Chapter 14

EQUATIONS

When you compare two numbers, you say that one number is greater or smaller or equal to the other number. Below are given some statements of comparison. The statements are incopmlete. Put the symbols =, > or < in the boxes to complete the statements.

ACTIVITY 1

Example:	(1)	3 + 5	>	7
	(2)	8 + 7		15
	(3)	4 + 6		11
	(4)	13 + 8		18
	(5)	23 + 7		30

How did you find out the appropriate symbol for the statements. Think about the reasons that you gave yourself.

These are two sides of the statements. Those towards the left of the box and those towards the right of the box. In the statements 3 + 5 is the left side & because it is greater than 7, so we wrote 3 + 5 > 7.

Select these statements for which you have used the = symbol and note them in your copy. Those statements which you have not noted down are statements of inequality. Let us, look at some more statements in which variables have been used for e.g. in $x + 5 = 13$, if $x = 5$, then substituting 5 in the place of x, we would get 5 + 5 = 10, whereas the right side = 13. Therefore, the statement that left hand side is equal to right hand side is not true

So, L.H.S. $\neq$ R.H.S., in the statement if $x = 8$, then both sides would become equal and the statement L.H.S. = R.H.S. would become true.

ACTIVITY 2

Below are given some statements. The value of x is given with them. Write down whether the statements are true or false according to the given values of x.

1) $x + 3 = 8$, if $x = 5$, then the statement is true.

2) $x - 2 = 4$, if $x = 7$, then the statement is ________.

3) $x + 2 = 10$, if $x = 8$, then the statement is _______.

4) $7 = 12 - x$, if $x = 3$, then the statement is _______.

5) $3 = x - 9$, if $x = 5$, then the statement is _______.

Find out whether these statements which are false, would become true for some value of *x*. If yes, then write down these values of *x* for the false statements in your notebooks. In that case, the above statements would stand true only when both the sides are equal to each other. These statements in which variables are included and both the sides are equal are known as equations.

These equivalent statements which have one or more than one algebraic numbers are called equations. Thus in all the variable and nonvariable elements of the equivalents symbol (=) represent. The Left Hand Side and all those variables and nonvariable elements on the right of the sign represent the Right Hand Side of the equation.

Why Equation ?

One day, Naresh asked his friends a question.

"Guavas have been kept in two baskets. The second basket contains two times of the guavas in the first basket. If 8 guavas are added to the first basket, the number of guavas in the second basket become equal to the number of guavas in the first basket. Can anyone tell the number of guavas in both the baskets?"

All the friends of Naresh began to think about the problem but they couldn't make anything out of it. Just then Anu said that the first basket contains 8 and the second basket contains 16 guavas. Naresh said, "The answer is correct, but how did you solve it?" Anu said, "I have read that if a number is added to the same number, then we get twice that number since the number of guavas in the first basket is 8, then 8 more guavas added to the first basket would make 16, which is the number of guavas kept in the second basket, so I got the answer."

Naresh said, "We can solve this by another method.

Guavas in the first basket.	Guavas kept in the second basket, twice the number of guavas in the 1^{st} basket.

Guavas (1^{st} basket) + 8 = 2 × guavas (2^{nd} basket)

8 added & 8 alone will make it two times (twice). Therefore, the first basket has 8 and the second basket has 16 guavas.

Farida remembered, "We have read about it in the lesson on 'Variables' that when we do not know any number, we can consider it as a variable."

Suppose, the first basket has ***x*** guavas. Then in the second basket we will have 2***x*** number of guavas.

Now, 8 guavas added to the first basket, will make ***x*** + 8 guavas in that basket which is equal to the number of guavas in the second basket. This means,

$$x + 8 = 2x$$

Naresh exclaimed, "Wow! that makes it an equation! Here if we put 8 as the value of ***x***, the statement will become true. This also means that all equations with unknown values or variables can easily be solved!" So, you have seen that equations are quite useful to help us find out values of unknown quantities. Now, let us understand how variables are formed:

How to Make Equations

Let us play a game. How old are you? Think of your own age. Add 5 to it. Multiply the sum by 2 and subtract 10 from the product. Now from this difference subtract your age. The answer will be your age!

