PRACTICE SET -4

1. The real roots of the equation $x^2 + 5|x| + 4 = 0$ are

a. – 1, 4 b. 1, 4
c. – 4, 4 d. None of these

2. If $\frac{1+\sqrt{3}i}{2}$ is a root of equation $x^4 - x^3 + x - 1 = 0$ then its real roots are

a. 1, 1 b. – 1, – 1
c. 1, – 1 d. 1, 2

3. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ -1 & 6 & 1 \end{bmatrix}$, then the adjoint of A is

a. $\begin{bmatrix} 2 & -5 & 32 \\ 0 & 1 & -6 \\ 0 & 0 & 2 \end{bmatrix}$ b. $\begin{bmatrix} -1 & 0 & 0 \\ -5 & -2 & 0 \\ 1 & -6 & 1 \end{bmatrix}$

c. $\begin{bmatrix} -1 & 0 & 0 \\ -5 & -2 & 0 \\ 1 & -6 & -1 \end{bmatrix}$ d. None of these

4. In the expansion of $\frac{1-2x+3x^2}{e^x}$, the coefficient of x^5 will be

a. $\frac{71}{120}$ b. $-\frac{71}{120}$
c. $\frac{31}{40}$ d. $-\frac{31}{40}$

5. The total number of seven digit numbers the sum of whose digits is even is

a. 9000000 b. 4500000
c. 8100000 d. None of these

6. If E and F are events with $P(E) \le P(F)$ and $P(E \cap F) > 0$, then

a. occurrence of E $\Rightarrow$ occurrence of F
b. occurrence of F $\Rightarrow$ occurrence of E
c. non-occurrence of E $\Rightarrow$ non-occurrence of F
d. None of the above implication holds

7. $\tan\left(\frac{\pi}{4}+\theta\right) - \tan\left(\frac{\pi}{4}-\theta\right) =$

a. $2\tan 2\theta$ b. $2\cot 2\theta$
c. $\tan 2\theta$ d. $\cot 2\theta$

8. $\tanh(x+y)$ equals

a. $\frac{\tanh x + \tanh y}{1 - \tanh x \tanh y}$ b. $\frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$

c. $\frac{\tanh x - \tanh y}{1 - \tanh x \tanh y}$ d. $\frac{\tanh x - \tanh y}{1 + \tanh x \tanh y}$

9. The length of the shadow of a pole inclined at 10° to the vertical towards the sun is 2.05 metres, when the elevation of the sun is 38°. The length of the pole is

a. $\frac{2.05 \sin 38^\circ}{\sin 42^\circ}$ b. $\frac{2.05 \sin 42^\circ}{\sin 38^\circ}$

c. $\frac{2.05 \cos 38^\circ}{\cos 42^\circ}$ d. None of these

10. If $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{when } x \ne 0 \\ 0, & \text{when } x = 0 \end{cases}$, then

a. $f(0+0) = 1$ b. $f(0-0) = 1$
c. f is continuous at $x = 0$ d. None of these

11. If $y = \sqrt{\frac{(x-a)(x-b)}{(x-c)(x-d)}}$, , then $\frac{dy}{dx} =$

a. $\frac{y}{2}\left[\frac{1}{x-a} + \frac{1}{x-b} - \frac{1}{x-c} - \frac{1}{x-d}\right]$

b. $y\left[\frac{1}{x-a} + \frac{1}{x-b} - \frac{1}{x-c} - \frac{1}{x-d}\right]$

c. $\frac{1}{2}\left[\frac{1}{x-a} + \frac{1}{x-b} - \frac{1}{x-c} - \frac{1}{x-d}\right]$

d. None of these

12. If $f(x) = \frac{x^2-1}{x^2+1}$, for every real number x, then the minimum value of f

a. Does not exist because f is unbounded
b. Is not attained even though f is bounded
c. Is equal to 1
d. Is equal to –1

13. $\int \frac{a^x}{\sqrt{1-a^{2x}}} dx =$

a. $\frac{1}{\log a} \sin^{-1} a^x + c$ b. $\sin^{-1} a^x + c$

c. $\frac{1}{\log a} \cos^{-1} a^x + c$ d. $\cos^{-1} a^x + c$

14. The area bounded by the circle $x^2 + y^2 = 4$, line $x = \sqrt{3}y$ and x – axis lying in the first quadrant, is

a. $\frac{\pi}{2}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{3}$ d. π

15. The general solution of $x^2 \frac{dy}{dx} = 2$ is

a. $y = c + \frac{2}{x}$ b. $y = c - \frac{2}{x}$

c. $y = 2cx$ d. $y = c - \frac{3}{x^2}$

16. The length of perpendicular from the point $(a\cos\alpha, a\sin\alpha)$ upon the straight line $y = x\tan\alpha + c, c > 0$ is

a. $c\cos\alpha$ b. $c\sin^2\alpha$

c. $c\sec^2\alpha$ d. $c\cos^2\alpha$

17. The circle passing through point of intersection of the circle $S = 0$ and the line $P = 0$ is

a. $S + \lambda P = 0$

b. $S - \lambda P = 0$ and $\lambda S + P = 0$

c. $P - \lambda S = 0$

d. All of these

18. Axis of a parabola is $y = x$ and vertex and focus are at a distance $\sqrt{2}$ and $2\sqrt{2}$ respectively from the origin. Then equation of the parabola is-

