CHAPTER

3 PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Syllabus

- *Pair of linear equations in two variables and graphical method of their solution, consistency/ inconsistency.*
- *Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically : by substitution, by elimination and by cross-multiplication method. Simple situational problems. Simple problems on equations reducible to linear equations.*

Chapter Analysis

List of Topics	2016			2017			2018
	Delhi	Outside Delhi	Foreign	Delhi	Outside Delhi	Delhi	Delhi & Outside Delhi
Word Problem based on rectangle	Summative Assessment-I						1 Q (2 Marks)

TOPIC-1
Graphical Solution of Linear Equations in Two Variables, Consistency/Inconsistency

Revision Notes

- **Linear Equation in two variables :** An equation in the form of $ax + by + c = 0$, where a, b and c are real numbers and a and b are not zero, is called a linear equation in two variables x and y.

  General form of a pair of linear equations in two variables is :

  $$a_1x + b_1y + c_1 = 0$$

  and $$a_2x + b_2y + c_2 = 0,$$

  where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers, such that

  $$a_1, b_1 \neq 0 \text{ and } a_2, b_2 \neq 0.$$

  e.g., $$3x - y + 7 = 0,$$

  and $$7x + y = 3$$

  are linear equations in two variables x and y.

  - There are two methods of solving simultaneous linear equations in two variables :
    (i) Graphical method and,
    (ii) Algebraic method.

1. Graphical Method :

(i) Express one variable (say y) in terms of the other variable x, $y = ax + b$, for the given equation.

(ii) Take three values of independent variable x and find the corresponding values of dependent variable y, take integral values only.

(iii) Plot these values on the graph paper in order to represent these equations.

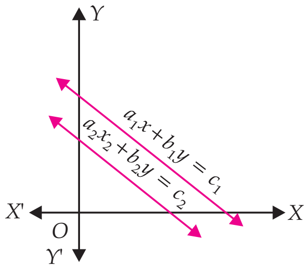
(iv) If the lines intersect at a distinct point, then point of intersection will be the unique solution for given equations. In this case, the pair of linear equations is consistent.

(v) If the lines representing the linear equations coincides, then system of equations has infinitely many solutions. In this case, the pair of linear equations is consistent and dependent.

(vi) If the lines representing the pair of linear equations are parallel, then the system of equations has no solution and is called inconsistent.

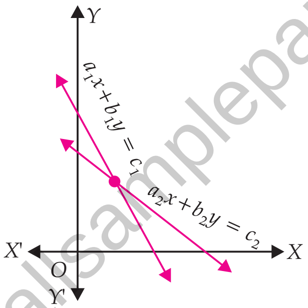
Parallel Lines :

(i) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations is inconsistent with no solution.



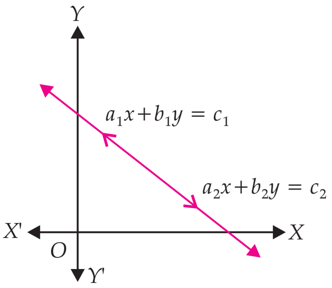
Intersecting Lines :

(ii) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations is consistent with a unique solution.



Coincident Lines :

(iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations is consistent with infinitely many solutions.



➢ If $\quad a_1x + b_1y + c_1 = 0$

and $\quad a_2x + b_2y + c_2 = 0$

is a pair of linear equations in two variables x and y such that :

Possibilities of solutions and Inconsistency :

Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation	Conditions for solvability
$x - 2y = 0$ $3x - 4y - 20 = 0$	$\frac{1}{3}$	$\frac{-2}{-4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution or Unique solution	System is consistent
$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	System is consistent
$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	System is inconsistent

Know the Formulae

- If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is a pair of linear equations in two variables x and y such that :

  (i) System has unique solution

  if $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

  (ii) System has infinite number of solutions

  if $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

  (iii) System has no solution

  if $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

How it is done on GREENBOARD ?

Q. From the given pair of linear equations $3x + ay = 50$ and $9x - 21y = 15$, find the value of a for them to be parallel

Sol. : Step I : Given $3x + ay = 50$ and $9x - 21y = 15$

$\Rightarrow a_1 = 3, b_1 = a, c_1 = -50$ and $a_2 = 9, b_2 = -21, c_2 = -15$

Step II : For lines to be parallel

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Step III :

$$\frac{3}{9} = \frac{a}{-21} \neq \frac{-50}{-15}$$

$$\Rightarrow \quad a = \frac{1}{3} \times (-21)$$

So, $a = -7$

Objective Type Questions

(1 mark each)

[A] Multiple Choice Questions :

Q. 1. Graphically, the pair of equations

$6x - 3y + 10 = 0$

$2x - y + 9 = 0$

represents two lines which are

(a) intersecting at exactly one point

(b) intersecting at exactly two points

(c) coincident

(d) parallel U **[NCERT Exemp.]**

Sol. Correct option : (d)

Explanation : Here, $\frac{a_1}{a_2} = \frac{6}{2} = 3$, $\frac{b_1}{b_2} = \frac{-3}{-1} = 3$, $\frac{c_1}{c_2} = \frac{10}{9}$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the system of linear equations is inconsistent (no solution) and graph will be a pair of parallel lines.

Q. 2. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have :

(a) a unique solution **(b) exactly two solutions**

(c) infinitely many solutions **(d) no solution**

U **[NCERT Exemp.]**

Sol. Correct option : (d)

Explanation : Here, $\frac{a_1}{a_2} = \frac{1}{-3} = -\frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{3}, \frac{c_1}{c_2} = \frac{5}{1}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the system of linear equations has no solution.

Q. 3. If a pair of linear equations is consistent, then the lines will be :

(a) parallel

(b) always coincident

(c) intersecting or coincident

(d) always intersecting

U **[NCERT Exemp.]**

Sol. Correct option : (c)

Explanation : Condition for consistency :

$\frac{a_1}{a_2} \uparrow \frac{b_1}{b_2}$ has unique solution (consistent), *i.e.,* intersecting at one point

or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(Consistent lines, coincident or dependent)

Q. 4. The pair of equations $x = a$ and $y = b$ graphically represents lines which are :

(a) parallel **(b) intersecting at (b, a)**

(c) coincident **(d) intersecting at (a, b)**

U **[NCERT Exemp.]**

Sol. Correct option : (d)

Explanation : ($x = a$) is the equation of a straight line parallel to the y-axis at a distance 'a' from it. Again, $y = b$ is the equation of a straight line parallel to the x-axis at a distance 'b' from it. So, the pair of equations $x = a$ and $y = b$ graphically represents lines which are intersecting at (a, b).

Q. 5. The pair of equations $y = 0$ and $y = -7$ has :

(a) one solution **(b) two solutions**

(c) infinitely many solutions **(d) no solution**

U **[NCERT Exemp.]**

Sol. Correct option : (d)

Explanation : We know that equation of the form $y = a$ is a line parallel to x-axis at a distance 'a' from it. $y = 0$ is the equation of the x-axis and $y = -7$ is the equation of the line parallel to the x-axis. So, these two equations represent two parallel lines. Therefore, there is no solution.

Q. 6. One equation of a pair of dependent linear equations is $-5x + 7y = 2$. The second equation can be :

(a) $10x + 14y + 4 = 0$ **(b) $-10x - 14y + 4 = 0$**

(c) $-10x + 14y + 4 = 0$ **(d) $10x - 14y = -4$**

R **[NCERT Exemp.]**

Sol. Correct option : (d)

Explanation : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{k}$ ...(i)

Given equation of line is, $-5x + 7y - 2 = 0$

Here, $a_1 = -5, b_1 = 7, c_1 = -2$

From Eq. (i), $-\frac{5}{a_2} = \frac{7}{b_2} = -\frac{2}{c_2} = \frac{1}{k}$

$\Rightarrow a_2 = -5k, b_2 = 7k, c_2 = -2k$

where, k is any arbitrary constant.

Putting $k = 2$, then $a_2 = -10$, $b_2 = 14$ and $c_2 = -4$

$\therefore$ The required equation of line becomes,

$a_2x + b_2y + c_2 = 0$

$\Rightarrow -10x + 14y - 4 = 0$

$\Rightarrow \quad 10x - 14y + 4 = 0$

[B] Very Short Answer Type Questions :

Q. 1. For what value of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines ?

R **[NCERT Exemp.]**

Sol. $3x - y = -18$ ...(i)

$6x - ky = -16$ ...(ii)

For coincident lines,

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{6} = \frac{-1}{-k} = \frac{-8}{-16}$

$\Rightarrow \frac{1}{2} = \frac{1}{k} = \frac{1}{2}$

So, $k = 2$.

Q. 2. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then find the value of k.

R **[NCERT Exemp.]**

Sol. For parallel lines (or no solution)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$

$\Rightarrow 4k = 15$

$\Rightarrow \quad k = \frac{15}{4}$

Q. 3. Find the value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions. R **[NCERT Exemp.]**

Sol. For infinitely many solutions,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{c}{6} = \frac{-1}{-2} = \frac{2}{3}$

$2c = 6 \Rightarrow c = 3$

$3c = 12 \Rightarrow c = 4$

As from the ratios, values of c are not common. So, there is no common value of c for which lines have infinitely many solutions.

Q. 4. If $am = bl$, then find whether the pair of linear equations $ax + by = c$ and $lx + my = n$ has no solution, unique solution or infinitely many solutions.

Sol. Since, $am = bl$

$$\therefore \quad \frac{a}{l} = \frac{b}{m} \neq \frac{c}{n}$$

So, $ax + by = c$ and $lx + my = n$ has no solution. **1**

Q. 5. Two lines are given to be parallel. The equation of one of the lines is $4x + 3y = 14$, then find the equation of a second line. A

Sol. The equation of one line is $4x + 3y = 14$. We know that if two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or $$\frac{4}{a_2} = \frac{3}{b_2} \neq \frac{-14}{c_2} \Rightarrow \frac{a_2}{b_2} = \frac{4}{3} \Rightarrow \frac{12}{9}$$

Hence, one of the possible, second parallel line is $12x + 9y = 5$. **1**

[C] True/False :

Q. 1. For all real values of c, the pair of equations

$x - 2y = 8$

$5x - 10y = c$

have a unique solution. Justify whether it is true or false. [NCERT Exemp.]

Sol. False,

$$x - 2y = 8 \quad \text{...(i)}$$

$$5x - 10 = c \quad \text{...(ii)}$$

$$\therefore \frac{a_1}{a_2} = \frac{1}{5}, \frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-c} = \frac{8}{c}$$

As $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, so system of linear equations can never have unique solution.

Short Answer Type Questions-I

(2 marks each)

Q. 1. Find whether the lines represented by $2x + y = 3$ and $4x + 2y = 6$ are parallel, coincident or intersecting.

U [Board Term-1, 2016, Set MV98HN3]

Sol. Here, $a_1 = 2, b_1 = 1, c_1 = -3$

and $a_2 = 4, b_2 = 2, c_2 = -6$

If $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2},$$

then the lines are coincident. **1**

Clearly, $$\frac{2}{4} = \frac{1}{2} = \frac{3}{6}$$ **1**

Hence, lines are coincident.

