Class: XII
SESSION : 2022-2023
SUBJECT: Mathematics SAMPLE
QUESTION PAPER - 16
with SOLUTION

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

Section A

1. The direction ratios of two lines are a, b, c and (b - c), (c - a), (a - b) respectively. The angle between these lines is **[1]**

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$

c) $\frac{\pi}{3}$ d) $\frac{3\pi}{4}$

2. If $\vec{a}=(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{b}=(3\hat{i}-\hat{j}+2\hat{k})$ then the angle between $(2\vec{a}+\vec{b})$ and $(\vec{a}+2\vec{b})$ is **[1]**

a) $\cos^{-1}\left(\frac{31}{50}\right)$ b) none of these

c) $\cos^{-1}\left(\frac{21}{40}\right)$ d) $\cos^{-1}\left(\frac{11}{30}\right)$

3. $\int \frac{x+3}{(x+4)^2} e^x dx =$ **[1]**

a) $\frac{e^x}{x+3}+c$ b) $\frac{e^x}{(x+4)^2}+C$

c) $\frac{e^x}{x+4}+C$ d) $\frac{1}{(x+4)^2}+c$

4. $\int$ x sin 2x dx = ? **[1]**

a) None of these b) $\frac{1}{2}$ x cos 2x - $\frac{1}{4}$ sin 2x + C

c) $\frac{1}{2}$ x cos 2x + $\frac{1}{4}$ sin 2x + C d) - $\frac{1}{2}$ x cos 2x + $\frac{1}{4}$ sin 2x + C

5. If two events are independent, then **[1]**

a) None of these b) they must be mutually exclusive

c) they must be mutually exclusive and the sum of their probabilities must be equal to 1 both are correct

d) the sum of their probabilities must be equal to 1

6. The area bounded by the curves $y^2 = 4x$ and $y = x$ is equal to **[1]**

a) $\frac{8}{3}$

b) $\frac{35}{6}$

c) none of these

d) $\frac{1}{3}$

7. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the probability of getting exactly one red ball is **[1]**

a) $\frac{45}{196}$

b) $\frac{15}{56}$

c) $\frac{15}{29}$

d) $\frac{135}{392}$

8. If $\vec{a} = (\hat{i} - 2\hat{j} + 3\hat{k})$ and $\vec{b} = (\hat{i} - 3\hat{k})$ and then $|\vec{b} \times 2\vec{a}| = ?$ **[1]**

a) $2\sqrt{23}$

b) $5\sqrt{17}$

c) $10\sqrt{3}$

d) $4\sqrt{19}$

9. The equation of the line passing through the points $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is **[1]**

a) $\vec{r} = \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$ - $t\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

b) $\vec{r} = \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right) + \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

c) None Of These

d) $\vec{r} = a_1(1-t)\hat{i} + a_2(1-t)\hat{j} + a_3(1-t)\hat{k} + t\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

10. The general solution of the DE $x\frac{dy}{dx} = y + x \tan \frac{y}{x}$ is **[1]**

a) $\sin\left(\frac{y}{x}\right) = C$

b) $\sin\left(\frac{y}{x}\right) = Cy$

c) none of these

d) $\sin\left(\frac{y}{x}\right) = Cx$

11. $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)dx = ?$ **[1]**

a) $\frac{\pi}{8}\log 2$

b) $\frac{\pi}{4}\log 2$

c) $\frac{\pi}{4}$

d) 0

12. If the area cut off from a parabola by any double ordinate is k times the corresponding rectangle contained by that double ordinate and its distance from **[1]**

the vertex, then k is equal to

a) $\frac{2}{3}$ b) 3

c) $\frac{1}{3}$ d) $\frac{3}{2}$

13. The function f(x) = tanx - x **[1]**

a) always increases b) never increases

c) always decreases d) sometimes increases and sometimes decreases.

14. If $A = \frac{1}{3}\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ satisfies $A^TA = I$, then x + y = **[1]**

a) -3 b) none of these

c) 0 d) 3

15. If A = $\begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and AB = I_3, then x + y equals **[1]**

a) -1 b) 0

c) none of these d) 2

16. The principal value of $\sin^{-1}(\sin\ \frac{3\pi}{4}\ ) = \ldots\ldots$ **[1]**

a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$

c) $\frac{5\pi}{4}$ d) $\frac{-\pi}{4}$

17. For the differential equation $\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right)$ + y = 0, which one of the following is not its solution? **[1]**

a) $4y = x^2$ b) y = -x - 1

c) y = x - 1 d) y = x

18. **Assertion (A):** $f(x) = 2x^3 - 9x^2 + 12x - 3$ is increasing outside the interval (1, 2). **[1]**
Reason (R): $f'(x) < 0$ for $x \in (1, 2)$.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

19. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists if. **[1]**

a) $\lambda = 2$

b) $\lambda \neq -2$

c) None of these

d) $\lambda \neq 2$

20. **Assertion (A):** If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then $x = \pm 6$. **[1]**

Reason (R): If A is a skew-symmetric matrix of odd order, then |A| = 0.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Using the principal values, write the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$. **[2]**

22. Find the general solution of the differential equation $\frac{dy}{dx} - 2y = \cos 3x$. **[2]**

23. Evaluate: $\begin{vmatrix} \cos 65^\circ & \sin 65^\circ \\ \sin 25^\circ & \cos 25^\circ \end{vmatrix}$. **[2]**

OR

Evaluate $\Delta = \begin{bmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{bmatrix}$

24. Show that $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$ **[2]**

25. An electronic assembly consists of two sub-systems say A and B. From previous testing procedures, the following probabilities are assumed to be known: **[2]**

P (A fails) = 0.2

P (B fails alone) = 0.15

P (A and B fail) = 0.15

Evaluate the following probabilities.

(1) $P\left(\overline{A}|\overline{B}\right)$

(2)P(A fails alone).