a. $(x - y)^2 = 8(x + y - 2)$ b. $(x + y)^2 = 2(x + y - 2)$

c. $(x - y)^2 = 4(x + y - 2)$ d. $(x + y)^2 = 2(x - y + 2)$

19. If $\vec{a}$ and $\vec{b}$ be unlike vectors, then $\vec{a} \cdot \vec{b} =$

a. $|\vec{a}||\vec{b}|$ b. $-|\vec{a}||\vec{b}|$

c. 0 d. None of these

20. $(\sim(\sim p)) \wedge q$ is equal to

a. $\sim p \wedge q$ b. $p \wedge q$

c. $p \wedge \sim q$ d. $\sim p \wedge \sim q$

21. The number of solutions of $\log 4(x-1) = \log 2(x-3)$ is:

a. 3 b. 1

c. 2 d. 0

22. The value of $0.2\dot{3}\dot{4}$ is

a. $\frac{232}{990}$ b. $\frac{232}{9990}$

c. $\frac{223}{990}$ d. $\frac{232}{9909}$

23. The sum of the coefficients of even power of x in the expansion of $(1 + x + x^2 + x^3)^5$ is

a. 256 b. 128

c. 512 d. 64

24. A line makes the same angle θ, with each of the x and z–axis. If the angle β, which it makes with y-axis is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals

a. $\frac{3}{5}$ b. $\frac{2}{3}$

c. $\frac{1}{5}$ d. None of these

25. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently, is

a. 40 b. 60

c. 80 d. 100

26. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1 z_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a

a. straight line parallel to x-axis

b. straight line parallel to y-axis

c. circle of radius 2

d. circle of radius $\sqrt{2}$

27. If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are)

a. –2 b. –1

c. 1 d. 2

28. If the sum of the first ten term of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + ...$, is $\frac{16}{5}m$, then m is equal to:

a. 102 b. 101

c. 100 d. 99

29. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope

numbered 2. Then the number of ways it can be done is

a. 264 b. 265

c. 53 d. 67

30. Let $\theta, \varphi \in [0, 2\pi]$ be such that $2\cos\theta(1-\sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi - 1$, $\tan(2\pi-\theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$ Then φ cannot satisfy.

a. $0 < \varphi < \frac{\pi}{2}$ b. $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$

c. $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ d. $\frac{3\pi}{2} < \varphi < 2\pi$

31. If $0 < x < 1$, then $\sqrt{1+x^2}\left[\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\}^2-1\right]^{1/2}$ is equal to

a. $\frac{x}{\sqrt{1+x^2}}$ b. x

c. $x\sqrt{1+x^2}$ d. $\sqrt{1+x^2}$

32. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to: (where c is a constant of integration)

a. 2x + sinx + 2sin2x + c b. x + 2sinx + 2sin2x + c

c. x + 2sinx + sin2x + c d. 2x + sinx + sin2x + c

33. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{1/3} + C$

where C is a constant of integration, then the function $f(x)$ is equal to:

a. $-\frac{1}{6x^3}$ b. $\frac{3}{x^2}$

c. $-\frac{1}{2x^2}$ d. $-\frac{1}{2x^3}$

34. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral $\int x\sqrt{\frac{2\sin(x^2-1)-\sin2(x^2-1)}{2\sin(x^2-1)+\sin2(x^2-1)}}dx$ is equal to: (where c is a constant of integration)

a. $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$ b. $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$

c. $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$ d. $\frac{1}{2}\log_e\left|\sec^2(x^2-1)\right|+c$

35. If $f(x) = \int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx, (x \geq 0)$ and f(0) = 0, then the value of f(1) is:

a. $-\frac{1}{2}$ b. $\frac{1}{2}$

c. $-\frac{1}{4}$ d. $\frac{1}{4}$

36. Let $n \geq 2$ be a natural number and $0 < \theta < \pi/2$. Then $\int \frac{(\sin^n\theta - \sin\theta)^{1/n}\cos\theta}{\sin^{n+1}\theta}d\theta$ is equal to:

(Where C is a constant of integration)

a. $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$

b. $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

c. $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

d. $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

37. If $\int x^5e^{-4x^3}dx = \frac{1}{48}e^{-4x^3}f(x)+C$, where C is a constant of integration, then f(x) is equal to:

a. $-4x^3-1$ b. $4x^3+1$

c. $-2x^3-1$ d. $-2x^3+1$

38. $\int\frac{\sqrt{1-x^2}}{x^4}dx = A(x)(\sqrt{1-x^2})^m+C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration then $(A(x))^m$ equals:

a. $\frac{-1}{3x^3}$ b. $\frac{-1}{27x^9}$

c. $\frac{-1}{9x^4}$ d. $\frac{1}{27x^6}$

39. If $\int\frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1}+C$, where C is a constant of integration, then f(x) is equal to:

a. $\frac{1}{3}(x+4)$ b. $\frac{1}{3}(x+1)$

c. $\frac{2}{3}(x+2)$ d. $\frac{2}{3}(x-4)$

40. Match the statement of Column I with those in Column II:

Column I	Column II
(A) The point $(\lambda, 2+\lambda)$ lines inside the circle	1. -1

$x^2+y^2=4$, then the value of λ can be	
(B) The point $(\lambda, \lambda+2)$ will lie outside the circle $x^2+y^2-2x+4y=0$, then the value of λ can be	2. $-\frac{1}{2}$
(C) Both the equation $x^2+y^2+2\lambda x+4=0$ and $x^2+y^2-4\lambda x+8=0$ represent real circles, then the value of λ can be	3. 1
	4. 3
	5. 5

a. A→1,2; B→1,2,3,4,5; C→4,5
b. A→2,3; B→4,5; C→1,2,3,4,5
c. A→1,4; B→3,5; C→1,2,3
d. A→4,5; B→1,2,3,4; C→3,4,5