[CBSE Marking Scheme, 2016]

Q. 2. Find whether the following pair of linear equations is consistent or inconsistent :

$3x + 2y = 8$

and $6x - 4y = 9$

U [Board Term-1, 2016, Set-ORDAWEZ]

Sol. Since, $$\frac{3}{6} \neq \frac{2}{-4}$$

i.e., $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$ **1**

Hence, the pair of linear equations is consistent. **1**

[CBSE Marking Scheme, 2016]

Q. 3. Is the system of linear equations $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ consistent ? Justify your answer. A [Board Term-1, 2012, Set–66]

Sol. We have, for the equation

$$2x + 3y - 9 = 0$$

$$a_1 = 2, b_1 = 3 \text{ and } c_1 = -9$$ ½

and for the equation, $4x + 6y - 18 = 0$

$$a_2 = 4, b_2 = 6 \text{ and } c_2 = -18$$ ½

Here, $$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

and $$\frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$$ ½

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$ ½

Hence, system is consistent and dependent.

Q. 4. Given the linear equation $3x + 4y = 9$. Write another linear equation in these two variables such that the geometrical representation of the pair so formed is :

(i) intersecting lines

(ii) coincident lines.

R [Board Term-1, 2016, Set-O4YP6G7]

Sol. (i) For intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, one of the possible equation is $3x - 5y = 10$ **1**

(ii) For coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, one of the possible equation is $6x + 8y = 18$ **1**

[CBSE Marking Scheme, 2016]

Q. 5. For what value of p does the pair of linear equations given below have unique solution ?

$4x + py + 8 = 0$ and $2x + 2y + 2 = 0$.

U [Board Term-1, 2012, Set– 44]

Sol. Given, $4x + py + 8 = 0$ ...(1)

$2x + 2y + 2 = 0$ ...(2)

Thus, the condition of unique solution is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ **1**

Hence, $\frac{4}{2} \neq \frac{p}{2}$ or $\frac{2}{1} \neq \frac{p}{2}$

$\therefore$ $p \neq 4$ **1**

Hence the value of p is other than 4, it may be 1, 2, 3, – 4, ... etc.

Q. 6. For what value of k, the pair of linear equations $kx - 4y = 3$, $6x - 12y = 9$ has an infinite number of solutions ? C + U [Board Term-1, 2012, Set–25]

Sol. Try yourself, Similar to Q. 7., of Short Answer Type Question-I.

Q. 7. For what value of k, $2x + 3y = 4$ and $(k + 2)x + 6y = 3k + 2$ will have infinitely many solutions ?

U [Board Term-1, 2012, Set–68]

Sol. We have, for the equation $2x + 3y - 4 = 0$

$a_1 = 2$, $b_1 = 3$ and $c_1 = -4$ ½

and for the equation $(k + 2)\,x + 6y - (3k + 2) = 0$

$a_2 = k + 2$, $b_2 = 6$ and $c_2 = -(3k + 2)$ ½

For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow$ $\frac{2}{k+2} = \frac{3}{6} = \frac{-4}{-(3k+2)}$

$\Rightarrow$ $3(k + 2) = 2 \times 6$

or, $3k + 6 = 12$	or, $3k + 2 = 8$
$\Rightarrow$ $6 = 3k$	or, $3k = 6$
Hence, $k = 2$	$k = 2$ **1**

Q. 8. For what value of 'k', the system of equations $kx + 3y = 1$, $12x + ky = 2$ have no solution.

C + U [Board Term-1, 2011, Set–A2, NCERT]

Sol. The given equations can be written as

$kx + 3y - 1 = 0$ and $12x + ky - 2 = 0$

Here, $a_1 = k$, $b_1 = 3$, $c_1 = -1$

and $a_2 = 12$, $b_2 = k$, $c_2 = -2$ **1**

The equation for no solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow$ $\frac{k}{12} = \frac{3}{k} \neq \frac{-1}{-2}$	or, $\frac{k}{12} = \frac{3}{k}$
$\therefore$ $\frac{3}{k} \neq \frac{1}{2}$	or, $k^2 = 36$
or, $k \neq 6$	or, $k = \pm 6$

Hence, $k = -6$ **1**

Q. 9. For what value of p will the following pair of linear equations have infinitely many solutions.

$(p - 3)x + 3y = p$

and $px + py = 12$

C + U [CBSE Sample Paper 2018]

Sol. Try yourself, Similar to Q. 7., of Short Answer Type Question-I.

Q. 10. Find the value (s) of k for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions. C + U [SQP 2017]

Sol. Try yourself, Similar to Q. 7., of Short Answer Type Question-I.

Short Answer Type Questions-II

(3 marks each)

Q. 1. Solve the pair of equations graphically :

$4x - y = 4$ and $3x + 2y = 14$

U [Board Term-1, 2014, Set–A]

Sol. Given, $4x - y = 4$

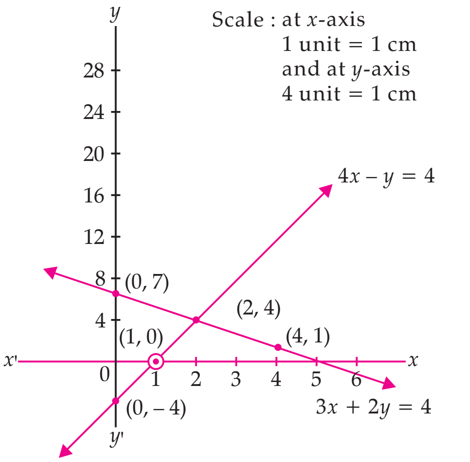
$\Rightarrow$ $y = 4x - 4$ **1**

x	0	1	2
y	– 4	0	4

and $3x + 2y = 14$

$\Rightarrow$ $y = \frac{14 - 3x}{2}$ **1**

x	0	2	4
y	7	4	1



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As obtained lines intersect each other at (2, 4), Hence, $x = 2$ and $y = 4$

AI **Q. 2. Determine the values of m and n so that the following system of linear equations have infinite number of solutions :**

$$(2m - 1)x + 3y - 5 = 0$$

and $3x + (n - 1)y - 2 = 0$ C + A

[Board Term-1, 2013, VKH6FFC; 2011, Set–66]

Sol. We have, for equation

$$(2m - 1)x + 3y - 5 = 0 \quad \text{...(i)}$$

$a_1 = 2m - 1$, $b_1 = 3$ and $c_1 = -5$ ½

and for equation

$$3x + (n - 1)y - 2 = 0 \quad \text{...(ii)}$$

$a_2 = 3$, $b_2 = (n - 1)$ and $c_2 = -2$ ½

For a pair of linear equations to have infinite number of solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

or $$\frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$$ 1

or $$2(2m - 1) = 15 \text{ and } 5(n-1) = 6$$

Hence, $$m = \frac{17}{4} \text{ and } n = \frac{11}{5}$$ 1

Q. 3. Find the value of α and β for which the following pair of linear equations have infinite number of solutions : $2x + 3y = 7$; $2ax + (a + b)y = 28$.

A **[Board Term-1, 2011, Set–25]**

Sol. Try yourself, Similar to Q. 2., of Short Answer Type Question-II.

Q. 4. Represent the following pair of linear equations graphically and hence comment on the condition of consistency of this pair.

$$x - 5y = 6, \; 2x - 10y = 12.$$

A **[Board Term-1, 2011, Set–5]**

Sol. Given, $$x - 5y = 6 \Rightarrow y = \frac{x-6}{5}$$

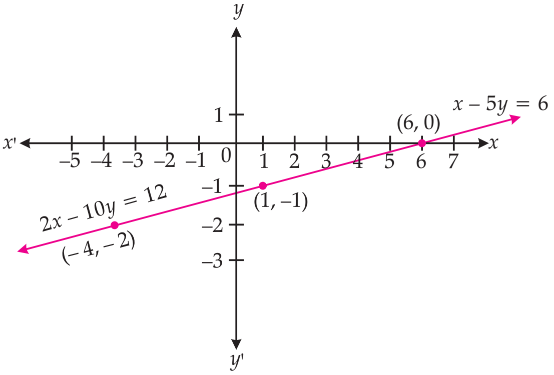
x	6	1	– 4
y	0	– 1	– 2

1

and $$2x - 10y = 12 \Rightarrow y = \frac{x-6}{5}$$

x	6	1	– 4
y	0	– 1	– 2

1



Since, the lines are coincident, so the system of linear equations is consistent with infinitely many solutions. 1

Q. 5. For what value of p will the following system of equations have no solution ?

$(2p - 1)x + (p - 1)y = 2p + 1$; $y + 3x - 1 = 0$.

A **[Board Term-1, 2011, Set–28]**

Sol. For $(2p - 1)x + (p - 1)y - (2p + 1) = 0$

$a_1 = 2p - 1$, $b_1 = p - 1$ and $c_1 = -(2p + 1)$ ½

and for $3x + y - 1 = 0$

$a_2 = 3$, $b_2 = 1$ and $c_2 = -1$ ½

The condition for no solution is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{2p-1}{3} = \frac{p-1}{1} \neq \frac{2p+1}{1}$$ ½

By $$\frac{2p-1}{3} = \frac{p-1}{1}$$

$$3p - 3 = 2p - 1$$

$$3p - 2p = 3 - 1$$

∴ $$p = 2$$ ½

From $$\frac{p-1}{1} \neq 2p + 1$$

We have $$p - 1 \neq 2p + 1 \Rightarrow 2p - p \neq -1 - 1$$

∴ $$p \neq -2$$

From $$\frac{2p-1}{3} \neq \frac{2p+1}{1}$$

⇒ $$2p - 1 \neq 6p + 3$$

⇒ $$4p \neq -4$$

∴ $$p \neq -1$$ 1

Hence, system has no solution when $p = 2$.

Q. 6. Find the value of k for which the following pair of equations have no solution :

$x + 2y = 3$, $(k - 1)x + (k + 1)y = (k + 2)$.

C + A **[Board Term-1, 2011, Set–52]**

Sol. Try yourself, Similar to Q. 5., of Short Answer Type Question-II.

Q. 7. Three lines $x + 3y = 6$, $2x - 3y = 12$ and $x = 50$ are enclosing a beautiful triangular park. Find the points of intersection of the lines graphically and the area of the park, if all measurements are in km.

