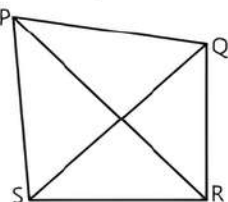


8

Quadrilaterals

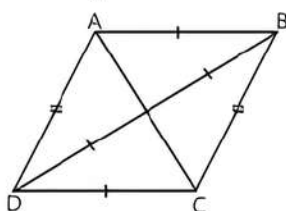
Fastrack® Revision

► **Quadrilateral:** A closed figure formed by four line segments (with no three points collinear). In quadrilateral PQRS; PQ, QR, RS and SP are sides; $\angle P$, $\angle Q$, $\angle R$ and $\angle S$ are the four angles; PR and QS are diagonals; Pairs (PQ, QR), (QR, RS), (RS, SP) and (SP, PQ) are adjacent sides; Pairs ($\angle Q$, $\angle S$) and ($\angle P$, $\angle R$) are of opposite angles.



The sum of the four angles of a quadrilateral is 360° .

► **Parallelogram:** A quadrilateral in which both opposite pairs of sides are parallel, is said to be parallelogram. In figure, ABCD is a parallelogram with $AB \parallel CD$ and $BC \parallel AD$.



► Properties of a Parallelogram

- (i) Opposite sides are equal and parallel.
- (ii) Opposite angles are equal.

- (iii) Diagonals bisect each other.
- (iv) Diagonals divided it into two congruent triangles.
- (v) The sum of the adjacent angles of a parallelogram is 180° .
- (vi) If a pair of opposite sides is parallel and equal then it is a parallelogram.

► **Mid-point Theorem:** The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is equal to half of it.

Converse of Mid-point Theorem: The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

Knowledge BOOSTER

1. A trapezium is not a parallelogram (as only one pair of opposite sides is parallel in a trapezium and we require both pairs to be parallel in a parallelogram).
2. The figure formed by joining the mid-points of the adjacent sides of a quadrilateral (or rhombus/square) is a parallelogram (or rectangle/square).



Practice Exercise



Multiple Choice Questions

Q 1. The consecutive angles of a parallelogram are:

- a. complementary
- b. supplementary
- c. equal
- d. None of the above

Q 2. If in a parallelogram its diagonals bisect each other and are equal, then it is a:

- a. square
- b. rectangle
- c. rhombus
- d. parallelogram

Q 3. The figure formed by joining the mid-points of the adjacent sides of a rhombus is a:

- a. rhombus
- b. square
- c. rectangle
- d. parallelogram

Q 4. Which of the following is not true for a parallelogram?

- a. Diagonals bisect each other
- b. Opposite sides are equal
- c. Opposite angles are equal
- d. Opposite angles are bisected by the diagonals

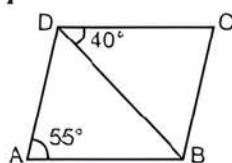
Q 5. If one angle of a parallelogram is 24 less than twice the smallest angle, then the largest angle of the parallelogram is:

- a. 68°
- b. 102°
- c. 112°
- d. 136°

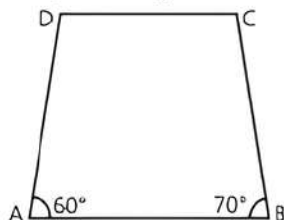
Q 6. If ABCD is a parallelogram with two adjacent angles $\angle A = \angle B$, then the parallelogram is a:

- a. rectangle
- b. rhombus
- c. trapezium
- d. kite

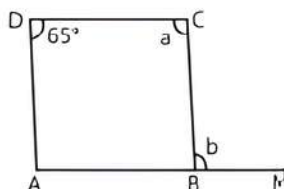
- Q 7. In the given figure, ABCD is a parallelogram in which $\angle BDC = 40^\circ$ and $\angle BAD = 55^\circ$, then $\angle CBD$ is equal to:



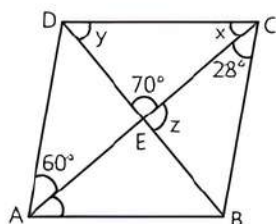
- a. 80° b. 70° c. 90° d. 85°
- Q 8. In the given figure $AB \parallel CD$, then measure $\angle C$ is:



- a. 120° b. 110° c. 115° d. 118°
- Q 9. If ABCD is a parallelogram in which $\angle ADC = 65^\circ$ and AB is produced to point M as shown in the figure. Then, $a + b$ is:



- a. 235° b. 230° c. 225° d. 0°
- Q 10. Diagonals of quadrilateral ABCD bisect each other. If $\angle A = 45^\circ$, then the value of $\angle B$ is:
- a. 90° b. 45°
c. 135° d. 120°
- Q 11. If angles A, B, C and D of a quadrilateral ABCD, taken in order are in the ratio 3 : 7 : 6 : 4, then ABCD is a:
- a. rhombus b. parallelogram
c. trapezium d. kite
- Q 12. The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O. If $\angle ACB = 32^\circ$ and $\angle AOB = 70^\circ$, then $\angle DBC$ is equal to:
- a. 24° b. 86° c. 38° d. 32°
- Q 13. In the given figure, ABCD is a parallelogram, the values of x and y are:



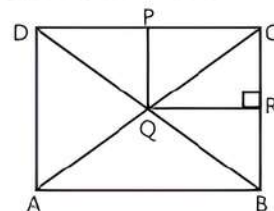
- a. $30^\circ, 75^\circ$ b. $32^\circ, 78^\circ$
c. $36^\circ, 74^\circ$ d. $35^\circ, 70^\circ$
- Q 14. In $\triangle ABC$, $AB = 6$ cm, $BC = 9$ cm and $AC = 8$ cm. If D and E are respectively the mid-point of AB and BC, then the length of DE is:
- a. 4 cm b. 6 cm c. 5 cm d. 4.5 cm



Assertion & Reason Type Questions

Directions (Q.Nos. 15-18): In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:

- a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.
- Q 15. Assertion (A): The opposite angles of a parallelogram are $(2x - 2)^\circ$ and $(52 - x)^\circ$. The measure of one of the angle is 34° . Reason (R): Opposite angles of a parallelogram are equal.
- Q 16. Assertion (A): In $\triangle ABC$, median AD is produced to E, such that $AD = DE$. Then, ABEC is a parallelogram. Reason (R): Diagonals AE and BC bisect each other at right angles.
- Q 17. Assertion (A): Diagonals AC and BD of a parallelogram ABCD intersect each other at point O. If $\angle BCA = 35^\circ$ and $\angle AOB = 65^\circ$, then $\angle DBC = 30^\circ$. Reason (R): The adjacent angles of a parallelogram is supplementary.
- Q 18. Assertion (A): ABCD and PQRC are rectangles and Q is a mid point of AC. Then $DP = PC$. Reason (R): The line segment joining the mid-point of any two sides of a triangle is parallel to the third side and equal to half of it.



Fill in the Blanks Type Questions

- Q 19. The angles of a parallelogram are equal.
- Q 20. The diagonals of a rectangle are and each other.
- Q 21. The quadrilateral formed by joining the mid-points of the consecutive sides of a square is a
- Q 22. The line segment joining the mid-points of the two sides of the triangle is to the third side.



True/False Type Questions

Q 23. All the angles of the quadrilateral are obtuse.

Solutions

- (b) supplementary
- (b) rectangle
- (c) rectangle
- (d) opposite angles are bisected by the diagonals
- (c) Let θ be the smallest angle of a parallelogram, then the other larger angle will be $2\theta - 24^\circ$.



TiP

Opposite angles of a parallelogram are equal.

The sum of all angles of a parallelogram is 360° .

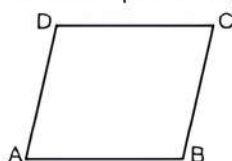
$$\therefore \theta + 2\theta - 24^\circ + \theta + 2\theta - 24^\circ = 360^\circ$$

$$\Rightarrow 6\theta - 48^\circ = 360^\circ$$

$$\Rightarrow \theta = \frac{408^\circ}{6} = 68^\circ$$

$$\begin{aligned} \therefore \text{The largest angle is } 2\theta - 24^\circ \\ = 2 \times 68^\circ - 24^\circ \\ = 136^\circ - 24^\circ = 112^\circ \end{aligned}$$

6. (a) Given, ABCD is a parallelogram.



Therefore $\angle A + \angle B = 180^\circ$

[Sum of adjacent angles of a parallelogram is 180°]

$$\Rightarrow \angle A + \angle A = 180^\circ \quad [\because \angle A = \angle B \text{ given}]$$

$$\Rightarrow \angle A = 90^\circ$$

$$\Rightarrow \angle B = 90^\circ$$

Hence, ABCD is a rectangle.

7. (d) Given, $\angle BDC = 40^\circ$



TiP

Opposite angles of a parallelogram are equal.

Here, $\angle BCD = \angle BAD = 55^\circ$

In $\triangle BCD$, use angle sum property of a triangle,

$$\angle BDC + \angle BCD + \angle CBD = 180^\circ$$

$$\Rightarrow 40^\circ + 55^\circ + \angle CBD = 180^\circ$$

$$\Rightarrow \angle CBD = 85^\circ$$

8. (b) Given $AB \parallel CD$, therefore the sum of two adjacent angles is 180° .

$$\Rightarrow \angle B + \angle C = 180^\circ$$

$$\Rightarrow 70^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 110^\circ$$

- Q 24. Out of four points A, B, C, D in plane, three of them are collinear. Then, a quadrilateral can be formed from these points.

9. (b) Given ABCD is a parallelogram, therefore sum of adjacent angles is 180° .

$$\text{i.e. } \angle D + \angle C = 180^\circ$$

$$65^\circ + a = 180^\circ$$

$$\Rightarrow a = 115^\circ$$

And opposite angles of a parallelogram are equal.

$$\therefore \angle B = \angle D$$

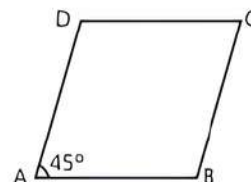
$$\angle B = 65^\circ$$

$$\therefore \angle CBM = 180^\circ - \angle B$$

$$= 180^\circ - 65^\circ = 115^\circ$$

$$\therefore a + b = 115^\circ + 115^\circ = 230^\circ$$

10. (c) Given, diagonals of a quadrilateral bisect each other, so it is a parallelogram.



