ICSE 2025 EXAMINATION

Sample Question Paper - 9

Mathematics

Time: 2 Hours **Max. Marks: 80**

General Instructions:

1. Answer to this Paper must be written on the paper provided separately
2. You will not be allowed to write during first 15 minutes.
3. This time is to be spent in reading the question paper.
4. The time given at the head of this Paper is the time allowed for writing the answers.
5. Attempt all questions from Section A and any four questions from Section B.
6. All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
7. Omission of essential working will result in loss of marks.
8. The intended marks for questions or parts of questions are given in brackets [ ].
9. Mathematical tables are provided.

SECTION-A

(Attempt all questions from this Section.)

QUESTION 1.
Choose the correct answers to the questions from the given options.

(Do not copy the questions, write the correct answer only.)

(i) ₹100 shares of a company are available in the market at a premium of ₹20. What is the rate of dividend given by the company when a man's return on his investment is 15% ?

(a) 17% (b) 16%
(c) 18.5% (d) 18%

Answer: (d) 18%

(ii) David opened a Recurring Deposit Account in a IndusInd bank and deposited ₹300 per month for two years. If he received ₹7,725 at the time of maturity, then the rate of interest per annum is:

(a) 7% (b) 7.5%

(c) 8% (d) 8.4%

Answer: (a) 7%

(iii) If $3 \le 3t - 18 \le 18$, then which one of the following is true?

(a) $8 < t \le 12$ (b) $8 \le t < 12$

(c) $8 < t+1 \le 13$ (d) $8 \le t+1 \le 13$

Answer: (c) $8 < t+1 \le 13$

(iv) $(x^2 + 1)^2 - x^2 = 0$ has

(a) Four real roots (b) Two real roots

(c) No real roots (d) One real root

Answer: (c) No real roots

(v) The duplicate ratio of $\sqrt{3}:5$ is:

(a) $3 : 25$ (b) $7 : \sqrt{3}$

(c) $2\sqrt{3} : 14$ (d) $(\sqrt{3} + 2) : 9$

Answer: (a) $3 : 25$

(vi) If $2x^3 + kx^2 - (5x - 3)x + 8$ is divide by $x+2$, the remainder is 30, then the value of k is:

(a) 8 (b) 9

(c) 10 (d) 11

Answer: (d) 11

(vii) If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $A^2 + xI = yA$, then the value of x is :

(a) 8 (b) 4

(c) -4 (d) -8

Answer: (a) 8

(viii) There are 60 terms is an AP of which the first term is 8 and the last term is 185. The 31^{st} term is

(a) 56 (b) 94

(c) 85 (d) 98

Answer: (d) 98

(ix) If P(α/3, 4) is the mid-point of the line segment joining the points Q(-6, 5) and R(-2, 3), then the value of α is

(a) – 4 (b) – 12
(c) 12 (d) – 6

Answer: (b) – 12

(x) The equation of a line is 2x + 3y = 6. It intersects the y-axis at A. Which of the following are the co- ordinates of A ?

(a) (0, 2) (b)(0, 3)
(c) (2, 0) (d)(3, 0)

Answer: (a) (0, 2)

(xi) **Assertion :** $\triangle ABC \sim \triangle DEF$ such that $\text{ar}(\triangle ABC) = 36\ \text{cm}^2$ and $\text{ar}(\triangle DEF) = 49\ \text{cm}^2$ then, $AB : DE = 6 : 7$.

Reason : If $\triangle ABC \sim \triangle DEF$, then $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Answer: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(xii) If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other is

(a) 3 cm (b) 6 cm
(c) 9 cm (d) 1 cm

Answer: (b) 6 cm

(xiii) A solid piece of iron in the form of a cuboid of dimensions 49cm × 33cm × 24cm, is molded to form a solid sphere. What is the radius of the sphere ?

(a) 21 cm (b) 32 cm
(c) 28 cm (d) 36 cm

Answer: (a) 21 cm

(xiv) An integer is chosen at random between 1 and 100.
Statement 1 : The probability that it is divisible by 8, is 6/49.
Statement 2 : The probability that it is not divisible by 8, is 43/49.

(a) Both the statement are true.
(b) Both the statement are false.
(c) Statement 1 is true and statement 2 is false.
(d) Statement 1 is false and statement 2 is true.

Answer: (a) Both the statement are true.

(xv) An event is very unlikely to happen. Its probability is closest to

(a) 0.0001
(b) 0.001
(c) 0.01
(d) 0.1

Answer: (a) 0.0001

QUESTION 2.

(i) The surface area of a solid metallic, sphere is 616 cm². It is melted and recast into smaller spheres of diameter 3.5 cm. How many such spheres can be obtained?

Answer:

The surface area of sphere = $4\pi r^2$

$4\pi r^2 = 616$

$r^2 = \frac{616 \times 7}{4 \times 22}$

r = 49

$r = \sqrt{49}$

r = 7 cm

Volume of big sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (7)^3$

Volume of small sphere = $\frac{4}{3}\pi\left(\frac{3.5}{2}\right)^3$

$\therefore$ No of smaller sphere = $\frac{\text{Volume of big sphere}}{\text{Volume of small sphere}}$

$= \frac{\frac{4}{3}\pi(7)^3}{\frac{4}{3}\pi\left(\frac{3.5}{2}\right)^3}$

= 64.

(ii) Kiran deposited ₹200 per month for 36 months in bank's recurring deposit account. If the bank pays interest at the rate of 11% per annum, find the amount she gets on maturity

Answer:

Amount deposited month (P) = Rs. 200

Period (n) = 36 months,

Rate (R) = 11% p.a.