AE **[Board Term-1, 2015, Set–MV98HN3]**

Sol. $$x + 3y = 6 \quad \text{...(i)}$$

and $$2x - 3y = 12 \quad \text{...(ii)}$$

For equation (i) : $y = \dfrac{6-x}{3}$

x	6	0
y	0	2

A(6, 0), B(0, 2)

Equation (ii) : $y = \dfrac{2x-12}{3}$

x	6	0
y	0	– 4

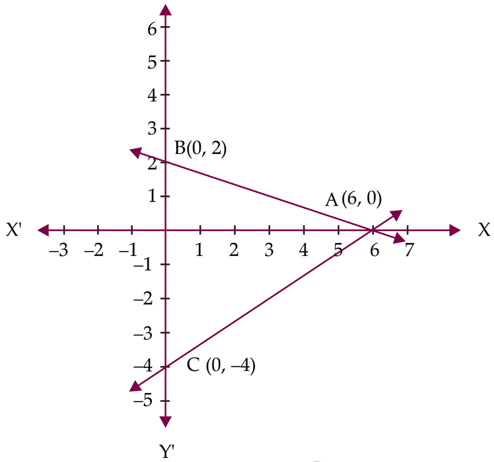
$A(6, 0)$, $C(0, -4)$ **1**

Vertices $A(6, 0)$, $B(0, 2)$ and $C(0, -4)$

$$\therefore \quad \text{Area of } \Delta ABC = \left(\frac{1}{2}\times 2\times 6+\frac{1}{2}\times 4\times 6\right)\text{ km}^2$$

$$= (6 + 12)\text{ km}^2$$

$$= 18\text{ km}^2 \quad \textbf{1}$$



1

Long Answer Type Questions

(4 marks each)

AI **Q. 1. For what values of m and n the following system of linear equations has infinitely many solutions.**
$3x + 4y = 12$
and $(m + n)x + 2(m - n)y = 5m - 1$
A [CBSE Comp. Set I/II/III 2018]

Sol. For infinitely many solutions,

$$\frac{3}{m+n} = \frac{4}{2(m-n)} = \frac{-12}{-(5m-1)} \quad \textbf{1}$$

$$\frac{3}{m+n} = \frac{4}{2(m-n)} \Rightarrow m - 5n = 0 \;...(1) \quad \textbf{1}$$

$$\frac{4}{2(m-n)} = \frac{12}{5m-1} \Rightarrow m - 6n = -1 \;...(2) \quad \textbf{1}$$

Solving (1) and (2), we get, $m = 5$ and $n = 1$ **1**

[CBSE Marking Scheme, 2018]

Commonly Made Error

- Many candidates are not able to solve this problem because of not having the basic idea of unique solution, infinitely many solutions and no solution.

Answering Tip

- Candidates must be familiar with all three condition for solvability like unique solution, infinitely many solutions and no solution.

Q. 2. For Uttarakhand flood victims two sections A and B of class X contributed ₹ 1,500. If the contribution of X-A was ₹ 100 less than that of X-B, find graphically the amounts contributed by both the sections. A [Board Term-1, 2016, Set-MV98HN3]

Sol. Let amounts contributed by two sections X-A and X-B be ₹ x and ₹ y.

$$x + y = 1{,}500 \quad ...(i)$$

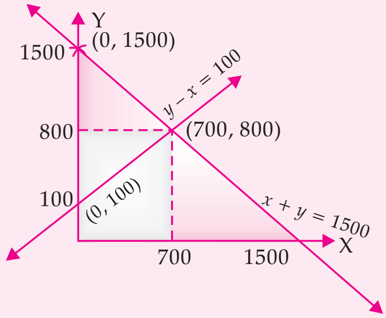
and $$y - x = 100 \quad ....(ii) \;\textbf{1}$$

From (i), $y = 1500 - x$

x	0	700	1,500
y	1,500	800	0

From (ii), $y = 100 + x$

x	0	700
y	100	800



Point of intersection = (700, 800) **1**

Hence, X-A contributed ₹700 and X-B contributed = ₹ 800. [CBSE Marking Scheme, 2016] **1**

Q. 3. Determine graphically whether the following pair of linear equations :

$$3x - y = 7$$

and $$2x + 5y + 1 = 0, \text{ has :}$$

(i) a unique solution
(ii) infinitely many solutions or
(iii) no solution. A [Board Term-1, 2015, Set–DDE - E]

Sol. From (i), $y = 3x - 7$

x	0	2	3
y	– 7	– 1	2

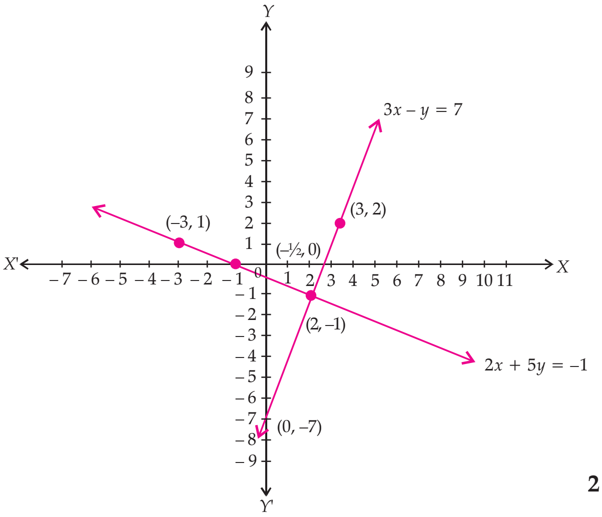
and from (ii), ½

$$2x + 5y + 1 = 0$$

$$\Rightarrow \quad y = \frac{-1-2x}{5}$$

x	2	–3
y	– 1	1

½



2

Since, point of intersection is (2, – 1). Hence, it has unique solution.

Hence, $x = 2$ and $y = -1$. 1

Q. 4. Draw the graphs of the pair of linear equations : $x + 2y = 5$ and $2x - 3y = -4$.

Also, find the points where the lines meet the x-axis. A [Board Term-1, 2015, Set–FHNQMGD]

Sol. Given, $x + 2y = 5$

$$\Rightarrow \quad y = \frac{5-x}{2}$$

x	1	3	5
y	2	1	0

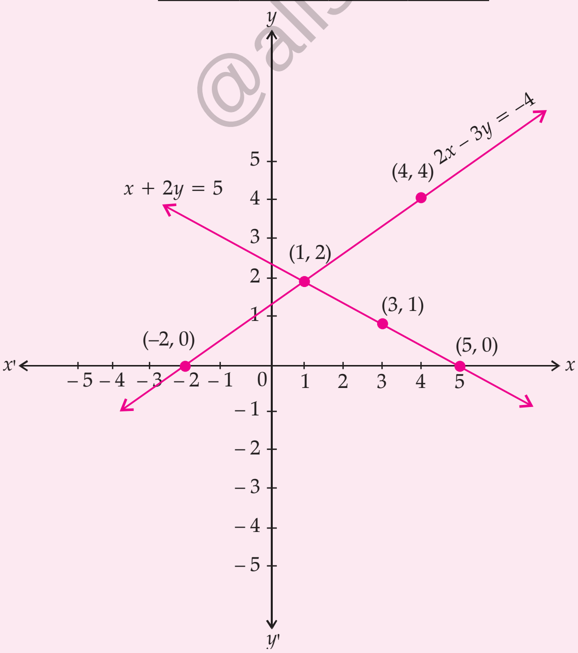
1

and $2x - 3y = -4$

$$\Rightarrow \quad y = \frac{2x+4}{3}$$

x	– 2	1	4
y	0	2	4

1



2

Thus, the lines meet x-axis at (5, 0) and (– 2 0) respectively. [CBSE Marking Scheme, 2015]

Q. 5. Solve graphically the pair of linear equations :

$3x - 4y + 3 = 0$ and $3x + 4y - 21 = 0$.

Find the co-ordinates of the vertices of the triangular region formed by these lines and x-axis. Also, calculate the area of this triangle.

A [Board Term-1, 2015, Set–DDE - E]

Sol. Given, $3x - 4y + 3 = 0$

$$\Rightarrow \quad y = \frac{3x+3}{4}$$

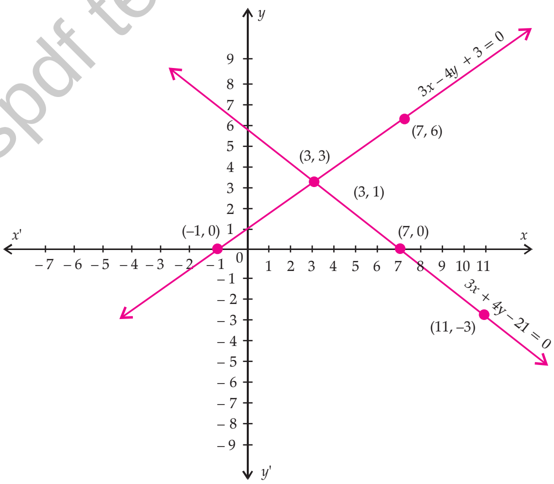
x	3	7	–1
y	3	6	0

and $3x + 4y - 21 = 0$

$$\Rightarrow \quad y = \frac{21-3x}{4}$$

x	3	7	11
y	3	0	–3

1



1

(i) These lines intersect each other at point (3, 3). Hence, $x = 3$ and $y = 3$. 1

(ii) The vertices of triangular region are (3, 3), (–1, 0) and (7, 0). 1

(iii) $$\text{Area of } \Delta = \frac{1}{2} \times 8 \times 3$$

Hence, Area of obtained Δ = 12 sq. units.

Q. 6. Solve the following pair of linear equations graphically :

$2x + 3y = 12$ and $x - y = 1$.

Find the area of the region bounded by the two lines representing the above equations and Y-axis.

A [Board Term-1, 2012, Set–58]

Sol. Given, $2x + 3y = 12$

$$\Rightarrow \quad y = \frac{12-2x}{3}$$

x	0	6	3
y	4	0	2

1

and $\quad x - y = 1$

$\Rightarrow \quad y = x - 1$

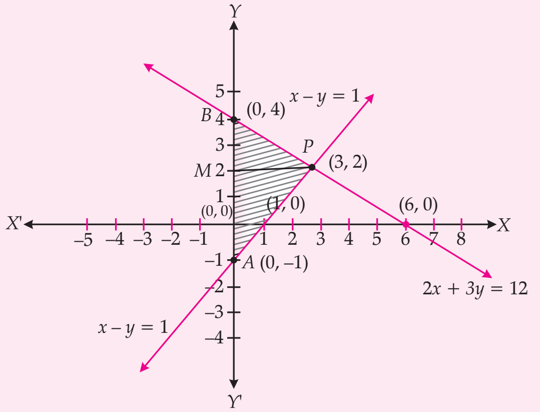
x	0	1	3
y	– 1	0	2

1

Plotting the above points and drawing lines joining them, we get the following graph.

Clearly, the two lines intersect at point $P(3, 2)$.
Hence, $x = 3$ and $y = 2$
Area of shaded triangle region = Area of ΔPAB

$$= \frac{1}{2} \times \text{base} \times \text{height}$$



1

$$= \frac{1}{2} \times AB \times PM$$

$$= \frac{1}{2} \times 5 \times 3$$

$= 7.5$ square unit. **1**

[CBSE Marking Scheme, 2012]

Q. 7. Solve the following pair of linear equations graphically :

$x + 3y = 6$ and $2x - 3y = 12.$

Also, shade the region bounded by the line $2x - 3y = 12$ and both the co-ordinate axes.

A **[Board Term-1 2013 FFC, 2012 Set-35, 48]**

Sol. Given, $\quad x + 3y = 6$ or $y = \dfrac{6 - x}{3}$ ...(i)

x	3	6	0
y	1	0	2

1

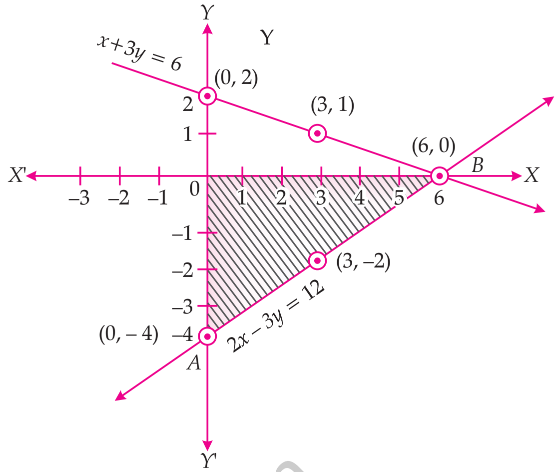
and $\quad 2x - 3y = 12$ ...(ii)

$$\Rightarrow \quad y = \frac{2x - 12}{3}$$

x	0	6	3
y	– 4	0	– 2

1

Plotting the above points and drawing lines joining them, we get the graphs of the equations $x + 3y = 6$ and $2x - 3y = 12$.