41. Match the statement of Column I with those in Column II:

Column I	Column II
(A) Two intersecting circles	1. have a common tangent
(B) Two mutually external circles	2. have a common normal
(C) Two circles, one strictly inside the other	3. do not have a common tangent
(D) Two branches of a hyperbola	4. do not have a common normal

a. A→1,2; B→1,2; C→2,3; D→2,3
b. A→2,3; B→4,1; C→3,2; D→1,3
c. A→1; B→3,2; C→2;1 D→4,2
d. A→4,3; B→1; C→3; D→2,3

42. Consider the following linear equations $ax+by+cz=0$, $bx+cy+az=0$, $cx+ay+bz=0$

Column I	Column II
(A) $a+b+c\neq 0$ and $a^2+b^2+c^2=ab+bc+ca$	1. the equations represent planes meeting only at a single point
(B) $a+b+c=0$ and $a^2+b^2+c^2\neq ab+bc+ca$	2. the equations represent the line $x=y=z$
(C) $a+b+c\neq 0$ and $a^2+b^2+c^2\neq ab+bc+ca$	3. the equations represent identical planes
(D) $a+b+c=0$ and $a^2+b^2+c^2=ab+bc+ca$	4. the equations represent the whole of the three dimensional space

a. A→3; B→1; C→4; D→2 b. A→3; B→2; C→1; D→4
c. A→1, B→3; C→2; D→4 d. A→4; B→1; C→3; D→2

43. Consider the lines $L_1:\frac{x-1}{2}=\frac{y}{-1}=\frac{z+3}{1}, L_2:\frac{x-4}{1}=\frac{y+3}{1}=\frac{z+3}{2}$ and the planes $P_1: 7x+y+2z=3$, $P_2: 3x+5y-6z=4$. Let $ax+by+cz=d$ the equation of the plane passing through the point of intersection of lines L_1 and L_2 and perpendicular to planes P_1 and P_2. Match Column I with Column II and select the correct answer using the code given below the Columns.

Column I	Column II
(A) $a=$	1. 13
(B) $b=$	2. –3
(C) $c=$	3. 1
(D) $d=$	4. –2

a. A→3; B→2; C→4; D→2 b. A→2; B→4; C→3; D→1
c. A→1; B→3; C→2, D→4 d. A→4; B→1; C→3; D→2

∴ $k_1=2$ and $k_2=1$

∴ Point of intersection (5, –2, –1)

∴ Equation of plane. $1\cdot(x-5)-3(y+2)-2(z+1)=0$

⇒ $x-3y-2z-13=0 \Rightarrow x-3y-2z=13$

∴ $a=1, b=-3, c=-2, d=13$

44. Let $f(x)=\frac{x}{(1+x^n)^{1/n}}$ for $n\geq 2$ and $g(x)=\underbrace{(f\circ f\circ\ldots\circ f)}_{f\text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2}g(x)\,dx$ equals

a. $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$ b. $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+K$

c. $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$ d. $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+K$

45. Let $f:[0,2]\rightarrow R$ be a function which is continuous on [0, 2] and is differentiable on (0, 2) with $f(0)=1$. Let

$F(x)=\int_0^{x^2} f(\sqrt{t})dt$ for $x\in[0,2]$. If $F'(x)=f'(x)$ for all $x\in(0,2)$, then $F(2)$ equals

a. e^2-1 **b.** e^4-1 **c.** $e-1$ **d.** e^4

46. The function $y=f(x)$ is the solution of the differential equation $\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^4+2x}{\sqrt{1-x^2}}$ in (–1,1) satisfying $f(0)=0$. Then $\int_{\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is

a. $\frac{\ }{3}\ \ \frac{\sqrt{3}}{2}$ **b.** $\frac{\ }{3}\ \ \frac{\sqrt{3}}{4}$ **c.** $\frac{\ }{6}\ \ \frac{\sqrt{3}}{4}$ **d.** $\frac{\ }{6}\ \ \frac{\sqrt{3}}{2}$

47. Let $O(0,0)$, $P(3,4)$, $Q(6,0)$ be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are

a. $\left(\frac{4}{3},3\right)$ **b.** $\left(3,\frac{2}{3}\right)$

c. $\left(3,\frac{4}{3}\right)$ **d.** $\left(\frac{4}{3},\frac{2}{3}\right)$

48. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line $2x-y=1$. The points of contact of the tangents on the hyperbola are.

a. $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ **b.** $\left(-\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

c. $3\sqrt{3},-2\sqrt{2}$ **d.** $-3\sqrt{3},2\sqrt{2}$

49. If the vectors $\overline{AB}=3\hat{i}+4\hat{k}$ and $\overline{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangles ABC, then the length of the median through A is

a. $\sqrt{18}$ **b.** $\sqrt{72}$

c. $\sqrt{33}$ **d.** $\sqrt{45}$

50. The integral $\int\frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$ is equal to:

(where C is a constant of integration)

a. $\frac{x^4}{(2x^4+3x^2+1)^3}+C$ **b.** $\frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$

c. $\frac{x^4}{6(2x^4+3x^2+1)^3}+C$ **d.** $\frac{x^{12}}{(2x^4+3x^2+1)^3}+C$

Answers and Solutions

1. **(d)** $x^2+5|x|+4=0$

$\Rightarrow |x|^2+5|x|+4=0$

$\Rightarrow |x|=-1,-4,$ which is not possible. Hence, the given equation has no real root.

2. **(c)** $x^4-x^3+x-1=0$

$\Rightarrow x^3(x-1)+1(x-1)=0$

$x-1=0$ or $x^3+1=0$

$\Rightarrow x=1,-1,\frac{1+\sqrt{3}i}{2},\frac{1-\sqrt{3}i}{2}$

So its real roots are 1 and -1.