Now amount deposited in 36 months = Rs. 200 x 36 = Rs 7200

Simple Interest(S.I.)

$= P\left(\frac{n(n+1)}{2}\right) \times \frac{1}{12} \times \frac{R}{100}$

$= 200\left(\frac{36(36+1)}{2}\right) \times \frac{1}{12} \times \frac{11}{100}$

$= \frac{200 \times 36 \times 37 \times 11}{2 \times 12 \times 100}$

= 1221

∴ Kiran will get maturity value

= Rs.7200 + 1221

= Rs.8421.

(iii) Prove the following identity $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2 \operatorname{cosec} A.$

Answer:

Let's solve this step by step:

1. Let's start with the left side of the equation:
   $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A}$
2. To add these fractions, we need a common denominator:
   $\frac{\sin^2 A+(1+\cos A)^2}{\sin A(1+\cos A)}$
3. Expand the numerator:
   $\frac{\sin^2 A+1+2\cos A+\cos^2 A}{\sin A(1+\cos A)}$
4. Recall that $\sin^2 A + \cos^2 A = 1$:
   $\frac{2+2\cos A}{\sin A(1+\cos A)}$
5. Factor out 2:
   $\frac{2(1+\cos A)}{\sin A(1+\cos A)}$
6. Cancel $(1 + \cos A)$:
   $\frac{2}{\sin A}$
7. Since $\frac{2}{\sin A} = 2 \operatorname{cosec} A$, we have proven that:
   $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2 \operatorname{cosec} A$

Therefore, the identity is proved.

QUESTION 3.

(i) Amit Kumar invests ₹36000 in buying ₹100 shares at ₹20 premium. The dividend is 15% per annum. Find

(i) The number of shares, he buys.

(ii) His yearly dividend.

(iii) The percentage return on his investment.

Given your answer correct to the nearest whole number.

Answer:

Investment = Rs.36000

Face value = Rs.100

Premium = Rs.20, dividend = 15%

(i) No. of shares

$= \frac{36000}{120}$

= 300

(ii) DIvidend

= 15% of (100 x 300)

= ₹4500

(iii) % Return

$= \frac{4500}{36000} \times 100$

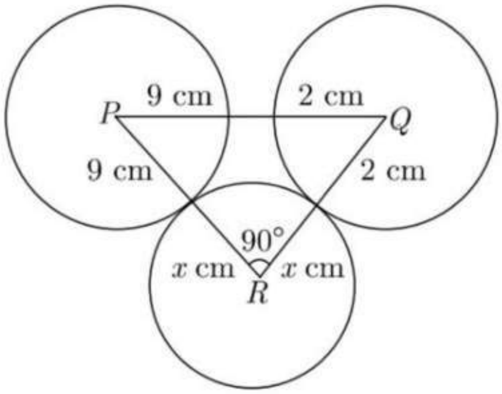
$= \frac{450}{36}$

= 12.5%

= 13%.

(ii) P and Q are centres of circles of radii 9 cm and 2 cm, respectively, PQ = 17 cm. R is the centre of a circle of radius x cm, which touches the above circles externally. Given that +PRQ = 90°, write an equation in x and solve it.

Answer: Let the circle with centre R touch the given two circles at A and B. Then, P, A, R are collinear and Q, B, R are collinear.



Since, ∠PRQ = 90°,

by Pythagoras theorem,

$PQ^2 = PR^2 + QR^2$

$\Rightarrow 17^2 = (9 + x)^2 + (2 + x)^2$

$\Rightarrow x^2 + 11x - 102 = 0$

$\Rightarrow (x + 17)(x - 6) = 0$

$\Rightarrow x = 6$ cm ....($x = -17$ is not possible).

(iii) The marks obtained by 120 students in a test are given below

Marks	Number of students
0-10	5
10-20	9
20-30	16
30-40	22
40-50	26
50-60	18
60-70	11
70-80	6
80-90	4
90-100	3

Draw an ogive for the given distribution on a graph sheet.

Use suitable scale for ogive to estimate the following

(i) The median.

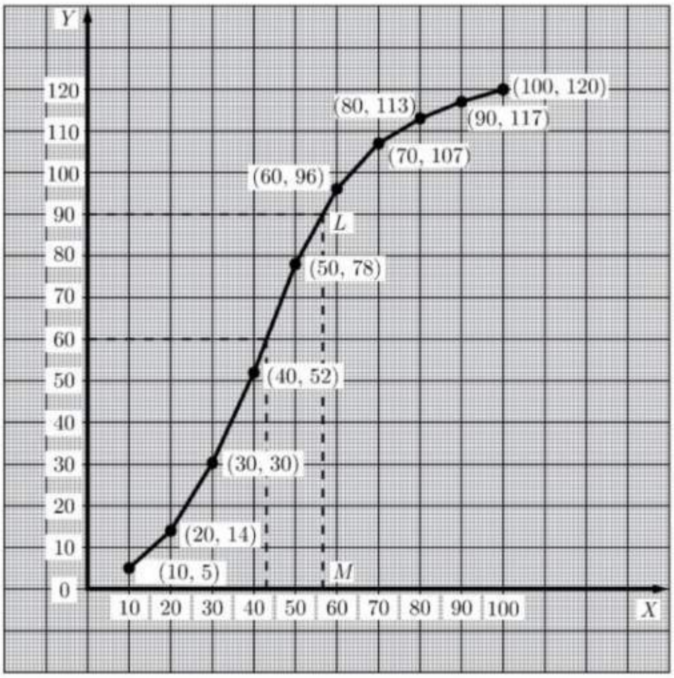
(ii) The number of students who obtained more than 75% marks in the test.