Therefore, the sum of co-interior angles is 180° .

$$\Rightarrow \angle A + \angle B = 180^\circ$$

$$\Rightarrow 45^\circ + \angle B = 180^\circ \Rightarrow \angle B = 135^\circ$$

11. (c) Let angles of a quadrilateral be $A = 3x$, $B = 7x$, $C = 6x$ and $D = 4x$.

Then, sum of all angles of a quadrilateral be 360° .

$$\therefore A + B + C + D = 360^\circ$$

$$\Rightarrow 3x + 7x + 6x + 4x = 360^\circ$$

$$\Rightarrow 20x = 360^\circ \Rightarrow x = 18^\circ$$

$$\therefore A = 3 \times 18 = 54^\circ \quad B = 7 \times 18 = 126^\circ$$

$$C = 6 \times 18 = 108^\circ \quad \text{and} \quad D = 4 \times 18 = 72^\circ$$

Here, we see that, neither pair of angles are equal nor sum of adjacent angles is 180° .

$$\text{But, here } \angle A + \angle B = 54^\circ + 126^\circ = 180^\circ$$

$$\text{and } \angle C + \angle D = 108^\circ + 72^\circ = 180^\circ$$

So, ABCD is a trapezium.

12. (c) Given, ABCD is a parallelogram and $\angle ACB = 32^\circ$, $\angle AOB = 70^\circ$.

$$\text{Also, } \angle AOB + \angle BOC = 180^\circ$$

[Linear pair]

$$\Rightarrow 70^\circ + \angle BOC = 180^\circ$$

$$\Rightarrow \angle BOC = 110^\circ$$

In $\triangle BOC$, use angle sum property of a triangle.

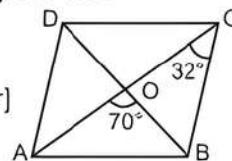
$$\angle BOC + \angle OCB + \angle OBC = 180^\circ$$

$$\Rightarrow 110^\circ + 32^\circ + \angle OBC = 180^\circ$$

$$[\because \angle OCB = \angle ACB = 32^\circ]$$

$$\Rightarrow \angle OBC = 38^\circ$$

$$\Rightarrow \angle DBC = 38^\circ \quad [\because \angle OBC = \angle DBC = 38^\circ]$$



13. (b) Given, ABCD is a parallelogram.
Therefore, $\angle DCB = \angle DAB$
[Opposite angles of a parallelogram are equal]

$$\therefore x + 28^\circ = 60^\circ$$

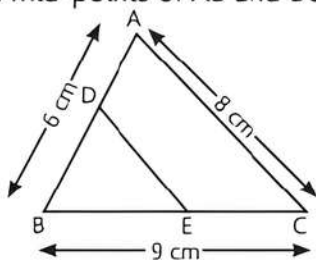
$$\Rightarrow x = 32^\circ$$

In $\triangle CDE$, using angle sum property of a triangle,
 $\angle CDE + \angle DEC + \angle DCE = 180^\circ$

$$\Rightarrow y + 70^\circ + 32^\circ = 180^\circ$$

$$\Rightarrow y = 78^\circ$$

14. (a) In $\triangle ABC$, we have
 $AB = 6$ cm, $BC = 9$ cm and $AC = 8$ cm. Since D and E are the mid-points of AB and BC respectively.



By mid-point theorem,

$$DE \parallel AC$$

$$\text{and } DE = \frac{1}{2}AC = \frac{8}{2}$$

$$= 4 \text{ cm}$$

15. (a) **Assertion (A):** Given opposite angles of a parallelogram are equal.

$$\therefore (2x - 2)^\circ = (52 - x)^\circ$$

$$\Rightarrow 2x + x = 52^\circ + 2^\circ \Rightarrow 3x = 54^\circ$$

$$\Rightarrow x = 18^\circ$$

Then the angles of a parallelogram are

$$(2x - 2)^\circ = (2 \times 18 - 2)^\circ = 34^\circ$$

$$\text{and } (52 - x)^\circ = (52 - 18)^\circ = 34^\circ$$

Hence, one of the angle of a parallelogram is 34°

So, Assertion (A) is true.

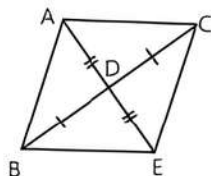
Reason (R): It is also true that opposite angles of a parallelogram are equal.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

16. (c) **Assertion (A):** Given in $\triangle ABC$, AD is median such that

$$AD = DE$$

$$\text{Also, } BD = DC.$$



It means in quadrilateral ABEC, diagonals AE and BC bisect each other at point D. Therefore, ABEC is a parallelogram.

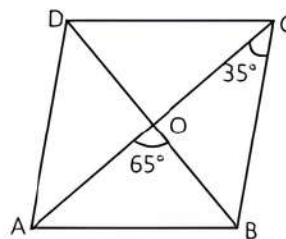
So, Assertion (A) is true.

Reason (R): In given figure diagonals are not right angled.

So, Reason (R) is false.

Hence, Assertion (A) is true but Reason (R) is false.

17. (b) **Assertion (A):** Given, ABCD is a parallelogram.



$$\Rightarrow \angle BCA = 35^\circ$$

$$\text{or } \angle BCO = 35^\circ$$

$$\angle BOA + \angle BOC = 180^\circ \quad [\text{Linear pair}]$$

$$\Rightarrow \angle BOC = 180^\circ - 65^\circ = 115^\circ$$

In $\triangle BOC$, use angle sum property of a triangle.

$$\angle OBC + \angle BOC + \angle BCO = 180^\circ$$

$$\Rightarrow \angle DBC + 115^\circ + 35^\circ = 180^\circ \quad [\because \angle OBC = \angle DBC]$$

$$\Rightarrow \angle DBC = 30^\circ$$

So, Assertion (A) is true.

Reason (R): It is true to say that adjacent angles of a parallelogram is supplementary.

Hence, both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

18. (b) **Assertion (A):** In right angled $\triangle ADC$, Q is the mid-point of AC such that $PQ \parallel AD$.

Therefore, P is the mid-point of DC.

[By converse of mid-point theorem]

$$DP = PC$$

So, Assertion (A) is true.

Reason (R): It is also true to say that the line segment joining the mid-point of any two sides of a triangle is parallel to the third side and equal to half of it.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

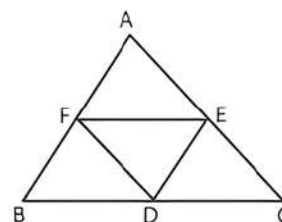
- | | |
|--------------|-------------------|
| 19. opposite | 20. equal, bisect |
| 21. square | 22. parallel |
| 23. False | 24. False |



Case Study Based Questions

Case Study 1

A metal marker has a triangular shaped metal. He welded another triangle on the mid-points of that metal, such that it appears like the following figure:



In the above figure, D, E and F are the mid-points of BC, AC and AB.

On the basis of the above information, solve the following questions:

- Q 1. DE is equal to:**
 a. AF b. $\frac{1}{2}AB$ c. BF d. All of these
- Q 2. If $FE = FD$, then which of the following relation is correct:**
 a. $AC = AB$ b. $\angle FED = \angle ECD$
 c. $BC = AC$ d. $\angle CAB = \angle AFD$
- Q 3. Which type of quadrilateral BDEF?**
 a. Parallelogram b. Square
 c. rectangle d. Trapezium
- Q 4. Identify the correct relation:**
 a. $FD = \frac{1}{2}AB$ b. $AE + FD = AC$
 c. $AB - DE = AC$ d. None of these
- Q 5. The sum of adjacent angles in a parallelogram is:**
 a. 90° b. 145°
 c. 180° d. None of these

Solutions

1. (d) \because D and E are the mid-points of side BC and AC.

\therefore By mid-point theorem, $DE = \frac{1}{2}AB$

But $\frac{1}{2}AB = AF = BF$

$\therefore DE = \frac{1}{2}AB = AF = BF$

So, option (d) is correct.

2. (c) $FD = \frac{1}{2}AC$ [By mid-point theorem]

and $FE = \frac{1}{2}BC$

Given, $FD = FE$

$\therefore \frac{1}{2}AC = \frac{1}{2}BC$

$AC = BC$

So, option (c) is correct.

3. (a) In quadrilateral BDEF; $FE = BD$ and $DE = BF$, so quadrilateral is a parallelogram.

So, option (a) is correct.

4. (b) $\because FD = \frac{1}{2}AC$ [By mid-point theorem] ... (1)

\because E is the mid-point of AC.

$\therefore AE = \frac{1}{2}AC$... (2)

Adding eqs. (1) and (2), we get

$$AE + FD = AC,$$

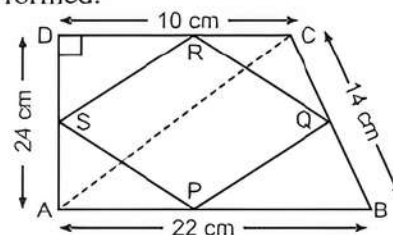
So, option (b) is correct.

5. (c) The sum of pair of adjacent angles in a parallelogram is 180° .

So, option (c) is correct.

Case Study 2

Person A has a quadrilateral shaped paper which he cut from a circular paper. Person B joined the mid-points of all sides and another quadrilateral was formed.



Above figure shows how the paper appears, side $AB = 22$ cm, $BC = 14$ cm, $CD = 10$ cm and $AD = 24$ cm.

On the basis of the above information, solve the following questions:

- Q 1. The measure of diagonal AC is:**
 a. 13 cm b. 30 cm c. 28 cm d. 26 cm
- Q 2. If $PQ \parallel AC$, then the measure of PQ is:**
 a. 15 cm b. 13 cm
 c. 17 cm d. 19 cm
- Q 3. Quadrilateral PQRS is which of type quadrilateral?**
 a. Rhombus b. Rectangle
 c. Parallelogram d. Trapezium
- Q 4. While proving quadrilateral is a rectangle, choose the correct option:**
 a. by showing opposite sides equal and each adjacent angle is 90°
 b. by proving diagonals are equal
 c. by proving all angles 90°
 d. by proving all of the above
- Q 5. In any quadrilateral, the sum of all angles is:**
 a. 250° b. 360° c. 290° d. 270°

Solutions

1. (d) $\because \triangle ADC$ is a right angled triangle.

$\therefore AC^2 = AD^2 + DC^2$ [Use Pythagoras theorem]

$$AC^2 = 24^2 + 10^2$$

$$AC^2 = 576 + 100$$

$$AC^2 = 676$$

$$AC = \sqrt{676}$$

$$AC = 26 \text{ cm}$$

So, option (d) is correct.

2. (b) In $\triangle ABC$, P and Q are the mid-points of side AB and BC.