3. **(d)** $A=\begin{bmatrix}1&0&0\\5&2&0\\-1&6&1\end{bmatrix}$

$\Rightarrow adj(A)=\begin{bmatrix}2&-5&32\\0&1&-6\\0&0&2\end{bmatrix}^T=\begin{bmatrix}2&0&0\\-5&1&0\\32&-6&2\end{bmatrix}.$

4. **(b)** $(1-2x+3x^2)e^{-x}=(1-2x+3x^2)\left\{1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots\right\}$

$\therefore$ The coefficient of $x^5=1\left(-\frac{1}{5!}\right)+(-2)\left(\frac{1}{4!}\right)+3\left(-\frac{1}{3!}\right)$

$=-\frac{1}{120}-\frac{1}{12}-\frac{1}{2}=-\frac{71}{120}.$

5. **(b)** Suppose $x_1x_2x_3x_4x_5x_6x_7$ represents *a* seven digit number. Then x_1 takes the value $1,2,3,\ldots,9$ and $x_2,x_3,\ldots x_7$ all take values 0, 1, 2, 3, …, 9. If we keep $x_1,x_2,\ldots,x_6$ fixed, then the sum $x_1+x_2+\ldots+x_6$ is either even or odd. Since x_7 takes 10 values 0, 1, 2, …,9, five of the numbers so formed will be even and 5 odd. Hence the required number of numbers

$=9.10.10.10.10.10.5=4500000.$

6. **(d)** It is given that $P(E)\le P(F)\Rightarrow E\subseteq F$

and $P(E\cap F)>0\Rightarrow E\subset F$ . . .(*ii*)

(a) Occurrence of $E\Rightarrow$ occurrence of F [from Eq.(*i*)]

(b) Occurrence of $F\Rightarrow$ occurrence of E [from Eq.(*ii*)]

(c) Non-occurrence of $E\Rightarrow$ non-occurrence of F [from Eq.(*i*)]

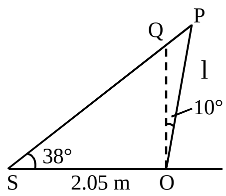
7. (a) $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)$

$$=\frac{1+\tan\theta}{1-\tan\theta}-\frac{1-\tan\theta}{1+\tan\theta}$$

$$=\frac{4\tan\theta}{1-\tan^2\theta}=2\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)=2\tan 2\theta$$

8. (b) It is understandable.

9. (a)



$$\frac{\sin 38^{\circ}}{l}=\frac{\sin(\angle SPO)}{2.05}$$

$$=\frac{\sin(180^{\circ}-38^{\circ}-90^{\circ}-10^{\circ})}{2.05}$$

$$\Rightarrow \quad l=\frac{2.5\sin 38^{\circ}}{\sin 42^{\circ}}$$

10. (c) $\lim_{x\to 0^+} f(x)=x^2\sin\frac{1}{x}$,

but $-1\le \sin\frac{1}{x}\le 1$ and $x\to 0$

Therefore, $\lim_{x\to 0^+} f(x)=0=\lim_{x\to 0^-} f(x)=f(0)$

Hence $f(x)$ is continuous at $x=0$.

11. (a) $y=\sqrt{\left[\frac{(x-a)(x-b)}{(x-c)(x-d)}\right]}$

$$\Rightarrow \quad \log y=\frac{1}{2}[\log(x-a)+\log(x-b)-\log(x-c)-\log(x-d)]$$

Differentiating w.r.t. x we get

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{2}\left[\frac{1}{(x-a)}+\frac{1}{(x-b)}-\frac{1}{(x-c)}-\frac{1}{(x-d)}\right]$$

Thus $\frac{dy}{dx}=\frac{y}{2}\left[\frac{1}{(x-a)}+\frac{1}{(x-b)}-\frac{1}{(x-c)}-\frac{1}{(x-d)}\right]$.

12. (d) $f(x)=\frac{x^2-1}{x^2+1}=\frac{x^2+1-2}{x^2+1}=1-\frac{2}{x^2+1}$

$\therefore$ $f(x)<1\ \forall x$

and ≥ -1 as $\frac{2}{x^2+1}\le 2$

$\therefore$ $-1\le f(x)<1$

Hence $f(x)$ has minimum value -1 and also there is no maximum value.

Alternate: $f'(x)=\frac{(x^2+1)2x-(x^2-1)2x}{(x^2+1)^2}=\frac{4x}{(x^2+1)^2}$

$$f'(x)=0\Rightarrow x=0$$

$$f''(x)=\frac{(x^2+1)^2\,4-4x.2(x^2+1)2x}{(x^2+1)^4}$$

$$=\frac{(x^2+1)4-16x(x)}{(x^2+1)^3}=\frac{-12x^2+4}{(x^2+1)^3}$$

$\therefore$ $f''(0)>0$

$\therefore$ There is only one critical point having minima.

