11. Circles

Exercise 11.1

1. Question

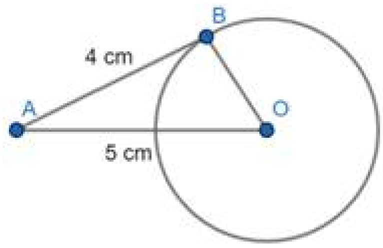
The length of a tangent from a point A at a distance 5cm from the centre of a circle is 4cm. Find the radius of the circle.

Answer

Let the centre of circle be O so that AO = 5 cm

Tangent is AB whose length is 4 cm

OB is radius as shown



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to AB

Hence $\angle ABO = 90°$

Consider ΔABO

Using Pythagoras theorem

$\Rightarrow AB^2 + OB^2 = AO^2$

$\Rightarrow 4^2 + OB^2 = 5^2$

$\Rightarrow 16 + OB^2 = 25$

$\Rightarrow OB^2 = 25 - 16$

$\Rightarrow OB^2 = 9$

$\Rightarrow OB = \pm 3$

As length cannot be negative

$\Rightarrow OB = 3$ cm

Hence length of radius is 3 cm

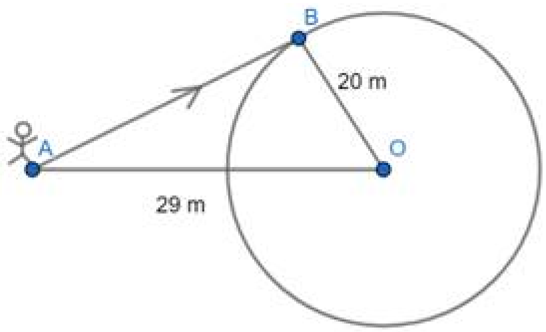
2. Question

Rajesh is 29 m away from the centre of a circular flower bed. Find the distance he has to cover to reach the flower bed along the tangential path if the radius of the flower bed is 20m.

Answer

Let the centre of circular flower bed be O and radius OB = 20 m

Let Rajesh is at point A, and he has to travel tangential path to reach flower bed which is AB as shown



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to AB

Hence $\angle ABO = 90°$

Consider ΔABO

Using Pythagoras theorem

$\Rightarrow AB^2 + OB^2 = AO^2$

$\Rightarrow AB^2 + 20^2 = 29^2$

$\Rightarrow AB^2 = 29^2 - 20^2$

Using $a^2 - b^2 = (a + b)(a - b)$

$\Rightarrow AB^2 = (29 - 20)(29 + 20)$

$\Rightarrow AB^2 = 9 \times 49$

$\Rightarrow AB = \sqrt{(9 \times 49)}$

$\Rightarrow AB = \pm (3 \times 7)$

⇒ AB = ±21

As length cannot be negative

⇒ AB = 21 m

Hence Rajesh has to cover 21 m to reach the flower bed along the tangential path.

3. Question

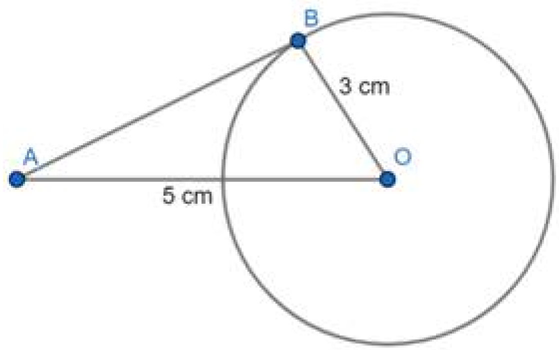
Find the length of the tangent drawn from a point, whose distance from the centre of a circle is 5c, and the radius of the circle is 3 cm.

Answer

Let A be the point at distance of 5 cm from the centre as AO = 5 cm

AB is the tangent at point B as shown

OB is the radius which is 3 cm



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to AB

Hence ∠ABO = 90°

Consider ΔABO

Using Pythagoras theorem

⇒ $AB^2 + OB^2 = AO^2$

⇒ $AB^2 + 3^2 = 5^2$

⇒ $AB^2 + 9 = 25$

⇒ $AB^2 = 25 - 9$

⇒ $AB^2 = 16$

$\Rightarrow$ AB = ±4

As length cannot be negative

$\Rightarrow$ AB = 4 cm

Hence length of tangent is 4 cm

4. Question

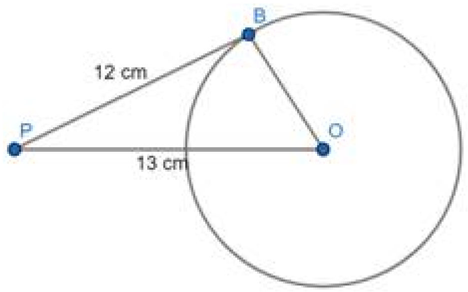
A point P is 13 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 12cm. Find the radius of the circle.

Answer

Let the centre of circle be O so that PO = 13 cm

Tangent is PB whose length is 12 cm

OB is radius as shown



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to PB

Hence $\angle PBO = 90°$

Consider ΔPBO

Using Pythagoras theorem

$\Rightarrow PB^2 + OB^2 = PO^2$

$\Rightarrow 12^2 + OB^2 = 13^2$

$\Rightarrow OB^2 = 13^2 - 12^2$

$\Rightarrow OB^2 = 169 - 144$

$\Rightarrow OB^2 = 25$

$\Rightarrow OB = \pm 5$

As length cannot be negative

⇒ OB = 5 cm

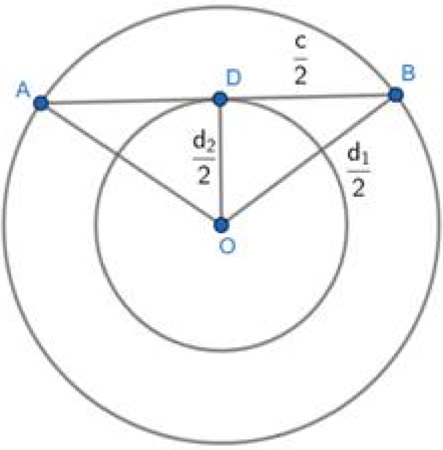
Hence length of radius is 5 cm

5. Question

If d_1, d_2 ($d_2<d_1$) are the diameters of two concentric circles and chord of one circle of length C is tangent to other circle, then prove that $d_2^2 = C^2 + d_1^2$.