(iii) The number of students who did not pass the test, if minimum marks required to pass is 40.

Answer:

Marks	No. of Students	Cumulative Frequency
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120

$$n = \frac{120}{2} = 60$$



1) Through marks 60, draw a line segment parallel to x-axis which meets the curve at A. From A, draw a line perpendicular to x-axis meeting at B. Median = 43

2) Through marks 75, draw a line segment parallel to y-axis which meets the curve at D. From D, draw a line perpendicular to y-axis which meets y-axis at 110.

Number of students getting more than 75% = 120 - 110 = 10 students

3) Through marks 40, draw a line segment parallel to y-axis which meets the curve at C. From C, draw a line perpendicular to y-axis which meets y-axis at 52.

The number of students who did not pass = 52.

SECTION-B

(Attempt any four questions.)

QUESTION 4.

(i) If $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$, find x and y when $A^2 = B$.

Answer:

Step 1: Calculate A^2

$$A^2 = A \cdot A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$$

Perform matrix multiplication:

$$A^2 = \begin{bmatrix} (3)(3) + (x)(0) & (3)(x) + (x)(1) \\ (0)(3) + (1)(0) & (0)(x) + (1)(1) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & 3x + x^2 \\ 0 & 1 \end{bmatrix}$$

Step 2: Equate A^2 with B

$$A^2 = \begin{bmatrix} 9 & 3x + x^2 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$$

Equating the corresponding elements:

1. From the top-right element:

$$3x + x^2 = 16$$

This simplifies to:

$$x^2 + 3x - 16 = 0$$

2. From the bottom-right element:

$$1 = -y$$

$$y = -1$$

Step 3: Solve for x

The equation $x^2 + 3x - 16 = 0$ is a quadratic equation. Solve it using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a = 1, b = 3, c = -16$$

Substitute the values:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-16)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9+64}}{2}$$

$$x = \frac{-3 \pm \sqrt{73}}{2}$$

Thus:

$$x = \frac{-3+\sqrt{73}}{2} \quad \text{or} \quad x = \frac{-3-\sqrt{73}}{2}$$

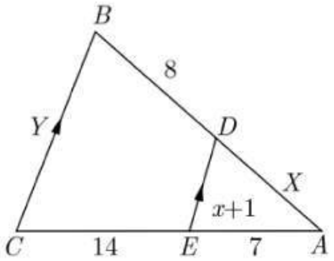
Final Answer:

$$x = \frac{-3 \pm \sqrt{73}}{2}, \quad y = -1$$

(ii) In ΔABC, line DE and BC are parallel

(i) Prove that $\Delta ADE \sim \Delta ABC$.

(ii) If $AD = x$, $DE = x+1$, $BC = y$, $AE = 7$, $BD = 8$, and $CE = 14$, find x and y.



Answer: (i)

To prove that △ADE ~ △ABC, we need to show that these triangles are similar. We'll do this by demonstrating that they have two angles equal, which is sufficient for similarity in triangles. Let's proceed step by step:

1. Given: In △ABC, line DE is parallel to BC.
2. Angle equivalence:
   a. ∠ADE = ∠ABC (Corresponding angles, as DE || BC)
   b. ∠DAE = ∠CAB (Common angle to both triangles)
3. Similarity conclusion:
   Since we have shown that two angles of △ADE are equal to two angles of △ABC (∠ADE = ∠ABC and ∠DAE = ∠CAB), we can conclude that:

   △ADE ~ △ABC

Therefore, we have proved that △ADE is similar to △ABC.

(ii)

Using the properties of similar triangles:
$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$

Given:

- AD = x
- DE = x + 1
- BC = y
- AE = 7
- BD = 8
- CE = 14

From the diagram:

- AB = AD + BD = x + 8
- AC = AE + CE = 7 + 14 = 21

Using the similarity ratio:
$\frac{x}{x+8} = \frac{x+1}{y} = \frac{7}{21}$

From $\frac{7}{21} = \frac{x}{x+8}$:

- 21x = 7(x + 8)
- 21x = 7x + 56
- 14x = 56
- x = 4

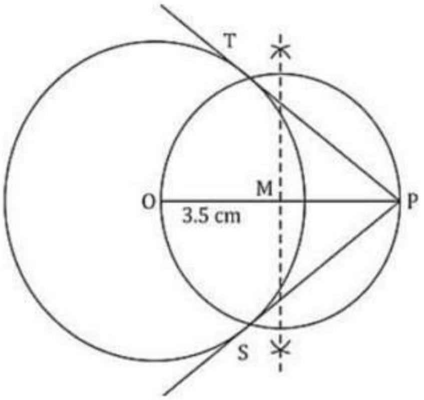
From $\frac{x+1}{y} = \frac{7}{21}$:

- 21(x + 1) = 7y
- 21(4 + 1) = 7y
- 105 = 7y
- y = 15

Therefore:
x = 4 and y = 15

(iii) Draw a circle of radius 3.5 cm. Mark a point P outside the circle at a distance of 6 cm from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent.

Answer:



Steps of construction:

i. Draw a line segment OP = 6 cm.

ii. With centre O and radius 3.5 cm, draw a circle.

iii. Draw the midpoint of OP.

iv. With centre M and diameter OP, draw a circle which intersect the circle at T and S.

v. Join PT and PS.
PT and PS are the required tangents. On measuring the length of PT = PS = 4.8 cm.

QUESTION 5

The sum of deviations of a set of values x_1, x_2, x_3, ……… x_n, measured from 50 is – 10 and the sum of deviations of the values from 46 is 70. Find the value of n and the mean.