Section C

26. Find the general solution of the differential equation: **[3]**

$\left(1 - x^2\right) \frac{dy}{dx} + xy = x\sqrt{1 - x^2}$

OR

Verify that $y^2 = 4ax$ is a solution of the differential equation $y = x\frac{dy}{dx} + a\frac{dx}{dy}$

27. Evaluate: $\int \frac{x}{\sqrt{x^2+x+1}}dx$ **[3]**

28. Evaluate the integral: $\int(2x+3)\sqrt{x^2+4x+3}dx$ **[3]**

OR

By using the properties of definite integrals, evaluate the integral $\int\limits_{-5}^{5}|x+2|\,dx$

29. If the function f(x) given by $f(x) = \begin{cases} 3ax+b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax-2b, & \text{if } x < 1 \end{cases}$ is continuous at x = 1, then find the values of a and b. **[3]**

30. Sketch the graph y = $|x-5|$. Evaluate $\int_0^1 |x-5|dx$. What does this value of the integral represent on the graph. **[3]**

31. If $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ are three vectors such that $|\vec{a}| = 5$, $|\vec{b}| = 12$ and $|\vec{c}| = 13$, and $\vec{a}+\vec{b}+\vec{c}=\vec{o}$ find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ **[3]**

OR

The vectors $\vec{a} = 3\hat{i} + x\hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + y\hat{k}$ are mutually $\perp$. Given $|\vec{a}| = \left|\vec{b}\right|$ find x and y.

Section D

32. Solve the Linear Programming Problem graphically: **[5]**
Maximize Z = 3x + 4y subject to the constraints: $x+y \leq 4, x \geq 0, y \geq 0$

33. Let n be a fixed positive integer. Define a relation R on Z as follows: **[5]**
(a, b) $\in$ R $\Leftrightarrow$ a - b is divisible by n. Show that R is an equivalence relation on Z.

OR

Let R be relation defined on the set of natural number N as follows:
R = {(x, y): x $\in$ N, y $\in$ N, 2x + y = 41}. Find the domain and range of the relation R.
Also verify whether R is reflexive, symmetric and transitive.

34. Differentiate $\sin^{-1}(4x\sqrt{1-4x^2})$ with respect to $\sqrt{1-4x^2}$, if $x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$ **[5]**

35. Show that the straight lines whose direction cosines are given by the equations al + bm + cn = 0 and $ul^2 + vm^2 + wn^2 = 0$ are perpendicular, if $a^2(v+w) + b^2(u+w) + c^2(u+v) = 0$ and, parallel, if $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$ **[5]**

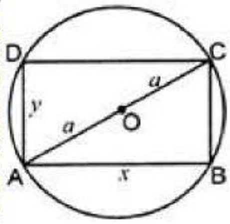
OR

$\overrightarrow{AB} = 3\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ are two vectors. The position vectors of the points A and C are $6\hat{i} + 7\hat{j} + 4\hat{k}$ and $-9\hat{j} + 2\hat{k}$, respectively. Find the position vector of a point P on the line AB and a point Q on the line CD such that $\overrightarrow{PQ}$ is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



(i) Find the perimeter of rectangle in terms of any one side and radius of circle.

(ii) Find critical points to maximize the perimeter of rectangle?

(iii) Check for maximum or minimum value of perimeter at critical point.

OR

If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region.

37. **Read the text carefully and answer the questions:** **[4]**

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.

Article	School	A	B	C
Fans		40	25	35
Mats		50	40	50
Plates		20	30	40

(i) Represent the sale of handmade fans, mats and plates by three schools A, B and C and the sale prices (in ₹) of given products per unit, in matrix form.

(ii) Find the funds collected by school A, B and C by selling the given articles.

(iii) If they increase the cost price of each unit by 20%, then write the matrix representing new price.

OR

Find the total funds collected for the required purpose after 20% hike in price.

38. **Read the text carefully and answer the questions:** **[4]**

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.

(i) Teacher ask Govind, what is the probability that tickets are drawn by Abhishek, shows a prime number on one ticket and a multiple of 4 on other ticket?

(ii) Teacher ask Girish, what is the probability that tickets drawn by Ankit, shows an even number on first ticket and an odd number on second ticket?

SOLUTION

Section A

1. **(a)** $\frac{\pi}{2}$

Explanation: Let's consider the first parallel vector to be $\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$ and second parallel vector be $\vec{b} = (b - c)\hat{i} + (c - a)\hat{j} + (a - b)\hat{k}$

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{a^2 + b^2 + (c)^2}$

$= \sqrt{a^2 + b^2 + c^2}$

$|\vec{b}| = \sqrt{(b - c)^2 + (c - a)^2 + (a - b)^2}$

$= \sqrt{2\left(a^2 + b^2 + c^2 - ab - bc - ca\right)}$

$$\Rightarrow \cos\alpha = \frac{(a\hat{i} + b\hat{j} + c\hat{k}) \cdot ((b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k})}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)} \times \sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \cos\alpha = \frac{ab-ac+bc-ba+ca-cb}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)} \times \sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \cos\alpha = \frac{0}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)} \times \sqrt{a^2+b^2+c^2}}$$

$\Rightarrow \alpha = \cos^{-1}(0)$

$\therefore \alpha = \frac{\pi}{2}$

2. **(a)** $\cos^{-1}\left(\frac{31}{50}\right)$

Explanation: Given vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ *and* $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

$(2\vec{a} + \vec{b}) = (5\hat{i} + 3\hat{j} - 4\hat{k})$ and $(\vec{a} + 2\vec{b}) = (7\hat{i} + 0\hat{j} + \hat{k})$

$$\cos\theta = \frac{(5\times7+3\times0-4\times1)}{\sqrt{50}\times\sqrt{50}} = \frac{31}{50} \Rightarrow \theta = \cos^{-1}\left(\frac{31}{50}\right)$$

3. **(c)** $\frac{e^x}{x+4} + C$

Explanation: $I = \int \frac{x+3}{(x+4)^2} e^x dx$

$$I = \int \left(\frac{x+4-1}{(x+4)^2} \right) e^x dx$$

$$I = \int \left(\frac{1}{x+4} - \frac{1}{(x+4)^2} \right) e^x dx$$

$$f(x) = \frac{1}{x+4} \Rightarrow f'(x) = -\frac{1}{(x+4)^2}$$

$$\Rightarrow I = \frac{e^x}{x+4} + c$$

4. **(d)** - $\frac{1}{2}$ x cos 2x + $\frac{1}{4}$ sin 2x + C

Explanation: $I = \int x \sin 2x \, dx$

By using IBP Formula we get,

$$\text{I} = x\int sin2x \, dx - \int((\frac{d}{dx}x)\int sin2x)dx$$

$$= -\frac{x}{2}cos2x + \frac{1}{4}sin2x + c$$

5. **(a)** None of these

Explanation: If two events A and B are independent, then we know that

$P(A \cap B) = P(A) \cdot P(B), P(A) \neq 0, P(B) \neq 0$

Since, A and B have a common outcome.

