Chapter 6. Linear Inequalities

Question-1

Solve the inequation: $3x - 7 > x + 3$

Solution:

$3x - 7 > x + 3$

$3x - 7 + 7 > x + 3 + 7$

$3x > x + 10$

$3x - x > x + 10 - x$

$2x > 10$

$x > 5$

Question-2

Solve the inequation: $x + 12 < 4x - 2$

Solution:

$x + 12 < 4x - 2$

$x + 12 - 12 < 4x - 2 - 12$

$x < 4x - 14$

$x - 4x < 4x - 14 - 4x$

$-3x < -14$

$-3x/-3 > -14/-3$

$x > 14/3$

Question-3

Solve the inequation: $4x - 7 < 3 - x$

Solution:

$4x - 7 < 3 - x$

$4x - 7 + 7 < 3 - x + 7$

$4x < 10 - x$

$4x + x < 10 - x + x$

$5x < 10$

$x < 2$

Question-4

Solve the inequation: $3x + 17 \leq 2(1 - x)$

Solution:

$3x + 17 \leq 2(1 - x)$

$3x \leq -2x - 15$

$3x + 2x \leq -2x - 15 + 2x$

$5x \leq -15$

$5x/5 \leq -15/5$

$x \leq -3$

Question-5

Solve the inequation: $-2x + 6 \geq 5x - 4$

Solution:

$-2x + 6 \geq 5x - 4$

$-2x \geq 5x - 10$

$-2x - 5x \geq 5x - 10 - 5x$

$-7x \geq -10$

$-7x/-7 \leq -10/-7$

$x \leq 10/7$

Question-6

Solve the inequation: $-(x - 3) + 4 > -2x + 5$

Solution:

$-(x - 3) + 4 > -2x + 5$

$-x + 7 > -2x + 5$

$-x + 7 - 7 > -2x + 5 - 7$

$-x > -2x - 2$

$x < 2x + 2$

$x - 2x < 2x + 2 - 2x$

$-x < 2$

$x > -2$

Question-7

Solve the inequation: $2(2x + 3) - 10 < 6(x - 2)$

Solution:

$2(2x + 3) - 10 < 6(x - 2)$

$4x + 6 - 10 < 6x - 12$

$4x - 4 < 6x - 12$

$4x - 4 + 4 < 6x - 12 + 4$

$4x < 6x - 8$

$4x - 6x < 6x - 8 - 6x$

$- 2x < - 8$

$- 2x/-2 > -8 /-2$

$x > 4$

Question-8

Solve the inequation: $2-3x \geq 2(x + 6)$

Solution:

$2 - 3x \geq 2x + 12$

$2 - 3x - 2 \geq 2x + 12 - 2$

$- 3x \geq 2x + 10$

$- 3x - 2x \geq 2x + 10 - 2x$

$- 5x \geq 10$

$- 5x/-5 \leq 10/-5$

$x \leq -2$

Question-8

Solve the inequation: $2-3x \geq 2(x + 6)$

Solution:

$2 - 3x \geq 2x + 12$

$2 - 3x - 2 \geq 2x + 12 - 2$

$- 3x \geq 2x + 10$

$- 3x - 2x \geq 2x + 10 - 2x$

$- 5x \geq 10$

$- 5x/-5 \leq 10/-5$

$x \leq -2$

Question-9

Solve the inequation: $37 - (3x + 5) \geq 9x - 8(x - 3)$

Solution:

$37 - 3x - 5 \geq 9x - 8x + 24$

$32 - 3x \geq x + 24$

$32 - 3x - 32 \geq x + 24 - 32$

$-3x \geq x - 8$

$-3x - x \geq x - 8 - x$

$-4x \geq -8$

$-4x/-4 \geq -8/-4$

$x \geq 2$

Question-10

Solve the inequation: $\frac{5x}{2} + \frac{3x}{4} \geq \frac{39}{4}$

Solution:

$\frac{5x}{2} + \frac{3x}{4} \geq \frac{39}{4}$

$10x + 3x \geq 39$

$13x \geq 39$

$13x/13 \geq 39/13$

$x \geq 3$

Question-11

Solve the inequation: $\frac{4 + 2x}{3} \geq \frac{x}{2} - 3$

Solution:

$\frac{4 + 2x}{3} \geq \frac{x}{2} - 3$

$2(4 + 2x) \geq 3x - 18$ (Multiplying by 6 both sides)

$8 + 4x \geq 3x - 18$

$8 + 4x - 8 \geq 3x - 18 - 8$

$4x \geq 3x - 26$

$4x - 3x \geq 3x - 26 - 3x$

$x \geq -26$

Question-12

Solve the inequation: $\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}$

Solution:

$\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}$

$9(x-2) \geq 25(2-x)$ (Multiplying by 15 both sides)

$9x - 18 \geq 50 - 25x$

$9x - 18 + 18 \geq 50 - 25x + 18$

$9x \geq 68 - 25x$

$9x + 25x \geq 68 - 25x + 25x$

$34x \geq 68$

$34x/34 \geq 68/34$

$x \geq 2$

Question-13

Solve the inequation: $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$

Solution:

$\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$

$15x < 20(5x - 2) - 12(7x - 3)$ (Multiplying by 60 both sides)

$15x < 100x - 40 - 84x + 36$

$15x < 16x - 4$

$15x - 16x < 16x - 4 - 16x$

$-x < -4$

$x > 4$

Question-14

Solve the inequation: $\frac{5-2x}{3} \leq \frac{x}{6} - 5$

Solution:

$\frac{5-2x}{3} \leq \frac{x}{6} - 5$

$2(5 - 2x) \leq x - 30$ (Multiplying by 6 both sides)

$10 - 4x \leq x - 30$

$10 - 4x - 10 \leq x - 30 - 10$

$-4x \leq x - 40$

$-4x - x \leq x - 40 - x$

$-5x \leq -40$

$-5x/-5 \leq -40/-5$

$x \geq 8$

Question-15

Solve the inequation: $\frac{1}{2}\left(\frac{3}{5}x+4\right) \geq \frac{1}{3}(x-6)$

Solution:

$\frac{1}{2}\left(\frac{3}{5}x+4\right) \geq \frac{1}{3}(x-6)$

$3\left(\frac{3}{5}x+4\right) \geq 2(x-6)$ (Multiplying by 6 both sides)

$3(3x+20) \geq 10(x-6)$ (Multiplying by 5 both sides)

$9x + 60 \geq 10x - 60$

$9x + 60 - 60 \geq 10x - 60 - 60$

$9x \geq 10x - 120$

$9x - 10x \geq 10x - 120 - 10x$

$-x \geq -120$

$x \leq 120$

Question-16

Solve the following system of inequations: x −2 > 0, 3x < 18

Solution:

x −2 > 0 .

3x < 18

x −2 > 0

x > 2...................(1)

3x < 18

x < 6 ....(2)

From (1) and (2), solutions of the given system are, therefore, given by 2 < x < 6

Hence the solution of the system is 2 < x < 6.

Question-17

Solve the following system of inequations:$5x + 1 > -24, 5x - 1 < 24$

Solution:

$5x + 1 > -24$

$5x - 1 < 24$

$5x + 1 > -24$

$5x > -25$

$x > -5$ ....(1)

$5x - 1 < 24$

$5x < 25$

$x < 5$....................(2)

From (1) and (2), solutions of the given system are, therefore, given by $-5 < x < 5$

Hence the solution of the system of is $-5 < x < 5$.

Question-18

Solve the following system of inequations:$x + 2 \leq 5, 3x - 4 > -2 + x$

Solution:

$x + 2 \leq 5$

$3x - 4 > -2 + x$

$x + 2 \leq 5$

$x \leq 3$ ....(1)

$3x - 4 > -2 + x$

$3x > 2 + x$

$2x > 2$

$x > 1$ ....(2)

From (1) and (2), solutions of the given system are, therefore, given by $1 < x \leq 3$.