Answer:

Sum of deviations from 50 is -10

$$\Rightarrow \sum(x_i - 50) = -10$$

$$\sum(x_i - 50) = -10$$

$$\sum x_i - 50n = -10$$

y−50n=−10.....(1)

Sum of deviations from 46 is 70

$$\Rightarrow \sum(x_i - 46) = 70$$

$$\sum x_i - 46\sum 1 = 70$$

$$\sum x_i - 46n = 70$$

$\therefore y - 46n = 70.....(2)$ Solving (1) and (2), we get

$4n = 80$ i.e. $n = 20$

Putting value of n in (1), we get

$y = 990$

$$\text{Mean } = \sum \frac{x_i}{n} = \frac{y}{n} = \frac{990}{20} = 49.5$$

(ii) Rate of GST for different categories are given below :

Category	Rate of GST
Pharmaceutical Preparations	18%
Life Saving Drugs	5%
Toiletries	18%
Wooden Furniture	18%
Mobile Phones	12%
Consumer Durables	28%

Dr. Anurag is opening a new hospital. He bought the following items from different sellers given in table :

S. No.	Item	Price (Per Unit)	Quantity
1.	Insulin	₹600	50
2.	Hospital Beds	₹10000	10
3.	Air Conditioners	₹40000	10
4.	Reception Table	₹25000	1

Find the :

(a) Combined GST paid by Dr. Anurag

(b) Combined bill amount for all items including GST.

Answer:

GST Calculation:

1. **Insulin** (Life Saving Drug, GST: 5%):

$$\text{Price (Total)} = 600 \times 50 = 30000$$

$$\text{GST} = 5\% \times 30000 = 1500$$

2. **Hospital Beds** (Wooden Furniture, GST: 18%):

$$\text{Price (Total)} = 10000 \times 10 = 100000$$

$$\text{GST} = 18\% \times 100000 = 18000$$

3. **Air Conditioners** (Consumer Durables, GST: 28%):

$$\text{Price (Total)} = 40000 \times 10 = 400000$$

$$\text{GST} = 28\% \times 400000 = 112000$$

4. **Reception Table** (Wooden Furniture, GST: 18%):

$$\text{Price (Total)} = 25000 \times 1 = 25000$$

$$\text{GST} = 18\% \times 25000 = 4500$$

(a) Combined GST Paid:

$$\text{Total GST} = 1500 + 18000 + 112000 + 4500 = 136000$$

(b) Combined Bill Amount Including GST:

$$\text{Total Price Including GST} = (\text{Price} + \text{GST}) \text{ for all items}$$

$$\text{Total Amount} = (30000 + 1500) + (100000 + 18000) + (400000 + 112000) + (25000 + 4500)$$

$$\text{Total Amount} = 31500 + 118000 + 512000 + 29500 = 691000$$

Final Answer:

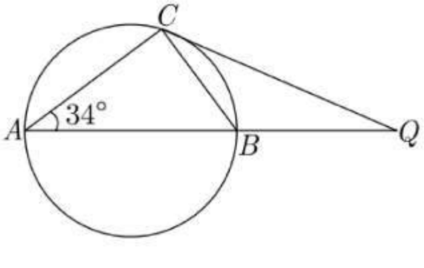
(a) **Combined GST paid** = ₹136,000

(b) **Combined bill amount** = ₹691,000

(iii) In the given figure, AB is a diameter. The tangent at C meets AB produced at Q, ∠CAB = 34°. Find

(i) ∠CBA

(ii) ∠CQA



Answer:

Let me solve this step by step:

(i) To find ∠CBA:

- Since AB is a diameter, ∠ACB is inscribed in a semicircle
- Therefore, ∠ACB = 90° (angle in semicircle property)
- In triangle CAB:
  - ∠CAB = 34° (given)
  - ∠ACB = 90°
  - ∠CBA = 180° - 90° - 34° = 56°

(ii) To find ∠CQA:

- The tangent at point C is perpendicular to the radius at point C
- Therefore, ∠QCB = 90°
- In triangle CQB:
  - ∠CQB = 34° (alternate angles, as tangent makes equal angles)
  - Therefore, ∠CQA = 34°

Final Answer:
(i) ∠CBA = 56°
(ii) ∠CQA = 34°

QUESTION 6.
(i) If a, b, c are in G.P., prove that $a^2 + b^2$, + ab + bc, $b^2 + c^2$ are also in G.P.

Answer:

a, b and c are in G.P.

$\therefore b^2 = ac.......(1)$

$(ab+bc)^2 = (ab)^2 + 2ab^2c + (bc)^2$

$\Rightarrow (ab+bc)^2 = (ab)^2 + ab^2c + ab^2c + (bc)^2$

$\Rightarrow (ab+bc)^2 = a^2b^2 + ac\,(ac) + b^2\left(b^2\right) + b^2c^2$ [Using (1)]

$\Rightarrow (ab+bc)^2 = a^2\left(b^2+c^2\right) + b^2\left(b^2+c^2\right)$

$\Rightarrow (ab+bc)^2 = \left(b^2+c^2\right)\left(a^2+b^2\right)$

Therefore , $\left(a^2+b^2\right), \left(b^2+c^2\right)$ and $(ab+bc)$ are also in G . P .

(ii) Arc of a Baby Swing : When Mackenzie's baby swing is started, the first swing (one way) is a 30 inch arc. As the swing slows down, each successive arc is 1.5 inch less than the previous one.

(i) Find the length of the tenth swing.

