

Sum of Odd Natural Numbers

Objective

To verify that the sum of first n odd natural numbers, $1 + 3 + 5 + \dots + (2n - 1) = n^2$ by paper activity.

Prerequisite Knowledge

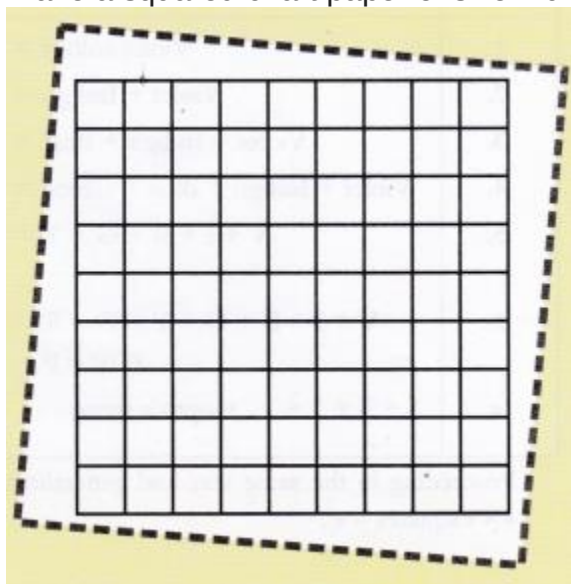
1. Natural numbers (i.e. counting numbers e.g., 1, 2, 3, 4, ...)
2. Odd natural numbers (Natural numbers which are not divisible by 2 i.e. 1, 3, 5, ...)
3. Even natural numbers (Natural numbers which are divisible by 2 i.e. 2, 4, 6, ...)
4. Formula to find the n^{th} term of an AP i.e.,
 $a_n = a + (n - 1)d$
where $a \rightarrow$ first term, $d \rightarrow$ common difference, $n \rightarrow$ no. of terms
5. For odd natural numbers 1, 3, 5, ..., term is
 $a_n = 1 + (n - 1) \cdot 2 = (2n - 1)$
6. Area of squares.

Materials Required

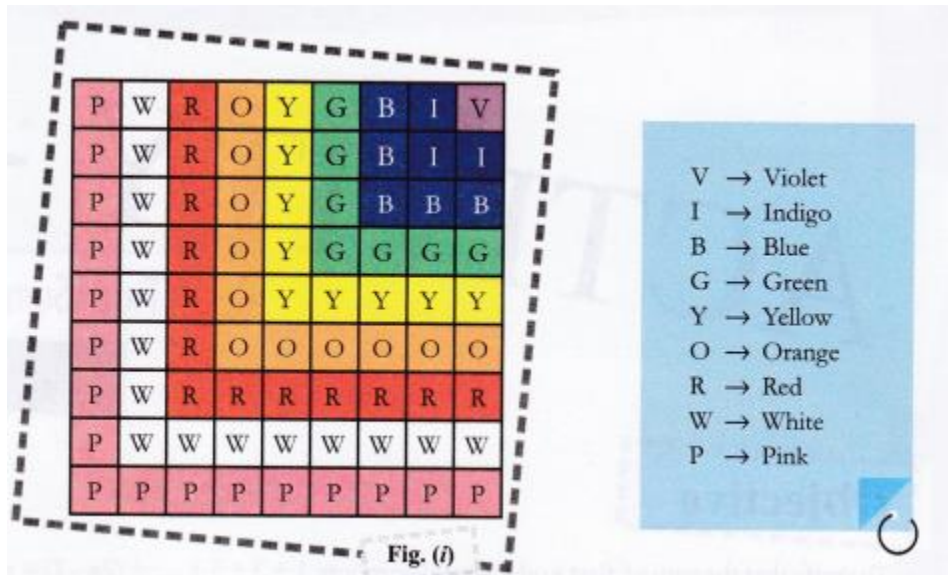
Squared papers, sketch pens, pencil, a pair of scissors, geometry box, fevicol, white drawing sheets

Procedure

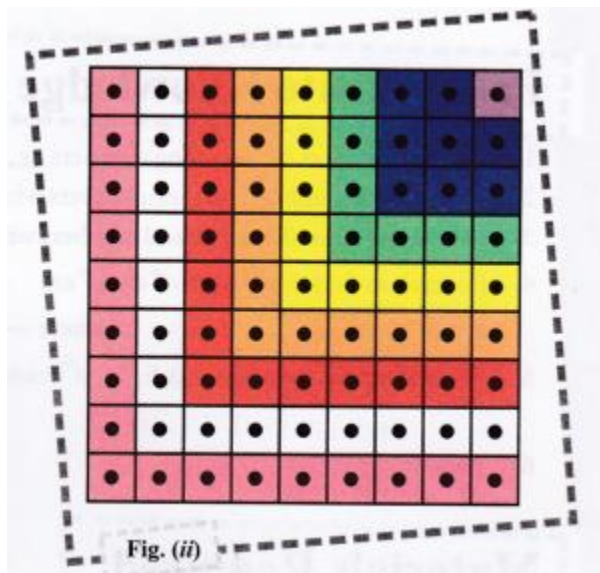
1. Take a squared chart paper of size n units $\times$ n units (Take $n = 9$).