Further, mutually exclusive events never have a common outcome.

In other words, two independents events having non-zero probabilities of occurrence cannot be mutually exclusive and conversely, i.e., two mutually exclusive events having non-zero probabilities of outcome cannot be independent.

6. **(a)** $\frac{8}{3}$

Explanation: The two curves $y^2 = 4x$ and $y = x$ meet where $x^2 = 4x$ i.e ..where $x = 0$ or $x = 4$. Moreover, the parabola lies above the line $y = x$ between $x = 0$ and $x = 4$. Hence, the required area is:

$$\int_0^4 \left(\sqrt{4x} - x\right)dx = \int_0^4 \left(2x^{\frac{1}{2}} - x\right)dx$$

$$= \left[\frac{2x^{\frac{3}{2}}}{3/2} - \frac{x^2}{2}\right]_0^4$$

$$= \frac{4}{3}\left(4^{\frac{3}{2}}\right) - \frac{16}{2} = \frac{32}{3} - 8 = \frac{8}{3}$$

7. **(b)** $\frac{15}{56}$

Explanation: Probability of getting exactly one red (R) ball

$= P_R \cdot P_B \cdot P_B + P_B \cdot P_R \cdot P_R + P_B \cdot P_B \cdot P_R$

$= \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{5}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{5}{6}$

$= \frac{15}{4 \cdot 7 \cdot 6} + \frac{15}{4 \cdot 7 \cdot 6} + \frac{15}{4 \cdot 7 \cdot 6}$

$= \frac{5}{56} + \frac{5}{56} + \frac{5}{56} = \frac{15}{56}.$

Which is the required solution

8. **(d)** $4\sqrt{19}$

Explanation: $2\vec{a} = (2\hat{i} - 4\hat{j} + 6\hat{k})$ and $\vec{b} = (\hat{i} - 3\hat{k})$

Now, $|\vec{b} \times 2\vec{a}| = \left\|\begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 3 \\ 2 & -4 & 6 \end{matrix}\right\| = |-12\vec{i} - 12\vec{j} - 4\vec{k}|$

$= \sqrt{(144) + (144) + 16} = \sqrt{304}$

$= 4\sqrt{19}$

9. **(d)** $\vec{r} = a_1(1-t)\hat{i} + a_2(1-t)\hat{j} + a_3(1-t)\hat{k} + t\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

Explanation: $\vec{r} = a_1(1-t)\hat{i} + a_2(1-t)\hat{j} + a_3(1-t)\hat{k} + t\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

Equation of the line passing through the points having position vectors

$a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is

$\vec{r} = \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right) + t\left\{\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) - \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)\right\}$, where t is a parameter

$= \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right) - t\left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right) + t\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

$= a_1(1 - t)\hat{i} + a_2(1 - t)\hat{j} + a_3(1 - t)\hat{k} + t\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

10. **(d)** $\sin\left(\frac{y}{x}\right) = Cx$

Explanation: Given DE: $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$

Now,Dividing both sides by x, we obtain $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$

Let y = v x Differentiating both sides,we get

$\frac{dy}{dx} = \mathrm{v} + \mathrm{x}\frac{dv}{dx}$

Now, our differential equation becomes,

$v + x\frac{dv}{dx} = v + \tan v$

On separating the variables, we obtain $\frac{dv}{\tan v} = \frac{dx}{x}$

Integrating both sides, we get, sin v = Cx

Substituting the value of v we get, $\sin\left(\frac{y}{x}\right) = Cx$

11. **(a)** $\frac{\pi}{8}\log 2$

Explanation: let $I = \int_0^{\frac{\pi}{4}}\log(1 + \tan x)dx$

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a - x) = 1$

$\therefore \mathrm{f(a - x)} = \log\left(1 + \tan\left(\frac{\pi}{4} - \mathrm{x}\right)\right)$

$= \log\left(1 + \frac{\left(\tan\frac{\pi}{4} - \tan x\right)}{1 + \tan\frac{\pi}{4}\tan x}\right)$

$= \log\left(1 + 1(1 - \tan x)\frac{1}{1+\tan x}\right.$

$= \log\frac{2}{1+\tan x}$

$\therefore \int_0^a f(a-x) = 1$

$= \int_0^{\frac{\pi}{4}} \log\frac{2}{1+\tan x}dx$

$= \int_0^{\frac{\pi}{4}} \log 2dx - \int_0^{\frac{\pi}{4}} (1 + \tan x)dx$

$\therefore I = \int_0^{\frac{\pi}{4}} \log 2dx - 1$

$\therefore 2I = \frac{\pi}{4}\log 2$

$\therefore I = \frac{\pi}{8}\log 2$

12. **(a)** $\frac{2}{3}$

Explanation: Required area :

$= 2\int_0^a \sqrt{4ax}dx$

$= k\alpha(2\sqrt{4a\alpha})$

$= \frac{8\sqrt{a}}{3}\alpha^{\frac{3}{2}}$

$= 4\sqrt{a}k\alpha^{\frac{3}{2}} \Rightarrow k = \frac{2}{3}$

13. **(a)** always increases

Explanation: We have, f(x) = tan x - x

$\therefore$ f'(x) = $\sec^2$ x - 1

$\Rightarrow f'(x) \geq 0, \forall x \in R$

So, f(x) always increases

14. **(b)** none of these

Explanation: We have, $A = \frac{1}{3}\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$

$$\Rightarrow A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & x \\ 1 & 1 & 2 \\ 2 & -2 & y \end{bmatrix}$$

Now, $A^TA = I$

$$\Rightarrow \begin{bmatrix} x^2+5 & 2x+3 & xy-2 \\ 3+2x & 6 & 2y \\ xy-6 & 2y & y^2+8 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

The corresponding elements of two equal matrices are not equal.
Thus, the matrix A is not orthogonal.

