EXERCISE : 5.1 (NCERT)

Express each of the complex Numbers given in the Exercises 1 to 10 in form of a+ib.

Q No 1 $(5i)\left(-\frac{3}{5}i\right)$

Sol $(5i)\left(-\frac{3}{5}i\right) = -5\times\frac{3}{5}i^2 = (-3)\times(-1) = 3$ $\quad [\because i^2=-1]$

$= 3+0i = a+ib$ where $a=3$, $b=0$

Q No 2 i^9+i^{19}

Sol $i^9+i^{19} = i^8\times i + i^{18}\times i = (i^2)^4\times i + (i^2)^9\times i$

$= (-1)^4\times i + (-1)^9\times i = i - i = 0 = 0+0i$

$= a+ib$ where $a=0$ and $b=0$

Q No 3 i^{-39}

Sol. $(i)^{-39} = \frac{1}{i^{39}} = \frac{1}{i^{38}\times i} = \frac{1}{(i^2)^{19}\times i} = \frac{1}{(-1)^{19} i} = -\frac{1}{i}$

$= -\frac{1}{i}\times\frac{i}{i} = \frac{-i}{i^2}\ \frac{-i}{-1} = +i = 0+i = a+ib$

Where $a=0$ and $b=1$

Q No 4 $3(7+i7)+i(7+i7)$

Sol $3(7+i7)+i(7+i7) = 21+21i+7i+7i^2$

$= 21+28i+7(-1) = 14+28i$

$= a+ib$ Where $a=14$, $b=28$

Q No 5 $(1-i)-(-1+i6)$

Sol $(1-i)-(-1+i6) = 1-i+1-i6 = 2-i7 = 2+i(-7)$

$= a+ib$ Where $a=2$ $b=-7$

Q No 6 : $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Sol. $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right) = \frac{1}{5}+i\frac{2}{5}-4-i\frac{5}{2}$

$= \left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right) = \left(\frac{1-20}{5}\right)+i\left(\frac{4-25}{10}\right)$

$= -\frac{19}{5}+i\left(-\frac{21}{10}\right) = a+ib$ where $a=-\frac{19}{5}$, $b=-\frac{21}{10}$

Q.No.7 $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

$= \left[\left(\frac{1}{3}+4\right)+i\left(\frac{7}{3}+\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

$= \frac{13}{3}+i\frac{8}{3}+\frac{4}{3}-i = \left(\frac{13}{3}+\frac{4}{3}\right)+i\left(\frac{8}{3}-1\right)$

$= \frac{17}{3}+i\left(\frac{5}{3}\right) = a+ib$ where $a=\frac{17}{3}$, $b=\frac{5}{3}$

Q.No.8 $(1-i)^4$

$= \left[(1-i)^2\right]^2 = \left[1+i^2-2i\right]^2 = \left[1+(-1)-2i\right]^2 \quad \left[\because i^2=-1\right]$

$= (-2i)^2 = 4i^2 = 4(-1) = -4+0i$

$= a+ib$ where $a=-4$, $b=0$

Q.No.9 $\left(\frac{1}{3}+3i\right)^3$

$= \left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left[\frac{1}{3}+3i\right] \quad \left[\because (a+b)^3 = a^3+b^3+3ab(a+b)\right]$

$= \frac{1}{27}+27i^3+3i\left(\frac{1}{3}+3i\right)$

$= \frac{1}{27}+27i\cdot i^2+i+9i^2 = \frac{1}{27}+27i(-1)+i+9(-1)$

$= \left(\frac{1}{27}-9\right)+i(-27+1) = -\frac{242}{27}-26i = a+ib$

where $a=-\frac{242}{27}$, $b=-26$.

Q.No.10 $\left(-2-\frac{1}{3}i\right)^3 = \left[(-2)+\left(-\frac{1}{3}i\right)\right]^3$

$= (-2)^3+\left(-\frac{1}{3}i\right)^3+3(-2)\left(-\frac{1}{3}i\right)\left[-2-\frac{1}{3}i\right]$

$= -8-\frac{1}{27}i^3+2i\left[-2-\frac{1}{3}i\right]$

$= -8-\frac{1}{27}i\cdot i^2-4i-\frac{2}{3}i^2 = -8-\frac{1}{27}i(-1)-4i-\frac{2}{3}(-1)$

$= -8+\frac{1}{27}i-4i+\frac{2}{3} = \left(-8+\frac{2}{3}\right)+i\left(\frac{1}{27}-4\right)$

$= \left(-\frac{22}{3}\right)+i\left(-\frac{107}{27}\right) = a+ib$ where $a=-\frac{22}{3}$, $b=-\frac{107}{27}$

Find the multiplicative inverse of the complex. Nos. given in exercise 11 to 13

Q.No. 11 $4-i3$

Sol. Let $z = 4-i3$

$\therefore$ Multiplicative inverse of $z = z^{-1} = \frac{1}{z}$

$$= \frac{1}{4-i3} = \frac{1}{4-i3} \times \frac{4+i3}{4+i3} = \frac{4+i3}{(4)^2-(i3)^2} = \frac{4+i3}{16-9i^2}$$

$$= \frac{4+i3}{16-9(-1)} = \frac{4+i3}{16+9} = \frac{4+i3}{25} = \frac{4}{25} + i\frac{3}{25}$$

Q.No 12 $\sqrt{5}+i3$

Sol. Let $z = \sqrt{5}+i3$

$\therefore$ Multiplicative inverse of $z = z^{-1} = \frac{1}{z}$

$$= \frac{1}{\sqrt{5}+i3} = \frac{1}{\sqrt{5}+i3} \times \frac{\sqrt{5}-i3}{\sqrt{5}-i3} = \frac{\sqrt{5}-i3}{(\sqrt{5})^2-(i3)^2} = \frac{\sqrt{5}-i3}{5-9i^2}$$

$$= \frac{\sqrt{5}-i3}{5-9(-1)} = \frac{\sqrt{5}-i3}{5+9} = \frac{\sqrt{5}-i3}{14} = \left(\frac{\sqrt{5}}{14}\right) + i\left(-\frac{3}{14}\right)$$

Q.No 13 $-i$

Let $z = -i$

$\therefore$ Multiplicative inverse of $z = z^{-1} = \frac{1}{z}$

$$= \frac{1}{-i} = \frac{1}{-i} \times \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = \frac{i}{1} = i = 0+i1$$

Q.No. 14: Express the following expression in the form of $a+ib$:

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

Sol.

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

$$= \frac{(3)^2-(i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i}$$

$$= \frac{9-5i^2}{2\sqrt{2}i} = \frac{9-5(-1)}{2\sqrt{2}i} = \frac{9+5}{2\sqrt{2}i} = \frac{14}{2\sqrt{2}i}$$

$$= \frac{7}{\sqrt{2}i} = \frac{7}{\sqrt{2}i} \times \frac{i}{i} = \frac{7i}{\sqrt{2}i^2} = \frac{7i}{\sqrt{2}(-1)}$$

$$= \frac{7i}{-\sqrt{2}} = 0+i\left(-\frac{7}{\sqrt{2}}\right)$$

$$= a+ib$$

Where $a=0$, $b=-\frac{7}{\sqrt{2}}$.

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