$\therefore PQ = \frac{1}{2}AC$ [By mid-point theorem]

$$PQ = \frac{1}{2} \times 26$$

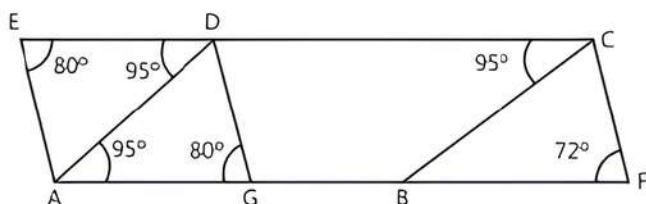
$$PQ = 13 \text{ cm}$$

So, option (b) is correct.

- (c) Since, P, Q, R and S are the mid-point of the sides AB, BC, CD and DA. Therefore, joining adjacent mid-point forms a parallelogram.
So, option (c) is correct.
- (d) by proving all of the above
So, option (d) is correct.
- (b) The sum of all angles in a quadrilateral is 360° .
So, option (b) is correct.

Case Study 3

A parallelogram shape park ABCD is in the middle of the city. Municipality decided to increase its area, so at the left side of park a triangle AED was added and on the right side triangle BFC was added. At point G on AB, municipality put a swing.



On the basis of the above information, solve the following questions:

- Prove that AGDE is a parallelogram when $ED \parallel AG$, $AE \parallel DG$.
- Find the value of $\angle ADC$.
- Find the value of $\angle DCF$.

Solutions

- Given, $AE \parallel DG$ and $ED \parallel AG$
 $\angle EDA = \angle DAG = 95^\circ$ and $\angle AGD = 80^\circ$
 In $\triangle AED$ and $\triangle DGA$,
 $\angle DGA = \angle AED$ [Each 80°]
 $\angle EDA = \angle DAG$ [Each 95°]
 $AD = DA$ [Common]
 $\therefore \triangle AED \cong \triangle DGA$ (by AAS congruence rule)
 $\therefore AE = DG$ [by CPCT]
 $ED = AG$ [by CPCT]
 If in a quadrilateral each opposite sides are equal, then the quadrilateral is a parallelogram.
 So, AGDE is a parallelogram. **Hence proved**

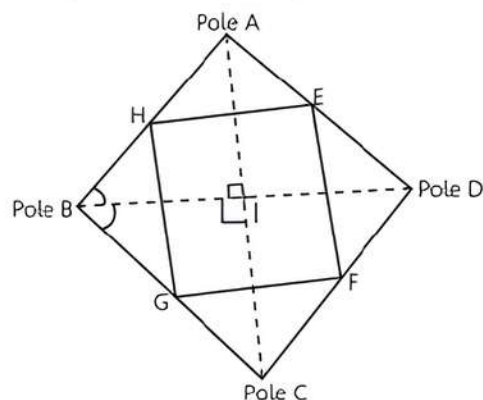
- In parallelogram ABCD, $\angle DAG + \angle ADC + \angle DCB + \angle ABC = 360^\circ$ [Sum of angles of a parallelogram]
 $95^\circ + \angle ADC + 95^\circ + \angle ABC = 360^\circ$
 $2\angle ADC + 190^\circ = 360^\circ$
 $[\because \angle ADC = \angle ABC]$
 $2\angle ADC = 360^\circ - 190^\circ$
 $2\angle ADC = 170^\circ$
 $\angle ADC = 85^\circ$
- In quadrilateral AFCD,
 $\angle DAG + \angle ADC + \angle AFC + \angle DCF = 360^\circ$
 [Angle sum property of a quadrilateral]
 $95^\circ + 85^\circ + 72^\circ + \angle DCF = 360^\circ$
 $\angle DCF = 360^\circ - 95^\circ - 85^\circ - 72^\circ$
 $\angle DCF = 108^\circ$

Case Study 4

Due to frequent robberies in the colony during night. The secretary with the members together decides to attach more lights besides the street light set by municipality. There are poles on which lights are attached.



These 4 poles are connected to each other through wire and they form a quadrilateral. Light from pole B focus light on mid-point G of wire between pole C and B, from pole C focus light on mid-point F of wire between pole C and pole D. Similarly pole D and pole A focus light on the mid-point E and H respectively.



On the basis of the above information, solve the following questions:

- If BD is the bisector of $\angle B$ then prove that I is the mid-point of AC.
- Prove that quadrilateral EFGH is a parallelogram.
- Is it true that every parallelogram is a rectangle?

Solutions

1. In $\triangle BIA$ and $\triangle BIC$,
 $\angle ABI = \angle CBI$ [\because BD is the bisector of $\angle B$]
 $BI = BI$ [Common]
 $\angle BIA = \angle BIC$ [Each 90°]
 $\therefore \triangle BIA \cong \triangle BIC$ [SAS congruence rule]
 $\therefore AI = CI$ [CPCT]

It means I is the mid-point of AC. **Hence proved**

2. Here, $HG = \frac{1}{2}AC$ [By mid-point theorem]

and $EF = \frac{1}{2}AC$ [By mid-point theorem]

$GH \parallel EF$ and $HG = EF$

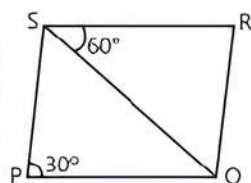
If in a quadrilateral opposite side is parallel and equal then the quadrilateral is a parallelogram. So, quadrilateral EFGH is a parallelogram.

3. It is false, because every parallelogram is not a rectangle.

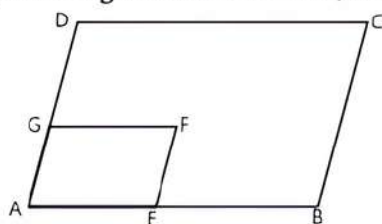
Very Short Answer Type Questions

- Q 1. Two consecutive angles of a parallelogram are in the ratio 1 : 3, then what will be the smaller angle?

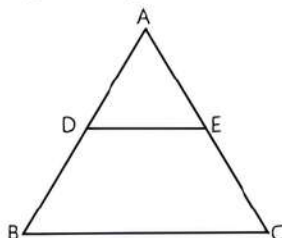
- Q 2. In the given figure, PQRS is a parallelogram in which $\angle QSR = 60^\circ$ and $\angle QPS = 30^\circ$. Find $\angle RQS$.



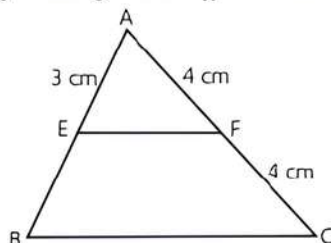
- Q 3. In the following figure, ABCD and AEFG are two parallelograms. If $\angle C = 55^\circ$, determine $\angle F$.



- Q 4. In $\triangle ABC$, D and E are the mid-points of AB and AC respectively. Find DE if $BC = 12$ cm.



- Q 5. In the given figure $EF \parallel BC$. Find BE.

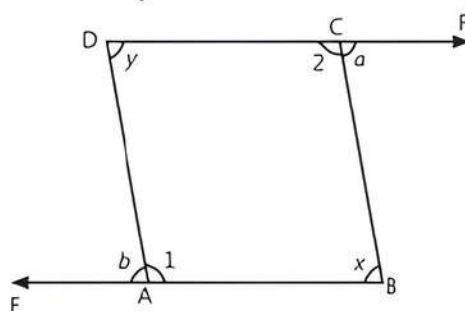


- Q 6. D, E, F are the mid-points of sides BC, CA and AB of $\triangle ABC$. If perimeter of $\triangle ABC$ is 12.8 cm, then what is the perimeter of $\triangle DEF$?



Short Answer Type-I Questions

- Q 1. Two opposite angles of a parallelogram are $(3x - 2)^\circ$ and $(63 - 2x)^\circ$. Find all the angles of a parallelogram.
- Q 2. In quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively, $\angle C = 60^\circ$ and $\angle D = 40^\circ$. Find $\angle AOB$.
- Q 3. In a parallelogram, show that the angle bisectors of two adjacent angles intersect at a right angle.
- Q 4. The angle between the two altitudes of a parallelogram through the vertex of an obtuse angle is 50° . Find the angles of a parallelogram.
- Q 5. A diagonal of a rectangle is inclined to one side of the rectangle at 35° . Find the acute angle between the diagonals.
- Q 6. The sides BA and DC of a quadrilateral ABCD are produced as shown in given figure. Prove that $x + y = a + b$.

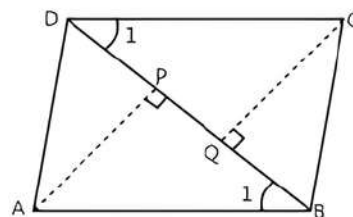


- Q 7. ABCD is a rhombus such that $\angle ACB = 55^\circ$, then find $\angle ADB$.

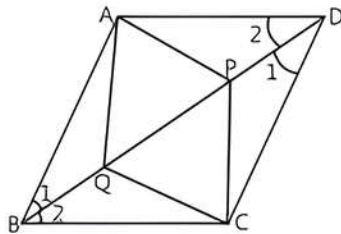


Short Answer Type-II Questions

- Q 1. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see figure). Show that:
 (i) $\triangle APB \cong \triangle CQD$ (ii) $AP = CQ$

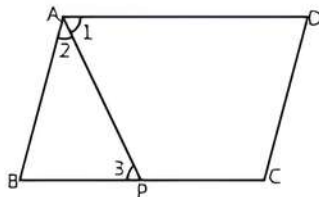


- Q 2. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that $DP = BQ$ (see figure). Show that:

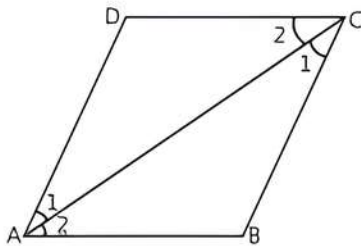


- (i) $\triangle APD \cong \triangle CQB$
- (ii) $AP = CQ$
- (iii) $\triangle AQB \cong \triangle CPD$

Q 3. P is the mid-point of side BC of parallelogram ABCD, such that $\angle 1 = \angle 2$ and $\angle 1 = \angle 3$. Prove that $AD = 2CD$.



- Q 4. Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see figure). Show that:
- (i) AC bisects $\angle C$
 - (ii) ABCD is a rhombus.