15. **(b)** 0

Explanation: $AB = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+0+0 & -2+2+0 & y+0+x \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence $x + y = 0$

16. **(a)** $\frac{\pi}{4}$

Explanation: $\sin^{-1}(\sin\frac{3\pi}{4}) = \sin^{-1}(\sin\left(\pi - \frac{\pi}{4}\right)$

$= \sin^{-1}(\sin\frac{\pi}{4}) = \frac{\pi}{4}$.

17. **(d)** y = x

Explanation: The given differential equation is

$\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0$ ...(i)

$y = x \Rightarrow \frac{dy}{dx} = -1$

From Eq. (i), $(1)^2 + x(1) + x = 1 \neq 0$

So, y = x is not a solution of Eq. (i).

18. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion: We have, $f(x) = 2x^3 - 9x^2 + 12x - 3$

$\Rightarrow f'(x) = 6x^2 - 18x + 12$

For increasing function, $f'(x) \geq 0$

$\therefore 6(x^2 - 3x + 2) \geq 0$

$\Rightarrow 6(x - 2)(x - 1) \geq 0$

$\Rightarrow x \leq 1$ and $x \geq 2$

$\therefore$ f(x) is increasing outside the interval (1, 2), therefore it is a true statement.

Reason: Now, $f'(x) < 0$

$\Rightarrow 6(x - 2)(x - 1) < 0$

$\Rightarrow 1 < x < 2$

$\therefore$ Assertion and Reason are both true but Reason is not the correct explanation of Assertion.

19. **(c)** None of these

Explanation: We have,

$$A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$$

A^{-1} exists if $|A| \neq 0$

Now $|A| = 2(6 - 5) - \lambda(-5) - 3(-2) = 8 + 5\lambda \neq 0$

$\Rightarrow 5\lambda \neq -8$

$\Rightarrow \lambda \neq \frac{-8}{5}$

So, A^{-1} exists if and only if $\neq \frac{-8}{5}$

20. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Both A and R are true but R is not the correct explanation of A.

Section B

21. We have, $\cos^{-1}\left(\frac{1}{2}\right) = \cos^{-1}\left(\cos\frac{\pi}{3}\right)$

$= \frac{\pi}{3}\left[\because \frac{\pi}{3} \in [0, \pi]\right]$

Also $\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}$ (-sin $\frac{\pi}{6}$)

$= \sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$

$= -\frac{\pi}{6}\left[\because -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - 2\left(-\frac{\pi}{6}\right)$

$= \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

22. The given differential equation is of the form $\frac{dy}{dx} + Py = Q$, where P = -2 and Q = cos 3x.

Thus, the given differential equation is linear.

IF $= e^{\int P\,dx} = e^{\int -2\,dx} = e^{-2x}$.

So, the required solution is

$y \times \text{IF} = \int\{Q \times \text{F}\}dx + C,$

i.e., $y \times e^{-2x} = \int e^{-2x}\cos 3x dx + C$

$= e^{-2x}\left[\frac{-2\cos 3x + 3\sin 3x}{\left\{(-2)^2 + 3^2\right\}}\right] + C$

$\left[\because \int e^{ax}\cos bx dx = e^{ax}\left\{\frac{a\cos bx + b\sin bx}{\left(a^2 + b^2\right)}\right\}\right]$

$\therefore y = \frac{(3\sin 3x - 2\cos 3x)}{13} + Ce^{2x}$, which is the required solution.

23. Given: $\begin{vmatrix} \cos 65^\circ & \sin 65^\circ \\ \sin 25^\circ & \cos 25^\circ \end{vmatrix}$

By directly opening this determinant we get

$\cos 65^\circ \times \cos 25^\circ - \sin 25^\circ \times \sin 65^\circ$

$= \cos(65^\circ + 25^\circ)$ $\because$ cosA cosB - sinA sinB = cos (A + B)

$= \cos 90^\circ$

$= 0$

OR

Expanding along R_1,we get,

$$\Delta = 0\begin{bmatrix} 0 & \sin\beta \\ -\sin\beta & 0 \end{bmatrix} - \sin\alpha\begin{vmatrix} -\sin\alpha & \sin\beta \\ \cos\alpha & 0 \end{vmatrix} - \cos\alpha\begin{vmatrix} -\sin\alpha & 0 \\ \cos\alpha & -\sin\beta \end{vmatrix}$$

$$= 0 - \sin\alpha(0 - \cos\alpha\sin\beta) - \cos\alpha(\sin\alpha\sin\beta - 0) = \sin\alpha\cos\alpha\sin\beta - \cos\alpha\sin\alpha\sin\beta = 0$$

24. We have,

$$(\vec{a} \times \vec{b})^2 = |\vec{a} \times \vec{b}|^2$$

$$\Rightarrow (\vec{a} \times \vec{b})^2 = \{|\vec{a}||\vec{b}|\sin\theta\}^2$$

$$\Rightarrow (\vec{a} \times \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2\sin^2\theta$$

$$\Rightarrow (\vec{a} \times \vec{b})^2 = \left\{|\vec{a}|^2|\vec{b}|^2\right\}\left(1 - \cos^2\theta\right)$$

$$\Rightarrow (\vec{a} \times \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta$$

$$\Rightarrow (\vec{a} \times \vec{b})^2 = (\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{b}) - (\vec{a} \cdot \vec{b})(\vec{a} \cdot \vec{b}) \; [\because \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta]$$

$$\Rightarrow (\vec{a} \times \vec{b})^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$$

