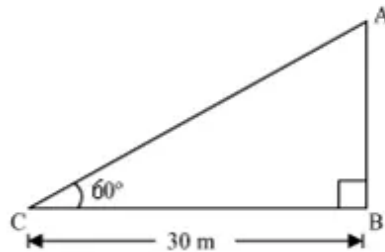


Chapter 22. Heights and Distances

Ex 22.1

Answer 2.



Let AB be the cliff and angle of elevation from point C (on ground) is 60° .

In $\triangle ABC$

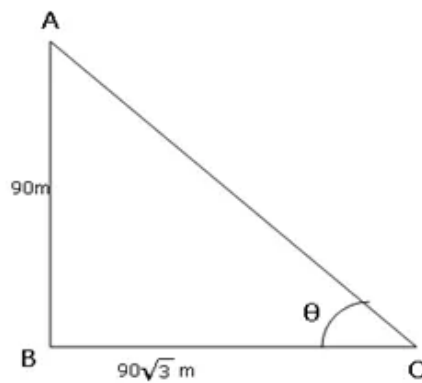
$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{AB}{30} = \sqrt{3}$$

$$AB = 30\sqrt{3} \text{ m}$$

So, height of tower is $30\sqrt{3}$ m.

Answer 3.



Let AB be the pole and BC be its shadow.

In $\triangle ABC$,

$$\tan \theta = \frac{AB}{BC}$$

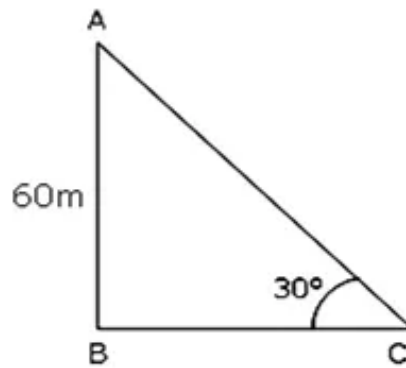
$$\Rightarrow \tan \theta = \frac{90}{90\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\text{But, } \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^\circ$$

Thus, the angle of elevation is 30° .

Answer 5.



Let AB be the tree of height 60 m and BC be its shadow.

In $\triangle ABC$

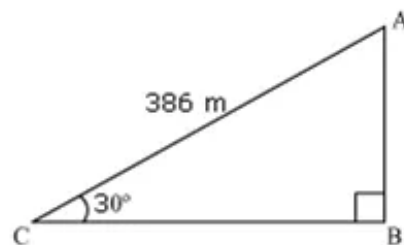
$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{60}{BC} = \frac{1}{\sqrt{3}}$$

$$BC = 60\sqrt{3} \text{ m}$$

So, height of tower is $60\sqrt{3}$ m.

Answer 6.



The plane takes off from point C on the ground. Let A be the final position of the plane.

In $\triangle ABC$

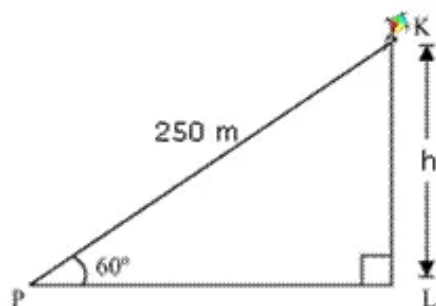
$$\frac{AB}{AC} = \sin 30^\circ$$

$$\frac{AB}{386} = \frac{1}{2}$$

$$h = \frac{386}{2} = 193$$

Thus, the required height of the aeroplane above the ground is 193 m.

Answer 7.



Let K be the kite and the string is tied to point P on ground.

In $\triangle KLP$

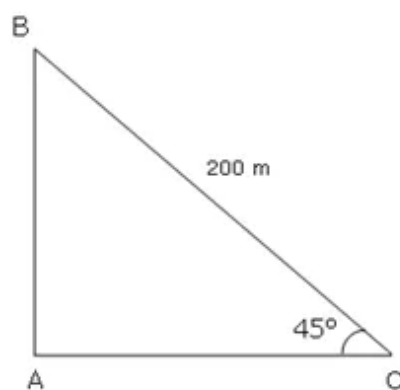
$$\frac{KL}{KP} = \sin 60^\circ$$

$$\frac{h}{250} = \frac{\sqrt{3}}{2}$$

$$h = \frac{250\sqrt{3}}{2} = 125\sqrt{3}$$

Thus, the perpendicular height of the kite is $125\sqrt{3}$ m.

Answer 8.



The topmost branch of the tree is at point B and C is the point on the ground to which the topmost branch is tied.

In $\triangle ABC$,

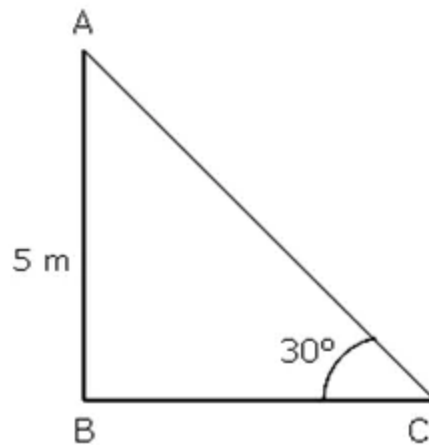
$$\frac{AC}{BC} = \cos 45^\circ$$

$$\frac{h}{200} = \frac{1}{\sqrt{2}}$$

$$h = \frac{200}{\sqrt{2}} = \frac{200\sqrt{2}}{2} = 100\sqrt{2} = 100 \times 1.414 = 141.4$$

Thus, the required distance is 141.4 m.

Answer 9.



Here, AC is the ladder and AB is the point which is 5 m above the ground.

In $\triangle ABC$,

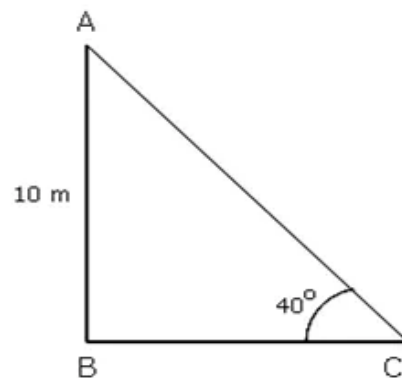
$$\sin 30^\circ = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{5}{AC}$$

$$\Rightarrow AC = 10$$

Thus, the length of the ladder is 10 m.

Answer 10.



Let AB be the pole and AC be the wire which runs from the top of the pole to the point on the ground where its other end is fixed.

In $\triangle ABC$,

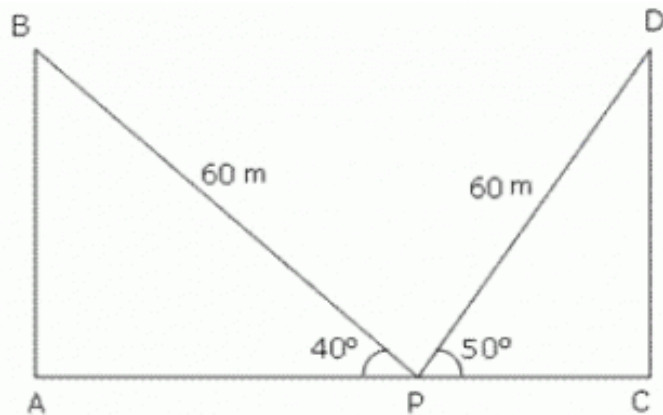
$$\sin 40^\circ = \frac{AB}{AC}$$

$$\Rightarrow 0.6428 = \frac{10}{AC}$$

$$\Rightarrow AC = \frac{10}{0.6428} = 15.6$$

Thus, the length of the wire is 15.6 m.

Answer 11.



Let AB and CD be two trees and P be a point on the street AC between the two trees.

PD and PB denotes the ladder at the two instants.

In $\triangle PCD$,

$$\cos 50^\circ = \frac{PC}{PD}$$

$$0.6428 = \frac{PC}{60}$$

$$\Rightarrow PC = 0.6428 \times 60 = 38.568$$

In $\triangle ABP$,

$$\cos 40^\circ = \frac{AP}{BP}$$

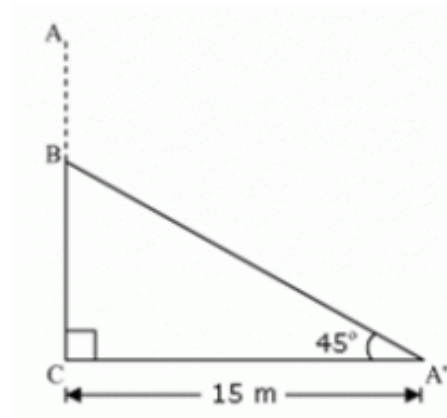
$$\Rightarrow 0.7660 = \frac{AP}{60}$$

$$\Rightarrow AP = 0.7660 \times 60 = 45.96$$

$$\therefore AC = AP + PC = 38.568 \text{ m} + 45.96 \text{ m} = 84.528 \text{ m} \approx 84.53 \text{ m}.$$

Thus, the width of the street is 84.53 m.