Hence $f(x)$ has least value at $x=0$.

$$f_{min}=f(0)=\frac{-1}{1}=-1.$$

13. (a) Put $a^x=t\Rightarrow a^x\log_e a\,dx=dt$, then

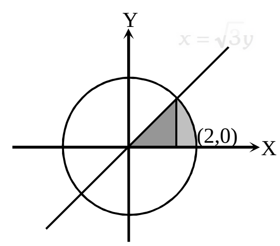
$$\int\frac{a^x}{\sqrt{1-a^{2x}}}dx=\frac{1}{\log_e a}\int\frac{dt}{\sqrt{1-t^2}}$$

$$=\frac{1}{\log_e a}\sin^{-1}(t)+c=\frac{\sin^{-1}(a^x)}{\log_e a}+c.$$

14. (c) Required area $=\int_0^{\sqrt{3}}\frac{x}{\sqrt{3}}dx+\int_{\sqrt{3}}^{2}\sqrt{4-x^2}\,dx$

$$=\frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}}+\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$=\frac{\sqrt{3}}{2}+\left[\pi-\frac{\sqrt{3}}{2}-\frac{2\pi}{3}\right]=\frac{\pi}{3}.$$



Trick: Area of sector made by an arc $=\frac{\theta^{\circ}R^2}{2}=\frac{\pi}{6}.\frac{4}{2}$

$$=\frac{\pi}{3}.$$

15. (b) $\frac{dy}{dx}=\frac{2}{x^2}\Rightarrow dy=\frac{2}{x^2}dx$, Now integrate it.

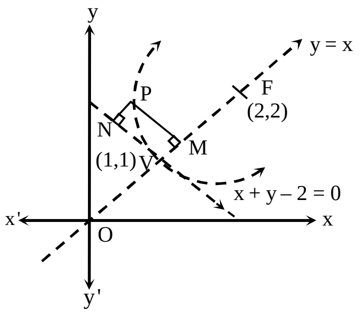
16. (a) Here, equation of line is $y=x\tan\alpha+c$, $c>0$

Length of the perpendicular drawn on line from point $(a\cos\alpha, a\sin\alpha)$

$$p=\frac{-a\sin\alpha+a\cos\alpha\tan\alpha+c}{\sqrt{1+\tan^2\alpha}};\ p=\frac{c}{\sec\alpha}=c\cos\alpha$$

17. (d) It is a fundamental concept.

18. (a)



Since, distance of vertex from origin is $\sqrt{2}$ and focus is $2\sqrt{2}$.

$\therefore$ V(1, 1) and F(2, 2) (i.e., lying on y = x)

where, length of latus rectum $= 4a = 4\sqrt{2}$ $(\because a = \sqrt{2})$

$\therefore$ By definition of parabola $PM^2 = (4a)(PN)$

Where, PN is length of perpendicular upon $x + y - 2 = 0$ (i.e., tangent at vertex).

$\Rightarrow \frac{(x-y)^2}{2} = 4\sqrt{2}\left(\frac{x+y-2}{\sqrt{2}}\right) \Rightarrow (x-y)^2 = 8(x+y-2)$

19. (b) $\vec{a}\cdot\vec{b} = -|\vec{a}||\vec{b}|$, $(\because \cos\theta = -1)$

20. (b) $(\sim(\sim p)) \wedge q = p \wedge q$.

21. (b) Given $\frac{\log(x-1)}{2\log 2} = \frac{\log(x-3)}{\log 2}$

$\Rightarrow (x-1) = (x-3)^2$

$\Rightarrow x = 2, 5$

But at $x = 2$, given log is not defined.

22. (a) $0.2\overline{34} = 0.2343434.....$

$0.2 + 0.034 + 0.00034 + 0.0000034 + ...$

$= 0.2 + \frac{34}{1000} + \frac{34}{100000} + \frac{34}{10000000} + ...\infty$

$= \frac{2}{10} + 34\left[\frac{1}{10^3} + \frac{1}{10^5} + \frac{1}{10^7} + ...\infty\right]$

$= \frac{2}{10} + 34\left[\frac{1/10^3}{1-1/1000}\right] = \frac{2}{10} + 34 \times \frac{1}{1000} \times \frac{100}{99}$

$= \frac{2}{10} + \frac{34}{990} = \frac{232}{990}$.

23. (c) $(1+x+x^2+x^3)^5 = (1+x)^5(1+x^2)^5$

$= (1+5x+10x^2+10x^3+5x^4+x^5)$

$\times(1+5x^2+10x^4+10x^6+5x^8+x^{10})$

Therefore the required sum of coefficients

$= (1+10+5).2^5 = 16 \times 32 = 512$

Note: $2^n = 2^5$ = Sum of all the binomial coefficients in the 2nd bracket in which all the powers of x are even.

24. (a) Here, $l = \cos\theta, m = \cos\beta, n = \cos\theta$, $(\because l = n)$

Now, $l^2 + m^2 + n^2 = 1$

$\Rightarrow 2\cos^2\theta + \cos^2\beta = 1$

$\Rightarrow 2\cos^2\theta = \sin^2\beta$ Given, $\sin^2\beta = 3\sin^2\theta$

$\Rightarrow 2\cos^2\theta = 3\sin^2\theta \Rightarrow 5\cos^2\theta = 3$,

$\therefore \cos^2\theta = \frac{3}{5}$.

25. (a) Total number of arrangements of word BANANA

$= \frac{6!}{3!2!} = 60$

The number of arrangements of words BANANA in which two N's appear adjacently $= \frac{5!}{3!} = 20$

Required number of arrangements = 60 – 20 = 40

26. (c) $\left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1 \Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$

$\Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$

$\Rightarrow z_1\bar{z}_1 + 4z_2\bar{z}_2 = 4 + z_1\bar{z}_1 z_2\bar{z}_2$

$\Rightarrow 4 + |z_1|^2|z_2|^2 - 4|z_2|^2 - |z_1|^2 = 0$

$\Rightarrow (|z_1|^2 - 1)\cdot(|z_2|^2 - 4) = 0$

But $|z_2| \neq 1$, $\therefore |z_2| = 2$

Hence, z lies on a circle of radius 2 centered at origin.