Hence, $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$

25. Event A fails and B fails denoted by $\bar{A}$ and $\bar{B}$ respectively.

$$\therefore P\left(\bar{A}\right) = 0.2 \text{ and P (A and B fails)} = 0.15$$

$$\Rightarrow P(A \cap \bar{B}) = 0.15$$

$$\therefore \text{ P}(B \text{ above}) = P\left(\bar{B}\right) - P(A \cap B)$$

$$\Rightarrow 0.15 = P\left(\bar{B}\right) - 0.15$$

$$\Rightarrow P\left(\bar{B}\right) = 0.30$$

i. $P\left(\bar{A}\mid\bar{B}\right)=\dfrac{P\left(\bar{A}\cap\bar{B}\right)}{P\left(\bar{B}\right)}=\dfrac{0.15}{0.30}=\dfrac{1}{2}=0.5$

ii. P (A fails alone) = P ($\bar{A}$ alone) = $P\left(\bar{A}\right)-P\left(\bar{A}\cap\bar{B}\right)$= 0.20 - 0.15 = 0.05

Section C

26. The given differential equation is,

$\left(1-x^2\right)\cdot\dfrac{dy}{dx}+xy=x\sqrt{1-x^2}$

$\Rightarrow\dfrac{dy}{dx}+\dfrac{x}{1-x^2}\cdot y=\dfrac{x}{\sqrt{1-x^2}}$

This is of the form $\dfrac{dy}{dx}+Py=Q$, where,

$P=\dfrac{x}{1-x^2}$ and $Q=\dfrac{x}{\sqrt{1-x^2}}$

Thus, the given differential equation is linear

Now, $IF=e^{\int P\,dx}$

$\Rightarrow IF=e^{\int\frac{x}{1-x^2}dx}$

$\Rightarrow I.F=e^{\frac{-1}{2}\int\frac{-2x}{1-x^2}dx}\Rightarrow I.F=e^{-\frac{1}{2}\log\left(1-x^2\right)}$

$\Rightarrow IF=\left(1-x^2\right)^{-\frac{1}{2}}\Rightarrow I\cdot F=\dfrac{1}{\sqrt{1-x^2}}$

Therefore the solution is given by

$(1.F)\cdot y=\int(1.F)Q+C$

$\Rightarrow\dfrac{1}{\sqrt{1-x^2}}\cdot y=\int\dfrac{1}{\sqrt{1-x^2}}\cdot\dfrac{x}{\sqrt{1-x^2}}dx+C$

$\Rightarrow\dfrac{y}{\sqrt{1-x^2}}=\int\dfrac{x}{1-x^2}dx+C\Rightarrow\dfrac{y}{\sqrt{1-x^2}}=-\dfrac{1}{2}\int\dfrac{-2x}{1-x^2}dx+C$

$$\Rightarrow \frac{y}{\sqrt{1-x^2}} = -\frac{1}{2}\log\left(1-x^2\right) + C$$

$$\Rightarrow y = -\frac{1}{2}\sqrt{1-x^2}\cdot\log\left(1-x^2\right) + C$$

OR

We have, $y^2 = 4ax$ ...(i)

Differentiating both sides of (i) with respect to x, we have,

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \frac{dy}{dx} = \frac{2a}{y} \text{ ...(ii)}$$

Differentiating both sides of (i) with respect to y, we have,

$$2y = 4a\frac{dx}{dy}$$

$$\Rightarrow \frac{dx}{dy} = \frac{y}{2a} \text{ ...(iii)}$$

$$\therefore x\frac{dy}{dx} + a\frac{dx}{dy} = x\left(\frac{2a}{y}\right) + a\left(\frac{y}{2a}\right) \text{ ...[Using (ii) and (iii)]}$$

$$\Rightarrow x\frac{dy}{dx} + a\frac{dx}{dy} = \frac{2ax}{y} + \frac{y}{2}$$

$$\Rightarrow x\frac{dy}{dx} + a\frac{dx}{dy} = \frac{y^2}{2y} + \frac{y}{2} \text{ ...[Using (i)]}$$

$$\Rightarrow x\frac{dy}{dx} + a\frac{dx}{dy} = \frac{y}{2} + \frac{y}{2}$$

$$\Rightarrow x\frac{dy}{dx} + a\frac{dx}{dy} = y$$

$$\Rightarrow y = x\frac{dy}{dx} + a\frac{dx}{dy}$$

Therefore, the given function is the solution to the given differential equation.

27. Let the given integral be, $I = \int \frac{x}{\sqrt{x^2+x+1}}dx$

Let $x = \lambda\frac{d}{dx}(x^2 + x + 1) + \mu$

$= \lambda(2x + 1) + \mu$

$\Rightarrow x = (2\lambda)x + \lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$

$\lambda + \mu = 0$

$$\Rightarrow \left(\frac{1}{2}\right) + \mu = 0$$

$$\Rightarrow \mu = -\frac{1}{2}$$

$$\text{So, } I = \int \frac{\frac{1}{2}(2x+1) - \frac{1}{2}}{\sqrt{x^2+x+1}} dx$$

$$= \frac{1}{2}\int \frac{(2x+1)}{\sqrt{x^2+x+1}} dx - \frac{1}{2}\int \frac{1}{\sqrt{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}} dx$$

$$= \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+1}} dx - \frac{1}{2}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}} dx$$

$$= \frac{1}{2} \times 2\sqrt{x^2+x+1} - \frac{1}{2}\log\left|x+\frac{1}{2}+\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}\right| + c$$ [Since,

$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c, \int \frac{1}{\sqrt{x^2-a^2}} dx = \log|x + \sqrt{x^2-a^2}| + c$]

$$\Rightarrow I = \sqrt{x^2+x+1} - \frac{1}{2}\log\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right| + c$$

28. Let the given integral be,

$I = \int (x+1)\sqrt{x^2+x+1}\, dx$

Also, $x + 1 = \lambda \frac{d}{dx}\left(x^2+x+1\right) + \mu$

$\Rightarrow x + 1 = \lambda(2x + 1) + \mu$

$\Rightarrow x + 1 = (2\lambda)x + \lambda + \mu$

Equating coefficient of like terms

$2\lambda = 1$

$$\Rightarrow \lambda = \frac{1}{2}$$

And

$\lambda + \mu = 1$

$\Rightarrow \frac{1}{2} + \mu = 1$

$\therefore \mu = \frac{1}{2}$

$$\therefore I = \frac{1}{2}\int (2x+1)\sqrt{x^2+x+1}dx + \frac{1}{2}\int \sqrt{x^2+x+1}dx$$

$$= \frac{1}{2}\int (2x+1)\sqrt{x^2+x+1}dx + \frac{1}{2}\int \sqrt{x^2+x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}dx$$

$$= \frac{1}{2}\int (2x+1)\sqrt{x^2+x+1}dx + \frac{1}{2}\int \sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$