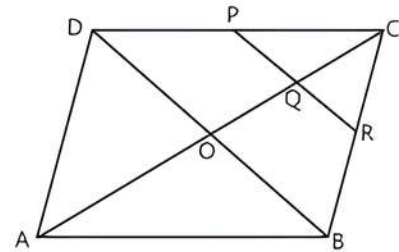


- Q 5. ABC is a triangle right-angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D such that $\angle ADM = \angle ACB$. Show that:
- (i) D is the mid-point of AC.
 - (ii) $MD \perp AC$.
 - (iii) $CM = MA = \frac{1}{2} AB$.

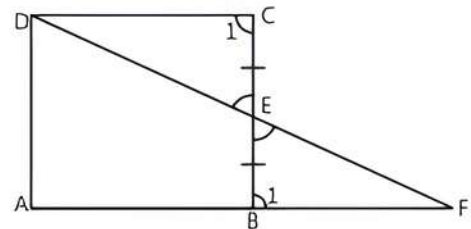
Q 6. D, E and F are the mid-points of the sides BC, CA and AB, respectively of an equilateral triangle ABC. Show that $\triangle DEF$ is also an equilateral triangle.

Q 7. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

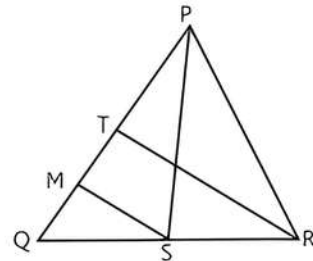
Q 8. In the following figure, ABCD is a parallelogram in which P is the mid-point of DC and Q is a point on AC, such that $CQ = \frac{1}{4} AC$. Also, PQ when produced meets BC at R. Prove that R is the mid-point of BC.



Q 9. In the figure, ABCD is a parallelogram and E is the mid-point of side BC. DE and AB are produced to meet at F. Prove that $AF = 2AB$.

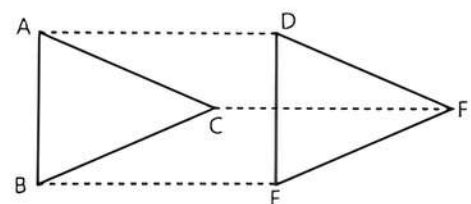


Q 10. In the following figure, PS and RT are medians of $\triangle PQR$ and $SM \parallel RT$. Prove that $QM = \frac{1}{4} PQ$.

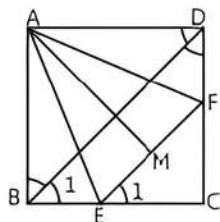


Long Answer Type Questions

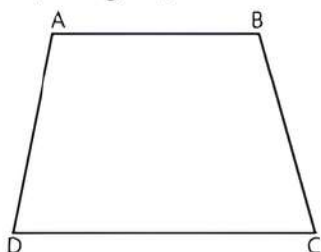
- Q 1. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.
- Q 2. In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively (see figure). Show that:
- (i) Quadrilateral ABED is a parallelogram.
 - (ii) Quadrilateral BEFC is a parallelogram.
 - (iii) $AD \parallel CF$ and $AD = CF$.
 - (iv) Quadrilateral ACFD is a parallelogram.
 - (v) $AC = DF$.
 - (vi) $\triangle ABC \cong \triangle DEF$.



Q 3. In the given figure, ABCD is a square and $EF \parallel BD$. M is the mid-point of EF. Prove that AM bisects $\angle BAD$.

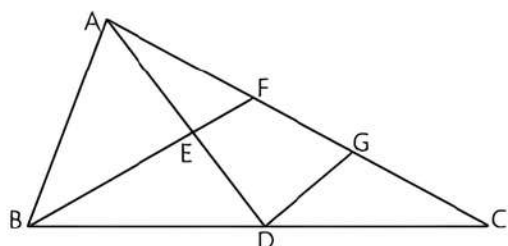


- Q 4. ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$ (see figure).

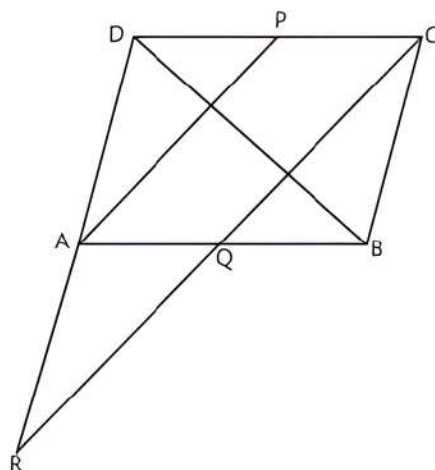


Show that:

- (i) $\angle A = \angle B$ (ii) $\angle C = \angle D$
 (iii) $\triangle ABC \cong \triangle BAD$
 (iv) Diagonal $AC =$ Diagonal BD
- Q 5. Prove that the bisector of the angles of a parallelogram encloses a rectangle.
- Q 6. In given figure, AD is the median of $\triangle ABC$. E is the mid-point of AD. $DG \parallel BF$. Prove that $AC = 3 AF$.



- Q 7. P is the mid-point of the side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q and DA produced to R. Prove that $DA = AR$ and $CQ = QR$.



- Q 8. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.
- Q 9. Two parallel lines l and m are intersected by a transversal p . Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.
- Q 10. E and F are respectively the mid-points of the non-parallel sides AD and BC of a trapezium ABCD.
 Prove that $EF \parallel AB$ and $EF = \frac{1}{2}(AB + CD)$.
- Q 11. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.
- Q 12. Prove that the line segment joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides and equal to half of their difference.

Solutions

Very Short Answer Type Questions

1. Let the consecutive angles be x and $(3x)$.

$$\therefore x + (3x) = 180^\circ$$

$$\Rightarrow 4x = 180^\circ$$

$$\Rightarrow x = 45^\circ$$

Hence, smaller angle is 45° .

2. Given, $\angle QSR = 60^\circ$ and $\angle QPS = 30^\circ$

Since, opposite angles of a parallelogram are equal.

$$\therefore \angle QRS = \angle QPS = 30^\circ$$

In $\triangle QRS$,

$$\angle QRS + \angle QSR + \angle RQS = 180^\circ$$

[Use angle sum property of a triangle]

$$\Rightarrow 30^\circ + 60^\circ + \angle RQS = 180^\circ$$

$$\Rightarrow 90^\circ + \angle RQS = 180^\circ$$

$$\Rightarrow \angle RQS = 90^\circ$$

3. Given, ABCD is a parallelogram.

$$\therefore \angle A = \angle C = 55^\circ \quad [\text{Opposite angles of a parallelogram}]$$

Also, AEFG is a parallelogram.

$$\therefore \angle F = \angle A = 55^\circ \quad [\text{Opposite angles of a parallelogram}]$$

4. Given, D is the mid-point of AB and E is the mid-point of AC.

By mid-point theorem,

$$DE = \frac{1}{2}BC = \frac{1}{2} \times 12 \text{ cm} = 6 \text{ cm}$$

5. Since $AF = FC = 4$ cm,
 \therefore F is the mid-point of AC.
 Also, $EF \parallel BC$
 By the converse of mid-point theorem,
 E is mid-point of AB.
 Hence, $BE = AE = 3$ cm

6. Given, perimeter of $\triangle ABC = 12.8$ cm
 Perimeter of $\triangle ABC$

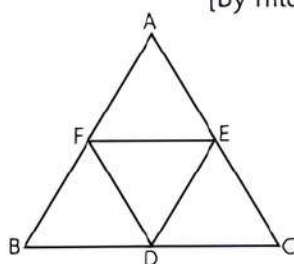
$$= AB + BC + CA = 12.8 \text{ cm}$$

Perimeter of $\triangle DEF$

$$= DE + EF + FD$$

$$= \frac{AB}{2} + \frac{BC}{2} + \frac{CA}{2}$$

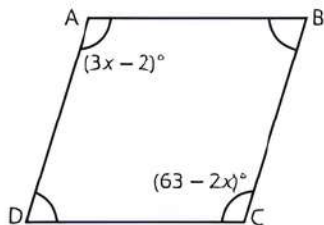
[By mid-point theorem]



$$= \frac{1}{2}(AB + BC + CA) = \frac{12.8}{2} = 6.4 \text{ cm}$$

Short Answer Type-I Questions

1. Since, opposite angles of a parallelogram are equal.



$$\therefore (3x - 2)^\circ = (63 - 2x)^\circ$$

$$\Rightarrow 5x = 65 \Rightarrow x = 13^\circ$$

$$\text{So, } \angle A = (3x - 2)^\circ = (3 \times 13 - 2)^\circ = (39 - 2)^\circ = 37^\circ$$

$$\angle B = 180^\circ - 37^\circ = 143^\circ \quad (\because \angle B = 180^\circ - \angle A)$$

$$\angle C = (63 - 2x)^\circ = (63 - 2 \times 13)^\circ$$

$$= 63 - 26 = 37^\circ$$

$$\text{and } \angle D = 180^\circ - 37^\circ = 143^\circ \quad [\because \angle D = 180^\circ - \angle C]$$

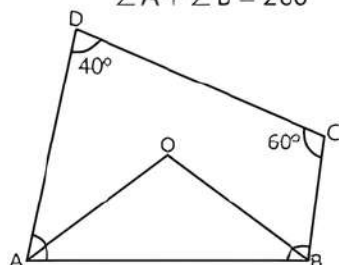
Hence, angles are $37^\circ, 143^\circ, 37^\circ$ and 143° .

