A and $\left|a^2 - b^2\right| \leq 5$. Write R as set of ordered pairs. Mention whether R is

i. reflexive
ii. symmetric
iii. transitive

Give reason in each case.

34. Three shopkeepers A, B and C go to a store to buy stationery. A purchases 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs 40 paise, a pen costs ₹1.25 and a pencil costs 35 paise. Use matrix multiplication to calculate each individual's bill. **[5]**

35. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base. **[5]**

OR

Show that the semi-vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$ or $\cos^{-1} \frac{1}{\sqrt{3}}$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let: E_1: represent the event when many workers were not present for the job;

E_2: represent the event when all workers were present; and

E: represent completing the construction work on time.

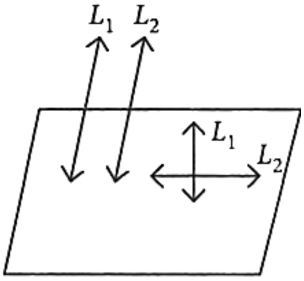
i. What is the probability that all the workers are present for the job? (1)
ii. What is the probability that construction will be completed on time? (1)
iii. What is the probability that many workers are not present given that the construction work is completed on time? (2)

OR

What is the probability that all workers were present given that the construction job was completed on time? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

If a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios of two lines say L_1 and L_2 respectively. Then $L_1 \parallel L_2$ iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and $L_1 \perp L_2$ iff $a_1a_2 + b_1b_2 + c_1c_2 = 0$.



i. Find the coordinates of the foot of the perpendicular drawn from the point A(1, 2, 1) to the line joining B(1, 4, 6) and C(5, 4, 4). (1)

ii. Find the direction ratios of the line which is perpendicular to the lines with direction ratios proportional to (1, -2, -2) and (0, 2, 1). (1)

iii. What is the relation between lines $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$ and $\frac{x-1}{1} = \frac{y+\frac{3}{2}}{\frac{3}{2}} = \frac{z+5}{2}$. (2)

OR

If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of L_1 and L_2 respectively, then what is the condition for L_1 parallel to L_2. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Sheetal rides her car at 25 km/hr. She has to spend ₹2 per km on diesel and if she rides it at a faster speed of 40 km/hr, the diesel cost increases to ₹ 5 per km. She has ₹ 100 to spend on diesel.

i. Formulate above information mathematically. (1)

ii. Represent the given information graphically. (1)

iii. Find the maximum distance covered by her in hour? (2)

OR

If Z = 6x - 9y be the objective function, then find maximum value of Z. (2)

Solution

Section A

1.

(c) nk

Explanation: $\because$ A = $[a_{ij}]_{n\times n}$

Trace of A, i.e., tr (A) = $\sum a_{ij}^{n} i$ = 1 = a_{11} + a_{22} + ... + a_{nm}

= k + k + k + k + k + ...(n times)

= k(n)

= nk

2.

(d) $\frac{1}{16}$

Explanation: $\frac{1}{16}$

3.

(d) -8

Explanation: -8

4.

(b) e^x cot e^x

Explanation: e^x cot e^x

y = log (sin e^x)

$\frac{dy}{dx} = \frac{d}{dx}$ log (sin e^x)

= $\frac{1}{sine^x}\frac{d}{dx}$ sin e^x

= $\frac{1}{sine^x}\cos e^x \frac{d}{dx}$ e^x

= $\cot e^x\left(e^x\right)$

= e^x cot e^x

5.

(d) < 0, 0, 1 >

Explanation: < 0, 0, 1 >

6.

(c) sec x + tan x

Explanation: We have,

$\frac{dy}{dx}$ + ysec x = tan x

Comparing with $\frac{dy}{dx}$ + Py = Q

P = sec x, Q = tan x

$\text{I. }F. = e^{\int \sec x dx} = e^{\log(\sec x + \tan x)} = \sec x + \tan x$

7. **(a)** q = 3p

Explanation: Since Z occurs maximum at (15, 15) and (0, 20), therefore, 15p + 15q = 0p + 20q ⇒ q = 3p.

8.

(b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Explanation: We know that the principal value branch of cosec^{-1} x is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

9. **(a)** (e - 1)

Explanation: $I = \int_0^{\frac{\pi}{4}} e^{\tan x} \sec^2 x dx$

Let, tan x = t,

Differentiating both side with respect to t

$\sec^2 x \frac{dx}{dt} = 1$

$\Rightarrow$ sec^{2}x dx = dt

At x = 0, t = 0

At $x = \frac{\pi}{4}$, t = 1

$I = \int_0^1 e^t dt$

$= e^{t_0^1}$

= e^1 - e^0

= e - 1

10.

(b) 3×3

Explanation: $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}_{3\times2} \left\{ \begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix}_{2\times3} - \begin{bmatrix} 0 & 1 & 23 \\ 1 & 0 & 21 \end{bmatrix}_{2\times3} \right\}$

$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}_{3\times2} \begin{bmatrix} -1 & -1 & -21 \\ 1 & 0 & -20 \end{bmatrix}_{2\times3}$

$= \begin{bmatrix} -2 & -1 & -1 \\ 2 & 0 & -40 \\ 1 & -2 & -102 \end{bmatrix}_{3\times3}$

11.

(b) an open half-plane containing the origin.

Explanation: The strict inequality represents an open half plane and it contains the origin as (0, 0) satisfies it.

12.