Now, let $x^2 + x + 1 = t$

$\Rightarrow (2x + 1)\, dx = dt$

Then,

$$I = \frac{1}{2}\int \sqrt{t}dt + \frac{1}{2}\left[\frac{x+\frac{1}{2}}{2}\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \right.$$

$$\left.\frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right|\right] + C$$

$$= \frac{1}{2}\times\frac{2}{3}t^{\frac{3}{2}} + \frac{1}{2}\left[\left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1} + \frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|\right] + C$$

$$= \frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{1}{2}\left[\left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1} + \frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|\right] + C$$

OR

Let $I = \int_{-5}^{5} |x+2|dx$ …(i)

Putting x + 2 = 0

$\Rightarrow x = -2 \in (-5, 5)$

$\therefore$ From eq. (i),

$$I = \int_{-5}^{-2} |x+2|dx + \int_{-2}^{5} |x+2|dx$$

$$= \int_{-5}^{-2} -(x+2)dx + \int_{-2}^{5} (x+2)dx$$

$$= -\left(\frac{x^2}{2} + 2x\right)_{-5}^{-2} + \left(\frac{x^2}{2} + 2x\right)_{-2}^{5}$$

$$= -\left[\left(\frac{4}{2} - 4\right) - \left(\frac{25}{2} - 10\right)\right] + \left[\left(\frac{25}{2} + 10\right) - \left(\frac{4}{2} - 4\right)\right]$$

$$= -\left(-2 - \frac{5}{2}\right) + \left(\frac{45}{2} + 2\right)$$

$$= 2 + \frac{5}{2} + \frac{45}{2} + 2$$

$$= 4 + 25 = 29$$

29. Given, $f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$

Since f(x) is continuos at x=1, therefore,

LHL = RHL = f(1)..........(i)

Now, LHL = $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (5ax - 2b)$

$= \lim_{h \to 0} [5a(1-h) - 2b]$

$= \lim_{h \to 0} (5a - 5ah - 2b)$= 5a - 2b

and RHL = $\lim_{x \to 1^+} (3ax + b) = \lim_{h \to 0} [3a(1+h) + b]$

$= \lim_{h \to 0} (3a + 3ah + b) = 3a + b$

Also, given that f(1) = 11

On substituting these values in Eq. (i), we get

5a - 2b = 3a + b = 11

$\Rightarrow$ 3a + b = 11............(ii)

and 5a - 2b = 11......(iii)

On subtracting 3 $\times$ Eq. (iii) from 5 $\times$ Eq. (ii), we get

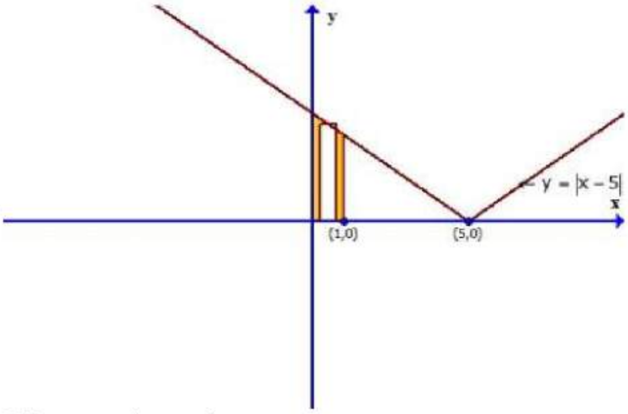
15a + 5b - 15a + 6b = 55 - 33

$\Rightarrow$ 11b = 22 $\Rightarrow$ b = 2

On putting the value of b in Eq. (ii), we get,

3a + 2 = 11 $\Rightarrow$ 3a = 9 $\Rightarrow$ a = 3

Hence, a = 3 and b = 2

30. 

We are given that
$y = |x-5|$ intersect x = 0 and x = 1 at (0, 5) and (1, 4)
Now, $y = |x-5|$
= - (x - 5) For all a $\in$ (0, 1)
Integration represents the area enclosed by the graph from x = 0 to x = 1
Now area denoted by A , is given by

$$A = \int_0^1 |y|\, dx$$

$$= \int_0^1 |x-5|\, dx$$

$$= \int_0^1 -(x-5)dx$$

$$= -\int_0^1 (x-5)dx$$

$$= -\left[\frac{x^2}{2} - 5x\right]_0^1$$

$$= -\left[\left(\frac{1}{2} - 5\right) - (0-0)\right]$$

$$= -\left(-\frac{9}{2}\right)$$

$$= \frac{9}{2}\text{sq} \cdot \text{ units}$$

31. If $|\vec{a}| = 5, |\vec{b}| = 12, |\vec{c}| = 13$
If $(\vec{a} + \vec{b} + \vec{c}) = 0$.......(i)
Find $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = ?$
Squaring the given equation (i) We get ,
$(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$
$\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{a} \cdot \vec{c} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c}$
$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$
25 + 144 + 169 + 2(x) = 0
338 + 2x = 0
2x = -338

$x = -169$

hence , the required term is equal to -169.