Answer

Let the two concentric circles have the centre O and let AB be the chord of an outer circle whose length is D and which is also tangent to the inner circle at point D as shown



The diameters are given as d_1 and d_2 hence the radius will be $\frac{d_1}{2}$ and $\frac{d_2}{2}$

In ΔOAB

⇒ OA = OB ...radius of the outer circle

Hence ΔOAB is an isosceles triangle

As radius is perpendicular to tangent OC is perpendicular to AB

OC is altitude from the apex, and in an isosceles triangle, the altitude is also the median

Hence $AD = DB = \frac{C}{2}$

Consider ΔODB

⇒ ∠ODB = 90° ...radius perpendicular to tangent

Using Pythagoras theorem

⇒ $OD^2 + BD^2 = OB^2$

$$\Rightarrow \frac{d_2^2}{2^2} + \frac{C^2}{2^2} = \frac{d_1^2}{2^2}$$

Multiply the whole by 2^2

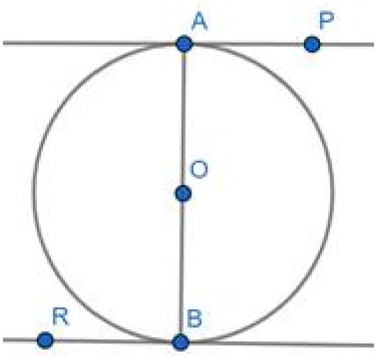
$\Rightarrow d_2{}^2 + C^2 = d_1{}^2$

Hence proved

6. Question

Prove that the line segment joining the point of contact of two parallel tangents to a circle is a diameter of the circle.

Answer



Let lines AP and BR are parallel tangents to circle having centre O

We have to prove that AB is the diameter

To prove AB as diameter, we have to prove that AB passes through O which means that points A, O and B are on the same line or collinear

OA is perpendicular to PA at A because the line from the centre is perpendicular to the tangent at the point of contact

PA || RB

Hence OA is also perpendicular to RB

⇒ OA perpendicular to PA and RB ...(i)

Similarly, OB is perpendicular to RB at B because the line from the centre is perpendicular to the tangent at the point of contact

PA || RB

Hence OB is also perpendicular to PA

⇒ OB perpendicular to PA and RB ...(ii)

From (i) and (ii) we can say that OA and OB can be same line or parallel lines, but we have a common point O which implies that OA and OB are same lines

Hence A, O, B lies on the same line, i.e. A, O and B are collinear

Thus AB passes through O

Hence AB is the diameter

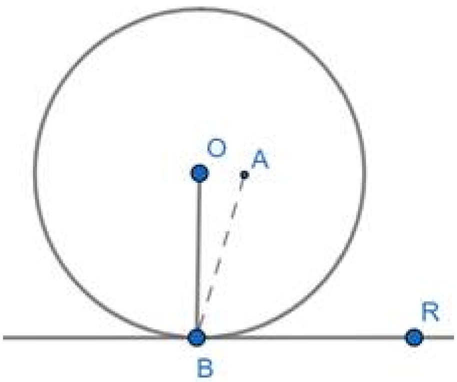
Hence, the line segment joining the point of contact of two parallel tangents to a circle is a diameter of the circle.

7. Question

Prove that the perpendicular at the point of contact of the tangent to a circle passes through the centre.

Answer

Let there be a circle with centre O and BR as tangent with the point of contact as B



Let AB be the line perpendicular to BR

⇒ $\angle ABR = 90°$ ...(i)

As OB is the radius of the circle and we know that radius is perpendicular to the tangent at the point of contact

OB is perpendicular to BR

⇒ $\angle OBR = 90°$ ...(ii)

Equation (i) and (ii) implies that

⇒ $\angle ABR = \angle OBR$

This is only possible iff A and O lie on the same line or A and O are the same points

Case 1: Suppose A and O are on the same line

If A and O are on the same line, then the perpendicular AB to tangent BR has passed through the centre

Case 2: suppose A and O are the same points

As O itself is the centre of the circle, and A and O are the same points hence the perpendicular to the tangent at the point of contact passes through the circle

In any scenario, the line has to pass through the centre.

Hence, the perpendicular at the point of contact of the tangent to a circle passes through the centre

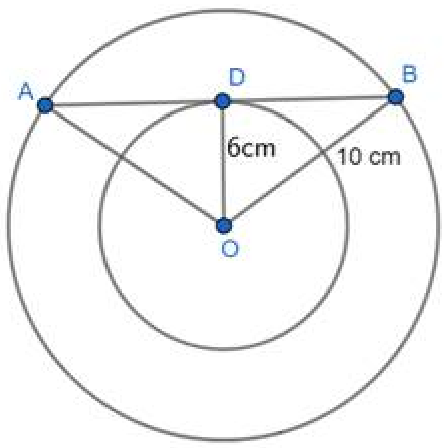
8. Question

Two concentric circles are of radii 10 cm, and 6 cm Find the length of the chord of the larger circle which touches the smaller circle.

Answer

Let the two concentric circles have the centre O and let AB be the chord of an outer circle whose length is D and which will also be tangent to the inner circle at point D because it is given that the chord touches the inner circle.