Naresh's solution	←	Teacher's Instructions	→	Anu's solution
12 years	←	Think of your age.	→	11 years
12 + 5 = 17 years	←	Add 5	→	11 + 5 = 16 years
17 × 2 = 34 years	←	Multiply by 2	→	2 × 16 = 32 years
34 - 10 = 24 years	←	Subtract 10	→	32 - 10 = 22 years
24 - 12 = 12 years	←	Subtract your age	→	22 - 11 = 11 years

Thus all students find that the age that they thought of in the beginning comes as the answer in the end. How did this happen? Let's find out.

Suppose, the age thought of is x years.

5 added to age	=	$x + 5$
Sum multiplied by 2	=	$2(x + 5) = 2x + 10$
10 subtracted	=	$2x + 10 - 10 = 2x$
Age thought of subtracted	=	$2x - x = x$

This means you are getting the number that you thought of in the beginning as your age.

As soon as Raju looked at the steps of the equation, he was excited and said, "Now I can also ask questions about making equations!" He asked, "If a number is multiplied by 2 and 5 is subtracted from its product, the result is 3, what will the equation be?"

Anu at once formed the equation.

Suppose, the number is x; multiplied by 2, we get $2x$, 5 subtracted from the product given $2x - 5$ which is equal to 3. This means the equation is

$$2x - 5 = 3$$

Anu said, "Now, I'm giving you an equation you'll have to write it in words?"

Equation : $7y - 5 = 9$

Hamida could at once think about it that 7 multiplied to any number and 5 subtracted from it gives 9.

Now, all children in the class started taking interest in framing & solving equations. It was fun.

EXERCISE 14.1

1. Identify which of these are equations:

(i)	$x - 4 = 10$	(vi)	$7 = 2x - 5$
(ii)	$x - 4 - 10$	(vii)	$3x - 2x = 2x$

(iii) $2y - 3 + 9$ (viii) $\frac{5}{x} = 3$

(iv) $5(2y - 3) = 15$ (ix) $4.5 + 3.2\boldsymbol{x} = z$

(v) $3\boldsymbol{x} + 4$ (x) $ly + lx = px$

2. Identify the L.H.S. and R.H.S. in the given equations
   - (i) $x - 5 = 9$
   - (ii) $2x - 3 = 7$
   - (iii) $2y = 9 - y$
   - (iv) $2y = 6$
   - (v) $15 = 2a + 5$
3. In the following statements use **"y"** for the unknown numbers & change them into equations.
   - (i) 3 subtracted from twice of the number gives 17.
   - (ii) The sixth part of the number is 7.
   - (iii) The difference of the number and 5 is 8.
   - (iv) 7 multiplied by the number and 5 subtracted gives 9.

4. Write the following equations as statements:
   - (i) $x - 6 = 9$
   - (ii) $7y - 14 = 0$
   - (iii) $\frac{2x}{3} = 6$
   - (iv) $\frac{x}{2} + 5 = 10$
   - (v) $38 - 2x = 4$

Solving Equations

In Activity 2, you have seen that each statement is true for only one value of x, e.g. In $x + 2 = 4$, if $x = 7$, the statement would become false because on putting the value of x as 7, L.H.S. will not be equal to R.H.S. Hence, the statement becomes true only when $x = 2$. This means that these kind of equations have only one solution.

ACTIVITY 3

In the table below are given some equations with x as the variable, decide whether the L.H.S. and R.H.S. of the equations are equivalent or not for the different values of x shown in the table.

S. No.	Equation	L. H. S. of the equation	R. H. S. of the equation	Whether L. H. S. is <, = or > R. H. S. for the different value of x					
				$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$
1.	$x + 3 = 5$	$x + 3$	5	<	<	=	>	>	>
2.	$x - 2 = 3$								
3.	$x + 3 = 2x$								
4.	$x + 4 = 4$								
5.	$3x + 1 = 3 + x$								
6.	$x + 5 = 10$								

In the above example, the value of x for which both L.H.S. and R.H.S. are equal, is the only correct solution for the equation. This method is known as the Trial and Error method.

Let us compare some of the characters of equations with the help of a physical balance.