Answer:

(i) Length of tenth swing:

- Initial arc length = 30 inches
- Common difference = -1.5 inches
- Using arithmetic sequence formula: $a_n = a_1 + (n-1)d$
- $a_{10} = 30 + (10-1)(-1.5)$
- Length of tenth swing = 16.5 inches

(ii) How far Mackenzie has travelled during the 10 swings ?

Answer:

(ii) Total distance traveled:

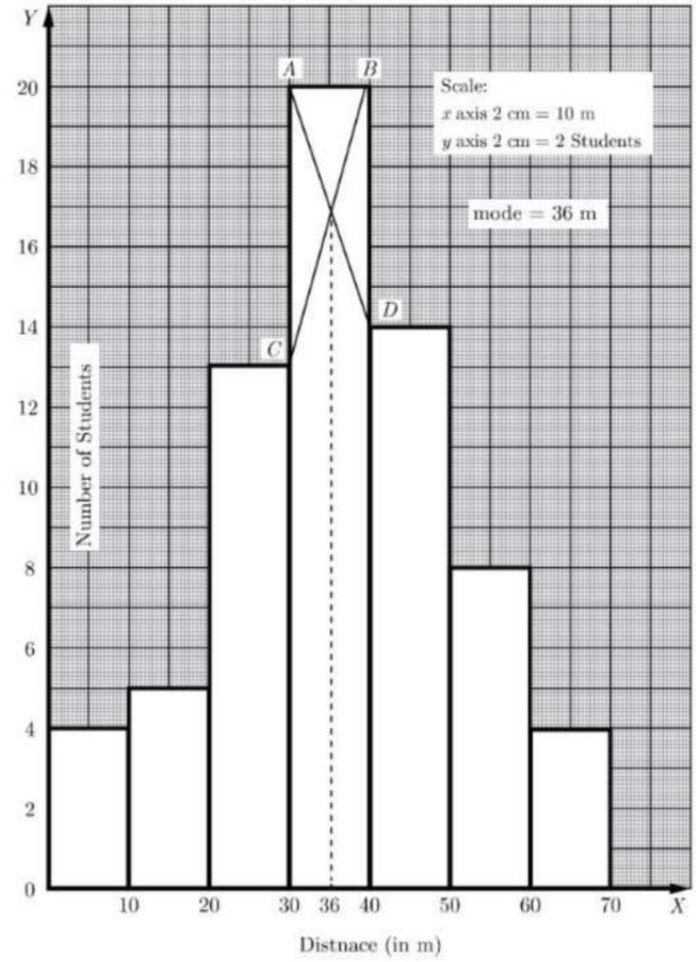
- Using sum of arithmetic sequence formula: $S_n = \frac{n}{2}[2a_1 + (n-1)d]$
- $S_{10} = \frac{10}{2}[2(30) + (10-1)(-1.5)]$
- Total distance = 232.5 inches

(iii) The following frequency distribution shows the distance (in meters) thrown by 70 students in a Javelin throw competition.

Distance (in m)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	6	7	13	20	14	7	3

Draw a histogram for the above data using a graph paper and locate the mode.

Answer: The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 36.
Hence, the required mode is 36 m.

QUESTION 7.
(i) The equation of a line is 3x+ 4y – 7 = 0. Find the

(i) slope of the line.

(ii) equation of a line perpendicular to the given line and passing through the intersection of the lines x – y + 2 = 0 and 3x + y – 10 = 0.

Answer:

(i) Slope of the line $3x + 4y - 7 = 0$:

Rearrange into slope-intercept form ($y = mx + c$):

$$4y = -3x + 7 \implies y = -\frac{3}{4}x + \frac{7}{4}$$

The slope (m) is:

$$m = -\frac{3}{4}$$

(ii) Equation of a line perpendicular to the given line and passing through the intersection of $x - y + 2 = 0$ and $3x + y - 10 = 0$:

Step 1: Find the point of intersection of the lines $x - y + 2 = 0$ and $3x + y - 10 = 0$:

$$x - y = -2 \quad (1)$$

$$3x + y = 10 \quad (2)$$

Add equations (1) and (2):

$$4x = 8 \implies x = 2$$

Substitute $x = 2$ into (1):

$$2 - y = -2 \implies y = 4$$

So, the point of intersection is $(2, 4)$.

Step 2: Slope of the given line is $m = -\frac{3}{4}$. The slope of a line perpendicular to it is:

$$m \qquad = \frac{4}{3}$$

Step 3: Equation of the line with slope $\frac{4}{3}$ passing through $(2, 4)$: Using point-slope form:

$$y - y_1 = m(x - x_1)$$

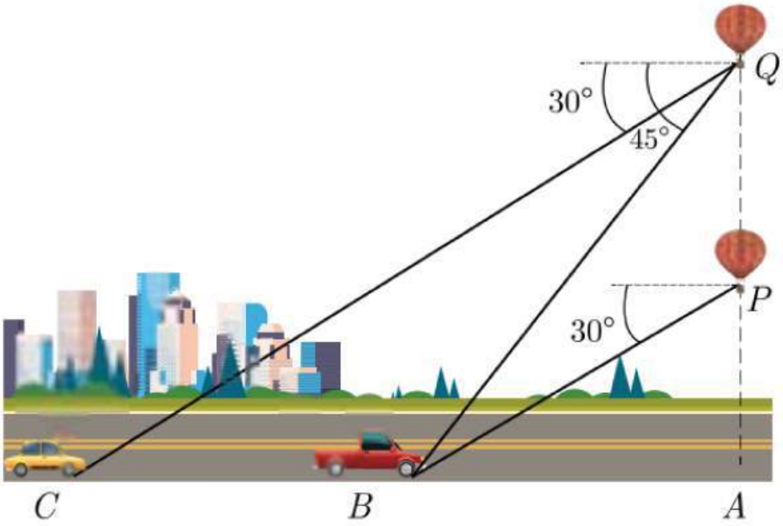
$$y - 4 = \frac{4}{3}(x - 2)$$

$$y - 4 = \frac{4}{3}x - \frac{8}{3}$$

$$3y - 12 = 4x - 8$$

$$4x - 3y + 4 = 0$$

(ii) A hot air balloon is a type of aircraft. It is lifted by heating the air inside the balloon, usually with fire.