(d) None of these

Explanation: Given vectors $4\vec{i} + 11\vec{j} + m\vec{k}.\ 7\vec{i} + 2\vec{j} + 6\vec{k}\ and\ \vec{i} + 5\vec{j} + 4\vec{k}$ are coplanar then

$\begin{vmatrix} 4 & 11 & m \\ 7 & 2 & 6 \\ 1 & 5 & 4 \end{vmatrix} = 0$

$\implies$ 4(8-30) - 11(28 - 6) + m(35 - 2) = 0

- 88 - 242 + 33 m = 0

- 330 + 33 m = 0

m = 10

13.

(d) (a^2 + b^2 + c^2 + d^2)

Explanation: Δ = (a + ib)(a - ib) + (c - id)(c + id) = (a^2 + b^2 + c^2 + d^2)

14.

(d) $\frac{7}{12}$

Explanation: Here, $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$

$P(B/A) + P(A/B) = \frac{P(B\cap A)}{P(A)} + \frac{P(A\cap B)}{P(B)}$

$= \frac{P(A)+P(B)-P(A\cup B)}{P(A)} + \frac{P(A)+P(B)-P(A\cup B)}{P(B)}$

$= \frac{\frac{3}{10}+\frac{2}{5}-\frac{3}{5}}{\frac{3}{10}} + \frac{\frac{3}{10}+\frac{2}{5}-\frac{3}{5}}{\frac{2}{5}}$

$= \frac{\frac{1}{10}}{\frac{3}{10}} + \frac{\frac{1}{10}}{\frac{2}{5}} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

15.

(d) e^y – e^x = $\frac{x^3}{3}$ + C

Explanation: We have, $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$\Rightarrow$ e^y dy = (e^x + x^2)dx

$\Rightarrow \int e^y dy = \int \left(e^x + x^2\right) dx$

$\Rightarrow$ e^y = e^x + $\frac{x^3}{3}$ + c

$\Rightarrow$ e^y - e^x = $\frac{x^3}{3}$ + c

16.

(d) $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

Explanation: The area of the parallelogram with adjacent sides AB and AC = $|\overrightarrow{AB} \times \overrightarrow{AC}|$. Hence, the area of the triangle with vertices A, B, C = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$.

17.

(b) -1

Explanation: For continuity left hand limit must be equal to right hand limit and value at the point.

Continuous at x = 2..

L.H.L = $\lim_{x \to 2^-}(kx + 5)$

$\Rightarrow \lim_{h \to 0}(k(2 - h) + 5)$

$\Rightarrow$ k(2 - 0) + 5 = 2k + 5

R.H.L = $\lim_{x \to 2^+}(x + 1)$

$\Rightarrow \lim_{h \to 0}(2 + h + 1)$

$\Rightarrow$ 2 + 0 + 1

= 3

As f(x) is continuous,we get

$\because$ 2k + 5 = 3

k = -1.

18.

(d) zero

Explanation: Since the lines intersect. Hence they have a common point in them. Hence the distance will be zero.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(c) A is true but R is false.

Explanation: Assertion is true because for each element a $\in$ A , |a - a| = 0 < 3, so (1, 1) $\in R$,(2,2) $\in R$, (3, 3) $\in R$, (4,4) $\in R$ therefore R is reflexive.

Reason is false because a relation R on the set A is said to be transitive if for (a, b) $\in$R and (b, c) $\in$R, we have (a, c) $\in$ R

Section B

21. Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

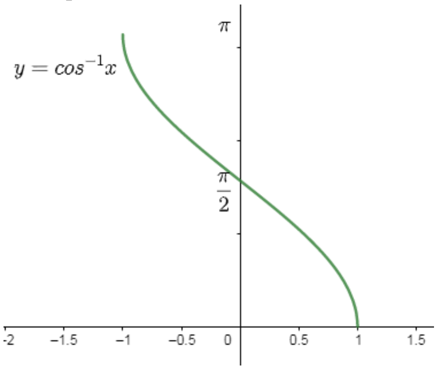
Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

OR

Principal value branch of $\cos^{-1} x$ is $[0, \pi]$ and its graph is shown here,



22. Given curve is,

$6y = x^3 + 2$

$\Rightarrow 6\frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt}$ ... (i)

Given: $\frac{dy}{dt} = 2 \cdot \frac{dx}{dt}$ ... (ii)

from (i) and (ii), $2\left(2\frac{dx}{dt}\right) = x^2 \cdot \frac{dx}{dt}$

$\Rightarrow x = \pm 2$

when x = 2, y = $\frac{5}{3}$; when x = –2, y = -1

Therefore, Points are $\left(2, \frac{5}{3}\right)$ and (-2, -1)

23. Given: $f(x) = \cos(2x + \frac{\pi}{4})$

$f'(x) = -2\sin(2x + \frac{\pi}{4})$

Now,

$x \in (\frac{3\pi}{8}, \frac{7\pi}{8})$

$\Rightarrow \quad \frac{3\pi}{8} < x < \frac{7\pi}{8}$

$\Rightarrow \quad \frac{3\pi}{4} < 2x < \frac{7\pi}{4}$

$\Rightarrow \quad \frac{\pi}{4} + \frac{3\pi}{4} < 2x + \frac{\pi}{4} < \frac{7\pi}{4} + \frac{\pi}{4}$

$\Rightarrow \quad \pi < 2x + \frac{\pi}{4} < 2\pi$

$\Rightarrow \quad \sin(2x + \frac{\pi}{4}) < 0$

$\Rightarrow \quad -2\sin(2x + \frac{\pi}{4}) > 0$

$\Rightarrow \quad f'(x) > 0$

Hence, f(x) is increasing on $(\frac{3\pi}{8}, \frac{7\pi}{8})$

OR

The function is f (x) = sin x

Then, f'(x) = cos x

Since for each x $\in \left(0, \frac{\pi}{2}\right)$, cos x > 0, we have $f'(x) > 0$

Therefore, f is strictly increasing in $\left(0, \frac{\pi}{2}\right)$……(1)

Now, The function is f (x) = sin x

Then, $f'(x) = cosx$

Since, for each $x \in \left(\frac{\pi}{2}, \pi\right)$, cos x < 0, we have $f'(x) < 0$

Therefore, f is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$……(2)

From (1) and (2),

It is clear that f is neither increasing nor decreasing in (0, π).

