Short Answer Type Questions – II

[3 marks]

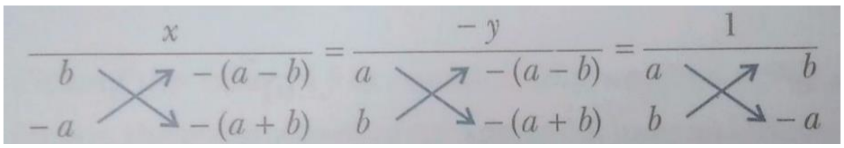
Que 1. Solve: $ax + by = a - b$ and $bx - ay = a + b$

Sol. The given system of equations may be written as

$$ax + by - (a - b) = 0$$

$$bx - ay - (a + b) = 0$$

By cross-multiplication, we have



$$\Rightarrow \quad \frac{x}{b \times -(a+b) - (-a) \times -(a-b)} = \frac{-y}{a \times -(a+b) - b \times -(a-b)} = \frac{1}{-a^2 - b^2}$$

$$\Rightarrow \quad \frac{x}{-b(a+b) - a(a-b)} = \frac{-y}{-a(a+b) + b(a-b)} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \quad \frac{x}{-b^2 - a^2} = \frac{-y}{-a^2 + b^2} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \quad \frac{x}{-(a^2 + b^2)} = \frac{y}{(a^2 + b^2)} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \quad x = -\frac{(a^2 + b^2)}{-(a^2 + b^2)} = 1 \text{ and } y = \frac{(a^2 + b^2)}{-(a^2 + b^2)} = -1$$

Hence, the solution of the given system of equations is $x = 1,\ y = -1$.

Que 2. Solve the following linear equations:

$$\mathbf{152x - 378y = -74 \text{ and } -378x + 152y = -604}$$

Sol. We have, $152x - 378y = -74$ …(i)

$-378x + 152y = -604$ …(ii)

Adding equation (i) and (ii), we get

$$\begin{array}{r} 152x - 378y = -74 \\ -378x + 152y = -604 \\ \hline -226x - 226y = -678 \end{array}$$

$\Rightarrow \quad -226(x+y) = -678$

$\Rightarrow \quad x+y = \frac{-678}{-226}$

$\Rightarrow \quad x+y = 3 \qquad \ldots(iii)$

Subtracting equation (ii) from (i), we get

$$\begin{array}{r} 152x - 378y = -74 \\ -378x + 152y = -604 \\ \hline 530x - 530y = 530 \end{array}$$

$\Rightarrow \quad x - y = 1 \qquad \ldots(iv)$

Adding equation (iii) and (iv), we get

$$\begin{array}{r} x + y = 3 \\ x - y = 1 \\ \hline 2x = 4 \end{array}$$

$\Rightarrow \quad x = 2$

Putting the value of x in (iii), we get

$$2 + y = 3 \quad \Rightarrow \quad y = 1$$

Hence, the solution of given system of equations is $x = 2,\ y = 1$

Que 3. Solve for x and y

$$\boldsymbol{\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2; x + y = 2ab}$$

Sol. We have, $\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2 \qquad \ldots(i)$

$x + y = 2ab \qquad \ldots(ii)$

Multiplying (ii) by b/a, we get

$$\frac{b}{a}x + \frac{b}{a}y = 2b^2 \qquad \ldots(iii)$$

Subtracting (iii) from (i), we get

$$\left(\frac{a}{b} - \frac{b}{a}\right) y = a^2 + b^2 - 2b^2 \quad \Rightarrow \quad \left(\frac{a^2 - b^2}{ab}\right) y = (a^2 - b^2)$$

$\Rightarrow \quad y = (a^2 - b^2) \times \frac{ab}{(a^2 - b^2)} \quad \Rightarrow \quad y = ab$

Putting the value of y in (ii), we get

$$x + ab = 2ab \quad \Rightarrow \quad x = 2ab - ab \quad \Rightarrow \quad x = ab$$

$\therefore \quad x = ab, y = ab$

Que 4. $(\boldsymbol{i})$ **For which values of $\boldsymbol{a}$ and $\boldsymbol{b}$ does the following pair of linear equations have and in finite number of solution?**

$$\mathbf{2x + 3y = 7}$$

$$(\boldsymbol{a - b})\boldsymbol{x} + (\boldsymbol{a + b})\boldsymbol{y} = \mathbf{3}\boldsymbol{a} + \boldsymbol{b} - \mathbf{2}$$

$(\boldsymbol{ii})$ **For which value of $\boldsymbol{k}$ will the following pair of linear equations have no solution?**

$$\mathbf{3x + y = 1}$$

$$(\mathbf{2}\boldsymbol{x} - \mathbf{1})\boldsymbol{x} + (\boldsymbol{x} - \mathbf{1})\boldsymbol{y} = \mathbf{2}\boldsymbol{k} + \mathbf{1}$$