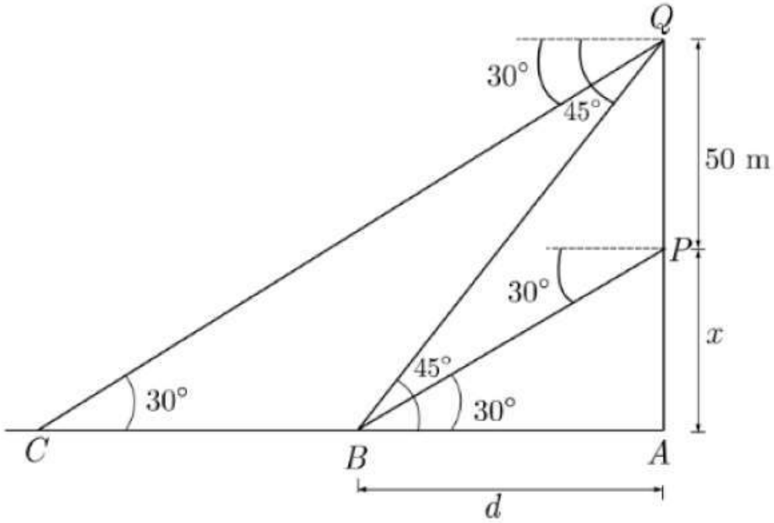


Lakshman is riding on a hot air balloon. After reaching at height x at point P, he spots a lorry parked at B on the ground at an angle of depression of 30°. The balloon rises further by 50 metres at point Q and now he spots the same lorry at an angle of depression of 45° and a car parked at C at an angle of depression of 30°.

(i) What is the relation between the height x of the balloon at point P and distance d between point A and B ?

Answer:

(i) We make the diagram as per given information.



From the figure:

- P is at a height x above point A,
- Angle of depression to B = 30°.

Using the tangent ratio:

$$\tan(30^\circ) = \frac{\text{Height at P}}{\text{Horizontal Distance to B}}$$

$$\tan(30^\circ) = \frac{x}{d}$$

We know that:

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

Thus:

$$\frac{1}{\sqrt{3}} = \frac{x}{d}$$

Rearranging for x:

$$x = \frac{d}{\sqrt{3}}$$

(ii) When balloon rises further 50 metres, then what is the relation between new height y and d ?

Answer:

(ii) In ΔBAQ,

$$\tan 45^\circ = \frac{AQ}{AB}$$

$$AB = AQ$$

$$d = y$$

(iii) What is the new height of the balloon at point Q ?

Answer:

(iii) The new height of the balloon at point Q:

The balloon rises an additional 50 meters from point P to point Q. Given that the initial height at P is x, the new height at Q is:

$$\text{New height at Q} = x + 50 = 100 \text{ meters}$$

(iv) What is the distance AB on the ground ?

Answer:

(iv) Distance AB on the ground:

At point Q, the angle of depression to B is 45°.

From trigonometry:

$$\tan(45^\circ) = \frac{\text{Height at Q}}{\text{Distance AB}}$$

We know:

$$\tan(45^\circ) = 1 \quad \text{and Height at Q} = x + 50.$$

Thus:

$$1 = \frac{x + 50}{AB}$$

Rearranging:

$$AB = x + 50.$$

(v) What is the distance AC on the ground ?

Answer:

(v) Distance AC on the ground:

At point Q, the angle of depression to C is 30°.

From trigonometry:

$$\tan(30^\circ) = \frac{\text{Height at Q}}{\text{Distance AC}}$$

We know:

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \quad \text{and Height at Q} = x + 50.$$

Thus:

$$\frac{1}{\sqrt{3}} = \frac{x + 50}{AC}$$

Rearranging:

$$AC = \sqrt{3} \cdot (x + 50).$$

QUESTION 8.

(i) Solve the following equation, write down the solution set and represent it on the real number line.

$10x - 5 \leq 13x + 10 < 25 + 10x, \qquad x \in Z$

Answer:

Step 1: Solve $10x - 5 \leq 13x + 10$:

$$10x - 5 \leq 13x + 10.$$

Simplify:

$$-5 - 10 \leq 13x - 10x \implies -15 \leq 3x.$$

Divide by 3:

$$x \geq -5.$$

Step 2: Solve $13x + 10 < 25 + 10x$:

$$13x + 10 < 25 + 10x.$$

Simplify:

$$13x - 10x < 25 - 10 \implies 3x < 15.$$

Divide by 3:

$$x < 5.$$

Step 3: Combine the results:

From Step 1 and Step 2:

$$-5 \leq x < 5.$$

Since $x \in \mathbb{Z}$ (integers), the solution set is:

$$x = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\}.$$

Step 4: Represent on the number line:

- Mark integers -5 to 4, including -5 (closed circle) and excluding 5 (open circle).

Final Answer:

Solution Set: $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$.