OR

$\vec{a}.\vec{b} = 0$ [$\because \vec{a} \perp \vec{b}$]

$\Rightarrow (3\hat{i} + x\hat{j} - \hat{k}).(2\hat{i} - \hat{j} + y\hat{k})$

$\Rightarrow$ 6 - x - y = 0

$\Rightarrow$ y + x = 6 .... (1)

$|\vec{a}| = |\vec{b}|$ [Given]

$3^2 + x^2 + 1 = 2^2 + 1^2 + y^2$

$y^2 - x^2 = 5$

(y - x) (y + x) = 5

6(y - x) = 5

$y - x = \frac{5}{6}$ ....(2)

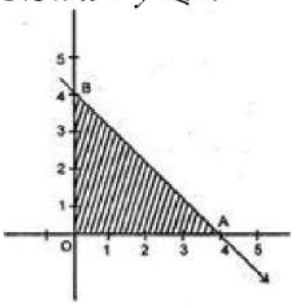
From(1) and (2), we get,

$x = \frac{31}{12}, y = \frac{41}{12}$

Section D

32. As $x \geqslant 0, y \geqslant 0$, therefore we shall shade the other inequalities in the first quadrant only.

Now $x + y \leqslant 4$



Let x + y = 4

$\Rightarrow \frac{x}{4} + \frac{y}{4} = 1$

Thus the line has 4 and 4 as intercepts along the axes. Now, (0, 0) satisfies the inequation, i.e., $0 + 0 \leqslant 4$. Therefore, shaded region OAB is the feasible solution.

Its corners are O (0, 0), A (4, 0), B (0, 4)

At O (0, 0) Z = 0

At A (4, 0) Z = 3 × 4 = 12

At B (0, 4) Z = 4 × 4 = 16

Hence, max Z = 16 at x = 0, y = 4.

33. R = {(a, b): a - b is divisible by n} on Z.

Now,

Reflexivity: Let a $\in$ Z

$\Rightarrow$ a - a = 0 × n

$\Rightarrow$ a - a is divisible by n

$\Rightarrow$ (a, a) $\in$ R
$\Rightarrow$ R is reflexive
Symmetric: Let (a, b) $\in$ R
$\Rightarrow$ a - b = np for some p $\in$ Z
$\Rightarrow$ b - a = n(-p)
$\Rightarrow$ b - a is divisible by n
$\Rightarrow$ (b, a) $\in$ R
$\Rightarrow$ R is symmetric
Transitive: Let (a, b) $\in$ R and (b, c) $\in$ R
$\Rightarrow$ a - b = np and b - c = nq for some p, q $\in$ Z
$\Rightarrow$ a - c = n(p + q)
$\Rightarrow$ a - c is divisible by n
$\Rightarrow$ (a, c) $\in$ R
$\Rightarrow$ R is transitive
Thus, R being reflexive, symmetric and transitive on Z.
Hence, R is an equivalence relation on Z

OR

Given that,
R = {(1, 39), (2, 37), (3, 35) .... (19, 3), (20, 1)}
Domain = {1,2,3,.......,20}
Range = {1,3,5,7......,39}
R is not reflexive as (2, 2) $\notin$ R as
$2 \times 2 + 2 \neq 41$
R is not symmetric
as (1, 39) $\in$ R but (39, 1) $\notin$ R
R is not transitive
as (11, 19) $\in$ R, (19, 3) $\in$ R
But (11, 3) $\notin$ R
Hence, R is neither reflexive, nor symmetric and nor transitive.

34. Let $\mathbf{u} = \sin^{-1}(4x\sqrt{1-4x^2})$
Put 2x = $\cos\theta$
$\Rightarrow u = \sin^{-1}(2 \times \cos\theta\sqrt{1-\cos^2\theta})$
$\Rightarrow u = \sin^{-1}(2\cos\theta\sin\theta)$
$\Rightarrow u = \sin^{-1}(\sin2\theta)$ ...(i)
Let $\mathbf{v} = \sqrt{1-4x^2}$ ....(ii)
Here,
$$x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$$
$$\Rightarrow 2x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \theta \in \left(\frac{3\pi}{4}, \pi\right)$$

So, from equation (i),

$$u = \pi - 2\theta\left[\text{ since , } \sin^{-1}(\sin\theta) = \pi - \theta,\ \text{if } \theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\right]$$

$$\Rightarrow u = \pi - 2\cos^{-1}(2x) \text{ [since , } 2x = \cos\theta]$$

Differentiate it with respect to x,

$$\frac{du}{dx} = 0 - 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$

$$\Rightarrow \frac{du}{dx} = \frac{2}{\sqrt{1-4x^2}}(2)$$

$$\Rightarrow \frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \text{ ....(iii)}$$

from equation (ii),

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

$$\text{but } x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$$

$$\therefore \frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$

$$\Rightarrow \frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \text{ ....(iv)}$$

Dividing equation (iii) by (iv)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{4x}$$

$$\therefore \frac{du}{dv} = \frac{1}{x}.$$

35. The given equations are

al + bm + cn = 0 ....(i)

and, $ul^2 + vm^2 + wn^2 = 0$ ..(ii)

From (i), we get

$n = -\left(\frac{al+bm}{c}\right)$

Substituting $n = -\left(\frac{al+bm}{c}\right)$ in (ii), we get

$ul^2 + vm^2 + w\,\frac{(al+bm)^2}{c^2} = 0$

$\Rightarrow\ (c^2u + a^2w)l^2 + 2abwlm + (c^2v + b^2w)m^2 = 0$

$\Rightarrow\ \left(a^2w + c^2u\right)\left(\frac{l}{m}\right)^2 + 2abw\left(\frac{l}{m}\right) + \left(b^2w + c^2v\right) = 0$ ....(iii)

This is a quadratic equation in $\frac{l}{m}$. So, it gives two values of $\frac{l}{m}$. Suppose the two values be $\frac{l_1}{m_1}$ and $\frac{l_2}{m_2}$.

$\therefore\quad \frac{l_1}{m_1}, \frac{l_2}{m_2} = \frac{b^2w+c^2v}{a^2w+c^2u} \Rightarrow \frac{l_1l_2}{b^2w+c^2v} = \frac{m_1m_2}{a^2w+c^2u}$ .....(iv)

Similarly, by making a quadratic equation in $\frac{m}{n}$, we obtain

$\frac{m_1m_2}{a^2w+c^2u} = \frac{n_1n_2}{a^2v+b^2u}$ ....(v)

From (iv) and (v), we get

$\frac{l_1l_2}{b^2w+c^2v} = \frac{m_1m_2}{a^2w+c^2u} = \frac{n_1n_2}{a^2v+b^2u} = \lambda$ (say)

$\Rightarrow\ l_1l_2 = \lambda\left(b^2w + c^2v\right), m_1m_2 = \lambda\left(a^2w + c^2u\right), n_1n_2 = \lambda\left(a^2v + b^2u\right)$

For the given lines to be perpendicular, we must have

$l_1\,l_2 + m_1\,m_2 + n_1\,n_2 = 0$

$\Rightarrow\ \lambda\left(b^2w + c^2v\right) + \lambda\left(a^2w + c^2u\right) + \lambda\left(a^2v + b^2u\right) = 0$

$\Rightarrow\ a^2\,(v + w) + b^2\,(u + w) + c^2\,(u + v) = 0$

For the given lines to be parallel, the direction cosines must be equal and so the roots of the equation (iii) must be equal.