Hence the solution of the system of is $1 < x \leq 3$.

Question-19

Solve the following system of inequations: 4x + 5 > 3x, -(x + 3) + 4 ≤ −2x + 5

Solution:

$4x + 5 > 3x$

$-(x + 3) + 4 \le -2x + 5$

$4x + 5 > 3x$

$4x > 3x - 5$

$x > -5$ ....(1)

$-(x + 3) + 4 \le -2x + 5$

$-x - 3 + 4 \le -2x + 5$

$-x + 1 \le -2x + 5$

$-x \le -2x + 4$

$x \le 4$ ....(2)

From (1) and (2), solutions of the given system are, therefore, given by $-5 < x \le 4$

Hence the solution of the system of is $-5 < x \le 4$.

Question-20

Solve the following system of inequations: $\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$, $\frac{7x-1}{3} - \frac{7x+2}{6} > x$

Solution:

$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$

$\frac{7x-1}{3} - \frac{7x+2}{6} > x$

$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$

$16x - 27 < 12x + 9$ (Multiplying by 12 both sides)

$16x < 12x + 36$

$4x < 36$

$x < 9$ ....(1)

$\frac{7x-1}{3} - \frac{7x+2}{6} > x$

$2(7x - 1) - (7x + 2) > 6x$ (Multiplying by 6 both sides)

$14x - 2 - 7x - 2 > 6x$

$7x - 4 > 6x$

$7x > 6x + 4$

$x > 4$ ....(2)

From (3) and (4), solutions of the given system are, therefore, given by $4 < x < 9$

Hence the solution of the system of is $4 < x < 9$.

Question-21

Solve the following system of inequations: 2(x + 1) < x + 5, 3(x + 2) > 2 − x.

Solution:
2(x + 1) < x + 5
3(x + 2) > 2 − x
2(x + 1) < x + 5
2x + 2 < x + 5
2x < x + 3
x < 3......................... (1)

3(x + 2) > 2 − x
3x + 6 > 2 − x
3x > -4 − x
4x > -4
x > -1 ....(2)

From (1) and (2), solutions of the given system are, therefore, given by -1 < x < 3
Hence the solution of the system of is -1 < x < 3.

Question-22

Solve the following system of inequations: 3x − 1 ≥ 5, x + 2 > -1

Solution:
3x − 1 ≥ 5
x + 2 > -1
3x − 1 ≥ 5
3x ≥ 5 + 1
3x ≥ 6
x ≥ 2 ....(1)

x + 2 > -1
x > -1 − 2
x > -3 ....(2)

From (1) and (2), solutions of the given system are, therefore, given by x ≥ 2
Hence the solution of the system of is x ≥ 2.

Question-23

Solve the following system of inequations: $3x - 7 > 2(x - 6)$, $6 - x > 11 - 2x$

Solution:

$3x - 7 > 2(x - 6)$

$6 - x > 11 - 2x$

$3x - 7 > 2x - 12$

$3x - 7 + 7 > 2x - 12 + 7$

$3x > 2x - 5$

$3x - 2x > 2x - 5 - 2x$

$x > -5$ ....(1)

$6 - x > 11 - 2x$

$6 - x - 6 > 11 - 2x - 6$

$-x > 5 - 2x$

$-x + 2x > 5 - 2x + 2x$

$x > 5$ ....(2)

From (1) and (2), solutions of the given system are, therefore, given by $x > 5$

Hence the solution of the system is $x > 5$.