The radius of inner circle OD = 6 cm and the radius of outer circle OB = 10 cm



In ΔOAB

⇒ OA = OB …radius of outer circle

Hence ΔOAB is isosceles triangle

As radius is perpendicular to tangent OC is perpendicular to AB

OC is altitude from apex and in isosceles triangle the altitude is also the median

Hence AD = DB

Hence AB = 2DB

Consider ΔODB

⇒ ∠ODB = 90° …radius perpendicular to tangent

Using Pythagoras theorem

⇒ $OD^2 + BD^2 = OB^2$

⇒ $6^2 + BD^2 = 10^2$

⇒ $36 + BD^2 = 100$

⇒ $BD^2 = 100 - 36$

⇒ $BD^2 = 64$

⇒ BD = ±8

As length cannot be negative

⇒ BD = 8 cm

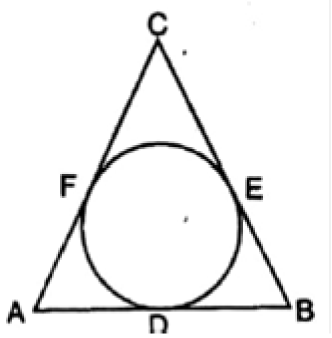
⇒ AB = 2 × 8 …since AB = 2BD

⇒ AB = 16 cm

9. Question

(i) A circle is inscribed in a △ABC having sides BC, CA and AB 16 cm, 20 cm and 24 cm respectively as shown in the figure Find AD, BE and CF.

(ii) If AF=4cm, BE=3cm, AC=11cm, then find BC.



Answer

i) Tangents drawn from external point are equal

AD and AF are tangents from point A

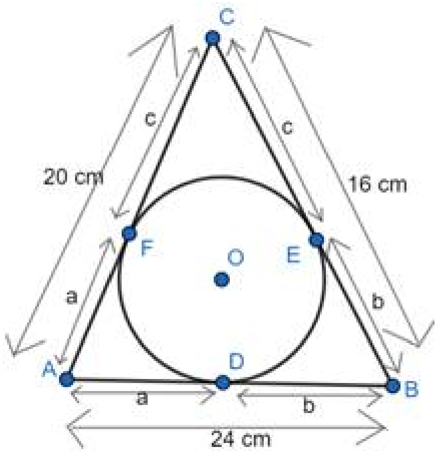
⇒ AD = AF = a

BF and BE are tangents from point B

⇒ BD = BE = b

CD and CE are tangents from point C

⇒ CF = CE = c



From figure

We have AC = AF + FC

⇒ 20 = a + c ...(i)

Also, AB = AD + DB

⇒ 24 = a + b ...(ii)

And CB = CE + EB

⇒ 16 = c + b ...(iii)

Add (i), (ii) and (iii)

⇒ 20 + 24 + 16 = a + c + a + b + c + b

⇒ 60 = 2(a + b + c)

⇒ a + b + c = 30 ...(iv)

Substitute (i) in (iv)

⇒ 20 + b = 30

⇒ b = 10

Substitute (ii) in (iv)

⇒ 24 + c = 30

⇒ c = 6

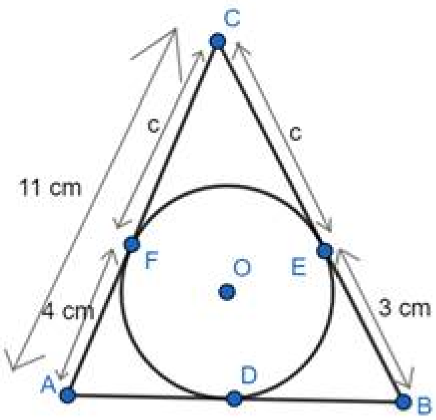
Substitute (iii) in (iv)

⇒ 16 + a = 30

⇒ a = 14

Hence AD = a = 14 cm, BE = b = 10 cm and CF = c = 6 cm

ii)



Tangents drawn from external point are equal

CF and CE are tangents from point C

⇒ CF = CE = c

From figure

AC = AF + FC

⇒ 11 = 4 + c

⇒ c = 7 cm

Hence EC = c = 7 cm

We have BC = BE + EC

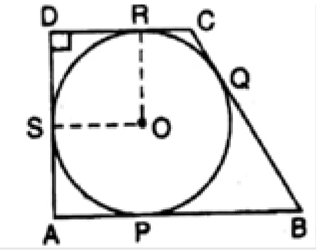
⇒ BC = 3 + 7

⇒ BC = 10 cm

Hence BC is 10 cm

10. Question

In the given figure, ABCD is a quadrilateral in which ∠D=90°. A circle C (O,r) touches the sides AB, BC, CD and DA at P,Q,R,S respectively, If BC =38 cm, CD=25 cm and BP=27 cm, find the value of r.



Answer

r is the radius which is OR = r

Consider quadrilateral DROS

⇒ ∠RDS = 90° …given

⇒ ∠DRO = 90° …radius is perpendicular to the tangent

⇒ DR = DS …tangents drawn from the same point are equal

As the adjacent angles are 90° and adjacent sides are same hence DROS is a square

Hence OR = DR = r …(i)

As tangents drawn from the same point are equal

BQ and BP are tangents drawn from B

⇒ BQ = BP⇒ BQ = 27 cm …BP is 27 cm given

From figure

⇒ BC = BQ + QC

⇒ 38 = 27 + QC …BC is 38 cm given

⇒ QC = 11 cm

CQ and CR are tangents drawn from C

⇒ CQ = CR …tangents from same point

⇒ CR = 11 cm

Again from figure

⇒ CD = CR + RD

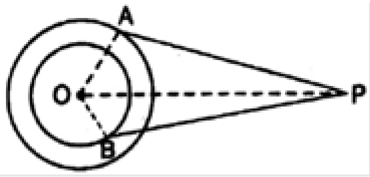
⇒ 25 = 11 + r …CD is 25 given and RD = r from (i)

⇒ r = 14 cm

Hence r radius is 14 cm

11. Question

In the given figure, O is the centre of two concentric circles of radii 4 cm and 6cm respectively. PA and PB are tangents to the outer and inner circle respectively. If PA=10cm, find the length of PB up to one place of decimal.