24. Clearly, $9x^2 + 6x + 5 = (3x + 1)^2 + (2)^2$

$\Rightarrow \int \frac{1}{9x^2+6x+5}dx = \int \frac{1}{(3x+1)^2+(2)^2}dx$

Let 3x + 1 = t

$\Rightarrow$ 3dx = dt

$\therefore \quad \int \frac{1}{(3x+1)^2+(2)^2}dx = \frac{1}{3}\int \frac{1}{t^2+2^2}dt$

$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right] + C$

$= \frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right) + C$

25. Let $A=\begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix} B=\begin{bmatrix}-1 & 1\\ -2 & 1\end{bmatrix} C=\begin{bmatrix}2 & -1\\ 0 & 4\end{bmatrix}$

Then The given equation becomes as

AXB = C

$\Rightarrow$X = A $^{-1}$CB $^{-1}$

now |A| = 35 – 14 = 21

and |B| = – 1 + 2 = 1

$\therefore A^{-1}=\frac{adj(A)}{|A|}=\frac{1}{21}\begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}$

and $B^{-1}=\frac{ad(B)}{|B|}=\frac{1}{1}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$

$\Rightarrow$ X = A $^{-1}$CB $^{-1}$ = $\frac{1}{21}\begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}\begin{bmatrix}2 & -1\\ 0 & 4\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$

= $\frac{1}{21}\begin{bmatrix}10+0 & -5-8\\ -14+0 & 7+12\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$

= $\frac{1}{21}\begin{bmatrix}10 & -13\\ -14 & 19\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$

= $\frac{1}{21}\begin{bmatrix}10-26 & -10+13\\ -14+38 & 14-19\end{bmatrix}$

Hence, $x=\frac{1}{21}\begin{bmatrix}-16 & 3\\ 24 & -5\end{bmatrix}$

Section C

26. $I=\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$

$I=\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\frac{\sin x}{\cos x}}}=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ...(1)

$I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}dx$

$\left[\because\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx\right]$

$I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$

$I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ...(2)

Adding (1) and (2), we get

$2I=\int_{\pi/6}^{\pi/3}1dx$

$=[x]_{\pi/6}^{\pi/3}$

$=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$

$I=\frac{\pi}{12}$

27. Let us define the following events:

A = First patient is allergic to weeds, B = Second patient is allergic to weeds C = Third patient is allergic to weeds, D = Fourth patient is allergic to weeds

Clearly, A, B, C, D are independent events such that

P(A) = P(B) = P(C) = P(D) = $\frac{60}{100}=\frac{3}{5}$

Therefore, required probability is given by,

= $P[(A\cap B\cap C\cap\bar{D})\cup(A\cap B\cap\bar{C}\cap D)\cup(A\cap\bar{B}\cap C\cap D)\cup(\bar{A}\cap B\cap C\cap D)]$

= $P(A\cap B\cap C\cap\bar{D})$ + $P(A\cap B\cap\bar{C}\cap D)$ + $P(A\cap\bar{B}\cap C\cap D)$ + $P(\bar{A}\cap B\cap C\cap D)$

= P(A) P(B) P(C) P($\bar{D}$)+P(A) P(B) P($\bar{C}$) P(D) + P(A) P($\bar{B}$) P(C) P(D) + P($\bar{A}$) P(B) P(C) P(D)

= $\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{2}{5}$ + $\frac{3}{5}\times\frac{3}{5}\times\frac{2}{5}\times\frac{3}{5}$ + $\frac{3}{5}\times\frac{2}{5}\times\frac{3}{5}\times\frac{3}{5}$ + $\frac{2}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}=\frac{216}{625}$

28. Let the given integral be ,

$I=\int\sec^{-1}\sqrt{x}dx$

Putting $\sqrt{x}=\sec\theta$

$\Rightarrow$ x = sec$^2\theta$

$\Rightarrow$ dx = 2 sec θ sec θ tan θ dθ

$= 2\sec^2\theta \tan\theta\, d\theta$
$\therefore I = 2\int \theta \sec^2\theta \tan\theta d\theta$
$= 2\int \theta \tan\theta \sec^2\theta d\theta$

Considering θ as first function and $\tan\theta \sec^2\theta$ as second function
$I = 2\left[\theta \frac{\tan^2\theta}{2} - \int 1 \frac{\tan^2\theta}{2} d\theta\right] \quad \left(\because \int \tan\theta \sec^2 \tan\theta d\theta = \frac{\tan^2\theta}{2}\right)$
$= \theta\tan^2\theta - \int \left(\sec^2\theta - 1\right) d\theta$
$= \theta\tan^2\theta - \tan\theta + \theta + C$
$= \theta\left(1+\tan^2\theta\right) - \tan\theta + C$
$= \theta\sec^2\theta - \sqrt{\sec^2\theta - 1} + C$
$= \sec^{-1}\sqrt{x}x - \sqrt{x-1} + C$
$= x\sec^{-1}\sqrt{x} - \sqrt{x-1} + C$