Question-24

Solve the following system of inequations: $-2 - \frac{x}{4} \leq \frac{1+x}{3}$, $3 - x < 4(x - 3)$

Solution:

$-2 - \frac{x}{4} \leq \frac{1+x}{3}$

$3 - x < 4(x - 3)$

$-2 - \frac{x}{4} \leq \frac{1+x}{3}$

$-24 - 3x \leq 4(1 + x)$ (Multiplying by 12 both sides)

$-24 - 3x \leq 4 + 4x$

$-3x \leq 4x + 28$

$-7x \leq 28$

$x \geq -4$ .... (1)

$3 - x < 4x - 12$

$-x < 4x - 15$

$-5x < -15$

$x > 3$ ....(2)

From (1) and (2), solutions of the given system are, therefore, given by $x > 3$

Hence the solution of the system is $x > 3$.

Question-25

Solve the following system of inequations:
5(2x - 7) − 3(2x + 3) ≤ 0, 2x + 19 ≤ 6x + 47

Solution:

$5(2x - 7) − 3(2x + 3) ≤ 0$

$2x + 19 ≤ 6x + 47$

$5(2x - 7) − 3(2x + 3) ≤ 0$

$10x - 35 − 6x - 9 ≤ 0$

$4x - 44 ≤ 0$

$4x ≤ 44$

$x ≤ 11$ .... (3)

$2x + 19 ≤ 6x + 47$

$2x ≤ 6x + 28$

$-4x ≤ 28$

$x ≥ -7$ .... (4)

From (1) and (2), solutions of the given system are, therefore, given by -7 ≤ x ≤ 11

Hence the solution of the system is -7 ≤ x ≤ 11.

Question-26

Solve the following system of inequations: 2x − 7 <11, 3x + 4 < - 5

Solution:

$2x − 7 <11$

$3x + 4 < - 5$

$2x − 7 <11$

$2x <18$

$x < 9$ ....(1)

$3x + 4 < - 5$

$3x < - 9$

$x < - 3$ ....(2)

From (1) and (2), solutions of the given system are, therefore, given by x < - 3

Hence the solution of the system is x < - 3.

Question-27

Solve the following system of inequations: 4 − 5x > -11, 4x + 11 ≤ -13

Solution:

$4 − 5x > -11$

$4x + 11 ≤ -13$

$4 − 5x > -11$

$− 5x > -15$

$x < 3$ .... (1)

$4x + 11 ≤ -13$

$4x ≤ - 24$

$x ≤ - 6$ .... (2)

From (1) and (2), solutions of the given system are, therefore, given by x ≤ - 6.

Hence the solution of the system is x ≤ - 6

Question-28

Solve the following system of inequations: 4x − 5 < 11, -3x − 4 ≥ 8.

Solution:

$4x − 5 < 11$

$-3x − 4 ≥ 8$

$4x − 5 < 11$

$4x < 16$

$x < 4$ ....(1)

$-3x − 4 ≥ 8$

$-3x ≥ 12$

$x ≤ -4$...............................(2)

From (1) and (2), solutions of the given system are, therefore, given by x ≤ -4.

Hence the solution of the system is x ≤ -4.

Question-29

Solve the following system of inequations: $5x - 7 < 3(x + 3)$, $1 - \frac{3x}{2} \geq x - 4$

Solution:

$5x - 7 < 3(x + 3)$

$1 - \frac{3x}{2} \geq x - 4$

$5x - 7 < 3(x + 3)$

$5x - 7 < 3x + 9$

$5x < 3x + 16$

$2x < 16$

$x < 8$............................(1)

$1 - \frac{3x}{2} \geq x - 4$ (Multiplying both sides by 2)

$2 - 3x \geq 2x - 8$

$- 3x \geq 2x - 10$

$- 5x \geq -10$

$x \leq 2$ ....(2)

From (1) and (2), solutions of the given system are, therefore, given by $x \leq 2$.

Hence the solution of the system is $x \leq 2$.