1

The two lines intersect each other at point B (6, 0).

Hence, $x = 6$ and $y = 0$.

Again, ΔOAB is the region bounded by the line $2x - 3y = 12$ and both the co-ordinate axes. **1**

Q. 8. Solve the following pair of linear equations graphically :

$$x - y = 1$$

and $\quad$ **$2x + y = 8.$**

Also, find the co-ordinates of the points where the lines represented by the above equation intersect y-axis. A **[Board Term-1 2012 Set 56]**

Sol. Given, $\quad x - y = 1$ or $y = x - 1$

x	2	3	– 1
y	1	2	–2

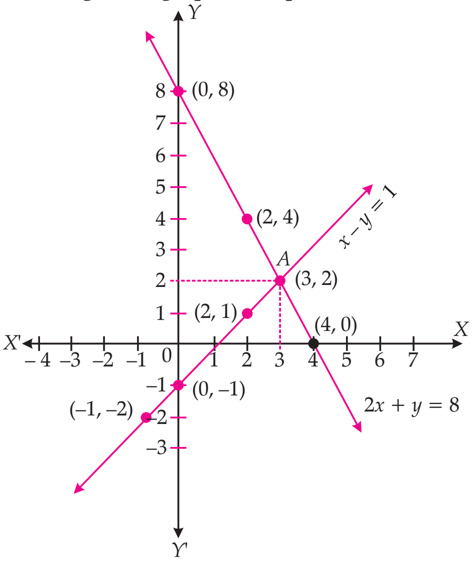
1

and $\quad 2x + y = 8$ or $y = 8 - 2x$

x	2	4	0
y	4	0	8

1

Plotting the above points and drawing a line joining them, we get the graphical representation.



1

The two lines intersect each other at point $A(3, 2)$.

$\therefore$ Solution of given equations is $x = 3$ and $y = 2$.

Again, $x - y = 1$ intersects y-axis at $(0, -1)$

and $2x + y = 8$ intersects y-axis at $(0, 8)$. **1**

Q. 9. Draw the graphs of the following equations :

$$2x - y = 1 \text{ and } x + 2y = 13$$

Find the solution of the equations from the graph and shade the triangular region formed by the lines and the Y-axis.

A **[Board Term-1, 2012 Set 52]**

Sol. Given, $2x - y = 1$

$\Rightarrow$ $y = 2x - 1$

x	0	1	3
y	– 1	1	5

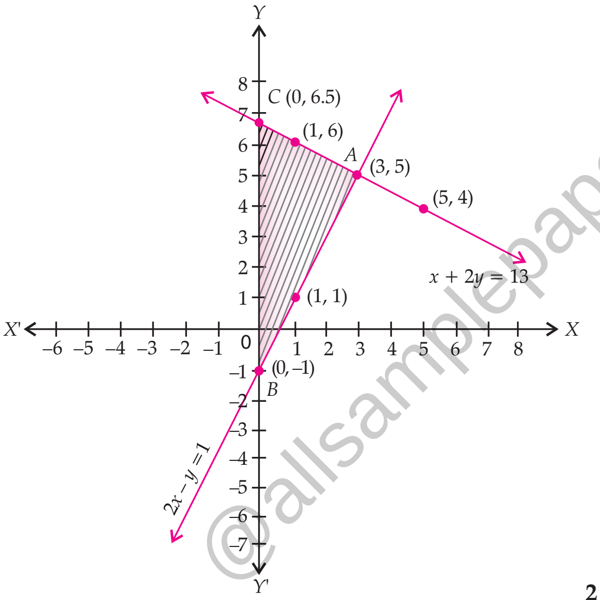
1

and $x + 2y = 13$

$\Rightarrow$ $y = \dfrac{13 - x}{2}$

x	1	3	5
y	6	5	4

1



2

Plotting the above points and drawing the lines joining them, we get the graph of above equations.
Two obtained lines intersect at point $A(3, 5)$.
Hence, $x = 3$ and $y = 5$.
ABC is the triangular shaded region formed by the obtained lines with the Y-axis.

Q. 10. Solve the following pair of equations graphically :

$$2x + 3y = 12 \text{ and } x - y - 1 = 0.$$

Shade the region between the two lines represented by the above equations and the X-axis. A **[Board Term-1, 2012, Set– 48]**

Sol. Given, $2x + 3y = 12$

$\Rightarrow$ $y = \dfrac{12 - 2x}{3}$

x	0	6	3
y	4	0	2

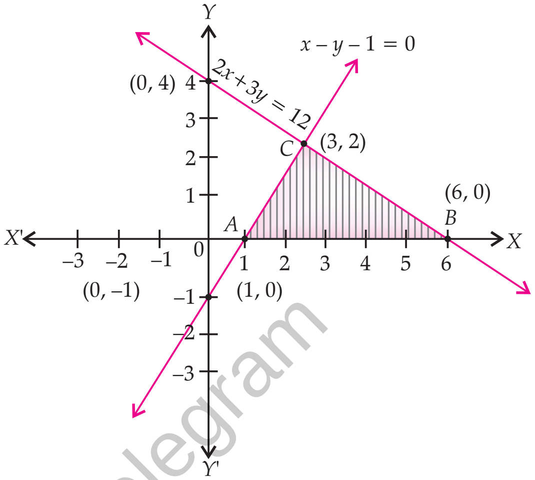
1

and $x - y = 1$

$\Rightarrow$ $y = x - 1$

x	0	1	3
y	–1	0	2

1



1

Plotting the above points and drawing the lines joining them, we get the above graph.

The two lines intersect each other at point $C(3, 2)$, Hence, $x = 3$ and $y = 2$. **1**

Thus, ΔABC is the region between the two lines represented by the given equations and the X-axis.

Q. 11. For what values of a and b does the following pair of linear equations have infinite number of solutions ?

$$2x + 3y = 7 \text{ and } a(x + y) - b(x - y) = 3a + b - 2$$

C + A **[Board Term-1, 2015, Set–CJTOQ]**

Sol. For equation $2x + 3y - 7 = 0$

$a_1 = 2, b_1 = 3$ and $c_1 = -7$

and $a(x + y) - b(x - y) = 3a + b - 2$

$ax + ay - bx + by = 3a + b - 2$

$(a - b)x + (a + b)y - (3a + b - 2) = 0$

$a_2 = a - b, b_2 = a + b$ and $c_2 = -(3a + b - 2)$

for infinite many solutions

or, $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

or, $$\frac{2}{a-b} = \frac{3}{a+b} = \frac{-7}{-(3a+b-2)}$$ **1**

$$\frac{2}{a-b} = \frac{7}{3a+b-2}$$

or, $2(3a + b - 2) = 7(a - b)$

or, $6a + 2b - 4 = 7a - 7b$

or, $a - 9b = -4$ ...(i) **1**

$$\frac{3}{a+b} = \frac{7}{3a+b-2}$$

or, $3(3a + b - 2) = 7(a + b)$

or, $9a + 3b - 6 = 7a + 7b$

or, $2a - 4b = 6$

or, $a - 2b = 3$ ...(ii) **1**

Subtracting eqn. (ii) from (i),

$$\begin{array}{r} a - 9b = -4 \\ a - 2b = 3 \\ - \quad + \qquad - \\ \hline -7b = -7 \end{array}$$

$\Rightarrow$ $b = 1$

On putting the value of b in eqn. (i), we get

$$a = 5$$ **1**

Hence, $a = 5$ and $b = 1$.

TOPIC-2
Algebraic Methods to Solve Pair of Linear Equations and Equations reducible to Linear Equations

Revision Notes

- **Algebraic Method :** We can solve the linear equations algebraically by **substitution method, elimination method** and **cross-multiplication method**.

1. Substitution Method :

(i) Find the value of one variable (say y) in terms of the other variable *i.e.*, x from either of the equations.

(ii) Substitute this value of y in other equation and reduce it to an equation in one variable.

(iii) Solve the equation so obtained and find the value of x.

(iv) Put this value of x in one of the equations to get the value of variable y.

2. Elimination Method :

(i) Multiply given equations with suitable constants, make either the x-coefficients or the y-coefficients of the two equations equal.

(ii) Subtract or add one equation from the other to get an equation in one variable.

(iii) Solve the equation so obtained to get the value of the variable.

(iv) Put this value in any one of the equation to get the value of the second variable.

Note :

(a) If in step (ii), we obtain a true equation involving no variable, then the original pair of equations has infinitely many solutions.

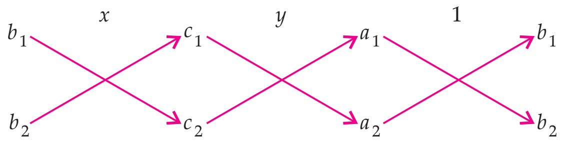
(b) If in step (ii), we obtain a false equation involving no variable, then the original pair of equations has no solution *i.e.*, it is inconsistent.

3. Cross-multiplication Method : If two simultaneous linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given, then a unique solution is given by :

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Then, $$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Note : To obtain the above result, following diagram may be helpful :



The arrows between the two numbers indicate that they are to be multiplied. The product with upward arrows are to be subtracted from the product with downward arrows.

- **Equations reducible to a pair of Linear Equations in two variables :** Sometimes, a pair of equations in two variables is not linear but can be reduced to linear form by making some suitable substitutions. Here, first we find the solution of new pair of linear equations and then find the solution for the given pair of equations.

Steps to be followed for solving word problems

S. No.	Problem type	Steps to be followed
1.	Age Problems	If the problem involves finding out the ages of two persons, take the present age of one person as x and of the other as y. Then, 'a' years ago, age of 1st person was '$x - a$' years and that of 2nd person was '$y - a$' and after 'b' years, age of 1st person will be '$x + b$' years and that of 2nd person will be '$y + b$' years. Formulate the equations and then solve them.
2.	Problems based on Numbers and Digits	Let the digit in unit's place be x and that in ten's place be y. The two-digit number is given by $10y + x$. On interchanging the positions of the digits, the digit in unit's place becomes y and in ten's place becomes x. The two digit number becomes $10x + y$. Formulate the equations and then solve them.
3.	Problems based on Fractions	Let the numerator of the fraction be x and denominator be y, then the fraction is $\frac{x}{y}$. Formulate the linear equations on the basis of conditions given and solve for x and y to get the value of the fraction.
4.	Problems based on Distance, Speed and Time	Speed $= \frac{\text{Distance}}{\text{Time}}$ or Distance = Speed × Time and Time $= \frac{\text{Distance}}{\text{Speed}}$. To solve the problems related to speed of boat going downstream and upstream, let the speed of boat in still water be x km/h and speed of stream be y km/h. Then, the speed of boat downstream $= (x + y)$ km/h and speed of boat upstream $= (x - y)$ km/h.
5.	Problems based on commercial Mathematics	For solving specific questions based on commercial mathematics, ● To the fare of 1 full ticket may be taken as ₹ x and the reservation charges may be taken as ₹ y, so that one full fare $= x + y$ and one half fare $= \frac{x}{2} + y$. ● To solve the questions of profit and loss, take the cost price of 1st article as ₹ x and that of 2nd article as ₹ y. ● To solve the questions based on simple interest, take the amount invested as ₹ x at some rate of interest and ₹ y at some other rate of interest.
6.	Problems based on Geometry and Mensuration	● Make use of angle sum property of a triangle ($\angle A + \angle B + \angle C = 180°$) in case of a triangle. ● In case of a parallelogram, opposite angles are equal and in case of a cyclic quadrilateral, opposite angles are supplementary.