Answer

Consider ΔPOA

OA = 6 cm …radius of the outer circle

PA = 10 cm …given

$\angle OAP = 90°$ …radius is perpendicular to the tangent

Hence ΔPOA is right-angled triangle

Using Pythagoras ⇒ $OA^2 + AP^2 = OP^2$

⇒ $6^2 + 10^2 = OP^2$

⇒ $36 + 100 = OP^2$

⇒ $OP^2 = 136$ …(i)

Consider ΔPBO

OB = 4 cm …radius of inner circle

$\angle OBP = 90°$ …radius is perpendicular to the tangent

Hence ΔPOB is right-angled triangle

Using Pythagoras ⇒ $OB^2 + BP^2 = OP^2$

Using (i)

⇒ $4^2 + BP^2 = 136$

⇒ $16 + BP^2 = 136$

⇒ $BP^2 = 120$

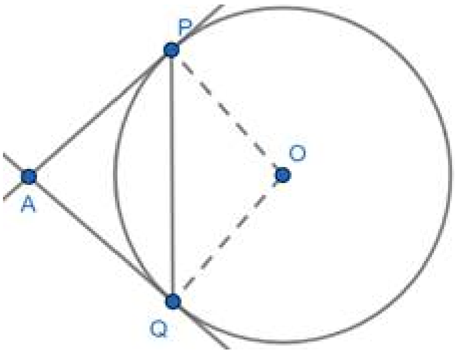
⇒ BP = 10.9 cm

Hence length of PB is 10.9 cm

12. Question

Show that the tangents at the extremities of any chord of a circle make equal angles with the chord.

Answer



Let the circle with centre O and chord PQ with tangents from point A as AP and AQ as shown

We have to prove that ∠APQ = ∠AQP

Consider ΔOPQ

⇒ OP = OQ ...radius

Hence ΔOPQ is an isosceles triangle

⇒ ∠OPQ = ∠OQP ...base angles of isosceles triangle ...(a)

As radius OP is perpendicular to tangent AP at point of contact P

⇒ ∠APO = 90°

From figure ∠APO = ∠APQ + ∠OPQ

⇒ 90° = ∠APQ + ∠OPQ

⇒ ∠APQ = 90° - ∠OPQ ...(i)

As radius OQ is perpendicular to tangent AQ at point of contact Q

⇒ ∠AQO = 90°

From figure ∠AQO = ∠APQ + ∠OPQ

⇒ 90° = ∠AQP + ∠OQP

⇒ ∠AQP = 90° - ∠OQP

Using (a)

⇒ ∠AQP = 90° - ∠OPQ ...(ii)

Using (i) and (ii), we can say that

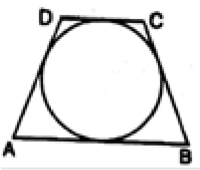
⇒ ∠APQ = ∠AQP

Hence proved

Hence, the tangents at the extremities of any chord of a circle make equal angles with the chord

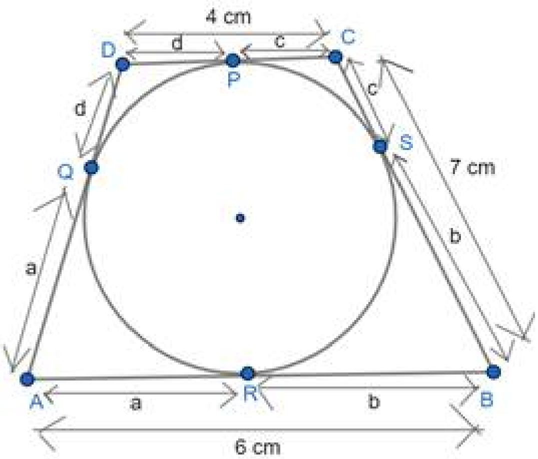
13. Question

In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose three sides are AB = 6cm, BC = 7cm, and CD = 4cm. Find AD.



Answer

Mark the touching points as P, Q, R and S as shown



As tangents from a point are of equal length we have

AQ = AR = a

BR = BS = b

CP = CS = c

DP = DQ = d

From figure

⇒ BC = BS + SC

⇒ 7 = b + c

⇒ b = 7 – c … BC is 7 cm given …(i)

Also,

⇒ DC = DP + PC

⇒ 4 = d + c

⇒ c = 4 – d … DC is 4 cm given …(ii)

And

⇒ AB = AR + RB

⇒ 6 = a + b … AB is 6 cm given

⇒ a = 6 – b

Using (i)

⇒ a = 6 – (7 – c)

Using (ii)

⇒ a = 6 – (7 – (4 – d))

⇒ a = 6 – (7 – 4 + d)

⇒ a = 6 – 7 + 4 – d

⇒ a + d = 3

⇒ AQ + QD = 3 …since AQ = a and QD = d

From figure AQ + QD = AD

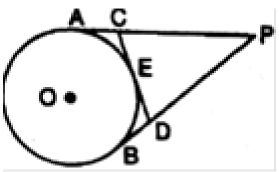
⇒ AD = 3 cm

Hence AD is 3 cm

14. Question

(i) From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at the point E and PA = 14 cm, find the perimeter of △ PCD.

(ii) If PA = 11cm, PD = 7 cm, then DE = ?



Answer

i) From P we have tangents PA and PB

Hence PA = PB …tangents from same point are equal …(a)

Point C is on PA

From C we have tangents CA and CE

⇒ CA = CE …tangents from same point are equal …(i)

Point D is on PB

From D we have two tangents DE and DB

⇒ DE = DB … tangents from same point are equal …(ii)

Consider ΔPCD

⇒ perimeter of ΔPCD = PC + CD + PD

From figure CD = CE + ED

⇒ perimeter of ΔPCD = PC + CE + ED + PD

Using (i) and (ii)

⇒ perimeter of ΔPCD = PC + CA + DB + PD

From figure we have

PC + CA = PA and DB + PD = PB

⇒ perimeter of ΔPCD = PA + PB

Using (a)

⇒ perimeter of ΔPCD = PA + PA

⇒ perimeter of ΔPCD = 2(PA)

PA is 14 cm given

⇒ perimeter of ΔPCD = 2 × 14

⇒ perimeter of ΔPCD = 28 cm

ii) PA = 11 cm …given

using (a)

PB = 11 cm

From figure

⇒ PB = PD + DB

Using (ii)

⇒ PB = PD + DE

⇒ 11 = 7 + DE …PD is 7 cm given

⇒ DE = 5 cm

Hence DE = 5 cm

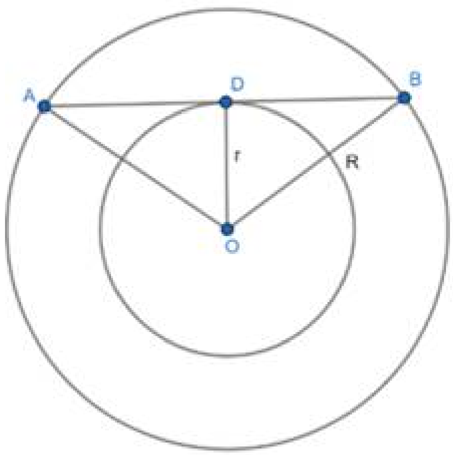
15. Question

In two concentric circles, prove that all chords of the outer circle which touch the inner arc of equal length.