2. As we know that, sum of all angles of a quadrilateral is 360° .

$$\therefore \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow \angle A + \angle B + 60^\circ + 40^\circ = 360^\circ$$

$$\Rightarrow \angle A + \angle B = 260^\circ$$



Divide both sides by 2, we get

$$\frac{1}{2} \angle A + \frac{1}{2} \angle B = 130^\circ$$

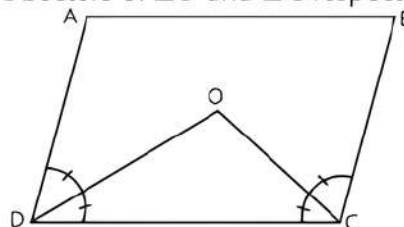
In $\triangle AOB$,

$$\frac{\angle A}{2} + \frac{\angle B}{2} + \angle AOB = 180^\circ$$

$$\Rightarrow 130^\circ + \angle AOB = 180^\circ$$

$$\Rightarrow \angle AOB = 50^\circ$$

3. Let ABCD be a parallelogram and DO and CO are the bisectors of $\angle D$ and $\angle C$ respectively.



In parallelogram ABCD,

$$\angle D + \angle C = 180^\circ$$

[Sum of two adjacent angles]

$$\Rightarrow \frac{1}{2} \angle D + \frac{1}{2} \angle C = \frac{1}{2} \times 180^\circ \quad [\text{Dividing by 2}]$$

$$\Rightarrow \angle ODC + \angle OCD = 90^\circ \quad \dots(1)$$

$[\because \text{DO and CO are the bisectors of } \angle D \text{ and } \angle C \text{ respectively}]$

In $\triangle DOC$,

$$\angle DOC + \angle ODC + \angle OCD = 180^\circ$$

[Angle sum property of a triangle]

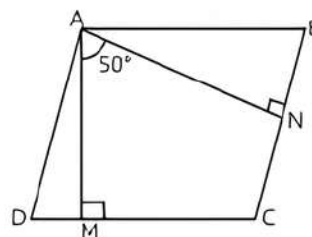
$$\Rightarrow \angle DOC + 90^\circ = 180^\circ \quad [\text{From eq. (1)}]$$

$$\Rightarrow \angle DOC = 180^\circ - 90^\circ$$

$$\Rightarrow \angle DOC = 90^\circ$$

So, the angle bisectors of two adjacent angles intersect at a right angle. **Hence proved**

4. In parallelogram ABCD, let $AM \perp DC$ and $AN \perp BC$.



In quadrilateral AMCN,

$$\angle A + \angle M + \angle C + \angle N = 360^\circ \quad [\text{Angle sum property of a quadrilateral}]$$

$$\therefore \angle A + \angle C = 360^\circ - 180^\circ = 180^\circ$$

$[\because \angle M + \angle N = 180^\circ]$

$$\Rightarrow 50^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 130^\circ$$

In parallelogram ABCD, $\angle A = \angle C = 130^\circ$

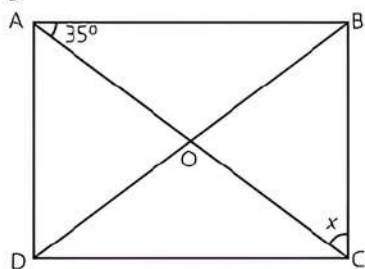


TiP

Adjacent angles of a parallelogram are supplementary.

$$\therefore \angle B = \angle D = 180^\circ - 130^\circ = 50^\circ$$

5. Given, ABCD is a rectangle and diagonals of a rectangle bisect each other.



$$\begin{aligned} \therefore AC &= BD \\ \Rightarrow \frac{1}{2}AC &= \frac{1}{2}BD \\ \Rightarrow OA &= OB \\ \Rightarrow \angle OBA &= \angle OAB \\ &\text{[Angles opposite to equal sides are equal]} \\ \Rightarrow \angle OBA &= 35^\circ \quad [\because \angle OAB = 35^\circ] \end{aligned}$$

$$\begin{aligned} \text{In } \triangle AOB, \angle AOB + \angle OAB + \angle OBA &= 180^\circ \\ &\text{[Angle sum property of a triangle]} \end{aligned}$$

$$\begin{aligned} \Rightarrow \angle AOB + 35^\circ + 35^\circ &= 180^\circ \\ \Rightarrow \angle AOB &= 180^\circ - 70^\circ = 110^\circ \end{aligned}$$

Therefore, $\angle AOB$ is an obtuse angle, but we have to find out acute angle between diagonals.

$$\begin{aligned} \text{So, } \angle AOD + \angle AOB &= 180^\circ \quad \text{[Linear pair]} \\ \Rightarrow \angle AOD + 110^\circ &= 180^\circ \\ \Rightarrow \angle AOD &= 180^\circ - 110^\circ \\ \Rightarrow \angle AOD &= 70^\circ \end{aligned}$$

COMMON ERROR

Sometimes the students do not read the question carefully. In haste they do a mistake of finding an acute angle instead of finding an obtuse angle.

6. From figure,

$$\begin{aligned} \angle 1 + b &= 180^\circ \quad \text{[Linear pair]} \\ \Rightarrow \angle 1 &= 180^\circ - b \quad \dots(1) \end{aligned}$$

Again,

$$\begin{aligned} \angle 2 + a &= 180^\circ \quad \text{[Linear pair]} \\ \Rightarrow \angle 2 &= 180^\circ - a \quad \dots(2) \end{aligned}$$

In quadrilateral ABCD,

$$\begin{aligned} \angle A + \angle B + \angle C + \angle D &= 360^\circ \\ &\text{[Sum of all interior angles in a quadrilateral]} \\ \Rightarrow \angle 1 + x + \angle 2 + y &= 360^\circ \quad \text{[From eqs. (1) and (2)]} \end{aligned}$$

$$\Rightarrow 180^\circ - b + x + 180^\circ - a + y = 360^\circ$$

$$\Rightarrow x + y = 360^\circ - 360^\circ + a + b$$

$$\Rightarrow x + y = a + b \quad \text{Hence proved}$$

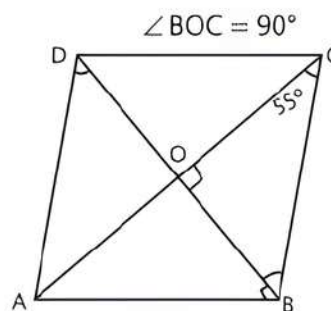
7.



TIP

Diagonals of a rhombus bisect each other at right angle.

Here,



In right-angled $\triangle BOC$,

$$\angle BOC + \angle OCB + \angle OBC = 180^\circ$$

[Angle sum property of a triangle]

$$\Rightarrow 90^\circ + 55^\circ + \angle OBC = 180^\circ$$

$$\Rightarrow 145^\circ + \angle OBC = 180^\circ$$

$$\Rightarrow \angle OBC = 180^\circ - 145^\circ$$

$$\Rightarrow \angle OBC = 35^\circ$$

$$\therefore \angle ADB = \angle OBC = 35^\circ$$

[Alternate interior angles as $AD \parallel BC$]

COMMON ERROR

Adequate practice is required of this type of question.

Short Answer Type-II Questions

1. **Given:** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on BD.

To Prove: (i) $\triangle APB \cong \triangle CQD$

(ii) $AP = CQ$

Proof: (i) In right-angled $\triangle APB$ and right-angled $\triangle CQD$,

$$\angle ABP = \angle CDQ \quad \text{[Given]}$$

$$AB = DC \quad \text{[Opposite sides of a parallelogram]}$$

$$\angle APB = \angle CQD \quad \text{[Each } 90^\circ]$$

$$\therefore \triangle APB \cong \triangle CQD \quad \text{[By ASA congruence rule]}$$

$$\text{(ii) } \therefore \triangle APB \cong \triangle CQD \quad \text{[Proved above]}$$

$$\therefore AP = CQ \quad \text{[By CPCT] Hence proved}$$

2. **Given:** ABCD is a parallelogram and P and Q are points on diagonal BD such that $DP = BQ$.

To Prove: (i) $\triangle APD \cong \triangle CQB$

(ii) $AP = CQ$

(iii) $\triangle AQB \cong \triangle CPD$

Proof: (i) In $\triangle APD$ and $\triangle CQB$,

$$AD = BC \quad \text{[Opposite sides of a parallelogram]}$$

$$\angle ADP = \angle CBQ \quad \text{[Given]}$$

$$DP = BQ \quad \text{[Given]}$$

$$\therefore \triangle APD \cong \triangle CQB \quad \text{[By SAS congruence rule]}$$

$$\text{(ii) } \therefore \triangle APD \cong \triangle CQB \quad \text{[Proved above]}$$

$$\therefore AP = CQ \quad \text{[By CPCT]}$$

(iii) In $\triangle AQB$ and $\triangle CPD$,

$$AB = CD$$

[Opposite sides of a parallelogram are equal]

$$\angle ABQ = \angle CDP \quad [\text{Given}]$$

$$BQ = DP \quad [\text{Given}]$$

$\therefore \triangle AQB \cong \triangle CPD$ [By SAS congruence rule]

3. **Given:** P is the mid-point of side BC of parallelogram ABCD such that $\angle 1 = \angle 2$.

To Prove: $AD = 2CD$

Proof: In parallelogram ABCD,

$$\angle 1 = \angle 3 \quad [\text{Given}]$$

$$\text{But } \angle 1 = \angle 2 \quad [\text{Given}]$$

$$\Rightarrow \angle 2 = \angle 3$$

$$\Rightarrow BP = BA$$

[Sides opposite to equal angles are equal]

But P is the mid-point of BC.

$$\text{So, } BP = \frac{1}{2}BC$$

$$\Rightarrow AB = \frac{1}{2}AD \quad [\because BP = BA \text{ and } BC = AD]$$

$$\Rightarrow CD = \frac{1}{2}AD \quad [\because AB = CD, \text{ opposite sides of a parallelogram}]$$

$$\Rightarrow AD = 2CD. \quad \text{Hence proved}$$

4. **Given:** A parallelogram ABCD in which diagonal AC bisects $\angle A$, i.e., $\angle DAC = \angle BAC$.

To Prove: (i) Diagonal AC bisects $\angle C$ i.e., $\angle DCA = \angle BCA$.

(ii) ABCD is a rhombus.

Proof: (i) In parallelogram ABCD,

$$\angle DAC = \angle BCA = \angle 1 \quad [\text{Given}]$$

$$\angle BAC = \angle DCA = \angle 2 \quad [\text{Given}]$$

$$\text{But } \angle DAC = \angle BAC \quad [\text{Given}]$$

$$\therefore \angle BCA = \angle DCA$$

So, AC bisects $\angle DCB$.

or AC bisects $\angle C$.

Hence proved

(ii) In $\triangle ABC$ and $\triangle ADC$,

$$\angle BAC = \angle DAC \quad [\text{Given}]$$

$$AC = AC \quad [\text{Common}]$$

$$\text{and } \angle BCA = \angle DCA \quad [\text{Proved above}]$$

$$\therefore \triangle ABC \cong \triangle ADC$$

[By ASA congruence rule]

$$\text{Thus, } BC = DC \quad [\text{By CPCT}]$$

$$\text{But } AB = DC$$

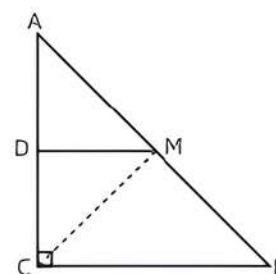
[\because Opposite sides of a parallelogram are equal]

$$\therefore AB = BC = DC = AD$$

So, ABCD is a rhombus.