Answer 12



Let AC was original tree. Due to storm it was broken into two parts. The broken part A' B is making 45° with ground.

In $\Delta A'BC$

$$\frac{BC}{A'C} = \tan 45^\circ$$

$$\frac{BC}{15} = 1$$

$$BC = 15$$

$$\frac{A'C}{A'B} = \cos 45^\circ$$

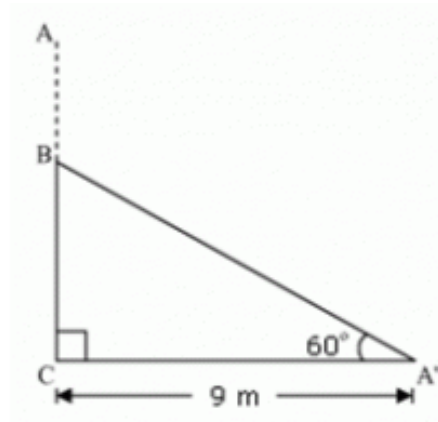
$$\frac{15}{A'B} = \frac{1}{\sqrt{2}}$$

$$A'B = 15\sqrt{2}$$

$$\text{Height of tree} = A'B + BC = 15 + 15\sqrt{2} = 15(1 + \sqrt{2}) = 15 \times 2.414 = 36.21$$

Hence, the height of tree was 36.21 m.

Answer 13.



Let AC was original tree. It was broken into two parts. The broken part BA' is making 60° with ground.

In $\triangle A'BC$

$$\frac{BC}{A'C} = \tan 60^\circ$$

$$\frac{BC}{9} = \sqrt{3}$$

$$BC = 9\sqrt{3}$$

$$\frac{A'B}{A'C} = \cos 60^\circ$$

$$\frac{A'B}{9} = \frac{1}{2}$$

$$A'B = 18$$

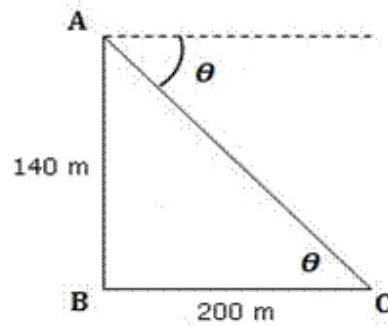
$$\text{Height of tree} = A'B + BC$$

$$= 9\sqrt{3} + 18 = 9 \times 1.732 + 18 = 15.588 + 18 = 33.588$$

Hence, the height of tree was $33.588 \text{ m} = 33.6 \text{ m}$ (approximately).

Answer 14.

Let AB be the pillar. Let the angle of depression be θ .



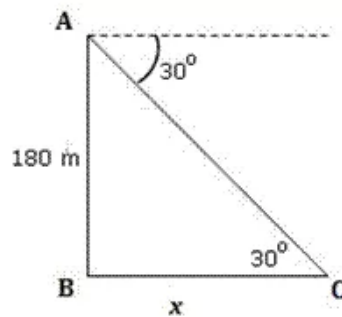
In triangle ABC,

$$\tan \theta = \frac{AB}{BC}$$

$$\Rightarrow \tan \theta = \frac{140}{200} = \frac{7}{10} = 0.7$$

We have: $\tan 35^\circ = 0.7$

Thus, the angle of depression is $\theta = 35^\circ$.

Answer 15.

Let AB be the tower and C be the position of the ship.

Let the distance of the boat from the foot of the observation tower be x .

In triangle ABC,

$$\tan \theta = \frac{AB}{BC}$$

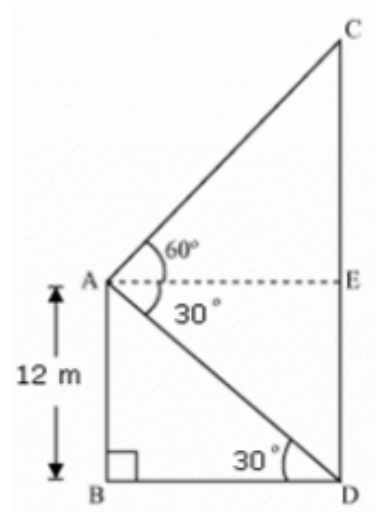
$$\Rightarrow \tan 30^\circ = \frac{180}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{180}{x}$$

$$\Rightarrow x = 180\sqrt{3} = 180 \times 1.732 = 311.76$$

Thus, the distance of the boat from the foot of the observation tower is 311.76 or 311.8 m.

Answer 16.



Let AB be the building and CD be the tower.

In $\triangle ABD$

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{12}{BD} = \frac{1}{\sqrt{3}}$$

$$BD = 12\sqrt{3}$$

In $\triangle ACE$

$$AE = BD = 12\sqrt{3}$$

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{CE}{12\sqrt{3}} = \sqrt{3}$$

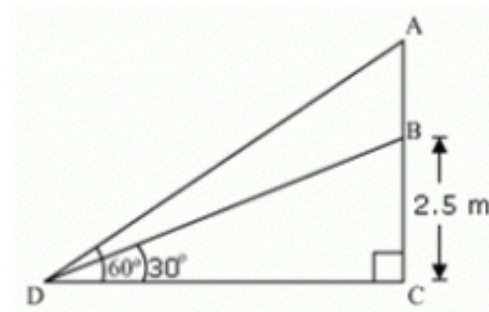
$$CE = 12\sqrt{3} \times \sqrt{3} = 12 \times 3 = 36$$

$$CD = CE + ED = 36 + 12 = 48$$

So, height of the tower is 48 m and its distance from the building

$$12\sqrt{3} \text{ m} = 12 \times 1.732 \text{ m} = 20.78 \text{ m (approximately)}.$$

Answer 17.



Let AB be the flagstaff, BC be the pole and D be the point on ground from where elevation angles are measured.

In $\triangle BCD$

$$\frac{BC}{CD} = \tan 30^\circ$$

$$\frac{BC}{CD} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}BC = CD \quad \dots (1)$$

In $\triangle ACD$

$$\frac{AB + BC}{CD} = \tan 60^\circ$$

$$\frac{AB + BC}{CD} = \sqrt{3}$$

$$AB + 2.5 = CD\sqrt{3} = 3BC \quad [\text{Using (1)}]$$

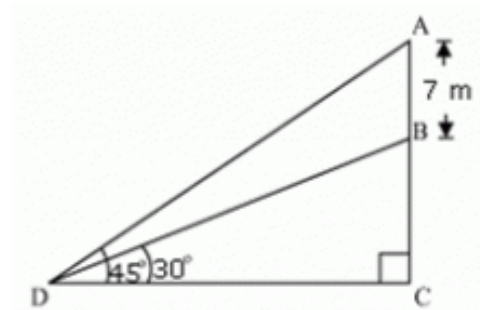
$$AB + 2.5 = 3 \times 2.5$$

$$AB + 2.5 = 7.5$$

$$AB = 5$$

Thus, the height of the flagstaff is 5 m.

Answer 18.



Let AB be the flagstaff, BC be the tower and D be the point on ground from where elevation angles are measured.

In $\triangle BCD$

$$\frac{BC}{CD} = \tan 30^\circ$$

$$\frac{BC}{CD} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}BC = CD$$

In $\triangle ACD$

$$\frac{AB + BC}{CD} = \tan 45^\circ$$

$$\frac{AB + BC}{\sqrt{3}BC} = 1$$

$$7 + BC = \sqrt{3}BC$$

$$BC(\sqrt{3} - 1) = 7$$

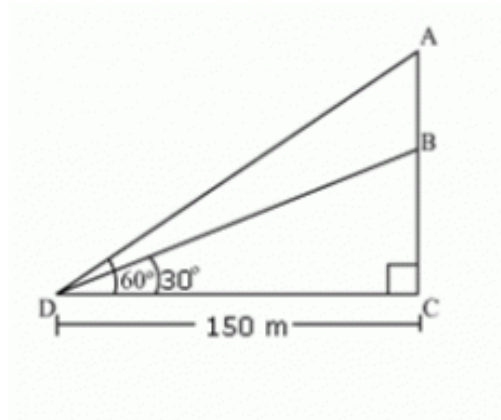
$$BC = \frac{(7)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{7(\sqrt{3} + 1)}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{7(\sqrt{3} + 1)}{2} = 3.5(\sqrt{3} + 1) = 3.5 \times 2.732 = 9.562$$

Thus, the height of the tower is 9.562 m = 9.56 m.