Answer

Let O be the centre of concentric circles with radius ‘r’ and ‘R’ (R>r) and AB be the chord which touches the inner circle at point D

We have to prove that AB has a fixed length



Consider ΔOAB

OA = OB …radius

Hence ΔOAB is an isosceles triangle

Radius OD is perpendicular to tangent AB at the point of contact D

Hence OD is the altitude, and we know that the altitude from the apex of the isosceles triangle is also the median

⇒ AD = BD …(a)

Now consider ΔODB

⇒ ∠ODB = 90° …radius is perpendicular to tangent

Using Pythagoras

$\Rightarrow OD^2 + BD^2 = OB^2$

The radius are OB = R and OD = r

$\Rightarrow r^2 + BD^2 = R^2$

$\Rightarrow BD^2 = R^2 - r^2$

$\Rightarrow BD = \sqrt{(R^2 - r^2)}$ ...(i)

From figure

$\Rightarrow AB = AD + BD$

Using (a)

$\Rightarrow AB = BD + BD$

$\Rightarrow AB = 2BD$

Using (i)

$\Rightarrow AB = 2\sqrt{(R^2 - r^2)}$

Here observe that AB only depends on R and r which are fixed radius of inner circle and outer circle.

And as the radius of both the circle will not change however one may draw the chord the radius will always be fixed.

And hence AB won't change AB is fixed length

Hence proved

Hence, in two concentric circles, all chords of the outer circle which touch the inner circle are of equal length.

Exercise 11.2

1. Question

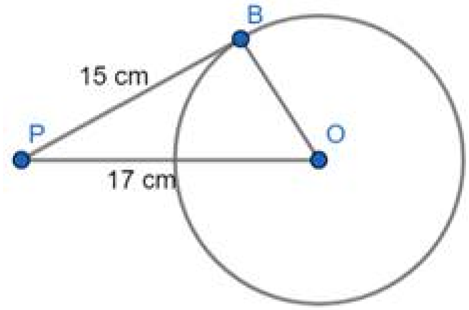
From a point P, the length of the tangent to a circle is 15 cm, and the distance of P from the centre of the circle is 17cm. Then what is the radius of the circle?

Answer

Let the centre of circle be O so that PO = 17 cm

Tangent is PB whose length is 15 cm

OB is radius as shown



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to PB

Hence ∠PBO = 90°

Consider ΔPBO

Using Pythagoras theorem

⇒ $PB^2 + OB^2 = PO^2$

⇒ $15^2 + OB^2 = 17^2$

⇒ $OB^2 = 17^2 - 15^2$

⇒ $OB^2 = 289 – 225$

⇒ $OB^2 = 64$

⇒ OB = ±8

As length cannot be negative

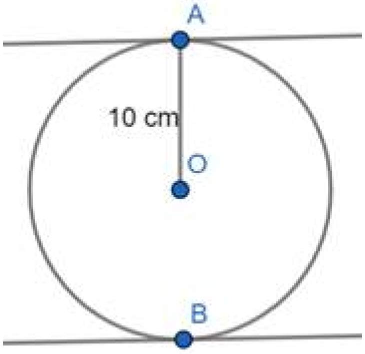
⇒ OB = 8 cm

Hence length of radius is 8 cm

2. Question

What is the distance between two parallel tangents of a circle of radius 10cm?

Answer



O is the centre of circle and tangents from point A and B are parallel

We know that the line joining point of contacts of two parallel tangents (here AB) passes through the centre

And as a line from the centre is perpendicular to tangent, hence that line(AB) will be the distance between parallel tangents

AB passes through centre O hence AB is also the diameter of the circle

Hence the distance between the two parallel tangents will be the diameter of the circle

Radius is given 10 cm

Hence diameter of circle = 2 × radius

Hence AB = 2 × 10

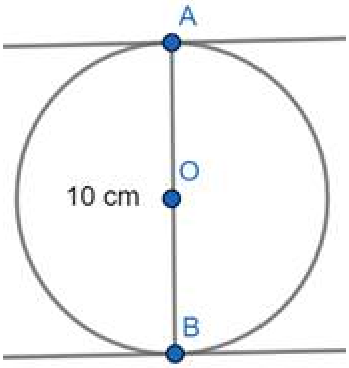
⇒ AB = 20 cm

Hence distance between parallel tangents is 20 cm

3. Question

If the distance between two parallel tangents of a circle is 10cm, what is the radius of the circle?

Answer



O is the centre of circle and tangents from point A and B are parallel

We know that the line joining point of contacts of two parallel tangents (here AB) passes through the centre

And as a line from the centre is perpendicular to tangent, hence that line(AB) will be the distance between parallel tangents

AB passes through centre O hence AB is also the diameter of the circle

Hence the distance between the two parallel tangents will be the diameter of the circle

The distance AB = 10 cm in diameter of the circle

Hence radius will be half of the diameter which is 5 cm

4. Question

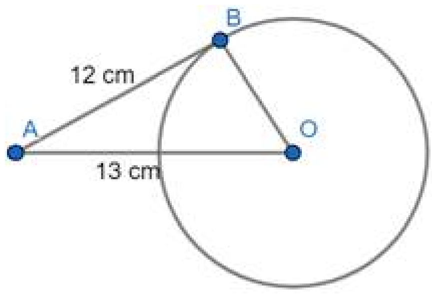
The length of the tangent from a point A at a distance of 13cm from the centre of the circle is 12cm. What is the radius of the circle?