2. Paste it on a white sheet.
3. Colour the internal squares with different 9 colours as shown in the fig. (i).



4. Put a black dot on each of the internal coloured squares as shown in fig.(ii)



Observation

S.No.	No. of shaded squares	No. of dots	Square	In terms of n^2
1.	Violet colour = 1	1	(1×1) squares	1^2
2.	Violet + Indigo = $1 + 3$	4	(2×2) squares	2^2
3.	Violet + Indigo + Blue = $1 + 3 + 5$	9	(3×3) squares	3^2
4.	Violet + Indigo + Blue + Green = $1 + 3 + 5 + 7$	16	(4×4) squares	4^2
5.	V + I + B + G + Y = $1 + 3 + 5 + 7 + 9$	25	(5×5) squares	5^2
$\vdots$				
9.	V + I + B + G + Y + O + R = $1 + 3 + 5 + 7 + 9$ + W + P = $11 + 13 + 15 + 17$	81	(9×9) squares	9^2
$\vdots$				
n	$1 + 3 + 5 + \dots$ + upto n terms	$n \times n$	$(n \times n)$ squares	n^2

Proceeding in the same way and generalising the result on the above table, we can say that the number of dots in $n \times n$ squares = n^2 .

Result

The sum of first n odd natural numbers is n i.e., $1 + 3 + 5 + 7 + \dots + (2n - 1) = n$

Learning Outcome

Through this activity students are able to find that the sum of first n odd natural numbers n^2 i.e., $\sum (2n - 1) = n^2$

Activity Time

- Find out the sum of first fifteen odd natural numbers with the help of above activity and verify the result using the formula $\sum (2n - 1) = n^2$
- Find the sum of $(11 + 13 + \dots + 21)$ using the result of this activity.

Viva Voce

Question 1:

What is the sum of first n natural numbers ?

Answer:

$$S_n = \frac{n(n+1)}{2}$$

Question 2:

Give the formula for sum of first $(n + 1)$ natural numbers.

Answer:

$$S_{n+1} = \frac{(n+2)(n+1)}{2}$$

Question 3:

What is the sum of first n multiples of 5 ?

Answer:

$$\frac{5 \times n(n+1)}{2}$$

Question 4:

What is the sum of $2 + 6 + 10 + 14 + 18 + \dots + 10$ terms ?

Answer:

200

[Hint: $2 [1 + 3 + 5 + \dots + 10 \text{ terms}] = 2 \times 10^2 = 200$]

Question 5:

What is the n th term of $1 + 3 + 5$?

Answer:

$$a_n = (2n - 1)$$

Question 6:

What is the n th term of $2 + 4 + 6$?

Answer:

$$a_n = 2n$$

Question 7:

Give the formula for the sum to n terms of an AP.

Answer:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Question 8:

Define odd numbers and which is the first odd natural number.

Answer:

Numbers which are not divisible by 2 are known as odd numbers. 1 is the first odd natural number.

Multiple Choice Questions

Question 1:

Find n for $\frac{n(n+1)}{2} = 55$

- (a) -11
- (b) 10
- (c) 11
- (d) None of these

Question 2:

Using, $\frac{n(n+1)}{2}$ evaluate $1 + 3 + 5 + 7 + 9$.

- (a) 24
- (b) 23
- (c) 25
- (d) 26

Question 3:

If n th term of an AP is $(2n + 1)$, find the sum of first n terms of the AP.

- (a) $n(n+2)$
- (b) $n(n-2)$
- (c) (n^2-2)
- (d) None of these

Question 4:

For AP $5 + 4 + 3 + 2 + 1 + 0 + (-1) + \dots$ what should be its last term so that sum of all terms is zero ?

- (a) -4
- (b) -7
- (c) -5
- (d) None of these

Question 5:

In AP $16 + 12 + 8 + 4 + 0 + (-4) + (-8) + (-12) + \dots$ S_7 is

- (a) 28
- (b) 16
- (c) 20
- (d) None of these

Question 6:

Common difference of an AP $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ is

- (a) $\sqrt{3}$
- (b) $\sqrt{2}$
- (c) $2\sqrt{3}$
- (d) 4

Question 7:

First term of an AP is p and its common difference is q then its 10th term is

- (a) $9p + q$
- (b) $p-9q$
- (c) $p + 9q$
- (d) None of these

Question 8:

The sum of first n positive integer S_n is

- (a) $\frac{n(n-1)}{2}$
- (b) $\frac{n(n+1)}{2}$
- (c) $\frac{n}{2}[2a + (n-1)d]$
- (d) $a+(n-1)d$

Question 9:

20th term of the series 4, 7, 10, is

- (a) 61
- (b) 60
- (c) 59
- (d) None of these

Question 10:

Choose the correct option : First four terms of the AP whose first term is -1 and common difference is $\frac{1}{2}$

- (a) $-1, -\frac{1}{2}, \frac{1}{2}, 0$
- (b) $-1, 0, -\frac{1}{2}, \frac{1}{2}$
- (c) $-1, -\frac{1}{2}, 0, \frac{1}{2}$
- (d) $0, -\frac{1}{2}, \frac{1}{2}, 1$

Answers

- 1. (b)
- 2. (c)
- 3. (a)
- 4. (c)
- 5. (a)
- 6. (b)
- 7. (c)
- 8. (b)
- 9. (a)
- 10. (c)