Answer 19.



Let BC be the length of unfinished tower. Let the tower be raised up point A so that the angle of elevation at point A is 60° . D is the point ground from where elevation angles are measured.

In $\triangle BCD$

$$\frac{BC}{CD} = \tan 30^\circ$$

$$\frac{BC}{CD} = \frac{1}{\sqrt{3}}$$

$$BC = \frac{CD}{\sqrt{3}}$$

$$BC = \frac{150}{\sqrt{3}} \quad \dots (1)$$

In $\triangle ACD$

$$\frac{AB + BC}{CD} = \tan 60^\circ$$

$$\Rightarrow \frac{AB + BC}{CD} = \sqrt{3}$$

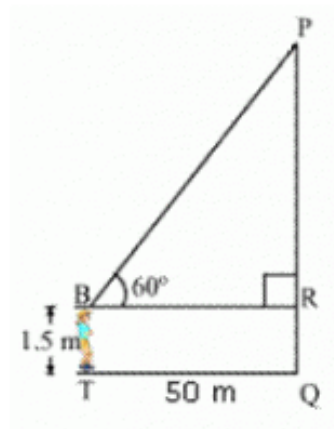
$$\Rightarrow \frac{AB + \frac{150}{\sqrt{3}}}{150} = \sqrt{3} \quad \dots (\text{using (1)})$$

$$\Rightarrow AB = 150\sqrt{3} - \frac{150}{\sqrt{3}} = \frac{150 \times 3 - 150}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{300}{\sqrt{3}} = \frac{300}{1.732} = 173.2\text{m}$$

Thus, the required height is 300 m.

Answer 20.



Let the position of the boy be at point T.

$$BR = TQ = 50 \text{ m}$$

$$RQ = BT = 1.5 \text{ m}$$

In $\triangle PRB$

$$\frac{PR}{BR} = \tan 60^\circ$$

$$\frac{PR}{50} = \sqrt{3}$$

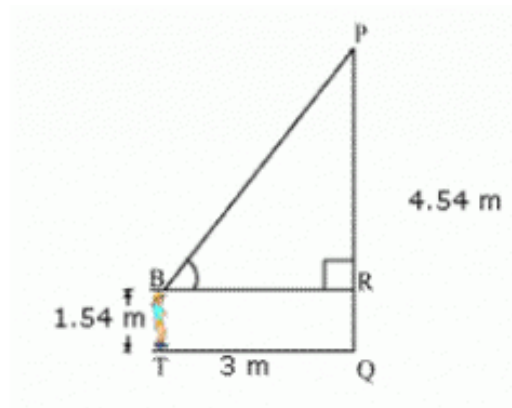
$$PR = 50\sqrt{3}$$

Height of the tower

$$= PQ = PR + RQ = 50\sqrt{3} + 1.5 = 50 \times 1.732 + 1.5 = 86.6 + 1.5 = 88.1 \text{ m}$$

Thus, the height of the tower is approximately 88 m.

Answer 21.



Let the position of the boy be at point T and P be the position of the sur

$$BR = TQ = 3 \text{ m}$$

$$PQ = 4.54 \text{ m}$$

$$BT = 1.54 \text{ m}$$

$$\therefore PR = 4.54 \text{ m} - 1.54 \text{ m} = 3 \text{ m}$$

In $\triangle PRB$

$$\frac{PR}{BR} = \tan \theta$$

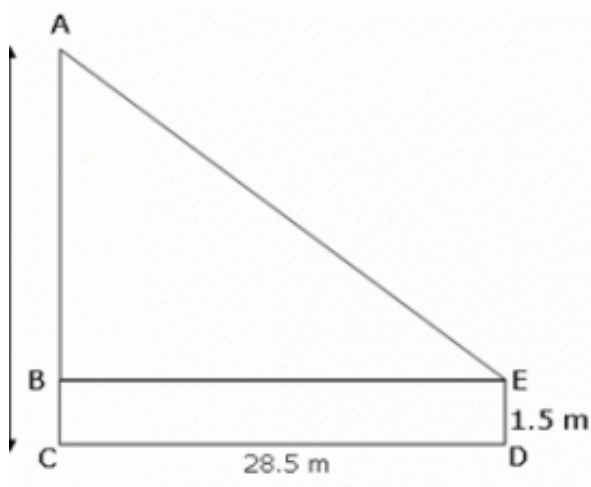
$$\frac{3}{3} = \tan \theta$$

$$\tan \theta = 1$$

We know that $\tan 45^\circ = 1$.

Thus, the angle of elevation is $\theta = 45^\circ$.

Answer 22.



Here, ED is the height of the observer and AC is the tower.

$$BE = CD = 28.5 \text{ m}$$

$$AB = AC - BC = 30 \text{ m} - 1.5 \text{ m} = 28.5 \text{ m}$$

In $\triangle ABE$,

$$\tan \angle ABE = \frac{AB}{BE}$$

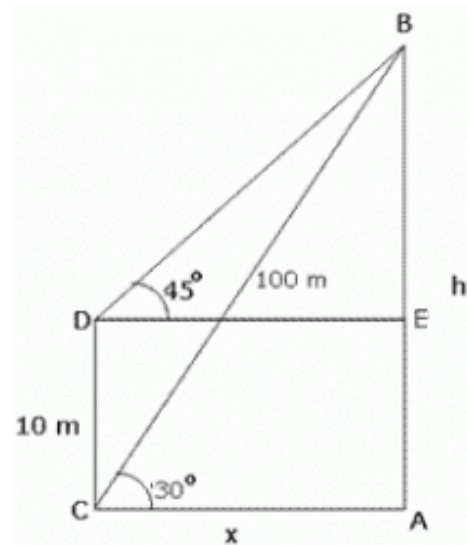
$$\Rightarrow \tan \angle ABE = \frac{28.5\text{m}}{28.5\text{m}} = 1$$

$$\text{But, } \tan 45^\circ = 1$$

$$\therefore \angle ABE = 45^\circ$$

Thus, the required angle of elevation is 45° .

Answer 23.



Let C be the position of the first boy and D be the position of the second boy who is standing on the roof of a 10 m high building.

Let B be the position of the kites of both the boys.

Let $AB = h$ and $CA = x$.

In $\triangle ABC$,

$$\begin{aligned}\sin 30^\circ &= \frac{h}{100} \\ \Rightarrow \frac{1}{2} &= \frac{h}{100} \\ \Rightarrow h &= 50 \quad \dots(1)\end{aligned}$$

In $\triangle BDE$,

$$\begin{aligned}\tan 45^\circ &= \frac{BE}{DE} \\ \Rightarrow 1 &= \frac{h-10}{x} \\ \Rightarrow x &= (h-10) \quad \dots(2)\end{aligned}$$

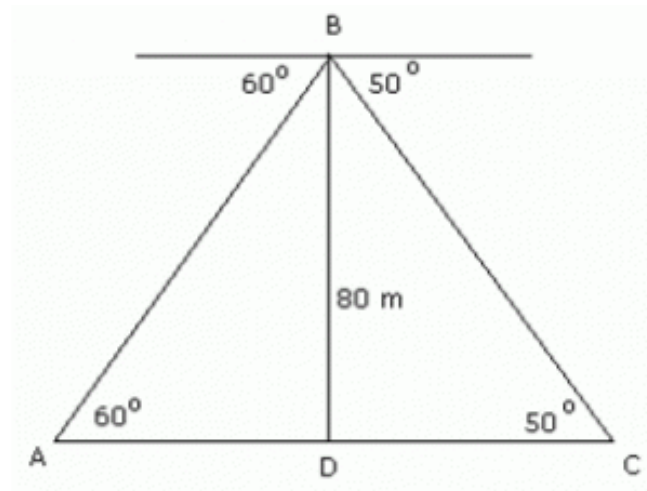
From (1) and (2),

$$x = 50 - 10 = 40$$

$$\begin{aligned}\sin 45^\circ &= \frac{BE}{BD} \\ \Rightarrow \frac{1}{\sqrt{2}} &= \frac{h-10}{BC} \\ \Rightarrow BC &= \sqrt{2}(50-10) = 40\sqrt{2}\end{aligned}$$

Thus, the required length of the string that the second boy must have $40\sqrt{2}$ m

Answer 25.