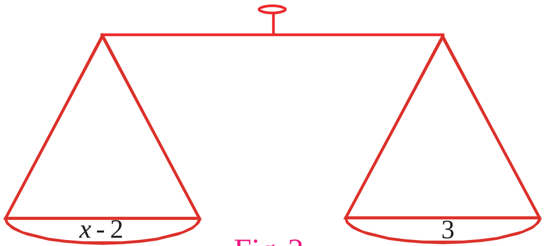

Fig 2

In figure 2, the equation $x - 2 = 3$ is $x - 2$ and the R.H.S. is 3. The balance is in a state of equilibrium. Now if we put some weight on the left pan of the balance, then to bring the balance into a state of equilibrium, we shall have to put the same amount of weight from the R.H.S. too. Similarly, if we take out some weight from the R.H.S., we will have to take out the same amount of weight from L.H.S. also to maintain the state of balance. This means if any operation is carried out on one side of an equation, the same operation has to take place on the other side of the equation too, then only we can maintain equilibrium in the equation and that is what an equation by its name means.

Therefore, in $x - 2 = 3$

knowing that $(- 2) + (2) = 0$, if we add 2 to the L.H.S. of the equation, only x will remain. Since 2 is added to the L.H.S., 2 is added to R.H.S. also.

Therefore, $x - 2 = 3$ is equivalent to

$x - 2 + 2 = 3 + 2$

or $x + (- 2)(+ 2) = 5$

or $x = 5$

Similarly, if $7x = 21$.

We know that if 7 is divided by 7, we get 1, so if $7x$ is divided by 7, we shall have x. Since L.H.S. is being divided by 7, the R.H.S. also will have to be divided by 7. This means

$7x = 21$ is the same as

$$\frac{7x}{7} = \frac{21}{7}$$

or $x = 3$

From the above examples, you have learnt that if a constant is added to both sides of an equation or is subtracted from both sides of equation or gets multiplied to or divided by a constant on both sides, these is no change in the state of equilibrium of the equation.

ACTICITY 4

In the table given below, which of the operations - addition, subtraction, multiplication or division will be carried out on both sides of the equations, so that the value of *x* is obtained. Fill in the table as shown in the example.

S.No	Equation	Which operation would remove the constant from the variable's side	Equation after theoperation	Value of *x* after the equation is solved
1.	x +3 = 5	3 subtracted	x + 3 - 3 = 5-3	x = 2
2.	x -5 = 7			
3.	2x = 6			
4.	x/3 = 5			
5.	x + 7 = 2			
6.	7 = z- 4			
7.	5 + x= 9			
8.	4 + x = 2			
9.	-7 = 3 + y			
10.	4 = 8y			

Solve the given equations:

(i) $x + 3 = 10$

(ii) $6 = y + 4$

(iii) $S + 6 = 15$

(iv) $7 + t = 25$

ACTIVITY 5

You have learnt how to solve simple equations, in which any operation has to be carried out once. Now let us solve some equations where two operations need to be carried out, in order to find a solution.

S.No	Equation	The operation that would remove the constant from the variable's side	Equation after the 1st operation	Operation for both sides that would remove	Equation after the 2nd operation	Value of x
1.	$2x + 3 = 9$	Subtracted 3	$2x + 3 - 3 = 9 - 3$	$2x = 6$; or dividing both side by 2	$\frac{2x}{2} = \frac{6}{2} = 3$	$x = 3$
2.	$18x - 11 = 61$					
3.	$\frac{x}{7} - 13 = 1$					
4.	$1 + \frac{x}{5} = 3$					
5.	$\frac{x}{4} - 5 = -6$					
6.	$0 = \frac{x}{14} - \frac{1}{7}$					

Practice

1. Solve the following equations -

i)	$3x + 8 = 20$	ii)	$4x + 10 = 30$
iii)	$5x - 7 = 8$	iv)	$6x - 7 = 11$
v)	$3x + \frac{21}{7} = 0$	vi)	$29 = 7x + 1$
vii)	$60 - 8x = -4$	viii)	$19x + 7 = 45$

You have learnt solving and making equations by now. Here are some problems about numbers. Try to solve them using equations.

Example 1.

If 5 is added to a number, the number becomes 20. What is the number?

Solution:

Suppose the number is x

According to the given problem:

$x + 5$

$x + 5 = 20$

5 subtracted on both sides

$x + 5 - 5 = 20 - 5$

$x = 15$

Verification:

L.H.S.= $x + 5$

$= 15 + 5$ (putting the value of x) $= 20 =$ R.H.S.

Example 2.

6 subtracted from a number makes it 10, what is the number ?