Answer

Let the centre of circle be O so that AO = 13 cm

Tangent is AB whose length is 12 cm

OB is radius as shown



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to AB

Hence $\angle ABO = 90°$

Consider ΔABO

Using Pythagoras theorem

$\Rightarrow AB^2 + OB^2 = AO^2$

$\Rightarrow 12^2 + OB^2 = 13^2$

$\Rightarrow OB^2 = 13^2 - 12^2$

⇒ $OB^2 = 169 - 144$

⇒ $OB^2 = 25$

⇒ OB = ±5

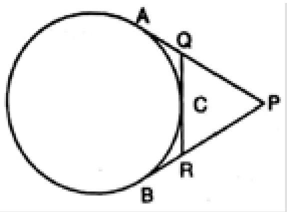
As length cannot be negative

⇒ OB = 5 cm

Hence length of radius is 5 cm

5 A. Question

In the given figure if PA=20 cm, what is the perimeter of ΔPQR.



Answer

From P we have tangents PA and PB

Hence PA = PB …tangents from same point are equal …(a)

Point Q is on PA

From Q we have tangents QA and QC

⇒ QA = QC …tangents from same point are equal …(i)

Point R is on PB

From R we have two tangents RC and RB

⇒ RC = RB … tangents from same point are equal …(ii)

Consider ΔPQR

⇒ perimeter of ΔPQR = PQ + QR + PR

From figure QR = QC + CR

⇒ perimeter of ΔPQR = PQ + QC + CR + PR

Using (i) and (ii)

⇒ perimeter of ΔPQR = PQ + QA + RB + PR

From figure we have

PQ + QA = PA and RB + PR = PB

⇒ perimeter of ΔPQR = PA + PB

Using (a)

⇒ perimeter of ΔPQR = PA + PA

⇒ perimeter of ΔPQR = 2(PA)

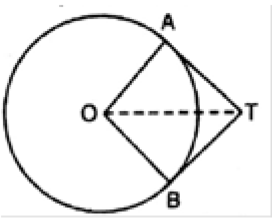
PA is 20 cm given

⇒ perimeter of ΔPQR = 2 × 20

⇒ perimeter of ΔPQR = 40 cm

5 B. Question

In the given figure if ∠ATO=40°, find ∠AOB.



Answer

∠ATO = 40° …given

From T we have two tangents TA and TB

We know that if we join point T and centre of circle O then the line TO divides the angle between tangents

⇒ ∠ATO = ∠OTB = 40° …(i)

∠OAT = ∠OBT = 90° …radius is perpendicular to tangent …(ii)

Consider quadrilateral OATB

⇒ ∠OAT + ∠ATB + ∠TBO + ∠AOB = 360°…sum of angles of quadrilateral

From figure ∠ATB = ∠ATO + ∠OTB

⇒ ∠OAT + ∠ATO + ∠OTB + ∠TBO + ∠AOB = 360°

Using (i) and (ii)

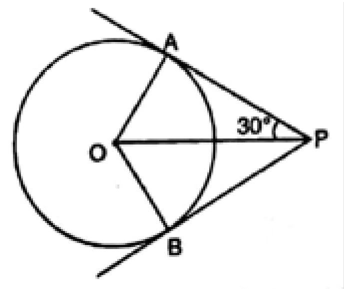
⇒ 90° + 40° + 40° + 90° + ∠AOB = 360°

⇒ 260° + ∠AOB = 360°

⇒ ∠AOB = 100°

6. Question

in the figure PA and PB are tangents to the circle. If ∠APO=30°, find ∠AOB



Answer

∠APO = 30° …given

From P we have two tangents PA and PB

We know that if we join point P and centre of circle O then the line PO divides the angle between tangents

⇒ ∠APO = ∠OPB = 30° …(i)

∠OAP = ∠OBP = 90° …radius is perpendicular to tangent …(ii)

Consider quadrilateral OAPB

⇒ ∠OAP + ∠APB + ∠PBO + ∠AOB = 360°…sum of angles of quadrilateral

From figure ∠APB = ∠APO + ∠OPB

⇒ ∠OAP + ∠APO + ∠OPB + ∠PBO + ∠AOB = 360°

Using (i) and (ii)

⇒ 90° + 30° + 30° + 90° + ∠AOB = 360°

⇒ 240° + ∠AOB = 360°

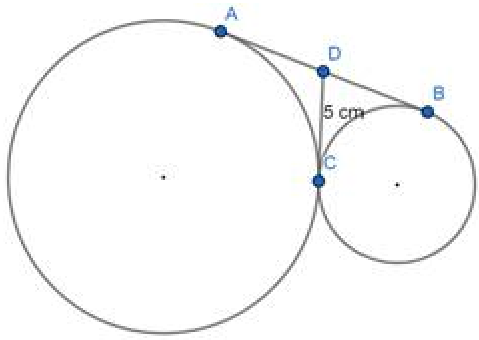
⇒ ∠AOB = 120°

Hence ∠AOB is 120°

7. Question

AB and CD are two common tangents of two circles which touch each other at C. If D lies on AB and CD=5 cm, then what is the length of AB.

Answer



DC and AB are tangents given to both circle

Point D is on AB which means DA and DB are also tangents to both circle

Now from point D, we have two tangents to bigger circle which are DA and DC

⇒ DA = DC …tangents from a point to a circle are equal

⇒ DA = 5 cm …DC is 5 cm given…(i)

Also from point D, we have two tangents to smaller circle which are DB and DC

⇒ DB = DC …tangents from a point to a circle are equal

⇒ DB = 5 cm …DC is 5 cm given…(ii)

Now as point D is on AB from the figure we can say that DA + DB = AB

⇒ AB = DA + DB

Using (i) and (ii)

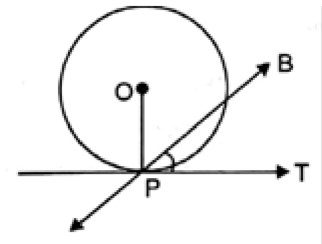
⇒ AB = 5 + 5

⇒ AB = 10 cm

Hence the length of tangent AB is 10 cm

8. Question

In the given figure, ∠BPT=50°. What is the measure of ∠OPB?