Let the position of the two persons be A and C. Let BD be the tower height 80 m.

In $\triangle BAD$,

$$\tan 60^\circ = \frac{BD}{AD}$$

$$\Rightarrow \sqrt{3} = \frac{80}{AD}$$

$$\Rightarrow AD = \frac{80}{\sqrt{3}}$$

$$\Rightarrow AD = \frac{80\sqrt{3}}{3} = \frac{80 \times 1.732}{3} = 46.19 \quad \dots(1)$$

In $\triangle BDC$,

$$\tan 50^\circ = \frac{BD}{DC}$$

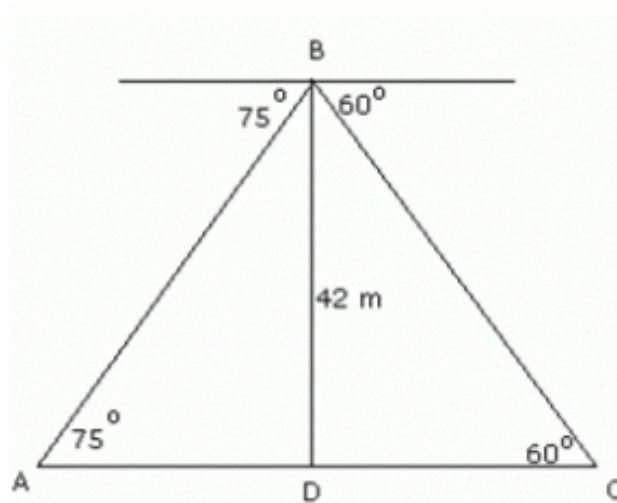
$$\Rightarrow 1.1918 = \frac{80}{DC}$$

$$\Rightarrow DC = \frac{80}{1.1918} = 67.13 \quad \dots(2)$$

$$\therefore AC = AD + DC = 46.19 \text{ m} + 67.13 \text{ m} = 113.32 \text{ m}$$

Thus, the horizontal distance between the two persons is 113.32 m.

Answer 26.



Let the position of the two cars be A and C. Let BD be the building height 42 m.

In $\triangle BAD$,

$$\tan 75^\circ = \frac{BD}{AD}$$

$$\Rightarrow 3.7321 = \frac{42}{AD}$$

$$\Rightarrow AD = \frac{42}{3.7321}$$

$$\Rightarrow AD = 11.25 \quad \dots(1)$$

In $\triangle BDC$,

$$\tan 60^\circ = \frac{BD}{DC}$$

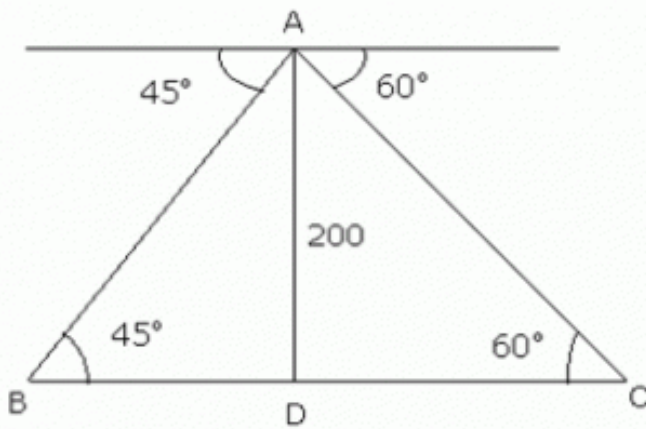
$$\Rightarrow \sqrt{3} = \frac{42}{DC}$$

$$\Rightarrow DC = \frac{42}{1.732} = 24.25 \quad \dots(2)$$

$$\therefore AC = AD + DC = 11.25 \text{ m} + 24.25 \text{ m} = 35.5 \text{ m}$$

Thus, the distance between the cars is 67.63 m.

Answer 27.



Let AD be the height of the aeroplane and $BC = x$ m be the width of the river.

Given: $AD = 200$ m

In $\triangle ABD$

$$\frac{AD}{BD} = \tan 45^\circ$$

$$\Rightarrow \frac{AD}{BD} = 1$$

$$\Rightarrow AD = BD$$

$$\Rightarrow BD = 200 \text{ m } (\because AD = 200 \text{ m})$$

Now,

In $\triangle ACD$

$$\frac{AD}{CD} = \tan 60^\circ$$

$$\Rightarrow \frac{AD}{CD} = \sqrt{3}$$

$$\Rightarrow CD = \frac{AD}{\sqrt{3}} = \frac{200}{\sqrt{3}}$$

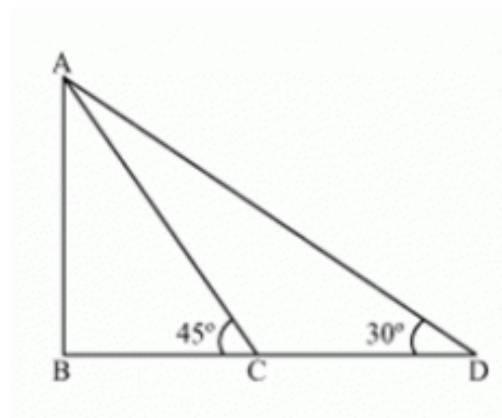
$$\Rightarrow BC = BD + CD = 200 + \frac{200}{\sqrt{3}} = 200 + 115.47$$

$$\Rightarrow BC = 315.4 \text{ m}$$

Thus, the width of the river is 315.4 m.

Answer 28.

Case 1: When the boats are on same side of the observation point.



Let the position of the two ships be C and D. Let A be the point of observation.

$$AB = 500 \text{ m}$$

In $\triangle ABC$,

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{500}{BC}$$

$$\Rightarrow BC = 500 \quad \dots(1)$$

In $\triangle ABD$,

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{500}{BD}$$

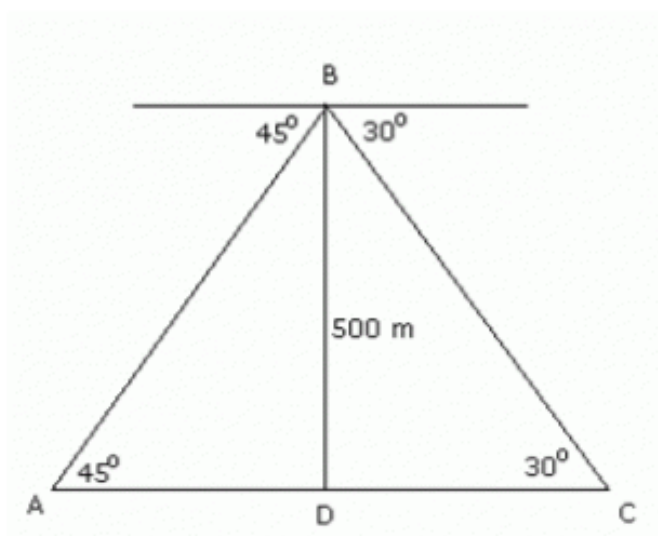
$$\Rightarrow BD = 500\sqrt{3} \quad \dots(2)$$

From (1) and (2),

$$CD = BD - BC = 500(\sqrt{3} - 1) = 500 \times 0.732 = 366$$

Thus, in this case, the distance between the boats is 366 m.

Case 2: When the boats are on different side of the observation point.



Let the position of the two ships be A and C. Let B be the point of observation.

In $\triangle BAD$,

$$\tan 45^\circ = \frac{BD}{AD}$$

$$\Rightarrow 1 = \frac{500}{AD}$$

$$\Rightarrow AD = 500 \quad \dots(1)$$

In $\triangle BDC$,

$$\tan 30^\circ = \frac{BD}{DC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{500}{DC}$$

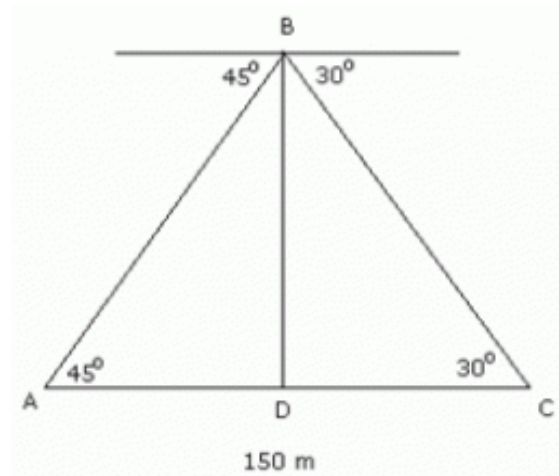
$$\Rightarrow DC = 500\sqrt{3} \quad \dots(2)$$

From (1) and (2),

$$AC = AD + DC = 500 (1 + \sqrt{3}) = 500 \times 2.732 = 1366$$

Thus, in this case, the distance between the boats is 1366 m.