OR

Let $I = \int_0^\pi \frac{x}{1-\cos\alpha\sin x}dx$. Then,
$I = \int_0^\pi \frac{(\pi - x)}{1-\cos\alpha\sin(\pi - x)}dx$
By using property of defnite integrals,
$\Rightarrow \quad I = \int_0^\pi \frac{\pi}{1-\cos\alpha\sin x}dx - \int_0^\pi \frac{x}{1-\cos\alpha\sin x}dx$
$\Rightarrow \quad I = \pi\int_0^\pi \frac{1}{1-\cos\alpha\sin x}dx - I$
$\Rightarrow \quad 2I = \pi\int_0^\pi \frac{1}{1-\cos\alpha\sin x}dx$
$\Rightarrow \quad 2I = \pi\int_0^\pi \frac{1+\tan^2 x/2}{\left(1+\tan^2 x/2\right) - 2\cos\alpha\tan x/2}dx$
$\Rightarrow \quad 2I = \pi\int_0^\pi \frac{\sec^2 x/2}{\tan^2 x/2 - 2\cos\alpha\tan x/2 + 1}dx$
$\Rightarrow \quad I = \frac{\pi}{2}\int_0^\pi \frac{\sec^2 x/2}{\tan^2 x/2 - 2\cos\alpha\tan x/2 + 1}dx$
Let $\quad \tan\frac{x}{2} = t.$ Then, $d\left(\tan\frac{x}{2}\right) = dt \Rightarrow \sec^2\frac{x}{2}dx = 2dt$
Also, $\quad x = 0 \Rightarrow t = \tan 0 = 0$ and $x = \pi \Rightarrow t = \tan\frac{\pi}{2} = \infty$
$\therefore \quad I = \frac{\pi}{2}\int_0^\infty \frac{2dt}{t^2 - 2t\cos\alpha + 1}$
$\Rightarrow \quad I = \pi\int_0^\infty \frac{1}{(t-\cos\alpha)^2 + \left(1-\cos^2\alpha\right)}dt$
$\Rightarrow \quad I = \pi\int_0^\infty \frac{1}{\sin^2\alpha + (t-\cos\alpha)^2}dt$
$\Rightarrow \quad I = \frac{\pi}{\sin\alpha}\left[\tan^{-1}\left(\frac{t-\cos\alpha}{\sin\alpha}\right)\right]_0^\infty$
$\Rightarrow \quad I = \frac{\pi}{\sin\alpha}\left[\tan^{-1}\alpha - \tan^{-1}(-\cot\alpha)\right]$
$\Rightarrow \quad I = \frac{\pi}{\sin\alpha}\left[\frac{\pi}{2} + \tan^{-1}(\cot\alpha)\right]$
$\Rightarrow \quad I = \frac{\pi}{\sin\alpha}\left[\frac{\pi}{2} + \tan^{-1}\left\{\tan\left(\frac{\pi}{2} - \alpha\right)\right\}\right] = \frac{\pi}{\sin\alpha}\left[\frac{\pi}{2} + \frac{\pi}{2} - \alpha\right] = \frac{\pi(\pi - \alpha)}{\sin\alpha}$

29. The given differential equation is,
$\frac{dy}{dx}$ + y tan x = 2x + x^2 tan x, y(0) = 1
It is a linear differential equation. Comparing it with,
$\frac{dy}{dx}$ + Py = Q
P = tan x, Q = 2 x + x^2 tan x
I.F. $= e^{\int pdx}$
$= e^{\int \tan x dx}$
= $e^{\log|\sec x|}$
Solution of the equation is given by,
y × (I.F.) = ∫ Q × (I.F.) dx + c
y sec x = ∫ (2x + x^2 tan x) sec dx + c
= ∫ 2x sec x dx + ∫ x^2 sec tan x dx + c
= ∫ 2x × sec x dx + [x^2 × ∫ sec x tan x dx - ∫ (2x ∫ sec tan x dx) dx] + c
y sec x = ∫ 2x sec dx + x^2 sec x - ∫ 2x sec x dx + c
y sec x = x^2 sec x + c ...(i)
Put x = 0, y = 1

1 = 0 + c

c = 1

Put c = 1 in equation (i),

y sec x = x^2 sec x + 1

y = x^2 + $\frac{1}{\sec x}$

y = x^2 + cos x

OR

The given differential equation is,

$\frac{dy}{dx} - y\tan x = e^x$

It is a linear differential equation.

Comparing it with $\frac{dy}{dx}$ + py = Q

P =-tan x, Q = e^x

I.F. = $e^{\int pdx}$

$= e^{-\int \tan x dx}$

$= e^{-\log\sec x}$

= cos x

Solution of the given equation is given by,

y (I.F.) = $\int Q \times (I.F)dx + c_1$

$y\cos x = \int e^x \cos x dx + c_1$

y cos x = I + c_1 ....(1)

$I = \int e^x \cos x dx$

Using equation by parts

$I = e^x \int \cos x dx - \int \left(e^x \int \cos x dx\right) dx + c_2$

$= e^x \sin x - \int e^x \sin x dx + c_2$

$= e^x \sin x - \left[e^x \left(\int \sin x dx\right) - \int \left(e^x \left(\int \sin x dx\right) dx\right] + c_2$

$I = e^x \sin x + e^x \cos x - \int e^x \cos x dx + c_2$

$I = e^x(\sin x + \cos x) - I + c_2$

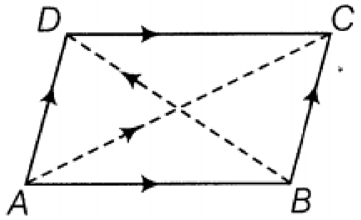
$2I = e^x(\sin x + \cos x) + c_2$

$I = \frac{1}{2}e^x(\sin x + \cos x) + c_3$

Using equation (i)

$y\cos x = \frac{1}{2}e^x(\sin x + \cos x) + c$

30. Let ABCD be the parallelogram.



According to the question,

$\overrightarrow{AB} = 2\hat{i} - 4\hat{j} - 5\hat{k}$ and $\overrightarrow{AD} = 2\hat{i} + 2\hat{j} + 3\hat{k}$.