Know the Formulae

1. If the equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are in the form

$$a_1x + b_1y = -c_1 \text{ and } a_2x + b_2y = -c_2$$

Then, we have by cross-multiplication

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

How it is done on GREENBOARD?

Q. Solve the following system of equations by substitution method.

$$x + 2y = 5 \text{ and } 2x + y = 4$$

Sol. : Step I : Given equations are $x + 2y = 5$ ...(i)

and $2x + y = 4$ ...(ii)

Step II : From $x + 2y = 5$, we get

$$x = 5 - 2y \quad ...(iii)$$

Step III : Putting this value of x in other equation $2x + y = 4$, we get

$$2(5 - 2y) + y = 4$$
$$10 - 4y + y = 4$$
$$3y = 10 - 4$$
$$y = \frac{6}{3} \Rightarrow 2$$

Setp IV : On putting this value of y in (iii)

Also $x = 5 - 2(2)$

$x = 1$

$\therefore$ $x = 1$ and $y = 2$.

Objective Type Questions

(1 mark each)

[A] Multiple Choice Questions :

Q. 1. A pair of linear equations which has a unique solution $x = 2, y = -3$ is :

(a) $x + y = -1$, $2x - 3y = -5$

(b) $2x + 5y = -11$, $4x + 10y = -22$

(c) $2x - y = 1$, $3x + 2y = 0$

(d) $x - 4y - 14 = 0$, $5x - y - 13 = 0$

U **[NCERT Exemp.]**

Sol. Correct option : (b) and (d)

Explanation :

(b) $2x + 5y = -11$ and $4x + 10y = -22$

Put $x = 2$ and $y = -3$ in both the equations,

$LHS = 2x + 5y \Rightarrow 2 \times 2 + (-3)$

$\Rightarrow 4 - 15 = -11 = \text{RHS}$

$LHS = 4x + 10y \Rightarrow 4(2) + 10(-3)$

$\Rightarrow 8 - 30 = -22 = RHS$

(d) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$

$x - 4y = 14$ and $5x - y = 13$

Put $x = 2$ and $y = -3$ in both the equations,

$LHS = x - 4y \Rightarrow 2 - 4(-3) \Rightarrow 2 + 12 = 14 = RHS$

$LHS = 5x - y \Rightarrow 5(2) - (-3) \Rightarrow 10 + 3 = 13 = RHS$

Q. 2. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(a) 35 and 15 **(b)** 35 and 20

(c) 15 and 35 **(d)** 25 and 25

C **[NCERT Exemp.]**

Sol. Correct option : (d)

Explanation : Let the number of ₹ 1 coins $= x$ and the number of ₹ 2 coins $= y$

So, according to the question,

$$x + y = 50 \quad ...(i)$$
$$x + 2y = 75 \quad ...(ii)$$

Subtracting equation (i) from (ii),

$$y = 25$$

Substituting value of y in (i),

$$x = 25$$

So, $y = 25$ and $x = 25$

Q. 3. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father are, respectively :

(a) 4 and 24 **(b)** 5 and 30

(c) 6 and 36 **(d)** 3 and 24

C **[NCERT Exemp.]**

Sol. Correct option : (c)

Explanation : Let the present age of father be x years and the present age of son be y years.

$\therefore$ According to the question,

$$x = 6y \quad ...(i)$$

Age of the father after four years $= (x + 4)$ years

Age of son after four years $= (y + 4)$ years

Now, according to the question,

$$x + 4 = 4(y + 4) \quad ...(ii)$$

$\Rightarrow$ $x + 4 = 4y + 16$

$\Rightarrow 6y - 4y = 16 - 4$ [From (i), $x = 6y$]

$\Rightarrow$ $2y = 12$

$\Rightarrow$ $y = 6$

$\therefore$ $x = 6 \times 6 = 36$ years

and $y = 6$ years

So, the present ages of the son and the father are 6 years and 36 years, respectively.

[B] Very Short Answer Type Questions :

Q. 1. If $x = a$ and $y = b$ is the solution of the pair of equations $x - y = 2$ and $x + y = 4$, find the values of a and b. U **[CBSE Compt Set I/II/III 2018]**

Sol. Solving for x and y and getting $x = 3$ and $y = 1$ ½

$\therefore a = 3$ and $b = 1$. ½

[CBSE Marking Scheme, 2018]

Short Answer Type Questions-I

(2 marks each)

AI **Q. 1. Solve the following pair of linear equations by cross multiplication method :**

$$x + 2y = 2$$
$$x - 3y = 7$$

U **[Board Term-1, 2016, Set-O4YP6G7]**

Sol.
$$x + 2y - 2 = 0$$
$$x - 3y - 7 = 0$$

Using the formula

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \quad 1$$

$$\frac{x}{-14-6} = \frac{y}{-2+7} = \frac{1}{-3-2}$$

$$\Rightarrow \frac{x}{-20} = \frac{y}{5} = \frac{-1}{5}$$

$$\Rightarrow \frac{x}{-20} = \frac{-1}{5}$$

$$\Rightarrow \frac{y}{5} = \frac{-1}{5}$$

$$\Rightarrow x = 4$$

and $y = -1$ 1

[CBSE Marking Scheme, 2016]

Q. 2. Solve the following pair of linear equations by substitution method :

$$3x + 2y - 7 = 0$$

and $4x + y - 6 = 0$

U **[Board Term-1, 2015, Set–CJTOQ]**

Sol. Given, $3x + 2y - 7 = 0$ ....(i)

and $4x + y - 6 = 0$ ....(ii)

From eqn. (ii), we have

$$y = 6 - 4x \quad ...(iii) \ ½$$

On putting this value of y in eqn. (i), we get

$$3x + 2(6 - 4x) - 7 = 0$$
$$3x + 12 - 8x - 7 = 0$$
$$5 - 5x = 0$$
$$5x = 5$$

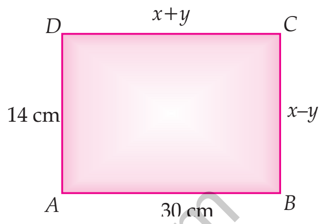
$\therefore$ $x = 1$ ½

Substituting this value of x in (iii), we get,

$$y = 6 - 4 \times 1$$
$$y = 2 \quad ½$$

Hence, values of x and y are 1 and 2 respectively. ½

AI **Q. 3. In Fig., ABCD is a rectangle. Find the values of x and y.**



[CBSE Delhi & O.D. 2018]

A **[Board Term-I, 2012, Set 30]**

Sol. Since, $AB = CD$ and $BC = AD$ 1

$\Rightarrow$ $x + y = 30$

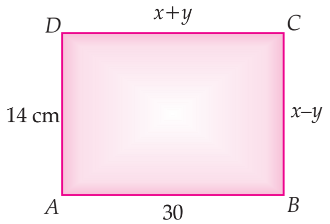
and $x - y = 14$

Solving to get $x = 22$ and $y = 8$. ½+½

[CBSE Marking Scheme, 2018]

Detailed Answer :

$ABCD$ is a rectangle



Since, $AB = CD$

or, $x + y = 30$ ...(i)

and $BC = AD$

or, $x - y = 14$ ...(ii) **1**

Adding equations (i) and (ii), we get

$$2x = 44$$
$$x = \frac{44}{2} = 22 \quad ½$$

Substituting $x = 22$ in equation (i), we get

$$22 + y = 30$$
$$y = 30 - 22 = 8$$

$\therefore$ $x = 22$ cm and $y = 8$ cm ½

Q. 4. Solve : $99x + 101y = 499$

and $101x + 99y = 501$.

A **[Board Term-1, 2012, Set–55]**

Sol. Given, $99x + 101y = 499$ …(i)

and $101x + 99y = 501$ …(ii)

Adding eqn. (i) and (ii), we get

$$200x + 200y = 1000$$

or, $x + y = 5$ ...(iii)

Subtracting eqn. (ii) from eqn. (i), we get ½

$$-2x + 2y = -2$$

or, $x - y = 1$ ...(iv) ½

Adding equations (iii) and (iv), we get

$$2x = 6$$

$\therefore$ $x = 3$ ½

Substituting the value of x in eqn. (iii), we get

$$3 + y = 5$$

$$y = 2$$ ½

Q. 5. Solve the following system of linear equations by substitution method :

$$2x - y = 2 \quad ...(i)$$

and $x + 3y = 15$ ...(ii)

U [Board Term-1, 2012, Set–50]

Sol. Try yourself, Similar to Q. No. 2 of Short Answer Type Question-I.

Q. 6. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Shristi paid ₹ 27 for a book kept for seven days, while Rekha paid ₹ 21 for the book she kept for five days. Find the fixed charge and the additional charge paid by them.

A; E [Board Term-1, 2015, Set–O4YP6G7]

Sol. Let fixed charges for reading book = ₹ x

Let additional charges per day = ₹ y

then

$$x + 4y = 27 \quad ...(i)$$

$$x + 2y = 21 \quad ...(ii)$$

On solving both the equations

$$x = ₹\ 15 \text{ and } y = ₹\ 3$$

Hence, Shristi paid additional charges = ₹ 12

Rekha paid additional charges = ₹ 6 **2**

Q. 7. The incomes of two persons A and B are in the ratio 8 : 7 and the ratio of their expenditures is 19 : 16 If their savings are ₹ 2550 per month, find their monthly income.

A; E [Board Term-1, 2016, Set–ORDAWEZ]

Sol. Let income of $A = 8x$ and

income of $B = 7x$.

Also their expenditures be $19y$ and $16y$.

$\Rightarrow$ $8x - 19y = 2550$ ...(i)

and $7x - 16y = 2550$ ...(ii)

Solving the equations

$$x = 1530 \text{ and } y = 510$$

$\therefore$ Salary of $A = 12240$

Salary of $B = 10710$ **2**

Short Answer Type Questions-II

(3 marks each)

Q. 1. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their respective ages.

A [Board Term-1, 2015, Set–WJQZQBN]

Sol. Let age of father and son be x and y respectively.