Answer 29.



Let the position of the two boats be at points A and C. Let BD be the lighthouse of height h .

Let $AD = x$. Then, $CD = 150 - x$

In $\triangle BAD$,

$$\tan 45^\circ = \frac{BD}{AD}$$

$$\Rightarrow 1 = \frac{h}{x}$$

$$\Rightarrow h = x \quad \dots(1)$$

In $\triangle BDC$,

$$\tan 30^\circ = \frac{BD}{DC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{150 - x}$$

$$\Rightarrow 150 - x = \sqrt{3}h \quad \dots(2)$$

From (1) and (2),

$$150 - h = \sqrt{3}h$$

$$150 = (\sqrt{3} + 1)h$$

$$h = \frac{150}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

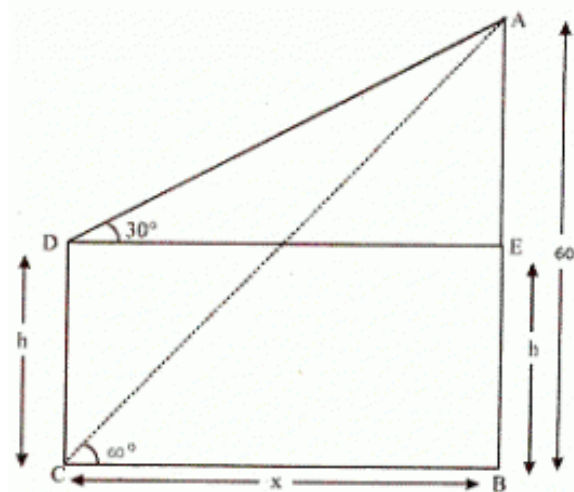
$$= \frac{150(\sqrt{3} - 1)}{3 - 1}$$

$$= 75(\sqrt{3} - 1)$$

$$= 75 \times 0.732 = 54.9$$

Thus, the height of the light house is 54.9 m.

Answer 30.



Let AB be the building. Then, $AB = 60$ m.

Let the height of the lamp post (CD) be h .

Let the distance between the building and the lamp post be x .

In $\triangle ACB$,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}} = 20\sqrt{3} = 20 \times 1.732 = 34.64 \dots (1)$$

Thus, the distance between the building and the lamp post is 34.64 m.

In $\triangle ADE$,

$$\tan 30^\circ = \frac{AE}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60 - h}{x}$$

$$\Rightarrow x = \sqrt{3}(60 - h) \dots (2)$$

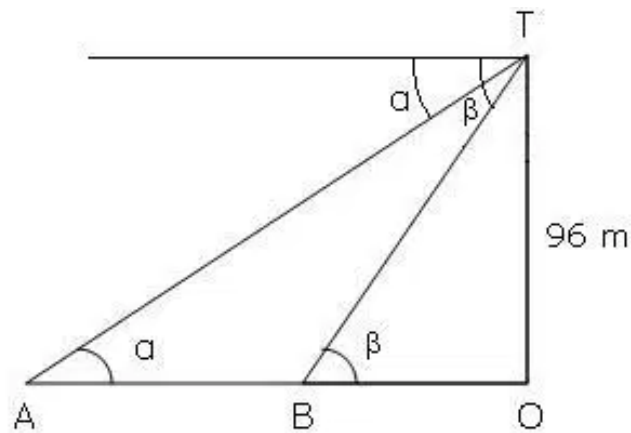
From (1) and (2):

$$\sqrt{3}(60 - h) = 20\sqrt{3}$$

$$60 - h = 20$$

$$h = 40$$

Answer 31.



In the figure, TO is the light house and A and B are the position of the two ships.

In $\triangle AOT$,

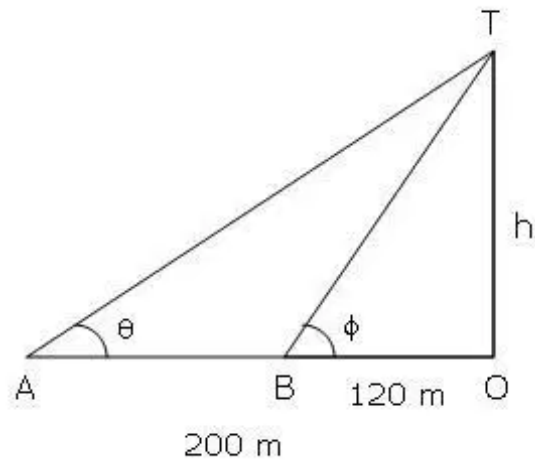
$$\begin{aligned}\frac{OT}{OA} &= \tan \alpha \\ \Rightarrow \frac{96}{OA} &= \frac{1}{4} \\ \Rightarrow OA &= 384\end{aligned}$$

In $\triangle BOT$,

$$\begin{aligned}\frac{OT}{OB} &= \tan \beta \\ \Rightarrow \frac{96}{OB} &= \frac{1}{7} \\ \Rightarrow OB &= 672\end{aligned}$$

\therefore Distance between the two ships = $AB = OA - OB = 384 - 672 = 288\text{m}$

Answer 32.



Let OT be the tower.

A and B be the two points from where the angle of elevation to the top of the tower is measured.

In $\triangle AOT$,

$$\begin{aligned}\frac{OT}{OA} &= \tan \theta \\ \Rightarrow \frac{h}{200} &= \frac{2}{5} \\ \Rightarrow h &= 80 \quad \dots(1)\end{aligned}$$

Thus, the height of the tower is 80 m.

In $\triangle BOT$,

$$\begin{aligned}\frac{OT}{OB} &= \tan \phi \\ \Rightarrow \frac{h}{120} &= \tan \phi \\ \Rightarrow \frac{80}{120} &= \tan \phi \quad [\text{Using (1)}] \\ \Rightarrow \frac{2}{3} &= \tan \phi\end{aligned}$$

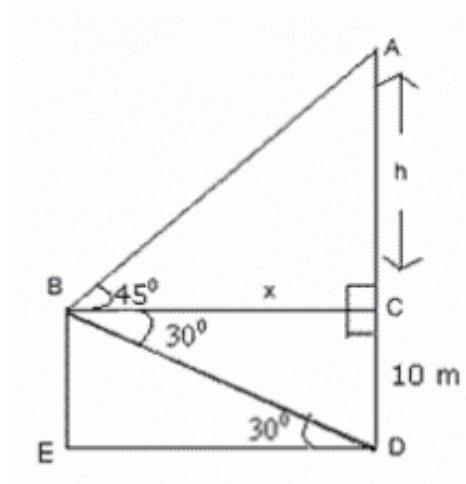
From the table, we get $\phi = 34^\circ$.

Answer 34.

Let B be the position of the man, D the base of the diff, x be the distance of diff from the ship and $h + 10$ be the height of the hill.

$\angle ABC = 45^\circ$ and $\angle DBC = 30^\circ$

Therefore, $\angle BDE = 30^\circ$



In $\triangle ABC$,

$$\tan 45^\circ = \frac{AC}{BC}$$

$$\Rightarrow \frac{h}{x} = 1$$

$$\Rightarrow h = x \quad (1)$$

In $\triangle BED$,

$$\tan 30^\circ = \frac{BE}{ED}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}$$

$$\Rightarrow x = 10\sqrt{3} = 10 \times 1.732 = 17.32$$

Thus, the distance of the diff from the ship is 17.32 m.

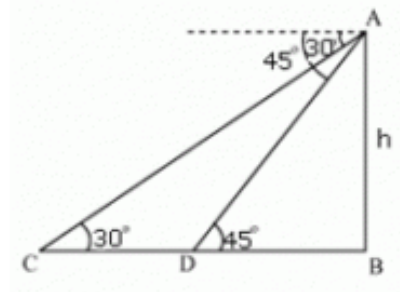
From (1),

$$h = x = 17.32$$

$$\therefore \text{Height of the diff} = 17.32 + 10 = 27.32$$

Thus, the height of the diff is 27.32 m.

Answer 38.



Let AB be the tower.

Initial position of car is C, which changes to D after 720 seconds.