By using parallelogram law of addition , we get

The diagonal $\overrightarrow{AC}$ is given by $\overrightarrow{AB} + \overrightarrow{AD} = 4\hat{i} - 2\hat{j} - 2\hat{k}$

The diagonal $\overrightarrow{BD}$ is given by $\overrightarrow{BC} + \overrightarrow{BA} = \overrightarrow{AD} - \overrightarrow{AB} = 6\hat{j} + 8\hat{k}$

Now, the unit vector along $\overrightarrow{AC}$ is given by

$\frac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = \frac{4\hat{i}-2\hat{j}-2\hat{k}}{\sqrt{16+4+4}}$

$= \frac{4\hat{i}-2\hat{j}-2\hat{k}}{\sqrt{24}}$

$= \frac{4\hat{i}-2\hat{j}-2\hat{k}}{2\sqrt{6}}$

$= \frac{1}{\sqrt{6}}(2\hat{i} - \hat{j} - \hat{k})$

the unit vector along $\overrightarrow{BD}$ is given by

$\frac{\overrightarrow{BD}}{|\overrightarrow{BD}|} = \frac{6\hat{j}+8\hat{k}}{\sqrt{36+64}}$

$= \frac{6\hat{j}+8\hat{k}}{10}$

$= \frac{1}{5}(3\hat{j} + 4\hat{k})$

Now, area of parallelogram ABCD

$= \frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{BD}|$

Here, $\overrightarrow{AC} \times \overrightarrow{BD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & 6 & 8 \end{vmatrix}$

$= \hat{i}(-16 + 12) - \hat{j}(32 - 0) + \hat{k}(24 - 0)$

$= -4\hat{i} - 32\hat{j} + 24\hat{k}$

$|\overrightarrow{AC} \times \overrightarrow{BD}| = \sqrt{(-4)^2 + (-32)^2 + (24)^2}$

$= \sqrt{4^2\left(1 + 8^2 + 6^2\right)}$

$= 4\sqrt{1 + 64 + 36}$

$= 4\sqrt{101}$

$\therefore$ Area of parallelogram ABCD = $\frac{1}{2} \times 4\sqrt{101}$

$= 2\sqrt{101}$ sq units.

OR

Given: Vectors $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$

We know that the cross-product of two vectors, $\vec{a} \times \vec{b}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$

Hence, vector $\vec{d}$ which is also perpendicular to both $\vec{a}$ and $\vec{b}$ is $\vec{d} = \lambda\left(\vec{a} \times \vec{b}\right)$ where $\lambda = 1$ or some other scalar.

Therefore, $\vec{d} = \lambda\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$

$= \lambda\left[\hat{i}\,(28 + 4) - \hat{j}\,(7 - 6) + \hat{k}\,(-2 - 12)\right]$

$\Rightarrow \vec{d} = 32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}$ ...(i)

Now given $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{c}.\vec{d} = 15$

$\vec{c}.\vec{d} = 15$

$= 2\,(32\lambda) + (-1)\,(-\lambda) + 4\,(-14\lambda) = 15$

$\Rightarrow 64\lambda + \lambda - 56\lambda = 15$

$\Rightarrow 9\lambda = 15$

$\Rightarrow \lambda = \frac{15}{9}$

$\Rightarrow \lambda = \frac{5}{3}$

Putting $\lambda = \frac{5}{3}$ in eq. (i), we get

$\vec{d} = \frac{5}{3}\left[32\hat{i} - \hat{j} - 14\hat{k}\right]$

$\Rightarrow \vec{d} = \frac{1}{3}\left[160\hat{i} - 5\hat{j} - 70\hat{k}\right]$

31. Given function is

f(x) = $\begin{cases} |x| + 3, & x \le -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \ge 3 \end{cases}$ = $\begin{cases} -x + 3, x \le -3 \\ -2x, -3 < x < 3 \\ 6x + 2, x \ge 3 \end{cases}$

First, we verify continuity at x = - 3 and then at x = 3

Continuity at x = -3

LHL = $\lim\limits_{x \to (-3)^-} f(x) = \lim\limits_{x \to (-3)^-}(-x + 3)$

$\Rightarrow \quad \text{LHL} = \lim\limits_{h \to 0}[-(-3 - h) + 3]$

$= \lim\limits_{h \to 0}(3 + h + 3)$

= 3 + 3 = 6

and RHL = $\lim_{x\to(-3)^+} f(x) = \lim_{x\to(-3)^+}(-2x)$

$\Rightarrow \quad RHL = \lim_{h\to0}[-2(-3+h)]$

$= \lim_{h\to0}(6-2h)$

$\Rightarrow$ RHL = 6

Also, f(-3) = value of f(x) at x = - 3

=-(-3) + 3

= 3 + 3 = 6

$\because$ LHL = RHL = f(-3)

$\therefore$f(x) is continuous at x = - 3 So, x = - 3 is the point of continuity.