Then, $x + y = 40$ ...(i) **1**

and $x = 3y$ ...(ii) **1**

By solving eqns. (i) and (ii), we get

$$x = 30 \text{ and } y = 10$$ **1**

Thus, the ages of father and son are 30 years and 10 years. [CBSE Marking Scheme, 2015]

Q. 2. Solve using cross multiplication method :

$$5x + 4y - 4 = 0$$

and $x - 12y - 20 = 0$

U [Board Term-1, 2015, Set–FHN8MGD]

Sol. Given : $5x + 4y - 4 = 0$ ...(i)

$$x - 12y - 20 = 0 \quad ...(ii)$$

By cross-multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$ **1**

$$\frac{x}{-80-48} = \frac{y}{-4+100} = \frac{1}{-60-4}$$

$$\frac{x}{-128} = \frac{y}{96} = -\frac{1}{-64}$$ **1**

$$\frac{x}{-128} = \frac{1}{-64} \text{ and } \frac{y}{96} = \frac{1}{-64}$$

Hence, $x = 2$ and $y = \frac{-3}{2}$ **1**

AI **Q. 3. A part of monthly hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay ₹ 3,000 as hostel charges whereas Mansi who takes food for 25 days has to pay ₹ 3,500 as hostel charges. Find the fixed charges and the cost of food per day.**

[Board Term-1, 2016, Set–MV98HN3]

A [Board Term-1, 2015, Set–FHN8MGD]

Sol. Let fixed charge be x and per day food cost be y

Then, $x + 20y = 3000$ ...(i)

and $x + 25y = 3500$ ...(ii) **1**

Subtracting (i) from (ii), we get

$$\begin{array}{r} x + 25y = 3500 \\ x + 20y = 3000 \\ - \quad - \quad\quad - \\ \hline 5y = 500 \end{array}$$ **1**

$$y = 100$$

Substituting this value of y in (i), we get

$$x + 20(100) = 3000$$
$$x = 1000$$

$\therefore \quad x = 1000$ and $y = 100$ **1**

Hence, fixed charge and cost of food per day are ₹ 1,000 and ₹ 100.

Q. 4. Solve for x and y :

$$\frac{x}{2} + \frac{2y}{3} = -1$$

and $$x - \frac{y}{3} = 3$$

U **[Board Term-1, 2015, Set–CJTOQ] [NCERT]**

Sol. Given, $$\frac{x}{2} + \frac{2y}{3} = -1$$

$$\frac{3x+4y}{6} = -1$$ **1**

or, $$3x + 4y = -6 \quad ...(i)$$

and $$\frac{x}{1} - \frac{y}{3} = 3$$

$$\frac{3x-y}{3} = 3$$

or, $$3x - y = 9 \quad ...(ii)$$

On subtracting eqn. (ii) from eqn. (i), **1**

$$3x + 4y = -6$$
$$3x - y = 9$$
$$- \quad + \qquad -$$
$$5y = -15$$

$\therefore$ $$y = -3$$

Putting $y = -3$ in eq (i), we get

$$3x + 4(-3) = -6$$
$$3x - 12 = -6$$
$$3x = 12 - 6$$
$$3x = 6$$

$\therefore$ $$x = 2$$

Hence, $x = 2$ and $y = -3$. **1**

Q. 5. Places A and B are 80 km apart from each other on a highway. A car starts from A and another from B at the same time. If they move in same direction they meet in 8 hours and if they move towards each other they meet in 1 hour 20 minutes. Find the speed of cars. A **[CBSE SQP 2018]**

Sol. Let the speed of car at A be x km/h **1**

And the speed of car at B y km/h

Case 1 $\quad 8x - 8y = 80$

or, $\quad x - y = 10$

Case 2 $\quad \frac{4}{3}x + \frac{4}{3}y = 80$

or, $\quad x + y = 60$ **1**

On solving, $x = 35$ and $y = 25$

Hence, speed of cars at A and B are 35 km/h and 25 km/h respectively. **1**

[CBSE Marking Scheme, 2018]

Detailed Answer :

Let the speed of the car 1 from A be x km/hr.

and speed of the car 2 from B be y km/hr. **1**

Same direction :

Distance covered by car 1 = 80 + (distance covered by car 2)

$$\Rightarrow \quad 8x = 80 + 8y$$
$$\Rightarrow \quad 8x - 8y = 80$$
$$\Rightarrow \quad x - y = 10 \quad ...(i)$$

Opposite direction :

Distance covered by car 1 + distance covered by car 2 = 80 km

$$\frac{4}{3}x + \frac{4}{3}y = 80$$

$$\Rightarrow \quad x + y = 60 \quad ...(ii)$$

Adding eq. (i) and (ii), we get

$$2x = 70$$
$$x = 35$$

substituting $x = 35$ in eq. (i)

$$y = 25$$

$\therefore$ Speed of the car 1 from A = 35 km/hr **1**

and speed of the car 2 of from B = 25 km/hr **1**

Commonly Made Error

- Some candidates, are not able to frame this word problem into equation.

Answering Tip

- Emphasis on solving such type of application based problem.

Q. 6. 2 men and 7 boys can do a piece of work in 4 days. It is done by 4 men and 4 boys in 3 days. How long would it take for one man or one boy to do it ? A **[Board Term-1, 2013, LK–59]**

Sol. Let one man can finish the work in x days and one boy can finish the same work in y days.

Then,work done by one man in one day $= \frac{1}{x}$

and work done by one boy in one day $= \frac{1}{y}$

According to the problem,

$$\frac{2}{x} + \frac{7}{y} = \frac{1}{4} \quad ...(i)$$

and $$\frac{4}{x} + \frac{4}{y} = \frac{1}{3} \quad ...(ii)$$

Let $\frac{1}{x}$ be a and $\frac{1}{y}$ be b, then

$$2a + 7b = \frac{1}{4} \quad \text{...(iii)}$$

and $$4a + 4b = \frac{1}{3} \quad \text{...(iv)} \ \mathbf{1}$$

On multiplying eqn. (iii) by 2 and subtract eqn. (iv) from it

$$10b = \frac{1}{6} \quad \mathbf{1}$$

or $$b = \frac{1}{60} \Rightarrow \frac{1}{y}$$

$$\Rightarrow \quad y = 60 \text{ days.}$$

Putting $b = \frac{1}{60}$ in equation (iii),

$$2a + \frac{7}{60} = \frac{1}{4}$$

$$\Rightarrow \quad 2a = \frac{1}{4} - \frac{7}{60}$$

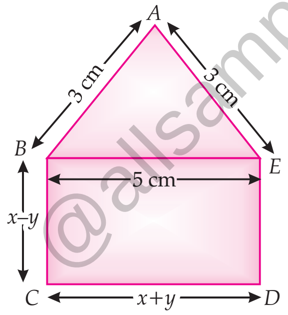
$$\Rightarrow \quad a = \frac{1}{15}$$

So $$\frac{1}{15} = \frac{1}{x}$$

$$\therefore \quad x = 15 \text{ days.}$$

Hence, one man can finish it in 15 days and boy in 60 days. **1**

Q. 7. In the figure below *ABCDE* is a pentagon with *BE* || *CD* and *BC* || *DE*. *BC* is perpendicular to *CD*. If the perimeter of ABCDE is 21 cm, find the values of x and y. A **[Board Term-I, 2011, Set– 45, 53]**



Sol. Since, $BC \parallel DE$ and $BE \parallel CD$ with $BC \perp CD$, $BCDE$ is a rectangle.

Since, $BE = CD, \therefore x + y = 5$ ...(i)

and $DE = BC \Rightarrow x - y$ **1**

Since, perimeter of $ABCDE$ is 21

$\therefore \; AB + BC + CD + DE + EA = 21$

$\Rightarrow 3 + (x - y) + (x + y) + (x - y) + 3 = 21$

$\Rightarrow \quad 6 + 3x - y = 21$

$\Rightarrow \quad 3x - y = 15$ ...(ii) **1**

Adding eqns. (i) and (ii), we get

$$4x = 20$$

$$\therefore \quad x = 5$$

On substituting the value of x in (i), we get

$$y = 0$$

$$\therefore \quad x = 5 \text{ and } y = 0. \quad \mathbf{1}$$

Q. 8. Solve for x and y :

$$\frac{x+1}{2} + \frac{y-1}{3} = 9 \text{ and } \frac{x-1}{3} + \frac{y+1}{2} = 8.$$

U **[Board Term-1, 2011, Set–52]**

Sol. Given, $$\frac{x+1}{2} + \frac{y-1}{3} = 9$$

$\Rightarrow \quad 3(x + 1) + 2(y - 1) = 54$

$\Rightarrow \quad 3x + 2y = 53$ ...(i)

and $$\frac{x-1}{3} + \frac{y+1}{2} = 8$$

$\Rightarrow \quad 2(x - 1) + 3(y + 1) = 48$

$\Rightarrow \quad 2x + 3y = 47$ ... (ii) **1**

Multiply eqn. (i) by 3, $\quad 9x + 6y = 159$

Multiply eqn. (ii) by 2, $\quad 4x + 6y = 94$

$\quad\quad\quad\quad\quad\quad\quad - \quad - \quad\quad -$

On subtracting $\quad 5x = 65$

$$\therefore \quad x = \frac{65}{5} \Rightarrow 13 \quad \mathbf{1}$$

Substituting the value of x in eqn. (ii),

$$2(13) + 3y = 47$$

$$3y = 47 - 26 \Rightarrow 21$$

$$\therefore \quad y = \frac{21}{3} \Rightarrow 7$$

Hence $\quad x = 13$ and $y = 7$. **1**

Q. 9. Solve for x and y :

$$\frac{6}{x-1} - \frac{3}{y-2} = 1 \text{ and}$$

$$\frac{5}{x-1} + \frac{1}{y-2} = 2, \text{ where } x \neq 1 \text{ and } y \neq 2.$$

U **[Board Term-1, 2011, Set–21]**

Sol. Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$

Then, given equations become

$$6p - 3q = 1 \quad \text{...(i)}$$

and $$5p + q = 2 \quad \text{...(ii)} \ \tfrac{1}{2}$$

Multiply eqn. (ii) by 3 and add in eqn. (i), we get

$$21p = 7$$

$$\therefore \quad p = \frac{7}{21} \Rightarrow \frac{1}{3} \quad \mathbf{1}$$

Putting this value of p in equation (i),

$$6\left(\frac{1}{3}\right) - 3q = 1$$

$$\Rightarrow \quad 2 - 3q = 1$$

$$\therefore \quad q = \frac{1}{3} \quad \mathbf{1}$$

Now, $$\frac{1}{x-1} = p \Rightarrow \frac{1}{3}$$

$$\Rightarrow \quad x - 1 = 3$$

$$\therefore \quad x = 4$$

and $$\frac{1}{y-2} = q \Rightarrow \frac{1}{3}$$

$\Rightarrow \quad y - 2 = 3$

$\therefore \quad y = 5$

Hence, $\quad x = 4$ and, $y = 5$. ½

Q. 10. Solve the following pair of equations for x and y :

$$\frac{a^2}{x} - \frac{b^2}{y} = 0 \text{ and } \frac{a^2b}{x} + \frac{b^2a}{y} = a + b,$$

where $x \neq 0$; $y \neq 0$.