Sol. (i) we have, $\quad 2x + 3y = 7 \quad \ldots(i)$

$$(a - b)x + (a + b)y = 3a + b - 2 \quad \ldots(ii)$$

Here, $\quad a_1 = 2, \quad b_1 = 3, \quad c_1 = 7$

And $\quad a_2 = a - b, \quad b_2 = a + b, \quad c_2 = 3a + b - 2$

For infinite number of solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \Rightarrow \quad \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

Now, $\frac{2}{a-b} = \frac{3}{a+b}$

$$\Rightarrow \quad 2a + 2b = 3a - 3b \quad \Rightarrow \quad 2a - 3a = -3b - 2b$$

$$\therefore \quad a = 5b$$

Again, we have

$$\frac{3}{a+b} = \frac{7}{3a+b-2} \quad \Rightarrow \quad 9a + 3b - 6 = 7a + 7b$$

$$\Rightarrow \quad 9a - 7a + 3b - 7b - 6 = 0 \quad \Rightarrow \quad 2a - 4b - 6 = 0 \quad \Rightarrow \quad 2a - 4b = 6$$

$$\Rightarrow \quad a - 2b = 3$$

Putting $a = 5b$ in equation (iv), we get

$$5b - 2b = 3 \quad or \quad 3b = 3 \quad i.e., \quad b = \frac{3}{3} = 1$$

Putting the value of b in equation (iii), we get $a = 5$ (1) = 5

Hence, the given system of equations will have an infinite number of solutions for $a = 5$ and $b = 1$.

(ii) We have, $\quad 3x + y = 1 \quad \Rightarrow \quad 3x + y - 1 = 0$

$$(2k-1)x+(k-1)y=2k+1$$

$\Rightarrow$ $$(2k-1)x+(k+1)y-(2k+1)=0$$

Here, $a_1=3,\ \ b_1=1,\ \ c_1=-1$

$$a_2=2k-1, b_2=k-1, c_2=-(2k+1)$$

For no solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2} \quad\Rightarrow\quad \frac{3}{2k-1}=\frac{1}{k-1}\neq\frac{1}{2k+1}$$

Now, $\frac{3}{2k-1}=\frac{1}{k-1} \quad\Rightarrow\quad 3k-3=2k-1$

$\Rightarrow$ $3k-2k=3-1 \quad\Rightarrow\quad k=2$

Hence, the given system of equations will have no solutions for $k=2$

Que 5. Find whether the following pair of linear equations has a unique solution. If yes, find the solution?

$$\boldsymbol{7x-4y=49 \ \ and \ \ 5x-6y=57}$$

Sol. We have, $7x-4y=49$ ...(i)

And $5x-6y=57$ ...(ii)

Here $a_1=7, b_1=-4, c_1=49$

$$a_2=5, b_2=-6, c_2=57$$

So, $\frac{a_1}{a_2}=\frac{7}{5}, \frac{b_1}{b_2}=\frac{-4}{-6}=\frac{2}{3}$

Since, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

So, system has a unique solution.

Multiply equation (i) by 5 and equation (ii) by 7 and subtract

$$\begin{array}{r} 35x-\ \ 20y=\ \ 245 \\ 35x-\ \ 42y=\ \ 399 \\ \hline 22y=-154 \end{array} \Rightarrow \qquad y=-7$$

Put $y=-7$ in equation (ii)

$$5x-6(-7)=57 \quad\Rightarrow\quad 5x=57-42 \quad\Rightarrow\quad x=3$$

Hence, $x=3$ and $y=-7$.

Que 6. Solve for x and y.

$$\frac{6}{x-1} - \frac{3}{y-2} = 1; \quad \frac{5}{x-1} + \frac{1}{y-2} = 2 \quad \textbf{Where } x \neq 1, y \neq 2$$

Sol. Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$

The given equations become

$$6p - 3q = 6 \qquad \ldots(i)$$

$$5p + q = 2 \qquad \ldots(ii)$$

Multiply equation (ii) by 3 and add in equation (i)

$$\begin{array}{r} 15p + \ 3q = \ 6 \\ 6p - \ 3q = \ 1 \\ \hline 21p \ = \ 7 \end{array} \qquad \Rightarrow \quad p = \frac{7}{21} = \frac{1}{3}$$

Putting this value in equation (i) we get

$$6 \times \frac{1}{3} - 3q = 1 \quad \Rightarrow \ 2 - 3q = 1 \quad \Rightarrow \ 3q = 1, \quad \Rightarrow \ q = \frac{1}{3}$$