Continuity at x = 3

LHL = $\lim_{x\to3^-} f(x) = \lim_{x\to3^-}[-(2x)]$

$\Rightarrow \quad LHL = \lim_{h\to0}[-2(3-h)]$

$= \lim_{h\to0}(-6+2h)$

$\Rightarrow$ LHL = -6

and RHL = $\lim_{x\to3^+} f(x) = \lim_{x\to3^+}(6x+2)$

$\Rightarrow \quad \text{RHL} = \lim_{h\to0}[6(3+h)+2]$

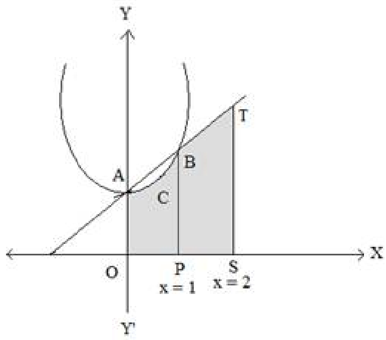
$\Rightarrow$ RHL = 20

$\because \quad \text{LHL} \neq \text{RHL}$

$\therefore$ f is discontinuous at x = 3 Now, as f (x) is a polynomial function for x < -3 , -3< x < 3 and x > 3 so it is continuous in these intervals.

Hence, only x = 3 is the point of discontinuity of f(x).

Section D

32. 

There are three curves respectively,

$C_1 = \{(x, y): 0 \leq y \leq x^2 + 1\}$

$C_2 = \{(x, y): 0 \leq y \leq x + 1\}$

$C_3 = \{(x,y): 0 \leq x \leq 2\}$

The points of intersection of, y = x^2 and y = x + 1y are A(0,1) and B(1,2).

The required area of shaded region = $\int_0^1 y_1 dx + \int_1^2 y_2 dx$

where $y_1 = x^2 + 1$ and $y_2 = x + 1$

$\therefore A = \int_0^1 (x^2+1)dx + \int_1^2 (x+1)dx$

$= [\frac{x^3}{3} + x]_0^1 + [\frac{x^2}{2} + x]_1^2$

$= (\frac{4}{3} - 0) + (4 - \frac{3}{2}) = \frac{8+24-9}{6} = \frac{23}{6}\ sq.units$

33. Given: f : W $\rightarrow$ W defined as $f(n) = \begin{cases} n-1, & if\ n\ is\ odd \\ n+1, & if\ n\ is\ even \end{cases}$

Injectivity: Let n, m be any two odd real numbers, then f(n) = f(m)

$\Rightarrow$ n - 1 = m - 1

$\Rightarrow$ n = m

Again, let n, m be any two even whole numbers, then f(n) = f(m)

$\Rightarrow$ n + 1 = m + 1
$\Rightarrow$ n = m
If n is even and m is odd, then n $\neq$ m.
Now n is even implies f(n) = n+1 and f(m) = m-1.
Therefore, f(n) $\neq$ f(m)
Similarly n is odd and m is even gives $f(n) \neq f(m)$
Therefore in all cases f is one-one.
Surjectivity: Let n be an arbitrary whole number.
If n is an odd number, then there exists an even whole number (n + 1) such that
f(n + 1) = n + 1 - 1 = n
If n is an even number, then there exists an odd whole number (n - 1) such that
f(n - 1) = n - 1 + 1 = n
Therefore, every n $\in$ W has its pre-image in W.
So, f : W $\rightarrow$ W is a surjective. Thus f is invertible and f^{-1} exists.
For f^{-1} : y = n - 1
$\Rightarrow$ n = y + 1 and y = n + 1 $\Rightarrow$ n = y - 1
$\therefore f^{-1}(n) = \begin{cases} n-1, & if\ n\ is\ odd \\ n+1, & if\ n\ is\ even \end{cases}$
Hence, $f^{-1}(y) = y$

OR

Given that
Let A = {1, 2, 3} and R = {(a, b) : a, b $\in$ A and $\left|a^2 - b^2\right| \leq 5$
Put a = 1, b = 1 $\left|1^2 - 1^2\right| \leq 5, (1, 1)$ is an ordered pair.
Put a = 1, b = 2 $\left|1^2 - 2^2\right| \leq 5, (1, 2)$ is an ordered pair.
Put a = 1, b = 3 $\left|1^2 - 3^2\right| > 5, (1, 3)$ is not an ordered pair.
Put a = 2, b = 1 $\left|2^2 - 1^2\right| \leq 5, (2, 1)$ is an ordered pair.
Put a = 2, b = 2 $\left|2^2 - 2^2\right| \leq 5, (2, 2)$ is an ordered pair.
Put a = 2, b = 3 $\left|2^2 - 3^2\right| \leq 5, (2, 3)$ is an ordered pair.
Put a = 3, b = 1 $\left|3^2 - 1^2\right| > 5, (3, 1)$ is not an ordered pair.
Put a = 3, b = 2 $\left|3^2 - 2^2\right| \leq 5, (3, 2)$ is an ordered pair.
Put a = 3, b = 3 $\left|3^2 - 3^2\right| \leq 5, (3, 3)$ is an ordered pair.
R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

i. For (a, a) $\in$ R
   $\left|a^2 - a^2\right| = 0 \leq 5$. Thus, it is reflexive.
ii. Let (a, b) $\in$ R
   $(a, b) \in R, \left|a^2 - b^2\right| \leq 5$
   $\left|b^2 - a^2\right| \leq 5$
   (b, a) $\in$ R
   Hence, it is symmetric
iii. Put a = 1, b = 2, c = 3
   $\left|1^2 - 2^2\right| \leq 5$
   $\left|2^2 - 3^2\right| \leq 5$
   But $\left|1^2 - 3^2\right| > 5$
   Thus, it is not transitive

34. Given the purchase details of three shopkeeper A, B, and C.
A: 12 dozen notebooks, 5 dozen pens, and 6 dozen pencils
B: 10 dozen notebooks, 6 dozen pens, and 7 dozen pencils
C: 11 dozen notebooks, 13 dozen pens, and 8 dozen pencils
Hence, the items purchased by A, B, and C can be represented in matrix X of order 3x3 where the rows denoting the each shopkeeper and columns denoting the number of dozens of items as –

$X = \begin{bmatrix} 12 & 5 & 6 \\ 10 & 6 & 7 \\ 11 & 13 & 8 \end{bmatrix}$

The price of each of the items is also given.