27. (a, d) adj P $= \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix} \Rightarrow |\text{adj } P| = 4$

We know, $|\text{adj } P| = |P|^{n-1}$ (where n is order of matrix)

$\Rightarrow |\text{adj } P| = |P|^2 \Rightarrow 4 = |P|^2 \Rightarrow |P| = \pm 2$

28. (b) $\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 + \left(\frac{24}{5}\right)^2 + ...$

$= \frac{8^2}{5^2} + \frac{12^2}{5^2} + \frac{16^2}{5^2} + \frac{20^2}{5^2} + \frac{24^2}{5^2} + ...$ $T_n = \frac{(4n+4)^2}{5^2}$

$S_n = \frac{1}{5^2}\sum_{n=1}^{10} 16(n+1)^2 = \frac{16}{25}\sum_{n=1}^{10}(n^2 + 2n + 1)$

$= \frac{16}{25}\left[\frac{10 \times 11 \times 21}{6} + \frac{2 \times 10 \times 11}{2} + 10\right] = \frac{16}{25} \times 505 = \frac{16}{5} m$

$\Rightarrow m = 101$

29. (c) Number of required ways

$= 5! - \{4\cdot 4\}! - {}^4C_2 \cdot 3! + {}^4C_3 \cdot 2! - 1\} = 53.$

30. (a, c, d,) Conditions:

$$-\tan(\theta) > 0 \Rightarrow \tan\theta < 0 \text{ and } -1 < \sin\theta < -\frac{\sqrt{3}}{2}$$

$$\therefore \quad \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right) \Rightarrow 0 < \cos\theta < \frac{1}{2}$$

$$\text{Also, } 2\cos\theta(1-\sin\phi) = \sin^2\theta\left(\frac{1}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)\cos\phi - 1$$

$$\Rightarrow \quad 2\cos\theta - 2\cos\theta\sin\phi = 2\sin\theta\cos\phi - 1$$

$$\Rightarrow \quad 1 + 2\cos\theta = 2\sin(\theta+\phi)$$

$$\Rightarrow \quad \sin(\theta+\phi) = \frac{1}{2} + \cos\theta$$

$$\Rightarrow \quad \frac{1}{2} < \sin(\theta+\phi) < 1 \Rightarrow \frac{\pi}{2} < \phi < \frac{4\pi}{3}$$

31. (c) $\sqrt{1+x^2}\left[(x\cos\cot^{-1}x + \sin\cot^{-1}x)^2 - 1\right]^{1/2}$

$$= \sqrt{1+x^2}\left[\left(x\cos\cos^{-1}\frac{x}{\sqrt{1+x^2}} + \sin\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$$

$$= \sqrt{1+x^2}\left[\left(\frac{x^2}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$$

$$= \sqrt{1+x^2}(x^2+1-1)^{1/2} = x\sqrt{1+x^2}.$$

32. (c) $\displaystyle\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$

$$= \int \frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx$$

$$= \int \frac{\sin 3x + \sin 2x}{\sin x}dx$$

$$= \int \frac{3\sin x - 4\sin^3 x + 2\sin x\cos x}{\sin x}dx$$

$$= \int (3 - 4\sin^2 x + 2\cos x)\,dx$$

$$= \int (3 - 2(1-\cos 2x) + 2\cos x)\,dx$$

$$= \int (1 + 2\cos 2x + 2\cos x)\,dx = x + 2\sin x + \sin 2x + c$$

33. (d) $\displaystyle\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{1/3} + C$

$$\int \frac{dx}{x^7\left(\frac{1}{x^6}+1\right)^{2/3}} = xf(x)(1+x^6)^{1/3} + C$$

$$\text{Let } t = \frac{1}{x^6} + 1 \quad dt = \frac{-6}{x^7}dx$$

$$-\frac{1}{6}\int \frac{dt}{t^{2/3}} = -\frac{1}{2}t^{1/3}$$

$$= -\frac{1}{2}\left(\frac{1}{x^6}+1\right)^{1/3} = -\frac{1}{2}\frac{(1+x^6)^{1/3}}{x^2}$$

$$\therefore \quad f(x) = -\frac{1}{2x^3}$$

34. (a) Put $(x^2 - 1) = 1$

$$\Rightarrow \quad 2x\,dx = dt$$

$$\therefore \quad I = \frac{1}{2}\int \sqrt{\frac{1-\cos t}{1+\cos t}}dt$$

$$= \frac{1}{2}\int \tan\left(\frac{t}{2}\right)dt$$

$$= \ln\left|\sec\frac{1}{2}\right| + c$$

$$I = \ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$$

35. (d) $\displaystyle\int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx$

$$= \int \frac{5x^{-6}+7x^{-8}}{\left(\frac{1}{x^7}+\frac{1}{x^5}+2\right)^2}dx$$

$$= \frac{1}{2+\frac{1}{x^5}+\frac{1}{x^7}} + C$$

$$\text{As } f(0) = 0, \quad f(x) = \frac{x^7}{2x^7+x^2+1}$$

$$f(1) = \frac{1}{4}$$

36. (c) $\displaystyle\int \frac{(\sin^n\theta - \sin\theta)^{1/n}\cos\theta}{\sin^{n+1}\theta}d\theta$

$$= \int \frac{\sin\theta\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{1/n}}{\sin^{n+1}\theta}d\theta$$