Answer

PT is tangent to circle and OP is radius

⇒ ∠OPT = 90° …radius is perpendicular to tangent

From figure

⇒ ∠OPT = ∠OPB + ∠BPT

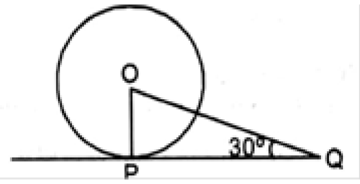
⇒ 90° = ∠OPB + 50° …∠BPT is 50° given

⇒ ∠OPB = 40°

Hence ∠OPB is 40°

9. Question

In the given figure, a measure of ∠POQ is…..



Answer

PQ is tangent to circle and OP is radius

⇒ ∠OPQ= 90° …radius is perpendicular to tangent

⇒ ∠PQO = 30° …given

Consider ΔOPQ

⇒ ∠OPQ + ∠PQO + ∠POQ = 180° …sum of angles of triangle

⇒ 90° + 30° + ∠POQ = 180°

⇒ 120° + ∠POQ = 180°

⇒ ∠POQ = 60°

Hence ∠POQ is 60°

10. Question

If all sides of a parallelogram touch a circle, then that parallelogram is….

Answer

Consider ABCD as a parallelogram touching the circle at points P, Q, R and S as shown

As ABCD is a parallelogram opposites sides are equal

⇒ AB = CD …(a)

⇒ AD = BC …(b)

AP and AS are tangents from point A⇒ AP = AS …tangents from point to a circle are equal…(i)

BP and BQ are tangents from point B⇒ BP = BQ …tangents from point to a circle are equal…(ii)

CQ and CR are tangents from point C⇒ CR = CQ …tangents from point to a circle are equal…(iii)

DR and DS are tangents from point D⇒ DR = DS …tangents from point to a circle are equal…(iv)

Add equation (i) + (ii) + (iii) + (iv)

⇒ AP + BP + CR + DR = AS + DS + BQ + CQ

From figure AP + BP = AB, CR + DR = CD, AS + DS = AD and BQ + CQ = BC

⇒ AB + CD = AD + BC

Using (a) and (b)

⇒ AB + AB = AD + AD

⇒ 2AB = 2AD

⇒ AB = AD

AB and AD are adjacent sides of parallelogram which are equal hence parallelogram ABCD is a rhombus

Hence if all sides of a parallelogram touch a circle then that parallelogram is a rhombus

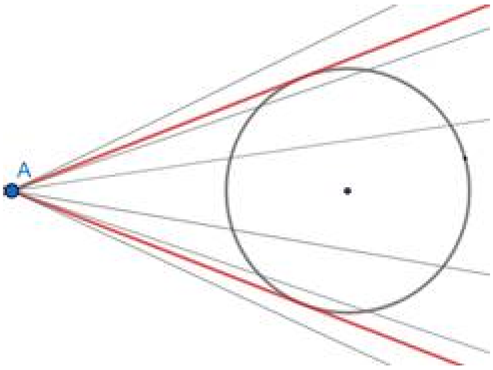
11. Question

From an external point…… tangents can be drawn to a circle.

Answer

We can see that only 2 tangents can be drawn from an external point (here A) to a circle

Every other line either don intersects the circle or intersects at two points hence there can be only 2 tangents from external point to a circle



12. Question

From an external point P two tangents PA and PB have been drawn. IFPA = 6cm, then what is the length of PB.

Answer

From P we have two tangents PA and PB

The tangents from an external point to a circle are equal

Hence PA = PB

PA = 6 cm given

Hence PB is also 6 cm

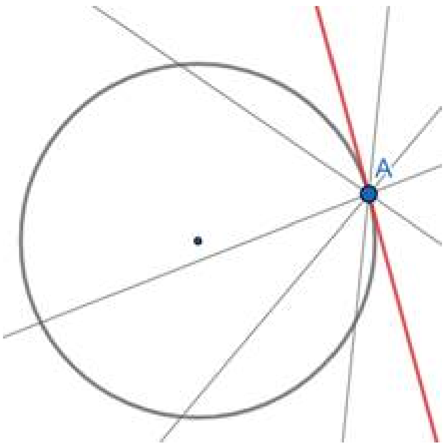
13. Question

How many tangents can be drawn at a point on a circle ?

Answer

Only one tangent can be drawn from a point on the circle

Every other line will intersect the circle at two points which won't be a tangent

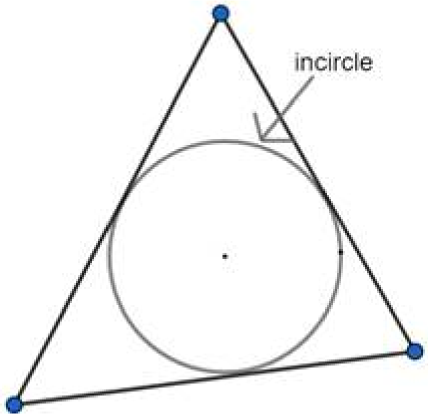


14. Question

What is the name of the circle touching the three sides of a triangle internally?

Answer

Touching three sides internally of a triangle which means the circle is inside the triangle hence it is called incircle of triangle



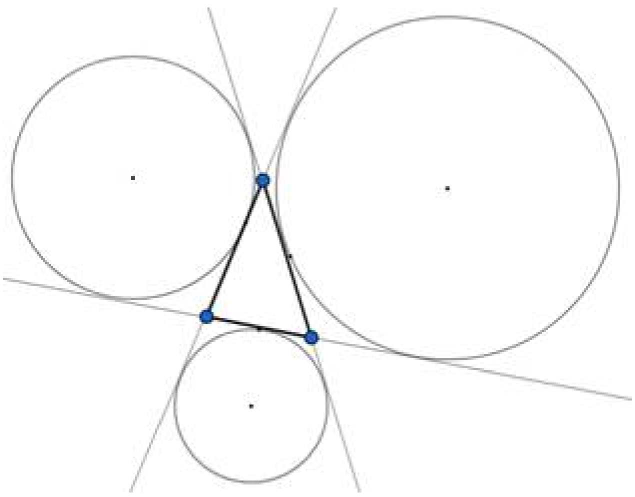
15. Question

How many excircles can be drawn to a triangle?