Now, $\frac{1}{x-1} = p = \frac{1}{3} \quad \Rightarrow \ x - 1 = 3 \quad \Rightarrow \ x = 4$

$\frac{1}{y-2} = q = \frac{1}{3} \quad \Rightarrow \ y - 2 = 3 \quad \Rightarrow \ y = 5$

Hence, $x = 4$ and $y = 5$

Que 7. Solve the following pair equations for x and y.

$$\frac{a^2}{x} - \frac{b^2}{y} = 0; \quad \frac{a^2 b}{y} = a + b, x \neq 0, y \neq 0.$$

Sol. $\frac{a^2}{x} - \frac{b^2}{y} = 0 \qquad \ldots(i)$

$\frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b$

Multiply equation (i) by a and adding to equation (ii)

$$\frac{a^2 a}{x} - \frac{b^2 a}{y} = 0 \qquad \Rightarrow \qquad \frac{a^2 b}{x} + \frac{b^2 a}{y} = (a + b)$$

$$\Rightarrow \ \frac{a^2}{x} - \frac{a^2 b}{x} = a + b \quad \Rightarrow \quad \frac{a^2}{x}(a + b) = a + b \quad \Rightarrow \quad x = \frac{a^2(a+b)}{a+b} = a^2$$

Putting the value of x in equation (i), we get

$$\frac{a^2}{a^2} - \frac{b^2}{y} = 0 \quad \Rightarrow \ 1 - \frac{b^2}{y} = 0 \quad \Rightarrow \ \frac{b^2}{y} = 1 \quad \Rightarrow \ y = b^2$$

Hence, $x = a^2, \ y = b^2$

Que 8. In $\Delta ABC, \angle A = x, \angle B = 3x, and\ \angle C = y$ if $3y - 5x = 30^o$ show that triangle is right angled.

Sol. $\angle A + \angle B + \angle C = 180^o$ (Sum of interior angles of ΔABC)

$x + 3x + y = 180^o$

$\Rightarrow$ $4x + y = 180^o$ $\dots(i)$

And $3y - 5x = 30^0$ (Given) $\dots(ii)$

Multiply equation (i) by 3 and subtracting from $eq.$ (ii), we get

$-17x = -510 \Rightarrow x = \frac{510}{17} = 30^o$

Then $\angle A = x = 30^o\ and\ \angle B = 3x = 3 \times 30^o = 90^o$

$\angle C = y = 180^o - (\angle A + \angle B) = 180^o - 120^o = 60^o$

$\therefore$ $\angle A = 30^o,\ \angle B = 90^o, \angle C = 60^o$

Hence ΔABC is right tringle right angled at B.

Que 9. In fig. 3.1. $ABCDE$ is a pentagon with $BE \parallel CD$ and $BC \parallel DE$. BC is perpendicular to CD. If the perimeter of $ABCDE$ is 21cm. Find the value of x and y.

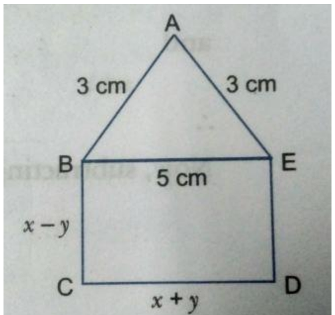


Fig. 3.1

Sol. Since $BC \parallel DE$ and $BE \parallel CD$ with $BC \perp CD$.

$BCDE$ is a rectangle.

$\therefore$ Opposite sides are equal

$i.e.,$ $BE = CD$ $\therefore x + y = 5$ $\dots(i)$

$DE = BC = x - y$

Since perimeter of $ABCDE$ is 21 cm.

$AB + BC + CD + DE + EA = 21$

$3 + x - y + x + y + x - y + 3 = 21$ $\Rightarrow$ $6 + 3x - y = 21$

$3x - y = 15$ $\dots(ii)$

Adding (i) and (ii), we get

$4x = 20$ $\Rightarrow$ $x = 5$

On putting the value of x in (i), we get $y = 0$

Hence, $x = 5$ and $y = 0$.

Que 10. Five years ago, A was thrice as old as B and ten years later, A shall be twice old as B. What are the present ages of A and B?

Sol. Let the present ages of B and A be x years and y Year respectively. Then

$B's$ age 5 years ago = $(x-5)$ years

And $A's$ age 5 years ago = $(y-5)$

$\therefore \quad (y-5) = 3\,(x-5) \quad \Rightarrow \quad 3x - y = 10 \quad \ldots(i)$

$B's$ age 10 years hence = $(x+10)$ years

$A's$ age 10 years hence = $(y+10)$ years

$\therefore \quad y + 10 = 2\,(x+10) \quad \Rightarrow \quad 2x - y = -10 \quad \ldots(ii)$

On subtracting (ii) from (i) we get $x = 20$

Putting $x = 20$ and (i) we get

$(3 \times 20) - y = 10 \quad \Rightarrow \quad y = 50$

$\therefore \quad x = 20$ and $y = 50$

Hence, $B's$ present age = 20 years and $A's$ present age = 50 Years.