Hence proved

5. **Given:** A triangle ABC, $\angle C = 90^\circ$, M is the mid-point of AB and BC \parallel DM.



To Prove: (i) D is the mid-point of AC.

(ii) $MD \perp AC$. (iii) $CM = MA = \frac{1}{2}AB$.

Construction: Join CM.

Proof: (i) In $\triangle ABC$,

M is the mid-point of AB. [Given]

$$BC \parallel DM \quad [\text{Given}]$$

\therefore D is the mid-point of AC.

[By converse of mid-point theorem]

(ii) Now, $\angle ADM = \angle ACB$ [Given]

$$\text{But } \angle ACB = 90^\circ \quad [\text{Given}]$$

$$\therefore \angle ADM = 90^\circ$$

$$\text{But } \angle ADM + \angle CDM = 180^\circ \quad [\text{Linear pair}]$$

$$\therefore \angle CDM = 180^\circ - 90^\circ = 90^\circ$$

$$\text{Hence, } MD \perp AC$$

(iii) Again, $AD = DC$... (1)

[D is the mid-point of AC]

Now, in $\triangle ADM$ and $\triangle CDM$,

$$AD = DC \quad [\text{From eq. (1)}]$$

$$\angle ADM = \angle CDM \quad [\text{Each } 90^\circ]$$

$$DM = DM \quad [\text{Common}]$$

$$\therefore \triangle ADM \cong \triangle CDM \quad [\text{By SAS congruence rule}]$$

$$\text{Thus, } MA = CM \quad [\text{By CPCT}] \dots (2)$$

Since M is mid-point of AB.

$$\therefore MA = \frac{1}{2}AB \quad \dots (3)$$

$$\text{So, } CM = MA = \frac{1}{2}AB \quad [\text{From eqs. (2) and (3)}]$$

Hence proved

6. **Given:** D, E and F are the mid-points of BC, CA and AB of $\triangle ABC$.

To Prove: $\triangle DEF$ is an equilateral triangle.

Proof: In $\triangle ABC$, F is the mid-point of AB and E is the mid-point of AC.

So, by mid-point theorem,

$$EF = \frac{1}{2}BC \quad \dots (1)$$

$$\text{Similarly, } DE = \frac{1}{2}AB \quad \dots (2)$$

$$\text{and } DF = \frac{1}{2}AC \quad \dots (3)$$

Since, $\triangle ABC$ is an equilateral triangle.

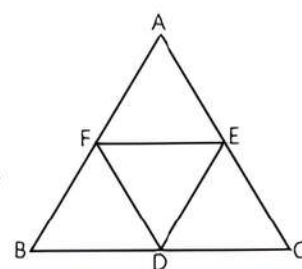
$$\therefore AB = BC = AC \quad [\text{All sides are equal}]$$

From eqs. (1), (2) and (3), we have

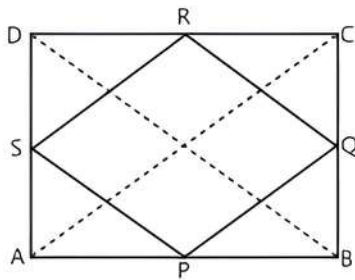
$$EF = DE = DF$$

So, $\triangle DEF$ is also an equilateral triangle.

Hence proved



7. **Given:** A rectangle ABCD in which P, Q, R, S are the mid-points of AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.



To Prove: PQRS is a rhombus.

Construction: Join AC and BD.

Proof: In $\triangle ABC$, P and Q are the mid-points of the sides AB and BC respectively.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \quad \dots(1)$$

[By mid-point theorem]

Similarly, in $\triangle ADC$,

$$SR \parallel AC \text{ and } SR = \frac{1}{2}AC \quad \dots(2)$$

From eqs. (1) and (2), we get

$$PQ \parallel SR \text{ and } PQ = SR \quad \dots(3)$$

Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is parallel and equal [From eq. (3)]

\therefore PQRS is a parallelogram.

R is the mid-point of DC and Q is the mid-point of CB.

$$\therefore RQ \parallel BD \text{ and } RQ = \frac{1}{2}BD \quad \dots(4)$$

[By mid-point theorem]

S is the mid-point of AD and P is the mid-point of AB.

$$\therefore SP \parallel BD \text{ and } SP = \frac{1}{2}BD \quad \dots(5)$$

[By mid-point theorem]

From eqs. (4) and (5), we get

$$RQ \parallel SP \text{ and } RQ = SP \quad \dots(6)$$

But $AC = BD$ [Diagonals of a rectangle]

$$\therefore PQ = QR \quad \text{[From eqs. (1) and (4)]}$$

Thus, $PQ = QR = PS = SR$

So, PQRS is a rhombus. **Hence proved**

8. **Given:** ABCD is a parallelogram in which P is the mid-point of DC and $CQ = \frac{1}{4}AC$.

To Prove: R is the mid-point of BC.

Proof: Since diagonals of a parallelogram bisect each other.

$$\therefore OC = OA$$

$$\text{i.e., } OC = OA = \frac{1}{2}AC \Rightarrow AC = 2OC$$

$$\text{But } CQ = \frac{1}{4}AC \quad \text{[Given]}$$

$$= \frac{1}{4} \times 2OC = \frac{1}{2}OC$$

Thus, Q is the mid-point of CO.

In $\triangle CDO$,

P is the mid-point of CD [Given]

and Q is the mid-point of CO. [Proved above]

$$\therefore PQ \parallel DO \quad \text{[By mid-point theorem]}$$

$$\text{or } PR \parallel DB$$

Now, in $\triangle COB$,

Q is the mid-point of OC. [Proved above]

$$\therefore QR \parallel OB \quad \text{[Proved above]}$$

So, R is the mid-point of CB. [By mid-point theorem]

Hence proved

9. **Given:** In parallelogram ABCD, E is the mid-point of side BC. Produce DE and AB meets at F.

To Prove: $AF = 2AB$.

Proof: In $\triangle DCE$ and $\triangle FBE$,

$$\angle DCE = \angle FBE = 1 \quad \text{[Given]}$$

$$CE = BE \quad \text{[Given]}$$

$$\angle DEC = \angle BEF \quad \text{[Vertically opposite angles]}$$

$$\therefore \triangle DCE \cong \triangle FBE \quad \text{[By ASA congruence rule]}$$

$$\text{Thus, } DC = FB \quad \text{[By CPCT]}$$

$$\begin{aligned} \text{Now, } AF &= AB + BF \\ &= AB + DC = AB + AB \quad [\because DC = AB] \\ &= 2AB \end{aligned}$$

$$\therefore AF = 2AB \quad \text{Hence proved}$$

10. **Given:** PS and RT are the medians of $\triangle PQR$, i.e., S and T are the mid-points of QR and PQ respectively, i.e., $SQ = SR = \frac{1}{2}QR$

$$\text{and } PT = QT = \frac{1}{2}PQ \text{ and } SM \parallel RT$$

To Prove: $QM = \frac{1}{4}PQ$.

Proof: In $\triangle QRT$, S is the mid-point of QR and $SM \parallel RT$.

So, M is the mid-point of QT.

[By converse of mid-point theorem]

$$\therefore QM = MT = \frac{1}{2}QT \quad \dots(1)$$

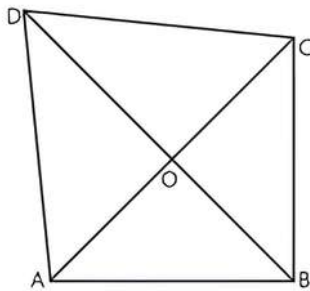
$$\text{But } QT = \frac{1}{2}PQ \quad \text{[Given]}$$

$$QM = \frac{1}{2} \times \frac{1}{2}PQ \quad \text{[From eq. (1)]}$$

$$\Rightarrow QM = \frac{1}{4}PQ \quad \text{Hence proved}$$

Long Answer Type Questions

1. **Given:** A quadrilateral ABCD, in which diagonals AC and BD are equal and bisect each other at right angles.



To Prove: ABCD is a square.

Proof: Since ABCD is a quadrilateral whose diagonals bisect each other, so it is a parallelogram. Also, its diagonals bisect each other at right angles, therefore, ABCD is a rhombus.

$\Rightarrow AB = BC = CD = DA$ [Sides of a rhombus]

In $\triangle ABC$ and $\triangle BAD$,

$$AB = AB \quad \text{[Common]}$$

$$BC = AD \quad \text{[Sides of a rhombus]}$$

$$AC = BD \quad \text{[Given]}$$

$$\therefore \triangle ABC \cong \triangle BAD \quad \text{[By SSS congruence rule]}$$

$$\text{Thus, } \angle ABC = \angle BAD \quad \text{[By CPCT]}$$

$$\text{But } \angle ABC + \angle BAD = 180^\circ \quad \text{[Consecutive interior angles of rhombus]}$$

$$\therefore \angle ABC = \angle BAD = 90^\circ$$

$$\Rightarrow \angle A = \angle B = \angle C = \angle D = 90^\circ \quad \text{[Opposite angles of a rhombus]}$$

\Rightarrow ABCD is a rhombus whose angles are of 90° each.

So, ABCD is a square.

Hence proved

2. **Given:** In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F.

To Prove: (i) ABED is a parallelogram.

(ii) BEFC is a parallelogram.

(iii) $AD \parallel CF$ and $AD = CF$

(iv) Quadrilateral ACFD is a parallelogram.

(v) $AC = DF$

(vi) $\triangle ABC \cong \triangle DEF$

Proof: (i) In quadrilateral ABED,

$$AB = DE \text{ and } AB \parallel DE. \quad \text{[Given]}$$

\Rightarrow ABED is a parallelogram.
[One pair of opposite sides is parallel and equal]

(ii) In quadrilateral BEFC,

$$BC = EF \text{ and } BC \parallel EF \quad \text{[Given]}$$

\Rightarrow BEFC is a parallelogram.