Representation on Number Line: Points from -5 to 4, with -5 as a closed point and 5 excluded.

(ii) If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, –3) and (3, 4). Find its centroid.

Answer:

Let P (1, 1), Q(2, −3), R(3, 4) be the mid-points of sides AB, BC and CA respectively of triangle ABC. Let $A(x_1, y_1), B(x_2, y_2)$ be the vertices of triangle ABC.

Then, P is the mid-point of BC

$$\Rightarrow \frac{x_1 + x_2}{2} = 1, \frac{y_1 + y_2}{2} = 1$$

$$\Rightarrow x_1 + x_2 = 2 \text{ and } y_1 + y_2 = 2 \ldots (1)$$

Q is the mid-point of BC

$$\Rightarrow \frac{x_2 + x_3}{2} = 2, \frac{y_2 + y_3}{2} = -3$$

$$\Rightarrow x_2 + x_3 = 4 \text{ and } y_2 + y_3 = –6 \ldots (2)$$

R is the mid-point of AC

$$\Rightarrow \frac{x_1 + x_3}{2} = 3, \frac{y_1 + y_3}{2} = 4$$

$$\Rightarrow x_1 + x_3 = 6 \text{ and } y1_1 + y_3 = 8 \ldots (3)$$

From (1), (2) and (3), we get

$$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 4 + 6 \text{ and } , y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 2–6 + 8$$

$$x_1 + x_2 + x_3 = 6 \text{ and } y_1 + y_2 + y_3 = 2 \ldots (4)$$

The coordinates of the centroid of ΔABC are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right) = \left(\frac{6}{3}, \frac{2}{3}\right)$$

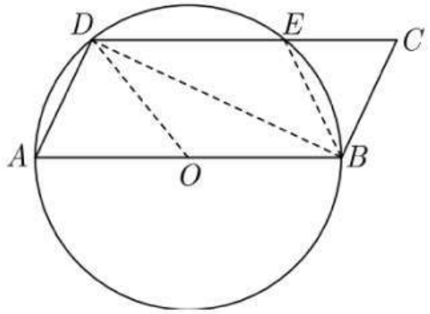
$$= \left(2, \frac{2}{3}\right)$$

(iii) In the circle with centre O, ABCD is a parallelogram. ∠BCD = 56°.

(i) Find ∠BED.

(ii) Find ∠AOD.

(iii) Find ∠ABD.



Answer:

(i) To find ∠BED:
In parallelogram ABCD, we know that ∠BCD = 56°. Since opposite angles in a parallelogram are equal, ∠DAB = 56° as well. In triangle BED, ∠BED is supplementary to ∠DAB, as DE is a straight line. Therefore:

∠BED = 180° - ∠DAB = 180° - 56° = 124°

(ii) To find ∠AOD:
ABCD is a cyclic quadrilateral (inscribed in the circle). In a cyclic quadrilateral, the angle at the center is twice the angle at the circumference subtended by the same arc. We know that ∠ABD = 56° (which we'll prove in part iii). Therefore:

∠AOD = 2 * ∠ABD = 2 * 56° = 112°

(iii) To find ∠ABD:
In a parallelogram, opposite angles are equal. Since ∠BCD = 56°, we can conclude:

∠ABD = ∠DAB = ∠BCD = 56°

QUESTION 9.

(i) If $(3a+2b):(5a+3b)=18:29$. Find $a:b$.

Answer:

Given:

$$\frac{(3a+2b)}{(5a+3b)}=\frac{18}{29}.$$

Step 1: Cross-multiply:

$$29(3a+2b)=18(5a+3b).$$

Expand both sides:

$$29 \cdot 3a + 29 \cdot 2b = 18 \cdot 5a + 18 \cdot 3b.$$

$$87a + 58b = 90a + 54b.$$

Step 2: Rearrange terms:

Move terms involving a and b to one side:

$$87a - 90a = 54b - 58b.$$

$$-3a = -4b.$$

Step 3: Simplify:

$$\frac{a}{b} = \frac{4}{3}.$$

Final Answer:

The ratio $a : b$ is:

$$a : b = 4 : 3.$$

(ii) A car covers a distance of 2592 km with a uniform speed. The number of hours taken for journey is one half the number representing the speed in km/hour.

Find the time taken to cover the distance.

Answer:

Distance covered = 2592 km

Let the speed of the car be x km/h

According to the question, Time =

$$\frac{\text{speed}}{2} = \frac{x}{2}h$$

we know that $\text{speed} = \frac{\text{distance}}{\text{time}}$

$$x = \frac{2592}{x/2}$$

$$x^2 = 5184$$

$$x = \sqrt{5184}$$

$$x = 72$$

Speed of the car $= 72$ km/h

Time taken to cover the given distance = 36 h

(iii) Draw a triangle LMN such that LM = 3 9. cm. MN = 7 2. cm and LN = 5 8. cm.

(i) Find the circumcentre of Δ LMN.

(ii) Find the incentre of Δ LMN.

Answer:

Steps of Construction :

Step 1 : Draw a line segment $MN = 7.2$ cm.

Step 2 : With centre M and radius 3.9 cm and with centre N and raduis 5.8 cm draw, arc's which intersect each other at L.

Step 3 : Join LM and LN. This is required $\triangle LMN$.