In $\triangle ADB$

$$\frac{AB}{DB} = \tan 45^\circ$$

$$\frac{AB}{DB} = 1$$

$$DB = AB$$

In $\triangle ABC$

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$

$$AB\sqrt{3} = AB + DC$$

$$DC = AB\sqrt{3} - AB = AB(\sqrt{3} - 1)$$

Time taken by car to travel DC distance (i.e. $AB(\sqrt{3} - 1)$) = 720 seconds

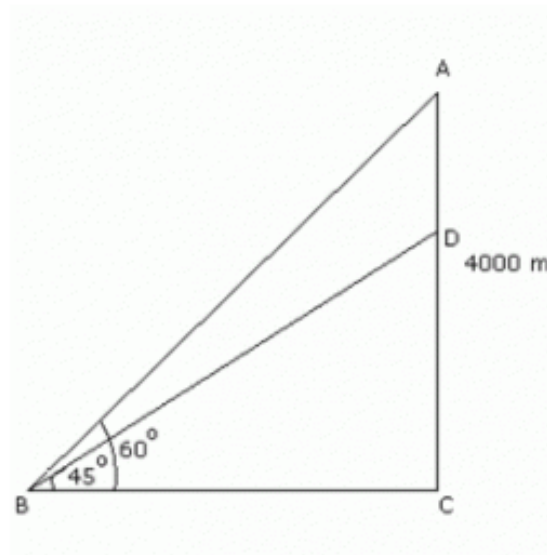
Time taken by car to travel DB distance (i.e. AB)

$$= \frac{720}{AB(\sqrt{3} - 1)} \times AB = \frac{720}{(\sqrt{3} - 1)} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{720(\sqrt{3} + 1)}{2} = 360(\sqrt{3} + 1) = 360 \times 2.732 = 983.52$$

Thus, the required time taken is 983.52 seconds = 984 seconds = 16 mins 24 secs.

Answer 40.



Let points A and D represent the position of the aeroplanes.

Aeroplane A is flying 4 km = 4000 m above the ground.

$$\angle ACB = 60^\circ, \angle DCB = 45^\circ$$

In $\triangle ABC$,

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\Rightarrow BC = \frac{4000}{\sqrt{3}}$$

In $\triangle DCB$,

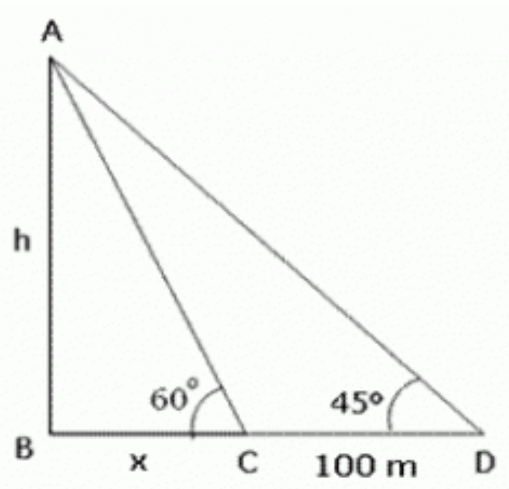
$$\frac{DB}{BC} = \tan 45^\circ$$

$$\Rightarrow DB = BC = \frac{4000}{\sqrt{3}}$$

$$\therefore AD = AB - BD$$

$$= 4000 - \frac{4000}{\sqrt{3}} = 4000 \left(1 - \frac{1}{\sqrt{3}} \right) = 4000 \times \frac{\sqrt{3} - 1}{\sqrt{3}} = 4000 \times \frac{0.732}{1.732} = 1690.53$$

Answer 41.



Let A be the position of the parachutist and C and D be the two observation points.

In $\triangle ABC$,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = \sqrt{3}x$$

In $\triangle ABD$,

$$\tan 45^\circ = \frac{AB}{BD}$$

$$\Rightarrow 1 = \frac{h}{x+100}$$

$$\Rightarrow x+100 = h$$

$$\Rightarrow x+100 = \sqrt{3}x$$

$$\Rightarrow x(\sqrt{3}-1) = 100$$

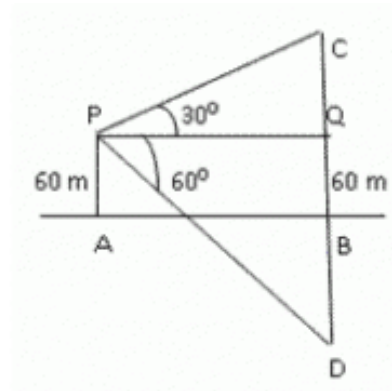
$$\Rightarrow x = 100 \times \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow x = 100 \times \frac{(\sqrt{3}+1)}{3-1} = 50(\sqrt{3}+1) = 50 \times 2.732 = 136.6$$

Thus, the distance of the point where he falls on the ground from the nearest observation point (C) is 136.6 m.

Height from which the parachutist fell

Answer 43.



Let C be the cloud and D be its reflection. Let the height of the cloud is h metres. $BC = BD = h$

$$BQ = AP = 60\text{m}$$

$$\therefore CQ = h - 60 \text{ and } DQ = h + 60$$

In $\triangle CQP$,

$$\frac{PQ}{CQ} = \cot 30^\circ$$

$$\Rightarrow \frac{PQ}{h - 60} = \sqrt{3}$$

$$\Rightarrow PQ = \sqrt{3}(h - 60) \dots (I)$$

In $\triangle DQP$,

$$\frac{PQ}{DQ} = \cot 60^\circ$$

$$\Rightarrow \frac{PQ}{h + 60} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow PQ = \frac{1}{\sqrt{3}}(h + 60) \dots (II)$$

From (I) and (II),

$$\Rightarrow \sqrt{3}(h - 60) = \frac{1}{\sqrt{3}}(h + 60)$$

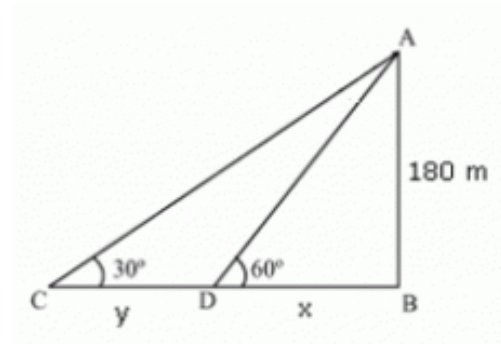
$$\Rightarrow 3h - 180 = h + 60$$

$$\Rightarrow 2h = 240$$

$$\Rightarrow h = 120$$

Thus, the height of the cloud is 120m.

Answer 44.



Let AB be the lighthouse.

Initial position of boat is C, which changes to D after 2 minutes.

In $\triangle ADB$

$$\begin{aligned}\frac{AB}{DB} &= \tan 60^\circ \\ \frac{180}{x} &= \sqrt{3} \\ x &= \frac{180}{\sqrt{3}}\end{aligned}$$

In $\triangle ABC$

$$\begin{aligned}\frac{AB}{BC} &= \tan 30^\circ \\ \frac{180}{x+y} &= \frac{1}{\sqrt{3}}\end{aligned}$$

$$180\sqrt{3} = x + y$$

$$180\sqrt{3} = \frac{180}{\sqrt{3}} + y$$

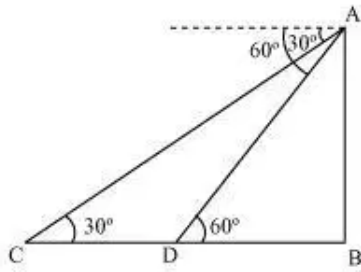
$$y = 180\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 180\left(\frac{2}{\sqrt{3}}\right) = \frac{360}{\sqrt{3}}$$

Time taken by car to travel DC distance $\left(\text{i.e. } \frac{360}{\sqrt{3}}\right) = 2 \text{ minutes} = 120 \text{ seconds}$

$$\text{Speed of the boat} = \frac{\text{Distance}}{\text{Time}} = \frac{\frac{360}{\sqrt{3}}}{120} = \frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3} = 1.732$$

Thus, the speed of the boat is 1.732 m/sec.

Answer 45.



Let AB be the cliff. Then, $AB = 450$ m

Initial position of boat is C, which changes to D after 3 minutes.