Solution :

Suppose the number is x

From the above statement, 6 subtracted from the number makes it $x - 6$, that is equal to 10.

This makes the equation : $x - 6 = 10$

6 added to both sides means

$$x - 6 + 6 = 10 + 6$$

here ($- 6 + 6 = 0$ and $10 + 6 = 16$)

or $x = 16$

Verification:

L.H.S. $= x - 6$ $= 16 - 6$ (putting the value of x)

$= 10$

= R.H.S.

Example 3.

7 added to twice of a number makes it 37. What will the number be?

Solution:

Suppose the number is x.

Twice the number would be $2x$.

According to the problem, 7 added to twice the number makes 37.

Step 1: Twice the number

$= 2x$

Step 2: 7 added to $2x$

$= 2x + 7$

Step 3: As per the problem

$2x + 7 = 37.$

(7 subtracted from both sides) $2x + 7 - 7 = 37 - 7$

$2x = 30$

(On dividing both side by 2) $\frac{2x}{2} = \frac{30}{2}$

$x = 15$

Verification:

L.H.S. $= 2x + 7$

$= 2 \times 15 + 7$ (on putting the value of x)

$= 30 + 7$

$= 37$

= R.H.S.

Example 4.

One third of a number gives 11. Find the number.

Solution:

Let the number be x.

$\frac{1}{3}$ the number would be $\frac{x}{3}$ which is equal to 11..

$\therefore \frac{x}{3} = 11$

To find out the value of x. 3 has to be removed from the denominator of the L.H.S. of the equation. For this 3 is multiplied to both sides of the eqation $\frac{x}{3} \times 3 = 11 \times 3$

$x = 33.$

Example 5.

The sum of the ages of Malati and her father is 49 years. If Malati is 12 years old. Find out how old is her father ?

Solution:

Suppose the age of Malati's father is x

The sum of both their ages would be

$x + 12.$

Given the sum of age of Malati & her father = 49 years.

Therefore, $x + 12 = 49$

$x + 12 - 12 = 49 - 12$ (On subtracting 12 from both sides of the eq^n.)

or $x + 0 = 37$

or $x = 37$

Thus Malati's father is 37 years old.

Verification:

Sum of Malati's age & her father's age

$12 + 37 = 49$ years.

Example 6.

Shivani has only 50 paise coins in her purse. If she has 25 rupees in her purse, how many coins are there in her purse?

Solution:

Suppose the number of coins in Shivani's purse is x.

The value of each coin = 50 paise.

or Value of each coin $= \frac{1}{2}$ rupee

$\therefore$ Value of all the coins $= \frac{1}{2}x$ rupees

According to the condition given on the equation

$$\frac{1}{2}x = 25$$

$$\frac{1}{2}x \times 2 = 25 \times 2 \text{ (On multiplying both sides by 2)}$$

$$x = 50$$

Therefore, Shivani's purse has 50 coins.

Verification:

The value of 50 coins $= 50 \times 50$
= 2500 paise
= 25 rupees.

EXERCISE 14.2

1. Solve the following equations.
   i) $x - 3 = -4$
   ii) $z - 8 = 0$
   iii) $3y = 9$
   iv) $16 = 3y + 7$
   v) $5 + \frac{x}{3} = 7$
   vi) $9z - 7 = 14$
2. Solve the given equations and verify your answer.
   i) $3(2 + x) = 12$
   ii) $10 - z = 6$

iii) $\frac{x}{5} = 15$

iv) $7 - 4y = 3$

3. Twice a number makes it 10, what is the number?
4. If 35 is added to twice a number, 85 is obtained. Find the number ?
5. How many 25 paise coins would make 10 rupees?
6. If 4 is subtracted from the half of a number, we get 6. What will that number be?
7. Uma has few meters of cloth. If she makes 4 curtains of 2 metres each, she still has 5 metres of cloth left. Find out how much cloth did she have in the beginning?

What Have We Learnt ?

To solve any problem with the help of equations, we will have to keep the following things in mind:

(i) Read the problem well and identify the known variables and the unknown variables.

(ii) Denote the unknown numbers/ variables by x, y, z etc.

(iii) Change every word of the problem (as far as possible) into a mathematical statement.

(iv) Identify the variables that are equal and make a proper equation.

(v) Solve the equation to find out the unknown variable.

(vi) Varify whether the solution satisfies the conditions and the equation that has been made.