Put $1-\frac{1}{\sin^{n-1}\theta}=t$

So $\frac{(n-1)}{\sin^{n}\theta}\cos\theta d\theta=dt$

Now $\frac{1}{n-1}\int (t)^{1/n}dt$

$=\frac{1}{(n-1)}\frac{(t)^{\frac{1}{n}+1}}{\frac{1}{n}+1}+C$

$=\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

37. (a) $\int x5e^{-4x^3}dx=\frac{1}{48}e^{-4x^3}f(x)+C,$

Put $x^3=t$

$3x^2dx=dt$

$\int x^3\cdot e^{-4x^3}\cdot x^2dx$

$\frac{1}{3}\int t\cdot e^{-4t}dt$

$\frac{1}{3}\left[t\cdot\frac{e^{-4t}}{-4}-\int\frac{e^{-4t}}{-4}dt\right]$

$-\frac{e^{-4t}}{48}[4t+1]+c$

$\frac{-e^{-4t}}{48}[4x^3+1]+c$

$f(x)=-1-4x^3$

(From the given options (a) is most suitable)

38. (b) $\int\frac{\sqrt{1-x^2}}{x^4}dx=A(x)(\sqrt{1-x^2})^m+C,$

$\int\frac{|x|\sqrt{\frac{1}{x^2}-1}}{x^4}dx,$

Put $\frac{1}{x^2}-1=t$

$\Rightarrow \frac{dt}{dx}=\frac{-2}{x^3}$

Case (i): $x\ge 0$

$-\frac{1}{2}\int\sqrt{r}dt \Rightarrow -\frac{t^{3/2}}{3}+C$

$\Rightarrow -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{3/2}$

$\Rightarrow \frac{(\sqrt{1-x^2})^3}{-3x^2}+C$

$A(x)=\frac{1}{3x^3}$ and m = 3

$(A(x))^m=\left(-\frac{1}{3x^3}\right)^3=-\frac{1}{27x^9}$

Case (ii): $x\le 0$

we get $\frac{\left(\sqrt{1-x^2}\right)^3}{-3x^3}+C$

$A(x)=\frac{1}{-3x^3}, m=3$

$(A(x))^m=\frac{-1}{27x^9}$

39. (a) $\sqrt{2x-1}=t$

$\Rightarrow 2x-1=t^2$

$\Rightarrow 2dx=2t.dt$

$\int\frac{x+1}{\sqrt{2x-1}}dx=\int\frac{\frac{t^2+1}{2}+1}{t}tdt$

$=\int\frac{t^2+3}{2}dt$

$=\frac{1}{2}\left(\frac{t^3}{3}+3t\right)=\frac{t}{6}(t^2+9)+c$

$=\sqrt{2x-1}\left(\frac{2x-1+9}{6}\right)+c$

$=\sqrt{2x-1}\left(\frac{x+4}{3}\right)+c$

$f(x)=\frac{x+4}{3}$

40. (a) $A\to 1,2$; $B\to 1,2,3,4,5$; $C\to 4,5$

(A) We have $\lambda^2+(\lambda+2)^2<4$

$\Rightarrow 2\lambda^2+4\lambda<0$; $\lambda(\lambda+2)<0$

$\Rightarrow -2<\lambda<0$

$\because -\frac{1}{2}\in(-2,0)$

and $-1\in(-2,0)$

(B) We have $\lambda^2+(\lambda+2)^2-2\lambda+4(\lambda+2)>0$

$\Rightarrow 2\lambda^2+6\lambda+12>0$

$\Rightarrow \lambda^2+3\lambda+6=0$

$$\Rightarrow \quad \left(\lambda+\frac{3}{2}\right)^2+\frac{15}{4}>0$$

$\therefore \quad \lambda \in R$

(C) For both circles $r > 0$

$\Rightarrow \quad \sqrt{\lambda^2-4}>0$

and $\sqrt{(2\lambda^2-8)}>0$

$\Rightarrow \quad \lambda^2 > 4$

and $\lambda^2 > 4$

$\Rightarrow \quad \lambda < -2$

and $\lambda > 2$

$\therefore \quad \lambda \in (-\infty, -2) \cup (2, \infty)$ (3,4)

41. (a) $A \to 1,2;\ B \to 1,2;\ C \to 2,3;\ D \to 2,3$

(A) When two circles are intersecting they have a common normal and common tangent.

(B) Two mutually external circles have a common normal and common tangent.

(C) When one circle lies inside of other, then they have a common normal but no common tangent.

(D) Two branches of a hyperbola have a common normal but no common tangent.

42. (b) $A \to 3,\ B \to 2,\ C \to 1,\ D \to 4$

Let $\triangle = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$$= -\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$$

(A) If $a+b+c \neq 0$ and $a^2+b^2+c^2 = ab+bc+ca$

$\Rightarrow \quad \triangle = 0$

and $a = b = c \neq 0$

$\Rightarrow$ the equations represent identical planes.

(B) $a+b+c = 0$ and $a^2+b^2+c^2 \neq ab+bc+ca$

$ax+by = (a+b)z$

$bx+cy = (b+c)z$

$\Rightarrow \quad (b^2-ac)y = (b^2-ac)z \Rightarrow y = z$

$\Rightarrow \quad ax+by+cy = 0 \Rightarrow ax = ay \Rightarrow x = y = z$.

(C) $a+b+c \neq 0$ and $a^2+b^2+c^2 \neq ab+bc+ca \quad \Rightarrow \Delta \neq 0$

$\Rightarrow$ the equation represent planes meeting at only one point.

(D) $a+b+c = 0$ and $a^2+b^2+c^2 = ab+bc+ca$

$\Rightarrow \quad a = b = c = 0$

$\Rightarrow$ the equations represent whole of the three dimensional space.

43. (a) $A \to 3,\ B \to 2,\ C \to 4,\ D \to 2$

$$L_1 : \frac{x-1}{2} = \frac{y-0}{-1} = \frac{z-(-3)}{1}$$

Normal of plane P : $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix}$

$= \hat{i}(-16) - \hat{j}(-42-6) + \hat{k}(32) \quad = -16\hat{i} + 48\hat{j} + 32\hat{k}$

DR's of normal $\vec{n} = \hat{i} - 3\hat{j} - 2\hat{k}$

Point of intersection of L_1 and L_2.