Answer

A circle is lying outside the triangle having one side of the triangle as tangent and also other two sides as tangent when they are extended.

There are three sides of a triangle hence there can be three excircles.



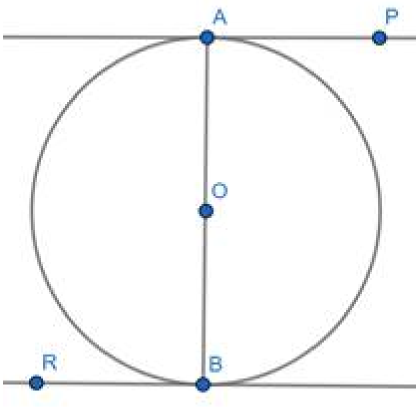
16. Question

What is the relation between the tangents at the extremities of a diameter of a circle?

Answer

The tangents at the extremities of diameter are parallel to each other

Proof:



Consider a circle with centre O and diameter AB having tangents as PA and RB as shown

∠OAP = 90° ...radius is perpendicular to the tangent at the point of contact

∠RBO = 90° ... radius is perpendicular to the tangent at the point of contact

⇒ ∠OAP = ∠RBO

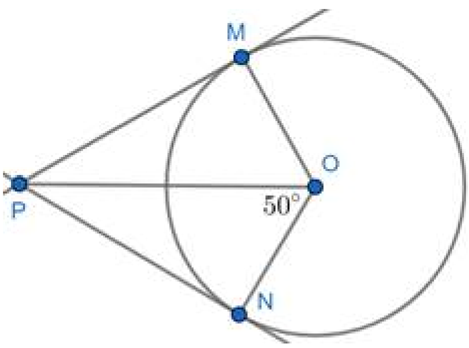
∠OAP and ∠RBO are alternate angles between lines PA and RB having transversal as AB

As alternate angles are equal lines, PA and RB are parallel

17. Question

O is the centre of a circle. From an external point, P two tangents PM and PN have been drawn which touch the circle at M and N. If ∠PON = 50°, then find the value of ∠MPN.

Answer



From P we have two tangents PM and PN

We know that if we join point P and centre of circle O then the line PO divides the angle between tangents

⇒ ∠MPO = ∠NPO ...(a)

Consider ΔPNO

⇒ ∠PON = 50° …given

As radius ON is perpendicular to tangent PN

⇒ ∠PNO = 90°

Now

⇒ ∠PON + ∠PNO + ∠NPO = 180° …sum of angles of triangle

⇒ 50° + 90° + ∠NPO = 180°

⇒ 140° + ∠NPO = 180°

⇒ ∠NPO = 40° …(i)

From figure

⇒ ∠MPN = ∠MPO + ∠NPO

Using (a)

⇒ ∠MPN = ∠NPO + ∠NPO

⇒ ∠MPN = 2∠NPO

Using (i)

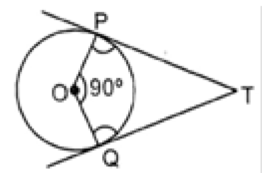
⇒ ∠MPN = 2 × 40°

⇒ ∠MPN = 80°

Hence ∠MPN is 80°

18 A. Question

In the given figure, two radii OP and OQ of a circle are mutually perpendicular. What is the degree measure of the angle between tangents drawn to the circle at P and Q?



Answer

a) we have to find ∠PTQ which is the angle between the tangents TP and TQ

∠OPT = ∠OQT = 90° …radius is perpendicular to tangent at point of contact

∠POQ = 90° ...given

Consider quadrilateral POQT

⇒ ∠OPT + ∠OQT + ∠POQ + ∠PTQ = 360° ...sum of angles of quadrilateral

⇒ 90° + 90° + 90° + ∠PTQ = 360°

⇒ 270° + ∠PTQ = 360°

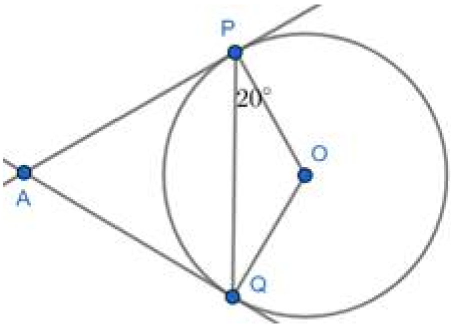
⇒ ∠PTQ = 90°

Hence angle between tangents is 90°

18 B. Question

Centre of the circle is O and AP, and AQ is tangent of the circle. If ∠OPQ=20°, then what is the value of ∠PAQ?

Answer



∠OPQ = 20° ...given

Radius OP is perpendicular to tangent PA at point of contact A

⇒ ∠APO = 90°

From figure ∠APO = ∠APQ + ∠OPQ

⇒ 90° = ∠APQ + 20°

⇒ ∠APQ = 70° ...(a)

Consider ΔAPQ

AP and AQ are tangents to circle from A

Tangents from a point to a circle are equal

⇒ AP = AQhence ΔAPQ is a isosceles triangle

⇒ ∠APQ = ∠AQP

Using (a)

$\Rightarrow \angle AQP = 70°$

Now

$\Rightarrow \angle APQ + \angle AQP + \angle PAQ = 180°$ …sum of angles of triangle

$\Rightarrow 70° + 70° + \angle PAQ = 180°$

$\Rightarrow 140° + \angle PAQ = 180°$

$\Rightarrow \angle PAQ = 40°$

Hence $\angle PAQ$ is $40°$