U [Board Term-1, 2011, Set–39]

Sol. Putting $p = \frac{1}{x}$ and $q = \frac{1}{y}$ in the given equations,

$$a^2p - b^2q = 0 \qquad \text{...(i)}$$

$$a^2bp + b^2aq = a + b \qquad \text{...(ii) } \mathbf{1}$$

On multiplying eqn. (i), by a

$$a^3p - b^2aq = 0 \qquad \text{...(iii)}$$

Adding eqn. (ii) and eqn. (iii), we get

$$(a^3 + a^2b)p = a + b$$

$$\Rightarrow \quad p = \frac{(a+b)}{a^2(a+b)} \Rightarrow \frac{1}{a^2} \qquad \mathbf{1}$$

Substituting the value of p in eqn. (i),

$$a^2\left(\frac{1}{a^2}\right) - b^2q = 0$$

$$\therefore \quad q = \frac{1}{b^2}$$

Now, $\quad p = \frac{1}{x} \Rightarrow \frac{1}{a^2}$

$$\therefore \quad x = a^2$$

and $\quad q = \frac{1}{y} \Rightarrow \frac{1}{b^2}$

$$\therefore \quad y = b^2$$

Hence, $x = a^2$ and $y = b^2$. **1**

Q. 11. Solve for x and y :

$$ax + by = \frac{a+b}{2}$$

$$3x + 5y = 4.$$

U [Board Term-1, 2011, Set–44]

Sol. Given, $\quad ax + by = \frac{a+b}{2}$

or $\quad 2ax + 2by = a + b \qquad$ ...(i)

and $\quad 3x + 5y = 4 \qquad$ ...(ii) **1**

Multiplying eqn. (i) by 5 and (ii) by $2b$, and subtracting,

$$10ax + 10by = 5a + 5b$$

$$6bx + 10by = 8b$$

$$\underline{- \quad - \quad -}$$

$$x(10a - 6b) = 5a - 3b \qquad \mathbf{1}$$

$$\therefore \quad x = \frac{5a-3b}{2(5a-3b)} = \frac{1}{2}$$

Substituting $x = \frac{1}{2}$ in eqn. (ii), we get

$$3\cdot\frac{1}{2} + 5y = 4$$

$$5y = 4 - \frac{3}{2}$$

$$y = \frac{5}{2\times 5} \Rightarrow \frac{1}{2}$$

Hence, $\quad x = \frac{1}{2}$ and $y = \frac{1}{2}$. **1**

Q. 12. Solve the following pair of equations for x and y :

$$4x + \frac{6}{y} = 15 \text{ and } 6x - \frac{8}{y} = 14$$

and also find the value of p such that $y = px - 2$.

U [Board Term-1, 2011, Set–60]

Sol. Let $\frac{1}{y} = a$, the given equations become

$$4x + 6a = 15 \qquad \text{...(i)}$$

and $\quad 6x - 8a = 14 \qquad$ ...(ii) ½

Multiply eqn. (i) by 4 and eqn. (ii) by 3 and adding,

$$16x + 24a = 60$$

$$\underline{18x - 24a = 42}$$

$$34x = 102$$

$$\therefore \quad x = \frac{102}{34} \Rightarrow 3 \qquad \text{½}$$

Substituting the value of x in eqn. (i),

$$4(3) + 6a = 15$$

$$\Rightarrow \quad 6a = 15 - 12 = 3$$

$$\Rightarrow \quad a = \frac{3}{6} \Rightarrow \frac{1}{2}$$

$$\because \quad a = \frac{1}{y} \Rightarrow \frac{1}{2}$$

$$\therefore \quad y = 2 \qquad \mathbf{1}$$

Hence, $\quad x = 3$ and $y = 2$

Again, $\quad y = px - 2$

$$\Rightarrow \quad 2 = p(3) - 2$$

$$\Rightarrow \quad 3p = 4$$

$$\therefore \quad p = \frac{4}{3}. \qquad \mathbf{1}$$

Q. 13. Find whether the following pair of linear equations has a unique solution. If yes, find the solution :

$$7x - 4y = 49 \text{ and } 5x - 6y = 57.$$

U [Board Term-1, 2011, Set–39]

Sol. Given, $\quad 7x - 4y = 49 \qquad$ ...(i)

and $\quad 5x - 6y = 57 \qquad$ ...(ii)

On comparing with the equation $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2x + c_2 = 0$

$$a_1 = 7, b_1 = -4, c_1 = -49$$

and $\quad a_2 = 5, b_2 = -6, c_2 = -57$

Since, $\quad \frac{a_1}{a_2} = \frac{7}{5}$ and $\frac{b_1}{b_2} = \frac{4}{6}$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \qquad \mathbf{1}$$

So, system has a unique solution.

Multiply eqn. (i) by 5 and eqn. (ii) by 7 and subtracting,

$$35x - 20y = 245$$

$$35x - 42y = 399 \qquad \mathbf{1}$$

$$\underline{- \quad + \quad -}$$

$$22y = -154$$

$$\Rightarrow \quad y = -7$$

Putting the value of y in eqn. (ii),

$$5x - 6(-7) = 57$$

$$\Rightarrow \quad 5x = 57 - 42 \Rightarrow 15$$

$$\therefore \quad x = 3$$

Hence, $x = 3$ and $y = -7$. **1**

Q. 14. Raghav scored 70 marks in a test, getting 4 marks for each right answer and losing 1 mark for each wrong answer. Had 5 marks been awarded for each correct answer and 2 marks been deducted for each wrong answer, then Raghav would have scored 80 marks. How many questions were there in the test?

A; E **[Board Term-1, 2015, Set–DDE-E] 4**

Sol. Let number of right answers be x.

Let number of wrong answers be y.

As per question

$$4x - y = 70 \quad \text{...(i)}$$

$$5x - 2y = 80 \quad \text{...(ii)}$$

2 × eq. (i) – eq. (ii),

$$8x - 2y = 140$$

$$5x - 2y = 80$$

$$\underline{- \quad + \quad -}$$

$$3x = 60 \quad \textbf{2}$$

$$\Rightarrow \quad x = 20$$

Substituting the value of x in eq (i) to get value of y,

$$4(20) - y = 70$$

$$\Rightarrow \quad 80 - y = 70$$

$$\therefore \quad y = 10 \quad \textbf{1}$$

Hence, total number of questions are = 20 + 10 = 30.

Q. 15. In a painting competition of a school a child made Indian national flag whose perimeter was 50 cm. Its area will be decreased by 6 square cm, if length is decreased by 3 cm and breadth is increased by 2 cm then find the dimension of flag.

A; E **[Board Term-1, 2015, Set–FHN8M9D] 4**

Sol. Let length of the flag be x cm and breadth of the flag be y cm

$$2(x + y) = 50$$

or, $$x + y = 25 \quad \text{....(i)}$$

$$(x - 3)(y + 2) = xy - 6$$

or, $$xy + 2x - 3y - 6 = xy - 6$$

or, $$2x - 3y = 0 \quad \text{....(ii)}$$

On solving the eqns. (i) and (ii),

$$x = 15 \text{ cm and } y = 10 \text{ cm}$$

∴ Length of the flag = 15 cm

and Breadth of the flag = 10 cm **3**

Long Answer Type Questions (4 marks each)

Q. 1. 4 chairs and 3 tables cost ₹ 2100 and 5 chairs and 2 tables cost ₹ 1750. Find the cost of one chair and one table separately.

A **[Board Term-1, 2015, Set–WJQZQBN]**

Sol. Let cost of 1 chair be ₹ x and cost of 1 table be ₹ y

According to the question,

$$4x + 3y = 2100 \quad \text{...(i)}$$

and $$5x + 2y = 1750 \quad \text{...(ii)} \ \textbf{1}$$

Multiplying eqn. (i) by 2 and eqn. (ii) by 3, we get

$$8x + 6y = 4200 \quad \text{...(iii)}$$

$$15x + 6y = 5250 \quad \text{...(iv)} \ \textbf{1}$$

eqn. (iv) – eqn. (iii)

$$\Rightarrow \quad 7x = 1050$$

$$\therefore \quad x = 150 \quad \textbf{1}$$

Substituting the value of x in (i) we get $y = 500$

Thus, the cost of one chair and one table are ₹ 150 and ₹ 500 respectively.

[CBSE Marking Scheme, 2015] 1

AI **Q. 2. Solve the following pair of equations :**

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \text{ and } \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1.$$

U **[Board Term-1, 2015, Set–WJQZQBN]**

Sol. Substituting $\dfrac{1}{\sqrt{x}} = X$ and $\dfrac{1}{\sqrt{y}} = Y$

$$2X + 3Y = 2 \quad \text{....(i)}$$

and $$4X - 9Y = -1 \quad \text{....(ii)} \ \textbf{1}$$

Multiplying eqn. (i) by 3, and add in (ii), we get

$$4X - 9Y = -1$$

$$\underline{6X + 9Y = 6}$$

$$10X \quad = 5$$

$$10X = 5 \text{ or } X = \frac{5}{10} \Rightarrow \frac{1}{2}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{2} \quad \therefore \quad x = 4 \quad \textbf{1}$$

Putting the value of X in eqn. (i), we get

$$2\cdot\frac{1}{2}+3Y = 2$$

$$3Y = 2-1$$

$$Y = \frac{1}{3} \quad \mathbf{1}$$

$$\Rightarrow \quad Y = \frac{1}{3} \text{ or } \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow y = 9$$

Hence, $x = 4$ and $y = 9$. **1**

Q. 3. Solve for x and y :

$$\mathbf{2x - y + 3 = 0}$$

and $\mathbf{3x - 5y + 1 = 0}$

U [Board Term-1, 2015, Set–DDE-M]

Sol. Given, $2x - y + 3 = 0$ ...(i)

and $3x - 5y + 1 = 0$ ...(ii)

Multiplying eqn. (i) by 5, and subtracting (ii)

$$10x - 5y + 15 = 0$$
$$3x - 5y + 1 = 0$$
$$- \quad + \quad -$$

$$7x + 14 = 0 \quad \mathbf{1}$$

$$7x = -14$$

$$\therefore \quad x = \frac{-14}{7} = -2 \quad \mathbf{1}$$

Substituting the value of x in eqn. (i),

$$2(-2) - y + 3 = 0$$

$$-4 - y + 3 = 0 \quad \mathbf{1}$$

or, $-y - 1 = 0$

$\therefore$ $y = -1$

Hence, $x = -2$ and $y = -1$ **1**

Q. 4. The ratio of incomes of two persons is 11 : 7 and the ratio of their expenditures is 9 : 5. If each of them manages to save ₹ 400 per month, find their monthly incomes. A [Board Term-1, 2012, Set–38]

Sol. Let the incomes of two persons be $11x$ and $7x$.

Also the expenditures of two persons be $9y$ and $5y$.

$\therefore$ $11x - 9y = 400$ ...(i)

and $7x - 5y = 400$ ...(ii)

Multiplying eqn. (i) by 5 and eqn. (ii) by 9 and subtracting,

$$55x - 45y = 2{,}000 \quad \text{...(iii)}$$
$$63x - 45y = 3{,}600 \quad \text{...(iv)} \; \mathbf{1}$$
$$- \quad + \quad -$$

On subtracting, $-8x = -1600$

or $x = \dfrac{-1{,}600}{-8} \Rightarrow 200$ **1**

Hence, their monthly incomes are 11×200 $=$ ₹ 2200 and $7 \times 200 =$ ₹ 1400.