Question-30

Solve the following system of inequations: $2(2x + 3) - 10 < 6(x - 2)$,

$\frac{2x - 3}{4} + 6 \geq 2 + \frac{4x}{3}$

Solution:

$2(2x + 3) - 10 < 6(x - 2)$

$\frac{2x - 3}{4} + 6 \geq 2 + \frac{4x}{3}$

$2(2x + 3) - 10 < 6(x - 2)$

$4x + 6 - 10 < 6x - 12$

$4x - 4 < 6x - 12$

$4x < 6x - 8$

$- 2x < - 8$

$x > 4$ .... (1)

$\frac{2x - 3}{4} + 6 \geq 2 + \frac{4x}{3}$

$3(2x - 3) + 72 \geq 24 + 16x$ (Multiplying both sides by 12)

$6x - 9 + 72 \geq 24 + 16x$

$6x + 63 \geq 24 + 16x$

$6x \geq 16x - 39$

$-10x \geq -39$

x ≤ 39/10..............................(2)

From (1) and (2), the system has no solution.

Question-31

Represent the following inequation graphically in two dimensional plane and hence solve them: x − 2y + 4 ≤ 0

Solution:
We draw the graph of the equation x − 2y + 4 = 0

x	0	-4
y	2	0

Put x = 0
Then 0 − 2y + 4 ≤ 0

or -2y ≤ - 4
or y ≥ 2
Put x = 0, y = 0
Then 0 − 2(0) + 4 ≤ 0
or 4 ≤ 0, which is false.

Hence, half plane II is not the solution of the given inequation.

Therefore, the shaded half plane I is the solution region of the inequation including points on the line
x − 2y + 4 = 0.

Scale
1cm = 1unit along x-axis
1cm = 1unit along y-axis
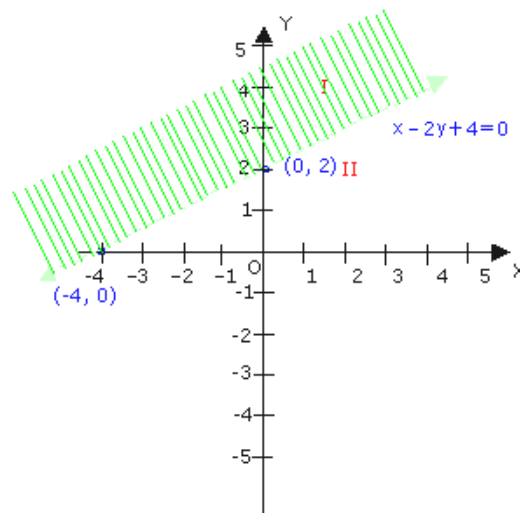
Question-32

Represent the following inequation graphically in two dimensional plane and hence solve them: 2x + y > 3

Solution:

We draw the graph of the equation $2x + y = 3$

X	0	3/2
Y	3	0

Put x = 0

Then $2(0) + y > 3$

or y > 3

Put x = 0, y = 0

Then $2(0) + 0 > 3$

or 0 > 3, which is false.