In $\triangle ADB$

$$\frac{AB}{DB} = \tan 60^\circ$$

$$\frac{450}{DB} = \sqrt{3}$$

$$DB = \frac{450}{\sqrt{3}}$$

In $\triangle ABC$

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{450}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$450\sqrt{3} = BD + DC$$

$$450\sqrt{3} = \frac{450}{\sqrt{3}} + DC$$

$$\begin{aligned} DC &= 450\sqrt{3} - \frac{450}{\sqrt{3}} = 450\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) \\ &= \frac{900}{\sqrt{3}} = \frac{900}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 300\sqrt{3} \end{aligned}$$

Time taken by car to travel DC distance (i.e., $300\sqrt{3}$) = 3 minutes

Time taken by car to travel DB distance $\left(\text{i.e., } \frac{450}{\sqrt{3}}\right)$

$$= \frac{3}{300\sqrt{3}} \times \frac{450}{\sqrt{3}} = \frac{450}{300} = 1.5$$

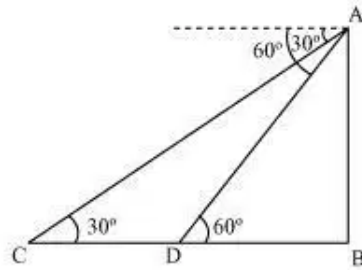
Thus, the time it will take to reach the shore is 1 min 30 secs.

$$\text{Speed of the boat} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{300\sqrt{3}}{3} = 100\sqrt{3} = 100 \times 1.732 = 173.2 \text{ m/min}$$

$$= \frac{173.2}{60} \text{ m/sec} = 2.9 \text{ m/sec}$$

Answer 46.



Let AB be the tower.

Initial position of ship is C, which changes to D after 3 minutes.

In $\triangle ADB$

$$\frac{AB}{DB} = \tan 60^\circ$$

$$\frac{AB}{DB} = \sqrt{3}$$

$$DB = \frac{AB}{\sqrt{3}}$$

In $\triangle ABC$

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$

$$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$$

$$\begin{aligned} DC &= AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) \\ &= \frac{2AB}{\sqrt{3}} \end{aligned}$$

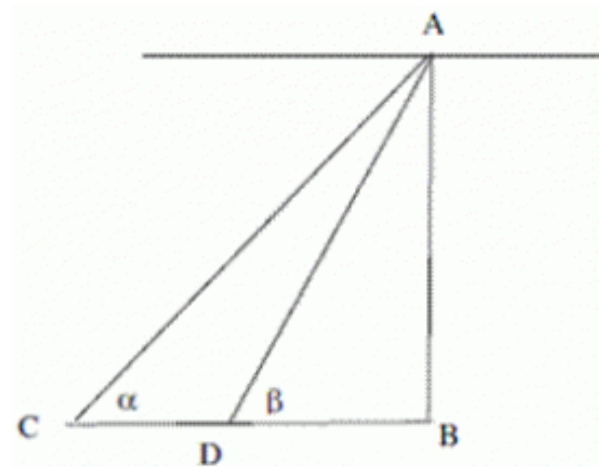
Time taken by car to travel DC distance $\left(\text{i.e., } \frac{2AB}{\sqrt{3}}\right) = 3 \text{ minutes}$

Time taken by car to travel DB distance $\left(\text{i.e., } \frac{AB}{\sqrt{3}}\right)$

$$= \frac{3}{\frac{2AB}{\sqrt{3}}} \times \frac{AB}{\sqrt{3}} = \frac{3}{2} = 1 \text{ min } 30 \text{ secs}$$

Thus, the total time taken is 3 minutes + 1 minute 30 seconds = minutes 30 seconds.

Answer 47.



In the figure, AB is the tower. A is the position of the man. C and D are the two positions of the truck.

Let the speed of the truck be x m/sec

$$\text{Distance } CD = \text{speed} \times \text{time} = 600x$$

In right triangle ABC,

$$\tan \alpha = \frac{h}{BC}$$

$$\text{It is given that } \tan \alpha = \frac{1}{\sqrt{5}}$$

$$BC = h\sqrt{5} \quad \dots (1)$$

In right triangle ABD,

$$\tan \beta = \frac{h}{BD}$$

$$\text{It is given that } \tan \beta = \sqrt{5}$$

$$h = \sqrt{5} BD$$

$$\text{Now, } CD = BC - BD$$

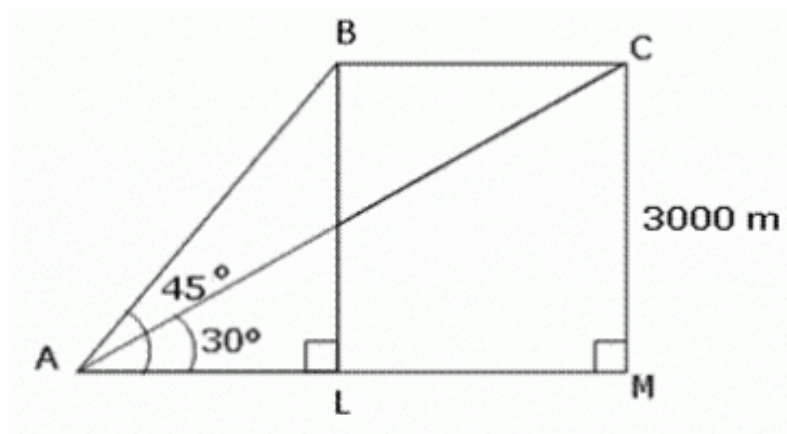
$$600x = 5BD - BD$$

$$BD = 150x$$

$$\text{Time taken} = \frac{150x}{x} = 150 \text{ seconds}$$

Thus, the time taken by the truck to reach the tower is 150 sec = 2 min 30 sec.

Answer 48.



Let A be the point of observation on the ground and B and C be the two positions of aeroplane. Let $BL = CM = 3000$ m.

In $\triangle ALB$,

$$\tan 45^\circ = \frac{BL}{AL}$$

$$\Rightarrow 1 = \frac{3000}{AL}$$

$$\Rightarrow AL = 3000$$

In $\triangle AMC$,

$$\tan 30^\circ = \frac{MC}{AM}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3000}{3000 + LM}$$

$$\Rightarrow 3000\sqrt{3} = (3000 + LM)$$

$$\Rightarrow LM = 3000(\sqrt{3} - 1)$$

$$\therefore BC = 3000(\sqrt{3} - 1)$$

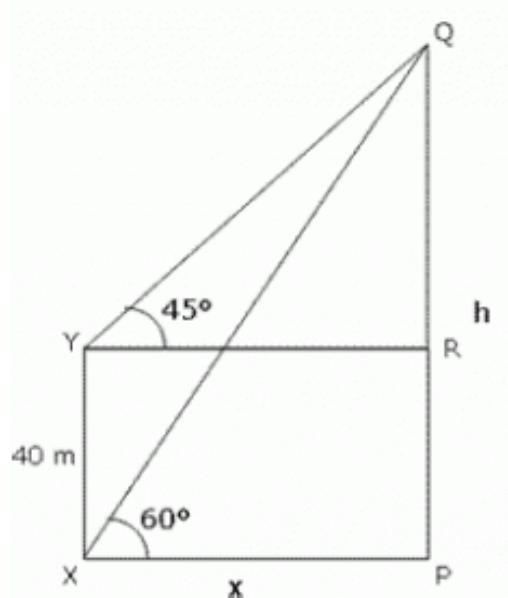
Now, time taken to travel distance $BC = 15$ seconds

$$\therefore \text{Speed of the aeroplane} = \frac{\text{Distance}}{\text{Time}} = \frac{3000(\sqrt{3} - 1)}{15} = 200 \times 0.732 = 146.4$$

Thus, the speed of the aeroplane is 146.4 m/sec

$$= 146.4 \times \frac{1}{\frac{1000}{3600}} \text{ km/hr} = 146.4 \times 3.6 \text{ km/hr} = 527.04 \text{ km/hr}$$

Answer 49.



In the figure, PQ is the tower.