Cost of one notebook = 40 paise

$\Rightarrow$ Cost of one dozen notebooks = 12 $\times$ 40 paise

$\Rightarrow$ Cost of one dozen notebooks = 480 paise

$\therefore$ Cost of one dozen notebooks = ₹ 4.80

Cost of one pen = ₹ 1.25

$\Rightarrow$ Cost of one dozen pens = 12 $\times$ ₹ 1.25

$\therefore$ Cost of one dozen pens = ₹ 15

Cost of one pencil = 35 paise

$\Rightarrow$ Cost of one dozen notebooks = 12 $\times$ 35 paise

$\Rightarrow$ Cost of one dozen notebooks = 420 paise

$\therefore$ Cost of one dozen notebooks = ₹ 4.20

Hence, the cost of purchasing one dozen of the items can be represented in matrix form with each row corresponding to an item as –

$Y = \begin{bmatrix} 4.80 \\ 15 \\ 4.20 \end{bmatrix}$

Now, the individual bill for each shopkeeper can be found by taking the product of the matrices X and Y.

The product is feasible because the order of X is 3x3 and that of Y is 3x1 ,which makes the number of columns of x is equal to the number of the columns of Y.The product is given as follows.

$XY = \begin{bmatrix} 12 & 5 & 6 \\ 10 & 6 & 7 \\ 11 & 13 & 8 \end{bmatrix} \begin{bmatrix} 4.80 \\ 15 \\ 4.20 \end{bmatrix}$

$\Rightarrow XY = \begin{bmatrix} 12 \times 4.80 + 5 \times 15 + 6 \times 4.20 \\ 10 \times 4.80 + 6 \times 15 + 7 \times 4.20 \\ 11 \times 4.80 + 13 \times 15 + 8 \times 4.20 \end{bmatrix}$

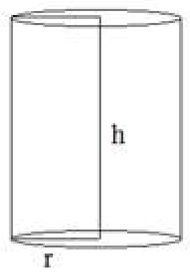
$\Rightarrow \begin{bmatrix} 57.60 + 75 + 25.20 \\ 48 + 90 + 29.40 \\ 52.80 + 195 + 33.60 \end{bmatrix}$

$\therefore XY = \begin{bmatrix} 157.80 \\ 167.40 \\ 281.40 \end{bmatrix}$

Thus, the bills of shopkeepers A, B and C are ₹ 157.80, ₹ 167.40 and ₹ 281.40 respectively.

35. Let radius of the cylinder = r

Height of the cylinder = h|



$s = 2\pi rh + 2\pi r^2$ (1)

$\Rightarrow \frac{s-2\pi r^2}{2\pi r} = h$

Now volume of cylinder is, $v = \pi r^2 h$

$\Rightarrow v = \pi . r^2 \left(\frac{s-2\pi r^2}{2\pi r}\right)$

$\Rightarrow v = \frac{1}{2}\left[sr - 2\pi r^3\right]$

Now, $\frac{dv}{dr} = \frac{1}{2}\left[s - 6\pi r^2\right]$

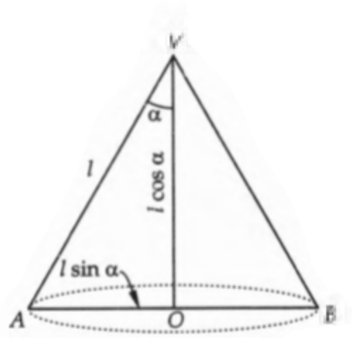
$\Rightarrow \frac{d^2v}{dr^2} = \frac{1}{2}[0 - 12\pi r]$

For maximum/minimum

$\frac{dv}{dr} = 0$
$\Rightarrow s = 6\pi r^2$
From equ (1)
$\Rightarrow 2\pi rh + \ 2\pi r^2 = 6\pi r^2$
$\Rightarrow r = \frac{h}{2}$
$\left[\frac{d^2v}{dr^2}\right]_{r=\frac{h}{2}} = \frac{1}{2}\left[0 - 12\pi \times \frac{h}{2}\right]$
$= -3\pi h < 0$
$\Rightarrow$ s is maximum at $r = \frac{h}{2}$
Hence h = 2r