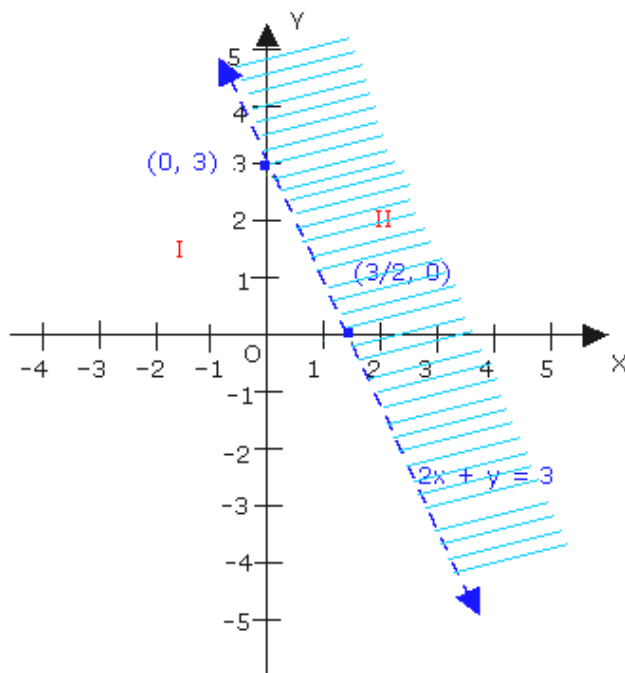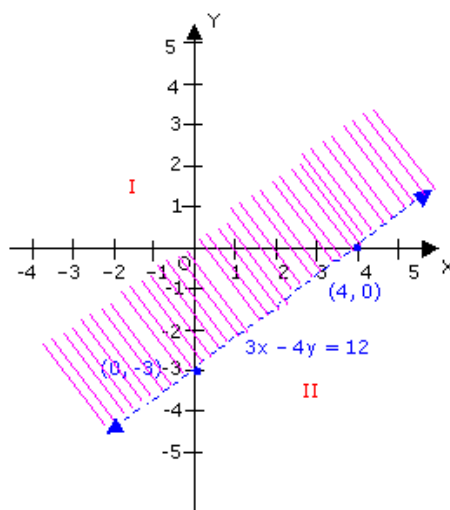Hence, half plane I is not the solution of the given inequation.

Therefore, the shaded half plane II is the solution region of the inequation excluding points on the line
2x + y = 3.

Scale

1cm = 1unit along x-axis

1cm = 1unit along y-axis

Question-33

Represent the following inequation graphically in two dimensional plane and hence solve them: $3x - 4y < 12$

Solution:

We draw the graph of the equation $3x - 4y = 12$

x	0	4
y	-3	0

Put x = 0

Then $3(0) - 4y < 12$

or $- y < 3$

or $y > -3$

Put x = 0, y = 0

Then $3(0) - 4(0) < 12$

or $0 < 12$, which is true.

Hence, half plane II is not the solution of the given inequation.

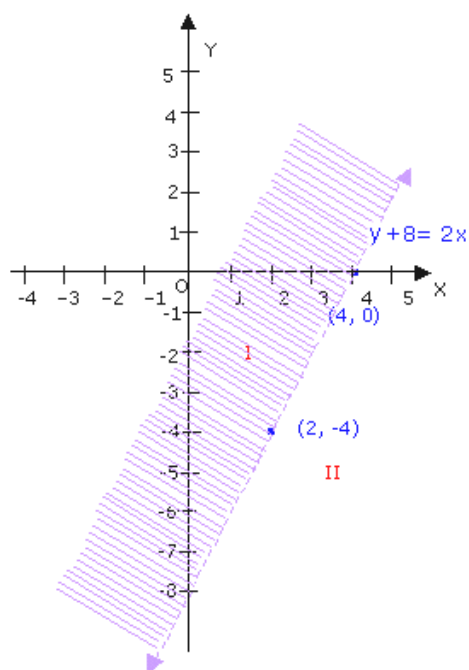Therefore, the shaded half plane I is the solution region of the inequation excluding points on the line

$3x - 4y = 12$.

Scale

1cm = 1unit along x-axis

1cm = 1unit along y-axis

Question-34

Represent the following inequation graphically in two dimensional plane and hence solve them: y + 8 ≥ 2x

Solution:

We draw the graph of the equation y + 8 = 2x

x	2	4
y	-4	0

Put x = 0

Then y + 8 ≥ 2x

or y + 8 ≥ 0

Put x = 0, y = 0

 Then 0 + 8 ≥ 2(0)

 or 8 ≥ 0, which is true.