[CBSE Marking Scheme, 2012] **2**

Q. 5. If 2 is subtracted from the numerator and 1 is added to the denominator, a fraction becomes $\frac{1}{2}$, but when 4 is added to the numerator and 3 is subtracted from the denominator, it becomes $\frac{3}{2}$. Find the fraction.

A [Board Term-1, 2012, Set–48]

Sol. Let the fraction be $\dfrac{x}{y}$.

According to the problem,

$$\frac{x-2}{y+1} = \frac{1}{2}$$

or, $2x - 4 = y + 1$

or, $2x - y = 5$ ...(i) **1**

Also, $\dfrac{x+4}{y-3} = \dfrac{3}{2}$

$\Rightarrow$ $2x + 8 = 3y - 9$

$\Rightarrow$ $2x - 3y = -17$ ...(ii) **1**

Subtracting eqn. (ii) from eqn. (i),

$$2y = 22$$

$\therefore$ $y = 11$ **1**

Substituting this value of y in eqn. (i),

$$2x - 11 = 5$$

$\therefore$ $x = 8$

Hence, fraction $= \dfrac{8}{11}$ **1**

Q. 6. If a bag containing red and white balls, half the number of white balls is equal to one-third the number of red balls. Thrice the total number of balls exceeds seven times the number of white balls by 6. How many balls of each colour does the bag contain ? A [Board Term-1, 2012, Set–55]

Sol. Let the number of red balls be x and white balls be y.

According to the question,

$$\frac{1}{2}y = \frac{1}{3}x \text{ or } 2x - 3y = 0 \quad \text{...(i)}$$

and $3(x + y) - 7y = 6$ **1**

or $3x - 4y = 6$ ...(ii)

Multiplying eqn. (i) by 3 and eqn. (ii) by 2 and then subtracting, we get

$$6x - 9y = 0 \quad \mathbf{1}$$
$$6x - 8y = 12$$
$$- \quad + \quad -$$
$$-y = -12$$

$\therefore$ $y = 12$

Substituting $y = 12$ in eqn. (i),

$$2x - 36 = 0$$

$\therefore$ $x = 18$ **1**

Hence, number of red balls = 18

and number of white balls = 12. **1**

[CBSE Marking Scheme, 2012]

Q. 7. A two digit number is obtained by either multiplying the sum of digits by 8 and then subtracting 5 or by multiplying the difference of digits by 16 and adding 3. Find the number.

A [Board Term-1, 2012, Set–48]

Sol. Let the digits of number be x and y.

$\therefore$ Number $= 10x + y$

According to the question,

$$8(x+y) - 5 = 10x + y$$

$$\Rightarrow \quad 2x - 7y + 5 = 0 \quad ...(i)$$

also,

$$16(x-y) + 3 = 10x + y$$

$$\Rightarrow \quad 6x - 17y + 3 = 0 \quad ...(ii) \; \mathbf{1}$$

On comparing the equation with $ax + by + c = 0$, we get

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

$$a_1 = 2, b_1 = -7, c_1 = 5$$

and

$$a_2 = 6, b_2 = -17, c_2 = 3$$

Then,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{(-7)(3)-(-17)(5)} = \frac{y}{(5)(6)-(2)(3)}$$

$$= \frac{1}{(2)(-17)-(6)(-7)} \quad \mathbf{1}$$

$$\Rightarrow \quad \frac{x}{-21+85} = \frac{y}{30-6} = \frac{1}{-34+42}$$

$$\Rightarrow \quad \frac{x}{64} = \frac{y}{24} = \frac{1}{8}$$

$$\Rightarrow \quad \frac{x}{8} = \frac{y}{3} = 1 \quad \mathbf{1}$$

Hence, $x = 8$ and $y = 3$

So, required number $= 10 \times 8 + 3 = 83$. **1**

Q. 8. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and the breadth is increased by 3 units. The area is increased by 67 square units if length is increased by 3 units and breadth is increased by 2 units. Find the perimeter of the rectangle.

A [Board Term-1, 2012, Set–48]

Sol. Let length of given rectangle be x and breadth be y

$\therefore$ Area of rectangle $= xy$

According to the first condition,

$$(x-5)(y+3) = xy - 9$$

$$\Rightarrow \quad 3x - 5y = 6 \quad ...(i) \; \mathbf{1}$$

According to the second condition,

$$(x+3)(y+2) = xy + 67$$

$$\Rightarrow \quad 2x + 3y = 61 \quad ...(ii) \; \mathbf{1}$$

Multiplying eqn. (i) by 3 and eqn. (ii) by 5 and then adding,

$$9x - 15y = 18$$

$$10x + 15y = 305$$

$$\therefore \quad x = \frac{323}{19} \Rightarrow 17 \quad \mathbf{1}$$

Substituting this value of x in eqn. (i),

$$3(17) - 5y = 6$$

$$\Rightarrow \quad 5y = 51 - 6$$

$$\therefore \quad y = 9 \quad \mathbf{1}$$

Hence, perimeter $= 2(x+y) \Rightarrow 2(17+9) \Rightarrow 52$ units.

Q. 9. Solve for x and y :

$2(3x - y) = 5xy$ and $2(x + 3y) = 5xy$.

U [Board Term-1, 2012, Set–25]

Sol. Given,

$$2(3x - y) = 5xy \quad ...(i)$$

and

$$2(x + 3y) = 5xy \quad ...(ii)$$

Divide eqns. (i) and (ii) by xy,

$$\frac{6}{y} - \frac{2}{x} = 5 \quad ...(iii)$$

and

$$\frac{2}{y} + \frac{6}{x} = 5 \quad ...(iv) \; \mathbf{1}$$

Let $\frac{1}{y} = a$ and $\frac{1}{x} = b$,

Then equations (iii) and (iv) become

$$6a - 2b = 5 \quad ...(v)$$

$$2a + 6b = 5 \quad ....(vi) \; \mathbf{1}$$

Multiplying eqn. (v) by 3 and then adding with eqn. (vi),

$$20a = 20$$

$$\therefore \quad a = 1$$

Substituting this value of a in eqn. (v),

$$b = \frac{1}{2} \quad \mathbf{1}$$

Now,

$$\frac{1}{y} = a \Rightarrow 1$$

$$\Rightarrow \quad y = 1$$

and

$$\frac{1}{x} = b \Rightarrow \frac{1}{2}$$

$$\Rightarrow \quad x = 2. \quad \mathbf{1}$$

Hence, $x = 2$ and $y = 1$.

Q. 10. The Present age of the father is twice the sum of the ages of his 2 children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

A [Board Term-1, 2012, Set–39]

Sol. Let the sum of the ages of the 2 children be x and the age of the father be y years.

$$\therefore \quad y = 2x$$

$$2x - y = 0 \quad ...(i) \; \mathbf{1}$$

and

$$20 + y = x + 40$$

$$x - y = -20 \quad ...(ii) \; \mathbf{1}$$

Subtracting (ii) from (i),

$$x = 20$$

From (i), $y = 2x \Rightarrow 2 \times 20 = 40$

$$y = 40 \quad \mathbf{1}$$

Hence, the age of the father = 40 years. **1**

[CBSE Marking Scheme, 2012

Q. 11. A fraction becomes $\frac{9}{11}$ if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator it becomes $\frac{5}{6}$. Find the fraction. A **[Board Term-1, 2012, Set 60]**

Sol. Let the fraction be $\frac{x}{y}$, then according to the question,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$11x + 22 = 9y + 18$$

$$\Rightarrow \quad 11x - 9y + 4 = 0 \quad \text{...(i) } \mathbf{1}$$

and $$\frac{x+3}{y+3} = \frac{5}{6}$$

$$\Rightarrow \quad 6x - 5y + 3 = 0 \quad \text{...(ii) } \mathbf{1}$$

On comparing with equation $ax + by + c = 0$.

we get $a_1 = 11, b_1 = -9, c_1 = 4,$

and $a_2 = 6, b_2 = -5$ and $c_2 = 3$

Now, $$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{(-9)(3)-(-5)(4)} = \frac{y}{(4)(6)-(11)(3)}$$

$$= \frac{1}{(11)(-5)-(6)(-9)} \quad \mathbf{1}$$

$$\Rightarrow \quad \frac{x}{-27+20} = \frac{y}{24-33} = \frac{1}{-55+54}$$

$$\Rightarrow \quad \frac{x}{-7} = \frac{y}{-9} = \frac{1}{-1}$$

Hence, $x = 7, y = 9$

$$\therefore \quad \text{Fraction} = \frac{7}{9} \quad \mathbf{1}$$

AI **Q. 12. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.** C + A **[Board Term-1, 2012 Set 48]**

Sol. Let the speed of the boat in still water be 'x' km/hr and speed of the stream be 'y' km/hr.

$\therefore$ Speed of boat up stream = $(x - y)$ km/hr.

Speed of boat down stream = $(x + y)$ km/hr.

$$\therefore \quad \frac{30}{x-y} + \frac{28}{x+y} = 7 \quad \tfrac{1}{2}$$

and $$\frac{21}{x-y} + \frac{21}{x+y} = 5 \quad \tfrac{1}{2}$$

Let $\frac{1}{x-y} = a$ and $\frac{1}{x+y} = b$, then

$$30a + 28b = 7 \quad \text{...(i)}$$

and $$21a + 21b = 5 \quad \text{...(ii)}$$

Multiplying eqn. (i) by 3 and eqn. (ii) by 4 and then subtracting,

$$90a + 84b = 21 \quad \text{...(iii)}$$

$$84a + 84b = 20 \quad \text{...(iv)}$$

$$- \quad - \quad -$$

$$6a = 1$$

$$\therefore \quad a = \frac{1}{6} \quad \tfrac{1}{2}$$

Putting this value of a in eqn (i),

$$30 \times \frac{1}{6} + 28b = 7$$

$$28b = 7 - 30 \times \frac{1}{6} \Rightarrow 2$$

$$\therefore \quad b = \frac{1}{14} \quad \tfrac{1}{2}$$

$$x + y = 14 \quad \text{...(iv)}$$

Now, $$a = \frac{1}{x-y} \Rightarrow \frac{1}{6}$$

$$\Rightarrow \quad x - y = 6 \quad \text{...(v) } \mathbf{1}$$

Solving eqns. (v) and (iv), we get

$$x = 10 \text{ and } y = 4$$

Hence, speed of the boat in still water = 10 km/hr and speed of the stream = 4 km/hr. **1**

Q. 13. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream. C + A **[Board Term-1, 2012 Set 48]**

Sol. Try yourself, Similar to Q. No. 12 of Long Answer Type Question.

Q. 14. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number.

C **[SQP 2017]**

Sol. Let the ten's and unit digit be y and x respectively.

So, the number is $10y + x$. ½

The number, when its digits are reversed, becomes $10x + y$.

So, $$7(10y + x) = 4(10x + y) \quad \tfrac{1}{2}$$

$$\Rightarrow \quad 70y + 7x = 40x + 4y$$

$$\Rightarrow \quad 70y - 4y = 40x - 7x$$

$$\Rightarrow \quad 2y = x \quad \text{...(i) } \mathbf{1}$$

and $$x - y = 3 \quad \text{...(ii) } \tfrac{1}{2}$$

From (i) and (ii), we get

$$y = 3 \text{ and } x = 6$$

Hence, the number is 36. **1½**

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