(iii) $BE = CF$ and $BE \parallel CF$ [BEFC is a parallelogram]

$$AD = BE \text{ and } AD \parallel BE$$

[ABED is a parallelogram]

$$\Rightarrow AD = CF \text{ and } AD \parallel CF$$

(iv) In quadrilateral ACFD, $AD = CF$ and $AD \parallel CF$

$$\Rightarrow \text{ACFD is a parallelogram.}$$

[One pair of opposite sides is parallel and equal]

(v) In parallelogram ACFD, $AC = DF$

[Opposite sides of parallelogram are equal]

(vi) In $\triangle ABC$ and $\triangle DEF$,

$$AB = DE \quad \text{[Given]}$$

$$BC = EF \quad \text{[Given]}$$

$$AC = DF \quad \text{[Proved above]}$$

$$\therefore \triangle ABC \cong \triangle DEF \quad \text{[By SSS congruence rule]}$$

Hence proved

3. **Given:** ABCD is a square and BD is a diagonal

$$\therefore \angle CBD = \angle CDB = \frac{1}{2} \times 90^\circ = 45^\circ$$

[\because Diagonal of a square bisect each angle at the vertex]

Also, given $\angle CEF = \angle CBD = 45^\circ$

and $\angle CFE = \angle CDB = 45^\circ$

[$\because \triangle CBD$ is a right angled triangle

$$\therefore \angle CDB = \angle CBD = 45^\circ]$$

$$\Rightarrow CE = CF$$

[\because Sides opposite to equal angles are equal]

$$\Rightarrow BC - CE = CD - CF \quad \text{[$\because BC = CD$]}$$

$$BE = DF \quad \dots(1)$$

Now, in $\triangle ABE$ and $\triangle ADF$,

$$AB = AD \quad \text{[Sides of a square]}$$

$$BE = DF \quad \text{[From eq. (1)]}$$

$$\angle ABE = \angle ADF \quad \text{[Each } 90^\circ]$$

So, $\triangle ABE \cong \triangle ADF$ [By SAS congruence rule]

Then, $AE = AF$ [By CPCT] $\dots(2)$

and $\angle BAE = \angle DAF$ $\dots(3)$

Now, in $\triangle AEM$ and $\triangle AFM$,

$$AE = AF \quad \text{[From eq. (2)]}$$

$$ME = MF \quad \text{[M is mid-point of EF]}$$

$$AM = AM \quad \text{[Common side]}$$

$\therefore \triangle AEM \cong \triangle AFM$ [By SSS congruence rule]

So, $\angle EAM = \angle FAM$ [By CPCT] $\dots(4)$

On adding eqs. (3) and (4), we get

$$\angle BAE + \angle EAM = \angle DAF + \angle FAM$$

$$\Rightarrow \angle BAM = \angle DAM$$

i.e., AM bisects $\angle BAD$.

Hence proved

4. **Given:** In trapezium ABCD, $AB \parallel DC$ and $AD = BC$.

To Prove: (i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

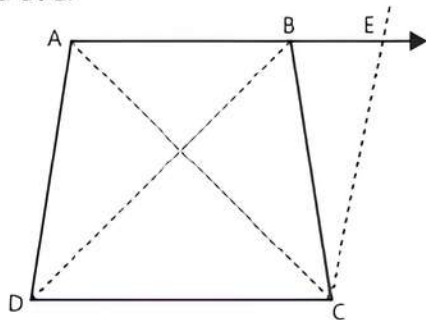
(iv) Diagonal AC = Diagonal BD



TiP

In parallelogram one pair of opposite sides is parallel and equal.

Construction: Join AC and BD. Extend AB and draw a line through C parallel to DA meeting AB produced at E.



Proof: (i) Since $AB \parallel DC$
 $\Rightarrow AE \parallel DC$... (1)
 and $AD \parallel CE$ [By construction] ... (2)
 $\Rightarrow ADCE$ is a parallelogram.

[Opposite pairs of sides are parallel]

$$\angle A + \angle E = 180^\circ \quad \dots (3)$$

[Co-interior angles]

$$\angle B + \angle CBE = 180^\circ \quad \text{[Linear pair]} \dots (4)$$

$$AD = EC \quad \dots (5)$$

[Opposite sides of a parallelogram are equal]

$$AD = BC \quad \text{[Given]} \dots (6)$$

$$\Rightarrow BC = EC \quad \text{[From eqs. (5) and (6)]}$$

$$\Rightarrow \angle E = \angle CBE \quad \dots (7)$$

[Angles opposite to equal sides are equal]

$$\therefore \angle B + \angle E = 180^\circ \quad \dots (8)$$

[From eqs. (4) and (7)]

Now, from eqs. (3) and (8),

$$\angle A + \angle E = \angle B + \angle E$$

$$\Rightarrow \angle A = \angle B$$

$$(ii) \quad \angle A + \angle D = 180^\circ \quad \text{[Consecutive interior angles]}$$

$$\angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle D = \angle B + \angle C$$

$$\Rightarrow \angle D = \angle C \quad [\because \angle A = \angle B]$$

$$\text{or } \angle C = \angle D$$

(iii) In $\triangle ABC$ and $\triangle BAD$,

$$AD = BC \quad \text{[Given]}$$

$$\angle A = \angle B \quad \text{[Proved above]}$$

$$AB = AB \quad \text{[Common]}$$

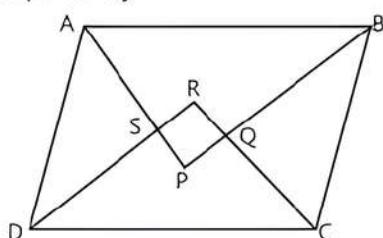
$$\therefore \triangle ABC \cong \triangle BAD \quad \text{[By SAS congruence rule]}$$

(iv) $\therefore \triangle ABC \cong \triangle BAD$

$$\therefore \text{Diagonal } AC = \text{Diagonal } BD \quad \text{[By CPCT]}$$

Hence proved

5. **Given:** ABCD is a parallelogram in which, AP, BP, CR and DR are the bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively.



To Prove: PQRS is a rectangle.

Proof: In parallelogram ABCD,

$$\angle A + \angle D = 180^\circ \quad \text{[Sum of co-interior angles as, } AB \parallel DC]$$

$$\Rightarrow \frac{1}{2} \angle A + \frac{1}{2} \angle D = \frac{180^\circ}{2} \quad \text{[Dividing by 2]}$$

$$\Rightarrow \angle DAS + \angle ADS = 90^\circ$$

In $\triangle ASD$,

$$\angle ASD + \angle DAS + \angle ADS = 180^\circ$$

[Angle sum property of a triangle]

$$\Rightarrow \angle ASD + 90^\circ = 180^\circ \quad \text{[From eq. (1)]}$$

$$\Rightarrow \angle ASD = 180^\circ - 90^\circ = 90^\circ$$

$$\text{Also, } \angle RSP = \angle ASD = 90^\circ$$

[Vertically opposite angles]

Similarly, $\angle PQR = \angle QPS = \angle SRQ = 90^\circ$

When each angle of a quadrilateral is 90° , then it is a rectangle.

So, PQRS is a rectangle.

Hence proved

6. **Given:** In $\triangle ABC$, AD is the median, E is the mid-point of AD and $DG \parallel BF$.

To Prove: $AC = 3AF$

Proof: In $\triangle ADG$,

F is the mid-point of AG.

[Converse of mid-point theorem]

$$\therefore AF = FG \quad \dots (1)$$

In $\triangle CBF$,

$BF \parallel DG$, D is the mid-point of BC.

So, G is the mid-point of FC

$$\therefore FG = GC \quad \dots (2)$$

From eq.(1) and (2), we get

$$AF = FG = GC$$

$$\therefore AC = AF + FG + GC$$

$$\Rightarrow AC = 3AF \quad \text{Hence proved}$$

7. **Given:** ABCD is a parallelogram and P is the mid-point of CD.

To Prove: $DA = AR$ and $CQ = QR$

Proof:

(i) In $\triangle DRC$,

P is the mid-point of DC and $AP \parallel RC$.

So, by converse of mid-point theorem,

A is the mid-point of DR.

$$\Rightarrow DA = AR$$

(ii) In $\triangle RDC$,

A is the mid-point of RD and $AQ \parallel DC$

[Opposite sides of a parallelogram]

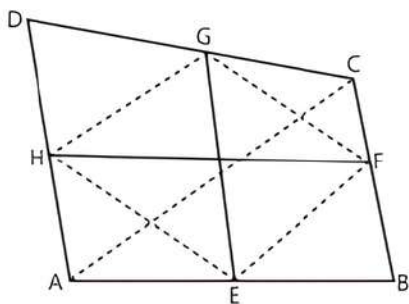
So, by converse of mid-point theorem,

Q is the mid-point of RC.

$$\Rightarrow RQ = QC.$$

Hence proved

8. **Given:** ABCD is a quadrilateral in which EG and FH are the line segments joining the mid-points of opposite sides.



To Prove: EG and FH bisect each other.

Construction: Join EF, FG, GH, HE and AC.

Proof: In $\triangle ABC$, E and F are mid-points of AB and BC respectively.

$$\therefore EF = \frac{1}{2}AC \text{ and } EF \parallel AC \quad \dots(1)$$

[By mid-point theorem]

In $\triangle ADC$, H and G are mid-points of AD and CD respectively.

$$\therefore HG = \frac{1}{2}AC \text{ and } HG \parallel AC \quad \dots(2)$$

[By mid-point theorem]

From eqs. (1) and (2), we get
 $EF = HG$ and $EF \parallel HG$

TiP
In quadrilateral if one pair of opposite sides is equal and parallel, then it is a parallelogram.

\therefore EFGH is a parallelogram.

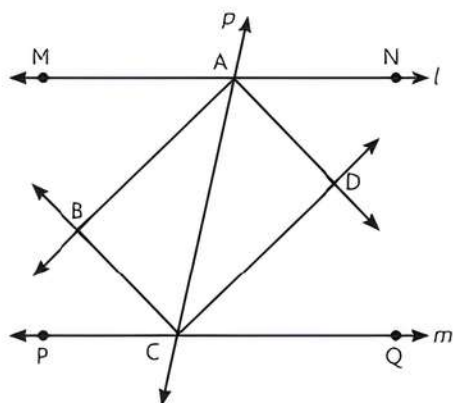
Now, EG and FH are diagonals of the parallelogram EFGH.

\therefore EG and FH bisect each other.

[Diagonals of a parallelogram bisect each other]

Hence proved

9. **Given:** $l \parallel m$ and p is a transversal, AB is the bisector of $\angle MAC$, AD is the bisector of $\angle NAC$, CB is the bisector of $\angle PCA$ and CD is the bisector of $\angle QCA$.



To Prove: ABCD is a rectangle.

Proof: Since, $l \parallel m$,

$$\angle MAC = \angle QCA$$

[Alternate interior angles]

$$\Rightarrow \frac{1}{2}\angle MAC = \frac{1}{2}\angle QCA \quad \text{[Dividing by 2]}$$

$$\Rightarrow \angle BAC = \angle DCA \quad [\because AB \text{ and } CD \text{ are the bisectors of } \angle A \text{ and } \angle C \text{ respectively}]$$

Since these are alternate interior angles, so

$$AB \parallel CD$$

Similarly, $AD \parallel BC$

So, quadrilateral ABCD is a parallelogram.