 Hence, half plane I is not the solution of the given inequation.
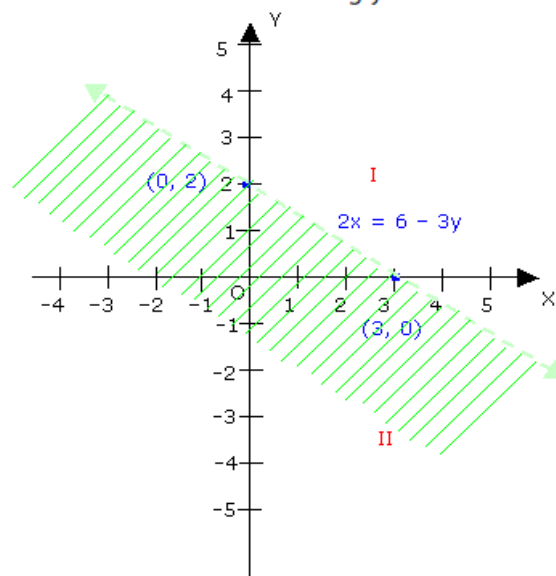
 Therefore, the shaded half plane II is the solution region of
 the inequation including points on the line

y + 8 = 2x.

Scale
1cm = 1unit along x-axis
1cm = 1unit along y-axis

Question-35

Represent the following inequation graphically in two dimensional plane and hence solve them: 2x ≤ 6 − 3y

Solution:

We draw the graph of the equation 2x = 6 − 3y

x	0	3
y	2	0

Put x = 0

Then 2(0) ≤ 6 − 3y

or y ≤ 2

Put x = 0, y = 0

Then 2(0) ≤ 6 − 3(0)

or 0 ≤ 6, which is true.

Hence, half plane I is not the solution of the given inequation.

Therefore, the shaded half plane II is the solution region of the inequation

including points on the line 2x = 6 − 3y

Scale

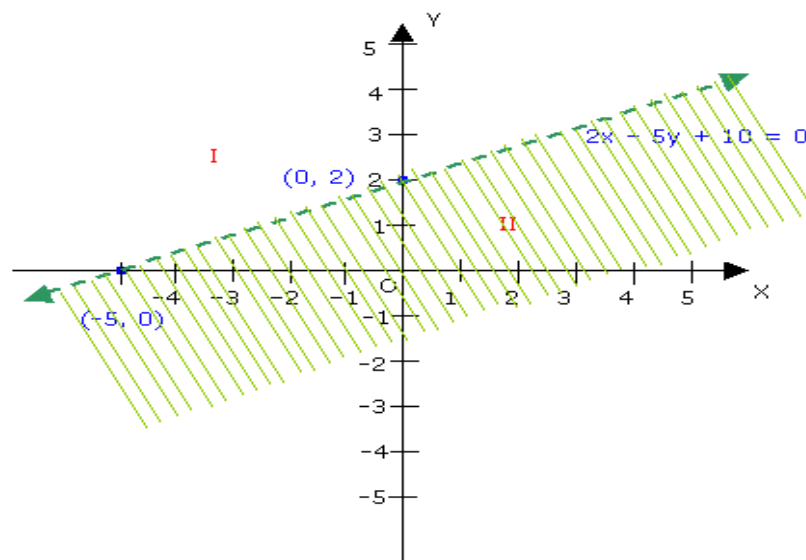1cm = 1unit along x-axis

1cm = 1unit along y-axis

Question-36

Represent the following inequation graphically in two dimensional plane and hence solve them: $0 \leq 2x - 5y + 10$

Solution:

Scale

1cm = 1unit along x-axis

1cm = 1unit along y-axis

We draw the graph of the equation $2x - 5y + 10 = 0$

x	0	-5
y	2	0

Put x = 0

Then $0 \leq 2(0) - 5y + 10$

or $-10 \leq -5y$

Put x = 0, y = 0

Then $0 \leq 2(0) - 5(0) + 10$

or $0 \leq 10$, which is false.

Hence, half plane I is not the solution of the given inequation.