OR

Let α be the semi-vertical angle of a cone VAB of given slant height l.
In ΔAOV,



cosα = $\frac{VO}{VA}$ and sin α = $\frac{OA}{VA}$
$\Rightarrow$ cos α = $\frac{VO}{l}$ and sin α = $\frac{OA}{l}$
$\Rightarrow$ VO = l cos a, OA = l sin α
Let V be the volume of the cone, Then,
V = $\frac{1}{3}\pi$ (OA)^2 (VO)
$\Rightarrow$ V = $\frac{1}{3}\pi$ (l sin α)^2 (l cos α)
$\Rightarrow$ V = $\frac{1}{3}\pi$l^3 sin$^2\alpha$ cos α
$\Rightarrow$ $\frac{dV}{d\alpha}$ = $\frac{\pi}{3}$l^3 (- sin^3 α + 2 sin α cos^2 α)
$\Rightarrow$ $\frac{dV}{d\alpha}$ = $\frac{\pi l^3}{3}$ sin α (-sin^2 α + 2 cos^2 α) ....(i)
The critical points of V are given by $\frac{dV}{d\alpha}$ = 0.
$\because$ $\frac{dV}{d\alpha}$ = 0
$\Rightarrow$ $\frac{\pi l^3}{3}$ sin α (-sin^2 α + 2 cos$^2\alpha$) = 0
$\Rightarrow$ 2 cos^2 α = sin^2 α
$\Rightarrow$ tan^2 α = 2 $\Rightarrow$ tan α = $\sqrt{2}$ [$\because$ a is acute $\because$ sin $\alpha \neq 0$]
$\because$ cos α = $\frac{1}{\sqrt{1+\tan^2\alpha}}$ = $\frac{1}{\sqrt{3}}$[$\because$ tan α = $\sqrt{21}$]
Differentiating (i) with respect to a, we get
$\frac{d^2V}{d\alpha^2}$ = $\frac{\pi l^3}{3}$ (- 3 sin^2 α cos α + 2 cos^3 α -4 sin^2 α cosα) = $\frac{\pi l^3}{3}$ cos^3 α (2-7 tan^2 α)
$\because \left(\frac{d^2V}{d\alpha^2}\right)_{\tan\alpha=\sqrt{2}}$ = $\frac{1}{3}\pi l^3$ $\left(\frac{1}{\sqrt{3}}\right)^3$(2 - 7 $\times$ 2) = $\frac{-4\pi l^3}{3\sqrt{3}}$ < 0.
Thus, V is maximum, when tan α= $\sqrt{2}$ or α = tan$^{-1}\sqrt{2}$ i.e. when the semi-vertical angle of the cone is tan$^{-1}\sqrt{2}$.

Section E

36. i. P(E$_2$) = 1 - P(E$_1$) = 1 – 0.65 = 0.35
ii. P(E) = P(E$_1$)$\cdot$P $\left(\frac{E}{E_1}\right)$ + P(E$_2$)$\cdot$P $\left(\frac{E}{E_2}\right)$
= 0.65 $\times$ 0.35 + 0.35 $\times$ 0.8
= 0.35 $\times$ 1.45
= 0.51

iii. $P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\cdot P\left(\frac{E}{E\,1}\right)}{P(E_1)\cdot P\left(\frac{E}{E\,1}\right)+P(E_2)\cdot P\left(\frac{E}{E\,2}\right)} = \frac{0.65\times0.35}{0.51} = 0.45$

OR

$P\left(\frac{E_2}{E}\right) = \frac{P(E_2)\cdot P\left(\frac{E}{E\,2}\right)}{P(E_1)\cdot P\left(\frac{E}{E\,1}\right)+P(E_2)\cdot P\left(\frac{E}{E\,2}\right)} = \frac{0.35\times0.8}{0.51} = 0.55$

37. i. Equation of line joining B and C is

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

$\frac{x-1}{4} = \frac{y-4}{0} = \frac{z-6}{-2} = \lambda$

Let coordinates of foot of perpendicular be D$(4\lambda + 1, 4, -2\lambda + 6)$ ...(i)

$\therefore$ D.R.'s of AD are $(4\lambda, 2, -2\lambda + 5)$.

Now, $4\,(4\lambda) + 0\,(2) + (-2)\,(-2\lambda + 5) = 0$

$\lambda = \frac{1}{2}$

Putting in eqn (i)

Coordinates of D are (3, 4, 5)

$\therefore$ Required coordinates are (3, 4, 5).

ii. Let a, b, c be the direction ratios of the required line. Since it is perpendicular to the lines whose direction ratios are (1, -2, -2) and (0, 2, 1) respectively.

$\therefore$ a - 2b - 2c = 0 ...(i)

a + 2b + c = 0 ...(ii)

On solving (i) and (ii) by cross-multiplication, we get

$\frac{a}{-2+4} = \frac{b}{0-1} = \frac{c}{2} \Rightarrow \frac{a}{2} = \frac{b}{-1} = \frac{c}{2}$

Thus, the direction ratios of the required line are (2, -1, 2)

iii. Direction ratio of given lines are (3, -2, 0) and (1, $\frac{3}{2}$, 2)

Now,

as $3\cdot1 + (-2)\cdot\left(\frac{3}{2}\right) + 0\cdot2 = 3 - 3 + 0 = 0$

$\therefore$ Given lines are perpendicular to each other.

OR

Since,direction ratio's are proportional to direction cosine's, therefore L_1 will be parallel to L_2, iff

$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

38. i. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr.

x, y ≥ 0

$\frac{x}{25} + \frac{y}{40} \leq 1$

2x + 5y $\leq$ 100

Z = x + y

ii. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr.

x, y ≥ 0

$\frac{x}{25} + \frac{y}{40} \leq 1$

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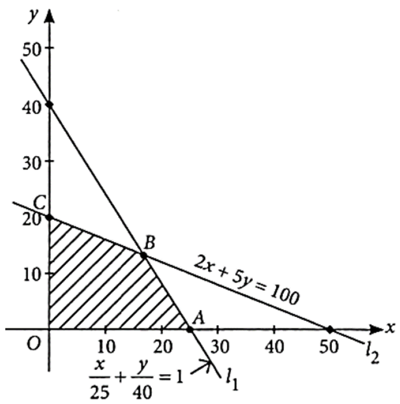
Z = x + y



iii. Here Z = x + y

Corner Points	Value of Z = x + y
(0, 0)	0
(25, 0)	25
$\left(\frac{50}{3}, \frac{40}{3}\right)$	30 ← Maximum
(0, 20)	20

Thus, max Z = 30 occurs at point $\left(\frac{50}{3}, \frac{40}{3}\right)$.

Hence maximum distance covered by Sheetal in 1 hour = 30 km

OR

Corner Points	Value of Z = 6x - 9y
(0, 0)	0
(25, 0)	150 ← Maximum
$\left(\frac{50}{3}, \frac{40}{3}\right)$	-20
(0, 20)	-180

Maximum value is 150.