In ΔPQX ,

$$\therefore \frac{h}{x} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow h = \sqrt{3}x \quad \dots(1)$$

In ΔQRY ,

$$\frac{h - 40}{x} = \tan 45^\circ = 1$$

$$\Rightarrow h = 40 + x \quad \dots(2)$$

From (1) and (2),

$$\sqrt{3}x = 40 + x$$

$$\Rightarrow (\sqrt{3} - 1)x = 40$$

$$\Rightarrow x = \frac{40}{\sqrt{3} - 1} = \frac{40(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{40}{2}(\sqrt{3} + 1) = 20(\sqrt{3} + 1)$$

$$\therefore h = 40 + 20(\sqrt{3} + 1) = 20\sqrt{3} + 60 = 20(\sqrt{3} + 3) = 20 \times 4.732 = 94.64$$

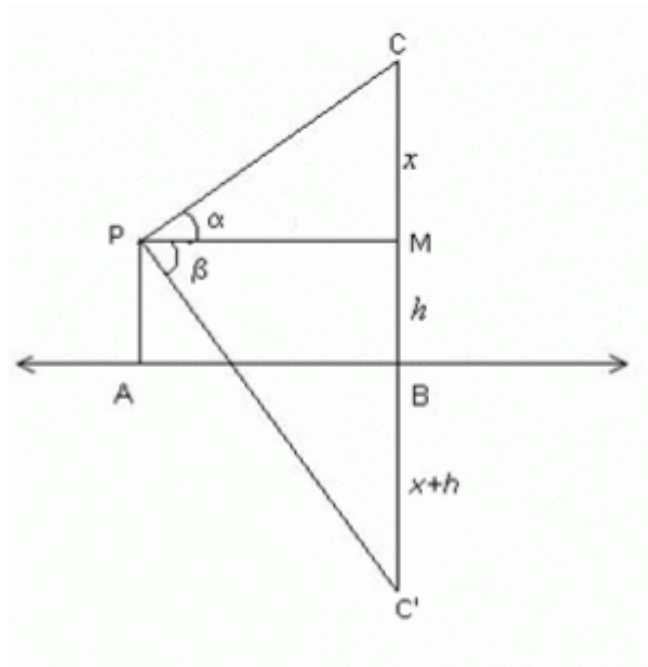
Thus, the height of the tower PQ is 94.64 m.

Again, in ΔPQX ,

$$\therefore \frac{h}{XQ} = \sin 60^\circ = \frac{1}{\sqrt{2}}$$

$$\Rightarrow XQ = \sqrt{2}h = 1.414 \times 94.64 = 109.3\text{m}$$

Answer 50.



Let AB be the surface of the lake and let P be an point of observation s.t. that $AP = h$ meters. Let c be the position of the cloud and C' be its reflection in the lake. Then $\angle CPM = \alpha$ and $\angle MPC = \beta$. Let $CM = x$. Then, $CB = CM + MB = CM + PA = x + h$

In $\triangle CPM$,

$$\tan \alpha = \frac{CM}{PM}$$

$$\Rightarrow \tan \alpha = \frac{x}{AB} \quad [\because PM = AB]$$

$$\Rightarrow AB = x \cot \alpha \quad \dots (1)$$

In $\triangle PMC'$,

$$\tan \beta = \frac{C'M}{PM}$$

$$\Rightarrow \tan \beta = \frac{x + 2h}{AB}$$

$$\Rightarrow AB = (x + 2h) \cot \beta \quad \dots (2)$$

From (1) and (2),

$$x \cot \alpha = (x + 2h) \cot \beta$$

$$\Rightarrow x \left(\frac{1}{\tan \alpha} - \frac{1}{\tan \beta} \right) = \frac{2h}{\tan \beta}$$

$$\Rightarrow x \left(\frac{\tan \beta - \tan \alpha}{\tan \alpha \tan \beta} \right) = \frac{2h}{\tan \beta}$$

$$\Rightarrow x = \frac{2h \tan \alpha}{\tan \beta - \tan \alpha}$$

Again, In $\triangle CPM$,

Answer 51.

Let P Q be h

QB be x

Given : AB = 1 mile

QB = x

AQ = (1 - x) mile

in ΔPAQ

$$\tan \alpha = \frac{PQ}{AQ}$$

$$\tan \alpha = \frac{h}{1-x}$$

$$1-x = \frac{h}{\tan \alpha} \quad \dots\dots\dots 1$$

In ΔPQB

$$\tan \beta = \frac{h}{x}$$

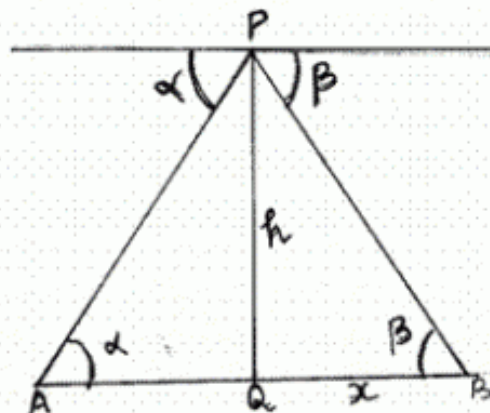
$$x = \frac{h}{\tan \beta}$$

Substitute for x in equation (1)

$$1 = \frac{h}{\tan \beta} + \frac{h}{\tan \alpha}$$

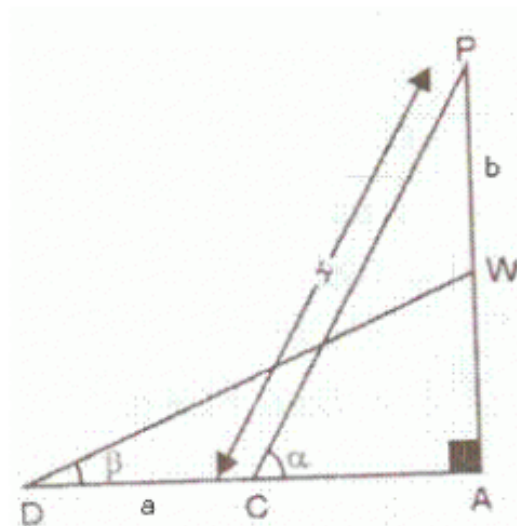
$$1 = h \left\{ \frac{1}{\tan \beta} + \frac{1}{\tan \alpha} \right\}$$

$$\frac{1}{h} = \frac{\tan \beta + \tan \alpha}{\tan \beta \tan \alpha}$$



Thus, the height in miles of aeroplane above the road is $\frac{\tan \alpha \tan \beta}{\tan \alpha + \tan \beta}$.

Answer 52.



Let CP and DW be the two positions of the ladder such that $CP = DW =$ (say).

$CD = a$, $PW = b$, $\angle ACP = \alpha$ and $\angle ADW = \beta$

In $\triangle APC$,

$$\frac{AC}{CP} = \cos \alpha \Rightarrow AC = x \cos \alpha \quad \dots(I)$$

In $\triangle ADW$,

$$\begin{aligned} \frac{AD}{DW} &= \cos \beta \Rightarrow \frac{AC + CD}{DW} = \cos \beta \\ \Rightarrow \frac{x \cos \alpha + a}{x} &= \cos \beta \quad \text{[using (I)]} \\ \Rightarrow x &= \frac{a}{\cos \beta - \cos \alpha} \quad \dots(II) \end{aligned}$$

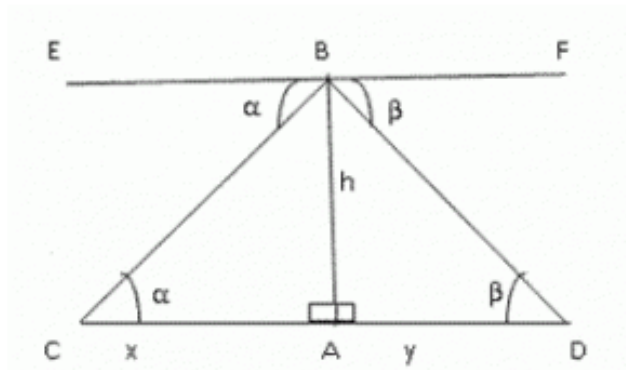
Again in $\triangle APC$, $\frac{AP}{CP} = \sin \alpha$

$$\Rightarrow AP = x \sin \alpha = \frac{a \sin \alpha}{(\cos \beta - \cos \alpha)} \quad \dots(III) \quad \text{[Using (II)]}$$

Again in $\triangle ADW$, $\frac{AW}{DW} = \sin \beta$

$$\Rightarrow AW = x \sin \beta = \frac{a \sin \beta}{(\cos \beta - \cos \alpha)} \quad \dots(IV)$$

Answer 53.



Let AB be the lighthouse of height h m. Let $AC = x$ and $AD = y$.

In $\triangle CAB$,

$$\frac{AB}{AC} = \tan \alpha$$

$$\tan \alpha = \frac{h}{x}$$

$$x = \frac{h}{\tan \alpha} \dots (i)$$

In $\triangle DAB$,

$$\frac{AB}{AD} = \tan \beta$$

$$\tan \alpha = \frac{h}{y}$$

$$y = \frac{h}{\tan \beta} \dots (ii)$$

Distance between the ships = $x + y$

$$= \frac{h}{\tan \alpha} + \frac{h}{\tan \beta}$$

$$= h \left(\frac{\tan \beta + \tan \alpha}{\tan \alpha \tan \beta} \right)$$