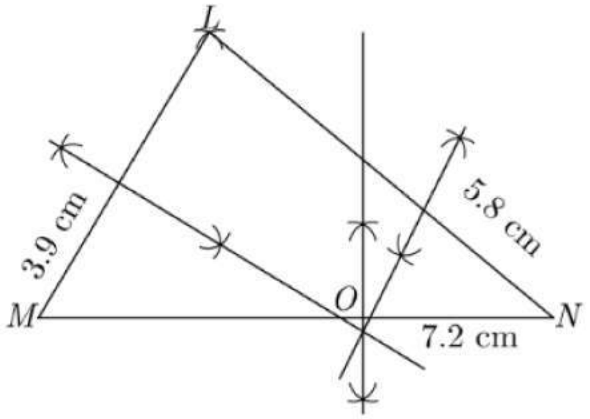
Step 4 : Draw the perpendicular bisector of the sides LM, MN and NL intersect at a point O. The O is the circumentre of the ΔLMN.

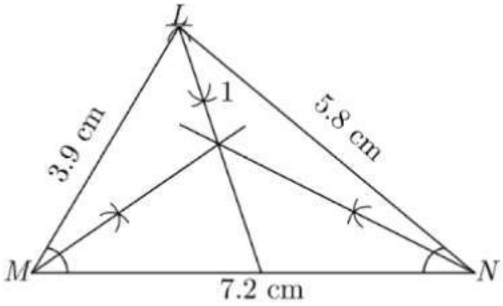
Step 5 : Draw a line segment $MN = 7.2$ cm.

Step 6 : With centre M and radius 3.9 cm and with centre N and raduis 5.8 cm draw arc's which intersect each other at L.

Step 7 : Join LM and LN.

Step 8 : Draw the angle bisectors of $\angle L$, $\angle M$ and $\angle N$ which intersect at I. I is the incentre of the ΔLMN.





QUESTION 10.

(i) Find a, if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$ leave the same remainder when divided by x + 3.

Answer: Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leaves the same remainder when divided by x + 3.

The given polynomials are $ax^3 + 3x^2 - 9$

and $2x^3 + 4x + a$

Let $p(x) = ax^3 + 3x^2 - 9$

and $q(x) = 2x^3 + 4x + a$

Given that p(x) and q(x) leave the same remainder when divided by (x + 3),

Thus by Remainder Theorem, we have

$p(-3) = q(-3)$

$\Rightarrow a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$

$\Rightarrow -27a + 27 - 9 = -54 - 12 + a$

$\Rightarrow -27a + 18 = -66 + a$

$\Rightarrow -27a - a = -66 - 18$

$\Rightarrow -28a = -84$

$\Rightarrow a = \frac{84}{28}$

$\therefore a = 3.$

(ii) Cards marked with numbers 1, 2, 3, 4, ..., 20 are well shuffled and a card is drawn at random. What is probability that the number of the cards is

(i) a prime number?

(ii) divisible by 3?

(iii) a perfect square?

Answer:

(i) Probability that the number is a prime number:

Prime numbers between 1 and 20 are:

$2, 3, 5, 7, 11, 13, 17, 19.$

Number of prime numbers = 8.

$$P(\text{prime}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{8}{20} = \frac{2}{5}.$$

(ii) Probability that the number is divisible by 3:

Numbers divisible by 3 between 1 and 20 are:

$3, 6, 9, 12, 15, 18.$

Number of such numbers = 6.

$$P(\text{divisible by } 3) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{6}{20} = \frac{3}{10}.$$

(iii) Probability that the number is a perfect square:

Perfect squares between 1 and 20 are:

$1, 4, 9, 16.$

Number of perfect squares = 4.

$$P(\text{perfect square}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{4}{20} = \frac{1}{5}.$$

(iii) Use a graph paper for this question.

(i) The point P(2, – 4) is reflected about the line x = 0 to get the image Q. Find the coordinates of Q.

Answer:

Take 1 cm = 1 unit on both axes.

(i) The reflection of $P(2, -4)$ about the line $x = 0$ (i.e. y-axis) is $Q(-2, -4)$.

(ii) Point Q is reflected about the line y = 0 to get the image R. Find the coordinates of R.

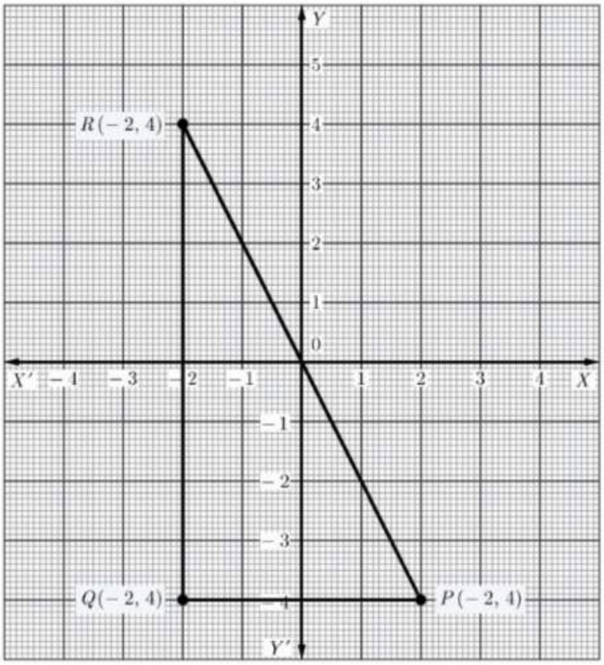
Answer:

(ii) The reflection of Q about the line $y = 0$ (i.e. x-axis) is $R(-2, 4)$.

(iii) Name the figure PQR.

Answer:

(iii) On joining all adjacent points, we get a right ΔPQR.



(iv) Find the area of figure PQR.

Answer:

iv. Area of ΔPQR = $\frac{1}{2} \times PQ \times QR$

= $\frac{1}{2} \times 4 \times 8$

= 16 sq. units