Que 11. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Sol. Let the numerator be x and denominator be y.

$\therefore$ Fraction = $\frac{x}{y}$

Now, according to question,

$\frac{x-1}{y} = \frac{1}{3} \quad \Rightarrow \quad 3x - 3 = y$

$\therefore \quad 3x - y = 3 \quad \ldots(i)$

And $\frac{x}{y+8} = \frac{1}{4} \quad \Rightarrow \quad 4x = y + 8$

$\therefore \quad 4x - y = 8 \quad \ldots(ii)$

Now, subtracting equation (ii) from (i), we have

$$\begin{array}{rrr} 3x- & y= & 3 \\ 4x- & y= & 8 \\ - & + & - \\ \hline & -x= & -5 \end{array}$$

$\therefore \quad x = 5$

Putting the value of x in equation (i), we have

$$3 \times 5 - y = 3 \quad \Rightarrow \quad 15 - y = 3 \quad \Rightarrow \quad 15 - 3 = y$$

$\therefore \quad y = 12$

Hence, the required fraction is $\frac{5}{12}$.

Que 12. Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{7x-2y}{xy} = 5$ $\qquad$ (ii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$\frac{8x+7y}{xy} = 15$ $\qquad$ $\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

Sol. (i) We have

$$\frac{7x-2y}{xy} = 5 \quad \Rightarrow \quad \frac{7x}{xy} - \frac{2y}{xy} = 5 \quad \Rightarrow \quad \frac{7}{y} - \frac{2}{x} = 5$$

Let $\frac{1}{y} = u$ $\quad$ *and* $\quad$ $\frac{1}{x} = v$

$$7u - 2v = 5$$

$$8u + 7v = 15$$

Multiplying (i) by 7 and (ii) by 2 and adding, we have

$$\begin{array}{rcrcr} 49u & - & 14v & = & 35 \\ 16u & + & 14v & = & 30 \\ \hline 65u & & & = & 65 \end{array}$$

$\therefore \quad u = \frac{65}{65} = 1$

Putting the value of u in equation (i), we have

$$7 \times 1 - 2u = 5 \quad \Rightarrow \quad -2v = 5 - 7 = -2$$

$\therefore \quad -2v = -2 \quad \Rightarrow \quad v = \frac{-2}{-2} = 1$

Here $u = 1 \quad \Rightarrow \quad \frac{1}{y} = 1 \quad \Rightarrow \quad y = 1$

And $v = 1 \quad \Rightarrow \quad \frac{1}{x} = 1 \quad \Rightarrow \quad x = 1$

Hence, the solution of given system of equations is $x = 1,\ y = 1$.

(ii) We have, $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$$

Let $\frac{1}{3x+y} = u$ and $\frac{1}{3x-y} = v$

We have, $u + v = \frac{3}{4}$ ...(i)

$$\frac{u}{2} - \frac{u}{2} = -\frac{1}{8} \quad \Rightarrow \quad \frac{u-v}{2} = -\frac{1}{8}$$

$\Rightarrow$ $u - v = -\frac{2}{8} = -\frac{1}{4}$

$\therefore$ $u - v = -\frac{1}{4}$

Adding (i) and (ii), we have

$$u + v = \frac{3}{4}$$

$$u - v = -\frac{1}{4}$$

$$2u = \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4}$$

$\Rightarrow$ $u = \frac{2}{4 \times 2} = \frac{1}{4}$ $\therefore$ $u = \frac{1}{4}$

Now putting the value of u in equation (i), we have

$$\frac{1}{4} + v = \frac{3}{4} \quad \Rightarrow \quad v = \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} \quad \Rightarrow \quad v = \frac{1}{2}$$

Here, $v = \frac{1}{4}$ $\Rightarrow$ $\frac{1}{3x+y} = \frac{1}{4}$ $\Rightarrow$ $3x + y = 4$ ...(iii)

And $v = \frac{1}{2}$ $\Rightarrow$ $\frac{1}{3x+y} = \frac{1}{2}$ $\Rightarrow$ $3x - y = 2$ ...(iv)

Now, adding (iii) and (iv), we have

$$3x + y = 2$$

$$3x - y = 2$$

$$6x = 6$$

$\therefore$ $x = \frac{6}{6} = 1$

Putting the value of x in equation (iii), we have

$$3 \times 1 + y = 4$$

$\Rightarrow$ $y = 4 - 3 = 1$

Hence, the solution of given system of equation is $x = 1,\ y = 1$