$\Rightarrow \quad 2K_1 + 1 = K_2 + 4$ and $-K_1 = K_2 - 3$

$\therefore \quad K_1 = 2$ and $K_2 = 1$

$\therefore$ Point of intersection (5, –2, –1)

$\therefore$ Equation of plane. $1\cdot(x-5) - 3(y+2) - 2(z+1) = 0$

$\Rightarrow \quad x - 3y - 2z - 13 = 0 \Rightarrow x - 3y - 2z = 13$

$\therefore \quad a = 1, b = -3, c = -2, d = 13$

44. (a) Here $ff(x) = \dfrac{f(x)}{[1+f(x)^n]^{1/n}} = \dfrac{x}{(1+2x^n)^{1/x}}$

$$\Rightarrow \quad fff(x) = \frac{x}{(1+3x^n)^{1/n}}$$

$$\Rightarrow \quad g(x) = (\underbrace{f \circ f \circ \ldots \circ f}_{n \text{ times}})(x) = \frac{x}{(1+nx^n)^{1/n}}$$

Hence $I = \int x^{n-2} g(x)\,dx = \int \dfrac{x^{n-1}dx}{(1+nx^n)^{1/n}}$

$$= \frac{1}{n^2}\int \frac{n^2 x^{n-1}dx}{(1+nx^n)^{1/n}} = \frac{1}{n^2}\int \frac{\frac{d}{dx}(1+nx^n)}{(1+nx^n)^{1/n}}dx$$

$$\therefore \quad I = \frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$$

45. (b) $F(0) = 0$

$F'(x) = 2xf(x) = f'(x)$

$f(x) = e^{x^2+c}$

$f(x) = e^{x^2}$ $(\because f(0) = 1)$

$F(x) = \displaystyle\int_0^{x^2} e^t dt$

$F(x) = e^{x^2} - 1 \quad (\because F(0) = 0) \quad \Rightarrow \quad F(2) = e^4 - 1$

46. (b) $\dfrac{dy}{dx} + \dfrac{x}{x^2-1}y = \dfrac{x^4+2x}{\sqrt{1-x^2}}$

This is a linear differential equation

I.F. = $e^{\int \frac{x}{x^2-1}dx} = e^{\frac{1}{2}\ln|x^2-1|} = \sqrt{1-x^2}$

$\Rightarrow$ Solution is $y\sqrt{1-x^2} = \int \frac{x(x^3+2)}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2}\, dx$

or $y\sqrt{1-x^2} = \int (x^4+2x)\, dx = \frac{x^5}{5} + x^2 + c$

$f(0) = 0 \Rightarrow c = 0 \Rightarrow f(x)\sqrt{1-x^2} = \frac{x^5}{5} + x^2$

Now, $\int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)dx = \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}dx$ (Using property)

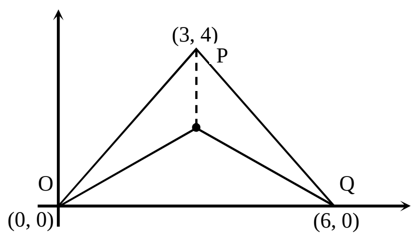
$= 2\int_0^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}dx = 2\int_0^{\pi/3} \frac{\sin^2\theta}{\cos\theta}\cos\theta d\theta$

(Taking $x = \sin\theta$)

$= 2\int_0^{\pi/3} \sin^2\theta d\theta = \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/3}$

$= 2\left(\frac{\pi}{6}\right) - 2\left(\frac{\sqrt{3}}{8}\right) = \frac{\pi}{3} - \frac{\sqrt{3}}{4}.$

47. (c) Since, $\triangle$ is isosceles, hence centroid is the desired point.



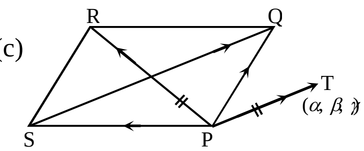
48. (a, b) Slope of tangent m = 2 Equation of tangent in slope form is

$y = mx \pm \sqrt{a^2m^2 - b^2}, y = 2x \pm 4\sqrt{2}$

and point of contact is $\left(-\frac{ma^2}{c}, \frac{-b^2}{c}\right)$

$\equiv \left(-\frac{2\times 9}{\pm 4\sqrt{2}}, -\frac{4}{\pm 4\sqrt{2}}\right) \equiv \left(\pm\frac{9}{2\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$

49. (c)



Area of base (PQRS) $= \frac{1}{2}|\overrightarrow{PR} \times \overrightarrow{SQ}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & -4 \end{vmatrix}$

$= \frac{1}{2}|-10\hat{i} + 10\hat{j} - 10\hat{k}| = 5|\hat{i} - \hat{j} + \hat{k}| = 5\sqrt{3}$

Height = proj. of PT on $\hat{i} - \hat{j} + \hat{k} = \left|\frac{1-2+3}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$

Volume $= (5\sqrt{3})\left(\frac{2}{\sqrt{3}}\right) = 10$ cu.units

50. (b) $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4}dx$

$\int \frac{\left(\frac{3}{x^3} + \frac{3}{x^5}\right)dx}{\left(2 + \frac{3}{x^2} + \frac{1}{x^4}\right)^4}$

Let $\left(2 + \frac{3}{x^2} + \frac{1}{x^4}\right) = t$

$-\frac{1}{2}\int \frac{dt}{t^4} = \frac{1}{6t^3} + C$

$\Rightarrow \frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$

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