Now, $\angle MAC + \angle NAC = 180^\circ$ [Linear pair]

$$\Rightarrow \frac{1}{2}\angle MAC + \frac{1}{2}\angle NAC = \frac{1}{2} \times 180^\circ$$

[Dividing by 2]

$$\Rightarrow \angle BAC + \angle DAC = 90^\circ \quad [\because AB \text{ and } AD \text{ are the bisectors of } \angle MAC \text{ and } \angle NAC]$$

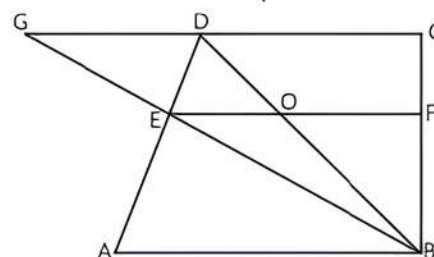
$$\Rightarrow \angle BAD = 90^\circ$$

A parallelogram with one angle 90° is called a rectangle.

So, ABCD is a rectangle.

Hence proved

10. **Given:** In trapezium ABCD in which $AB \parallel CD$. Also E and F are the mid-points of AD and BC.



Construction: Join BE and produce it to meet CD produced at G, also draw BD which intersects EF at O.

To Prove: $EF \parallel AB$ and $EF = \frac{1}{2}(AB + CD)$

Proof: In $\triangle GCB$, E and F are respectively the mid-points of GB and BC, then by mid-point theorem.

$$EF \parallel GC \text{ and } EF = \frac{1}{2}GC$$

$$GC \parallel AB \text{ or } CD \parallel AB \quad \text{[Given]}$$

$$\therefore EF \parallel AB$$

In $\triangle ADB$, E is the mid-point of AD and $AB \parallel EO$.

TiP
The line drawn through the mid-point of one side of a triangle, parallel to another side bisect the third side.

By converse of mid-point theorem, O is the mid-point of BD side.

$$\therefore EO = \frac{1}{2}AB \quad \dots(1)$$

In $\triangle BDC$, OF \parallel DC and O and F are the mid-points of BD and BC.

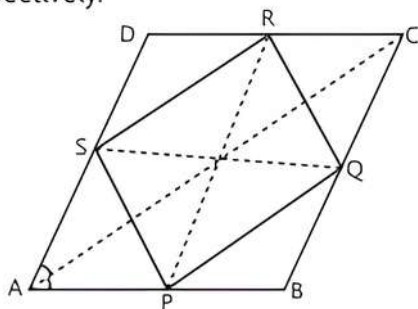
$$\therefore OF = \frac{1}{2}DC \quad \dots(2)$$

On adding eqs. (1) and (2), we get

$$EO + OF = \frac{1}{2}AB + \frac{1}{2}DC$$

$$EF = \frac{1}{2}(AB + CD) \quad \text{Hence proved}$$

11. **Given:** ABCD is a rhombus in which P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively.



To Prove: PQRS is a rectangle.

Construction: Join AC, PR and SQ.

Proof: In $\triangle ABC$,

P is the mid-point of AB and

Q is the mid-point of BC.

$$\Rightarrow PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots(1) \quad [\text{Given}]$$

[By mid-point theorem]

Similarly, in $\triangle DAC$,

$$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \dots(2)$$

From eqs. (1) and (2), we get

$$PQ \parallel SR \text{ and } PQ = SR$$

\Rightarrow PQRS is a parallelogram.

[One pair of opposite sides is parallel and equal]

Since, ABQS is a parallelogram.

$$\Rightarrow AB = SQ \quad \dots(3) \quad [\text{Opposite sides of a parallelogram}]$$

Similarly, PBCR is a parallelogram.

$$\Rightarrow BC = PR \quad \dots(4)$$

From eqs. (3) and (4)

$$\text{Thus, } SQ = PR \quad [\because AB = BC]$$

Since, SQ and PR are diagonals of parallelogram PQRS, which are equal.

\Rightarrow PQRS is a rectangle.

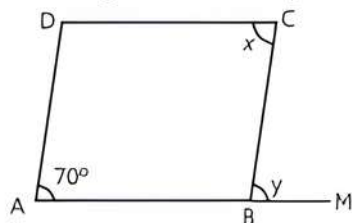
Hence proved



Chapter Test

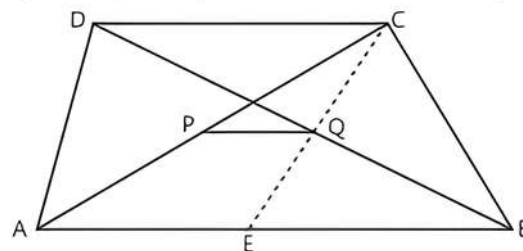
Multiple Choice Questions

- Q 1. If ABCD is a parallelogram in which $\angle DAB = 70^\circ$ and AB is produced to point M as shown in the figure. Then $x + y$ is:



- a. 70° b. 140° c. 120° d. 130°
- Q 2. In the given figure, ABCD is a parallelogram in which $\angle BDC = 50^\circ$ and $\angle BAD = 40^\circ$. Then $\angle CBD$ is equal to:
- a. 80° b. 70° c. 90° d. 50°

12. **Given:** ABCD is a trapezium with $AB \parallel DC$ and P, Q are the mid-points of CA and DB respectively.



To Prove: $PQ \parallel AB \parallel DC$ and

$$PQ = \frac{1}{2} (AB - DC)$$

Construction: Join CQ and produce it to meet AB at E.

Proof: In $\triangle CDQ$ and $\triangle EBQ$,

$$DQ = BQ \quad \dots(1) \quad [\text{Given}]$$

$$\angle DCQ = \angle BEQ$$

[Alternate interior angles since, $\angle DCQ = \angle DCE$ and $\angle BEQ = \angle BEC$]

$$\angle CDQ = \angle EBQ \quad [\text{Alternate interior angles}]$$

$$\therefore \triangle CDQ \cong \triangle EBQ \quad [\text{By AAS congruence rule}]$$

$$\text{Thus, } CQ = QE \quad [\text{By CPCT}]$$

$$DC = EB \quad [\text{By CPCT}]$$

Now, in $\triangle CEA$,

Q is the mid-point of CE [Proved above]

P is the mid-point of CA. [Given]

So, by mid-point theorem,

$$PQ \parallel AE \text{ and } PQ = \frac{1}{2} AE$$

$$\text{So, } PQ \parallel AB \parallel DC \quad [\because PQ \parallel AB \text{ and } AB \parallel DC]$$

$$\text{and } PQ = \frac{1}{2} AE = \frac{1}{2} (AB - EB) = \frac{1}{2} (AB - DC)$$

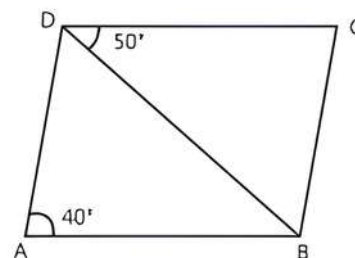
$$[\because EB = DC \text{ proved above}]$$

Hence proved

Assertion and Reason Type Questions

Directions (Q.Nos. 3-4) In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:

- a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).



- c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 3. Assertion (A): The opposite angles of a parallelogram are $(2x - 3)^\circ$ and $(54 - x)^\circ$. The measure of one of the angle is 52° .

Reason (R): Opposite angles of a parallelogram are equal.

Q 4. Assertion (A): Diagonals AC and BD of a parallelogram in ABCD intersect each other at point O. If $\angle DAC = 40^\circ$ and $\angle AOB = 60^\circ$, then $\angle DBC = 20^\circ$.

Reason (R): The adjacent angles of a parallelogram is supplementary.

Fill in the Blanks

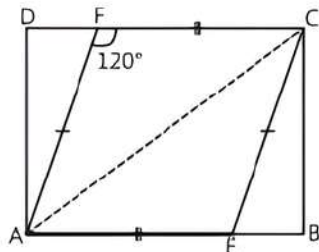
- Q 5.** The diagonals of a square bisect each other at
- Q 6.** The line segment joining the mid-points of the two sides of the triangle is to the third side.

True/False

- Q 7.** In a parallelogram, diagonals, bisect each other.
Q 8. The sum of the adjacent angles of a parallelogram is 90° .

Case Study Based Question

- Q 9.** An organisation was donated with some land for charity, which is in the shape of parallelogram AECF. The organisation was added some piece of land and converted it into a rectangular plot by adding triangular land $\triangle ADF$ and $\triangle BCF$.



On the basis of the above information, solve the following questions:

- (i) Write the measure of $\angle AFD$.
(ii) What is the measure of $\angle AEC$?

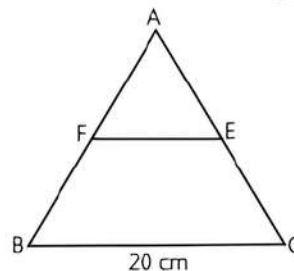
OR

In which criteria, $\triangle ADF$ and $\triangle CBF$ is congruent.

- (iii) What is the measure of $\angle FAE$?

Very Short Answer Type Questions

- Q 10.** In triangle ABC, if F and E are the mid points of AB and AC, then find the length of FE.



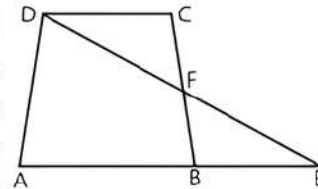
- Q 11.** Two consecutive angles of a parallelogram are in the ratio 2 : 3, then what will be the smaller angle?

Short Answer Type-I Questions

- Q 12.** A diagonal of a rectangle is inclined to one side of the rectangle at 45° . Find the angle between the diagonals.
Q 13. The angle between the two altitudes of a parallelogram through the vertex of an obtuse angle is 60° . Find the angles of a parallelogram.

Short Answer Type-II Questions

- Q 14.** In the figure, ABCD is a trapezium with $AB \parallel DC$. F is the mid-point of BC. DF and AB are produced to meet at E. Show that F is also the mid-point of DE.



- Q 15.** P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which $AC = BD$. Prove that PQRS is a rhombus.

Long Answer Type Question

- Q 16.** In a parallelogram ABCD, E and F are the mid points of sides AB and CD, respectively. Show that the line segments AF and EC trisect the diagonal BD.