Therefore, the shaded half plane II is the solution region of the inequation including points on the line
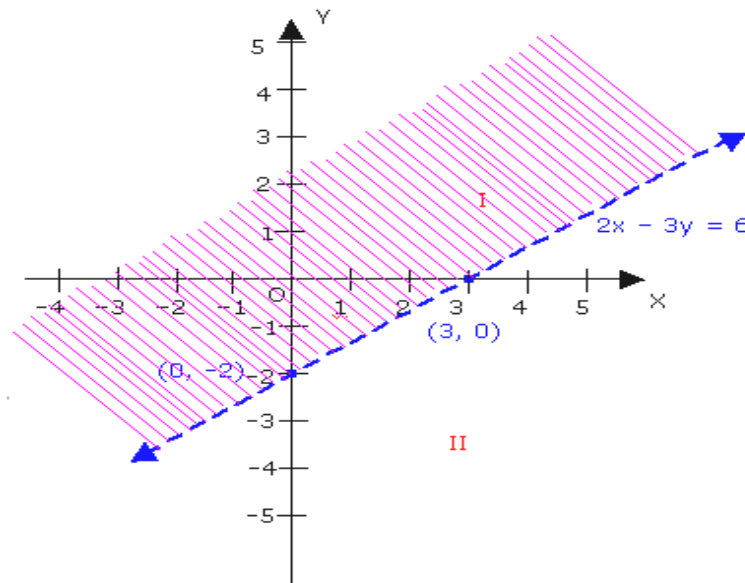
$2x - 5y + 10 = 0$.

Question-37

Represent the following inequation graphically in two dimensional plane and hence solve them: 2x − 3y < 6

Solution:

Scale

1cm = 1unit along x-axis

1cm = 1unit along y-axis

We draw the graph of the equation 2x − 3y = 6

x	0	3
y	-2	0

Put x = 0

Then $2(0) − 3y < 6$

or y < -2

Put x = 0, y = 0

Then $2(0) − 3(0) < 6$

or 0 < 6, which is true.

Hence, half plane II is not the solution of the given inequation.

Therefore, the shaded half plane I is the solution region of the inequation excluding points on the line

2x − 3y = 6
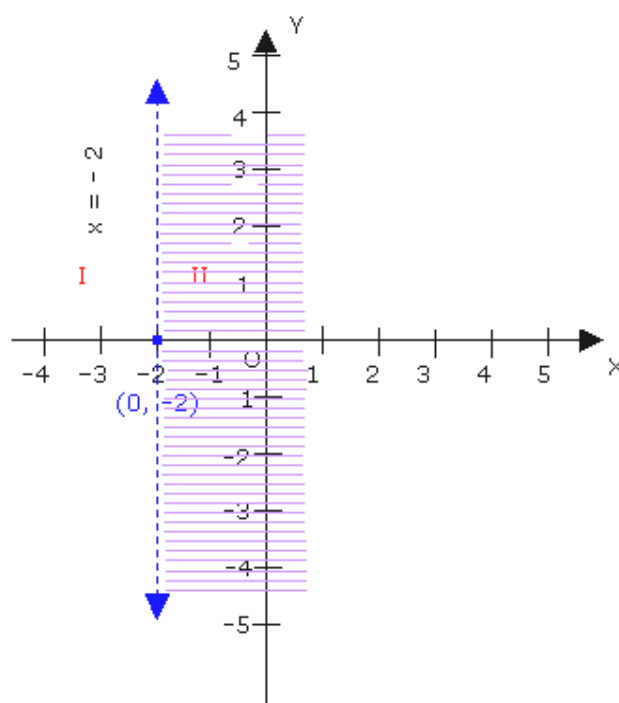
Question-38

Represent the following inequation graphically in two dimensional plane and hence solve them: x > - 2

Solution:

1cm = 1unit along x-axis

1cm = 1unit along y-axis

The graph of the equation x = - 2 is vertical line parallel to y − axis.

Put x = 0

Then 0 > -2, which is true.

Hence, the solution region is the shaded region on the right hand side of

the line x = -2 containing the origin.

Hence every point on the shaded region is the solution of the given

inequation.