$\therefore\ 4a^2\,b^2\,w^2 - 4(a^2w + c^2u)\,(b^2w + c^2v) = 0$ [On equating discriminant to zero]

$\Rightarrow\ a^2\,c^2\,vw + b^2\,c^2\,uw + c^4\,uv = 0$

$\Rightarrow\ a^2\,vw + b^2\,c^2\,uw + c^2uv = 0$

$\Rightarrow \dfrac{a^2}{u} + \dfrac{b^2}{v} + \dfrac{c^2}{w} = 0$ [Dividing throughout by uvw] Hence the required result is proved

OR

We have, $\vec{AB} = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$

Also, the position vectors of A and C are $6\hat{i} + 7\hat{j} + 4\hat{k}$ and $-9\hat{j} + 2\hat{k}$, respectively.

Since, $\vec{PQ}$ is perpendicular to both $\vec{AB}$ and $\vec{CD}$.

So, P and Q will be foot of perpendicular to both the lines through A and C.

Now, equation of the line through A and parallel to the vector $\vec{AB}$ is,

$\vec{r} = (6\hat{i} + 7\hat{j} + 4\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$

And the line through C and parallel to the vector $\vec{CD}$ is given by

$\vec{r} = -9\hat{j} + 2\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$ ..... (i)

Let $\vec{r} = (6i + 7j + 4\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$

and $\vec{r} = -9\hat{j} + 2\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$ .... (ii)

Let $P(6 + 3\lambda, 7 - \lambda, 4 + \lambda)$ is any point on the first line and Q be any point on second line is given by $(-3\mu, -9 + 2\mu, 2 + 4\mu)$.

$\therefore \vec{PQ} = (-3\mu - 6 - 3\lambda)\hat{i} + (-9 + 2\mu - 7 + \lambda)\hat{j} + (2 + 4\mu - 4 - \lambda)\hat{k}$

$= (-3\mu - 6 - 3\lambda)\hat{i} + (2\mu + \lambda - 16)\hat{j} + (4\mu - \lambda - 2)\hat{k}$

If $\vec{PQ}$ is perpendicular to the first line, then

$3(-3\mu - 6 - 3\lambda) - (2\mu + \lambda - 16) + (4\mu - \lambda - 2) = 0$

$\Rightarrow -9\mu - 18 - 9\lambda - 2\mu - \lambda + 16 + 4\mu - \lambda - 2 = 0$

$\Rightarrow -7\mu - 11\lambda - 4 = 0$ ..... (iii)

If $\vec{PQ}$ is perpendicular to the second line, then

$-3(-3\mu - 6 - 3\lambda) + (2\mu + \lambda - 16) + (4\mu - \lambda + 2) = 0$

$\Rightarrow 9\mu + 18 + 9\lambda + 4\mu + 2\lambda - 32 + 16\mu - 4\lambda - 8 = 0$

$\Rightarrow 29\mu + 7\lambda - 22 = 0$ ..... (iv)

On solving Eqs. (iii) and (iv), we get

$-49\mu - 77\lambda - 28 = 0$

$\Rightarrow 319\mu + 77\lambda - 242 = 0$

$\Rightarrow 270\mu - 270 = 0$

$\Rightarrow \mu = 1$

Using μ in Eq. (iii), we get

$-7(1) = -11\lambda - 4 = 0$

$\Rightarrow -7 - 11\lambda - 4 = 0$

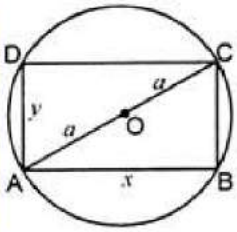
$\Rightarrow -11 - 11\lambda = 0$

$\Rightarrow \lambda = -1$

$\therefore \overrightarrow{PQ} = [-3(1) - 6 - 3(-1)]\hat{i} + [2(1) + (-1) - 16]\hat{j} + [4(1) - (-1) - 2]\hat{k}$
$= -6\hat{i} - 15\hat{j} + 3\hat{k}$

Section E

36. **Read the text carefully and answer the questions:**
A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)

(i) Let 'y' be the breadth and 'x' be the length of rectangle and 'a' is radius of given circle.
From fig $4a^2 = x^2 + y^2$
$\Rightarrow y^2 = 4a^2 - x^2$
$\Rightarrow y = \sqrt{4a^2 - x^2}$

Perimeter (P) $= 2x + 2y = 2\left(x + \sqrt{4a^2 - x^2}\right)$

(ii)
We know that $P = 2\left(x + \sqrt{4a^2 - x^2}\right)$

Critical points to maximize perimeter $\frac{dP}{dx} = 0$

$$\Rightarrow \frac{dp}{dx} = 2\left(1 + \frac{1}{2\sqrt{4a^2 - x^2}}(-2x)\right) = 0$$

$$2\left(\frac{\sqrt{4a^2 - x^2} - x}{\sqrt{4a^2 - x^2}}\right) = 0$$

$\Rightarrow \sqrt{4a^2 - x^2} = x$

$\Rightarrow 4a^2 - x^2 = x^2$
$\Rightarrow 2a^2 = x^2$
$\Rightarrow x = \pm\sqrt{2}a$
when $x = \sqrt{2}a$, $y = \sqrt{2}a$
when $x = -\sqrt{2}a$ not possible as 'x' is length critical point is $(\sqrt{2}a, \sqrt{2}a)$

(iii)

$$\frac{dp}{dx} = 2\left(1 + \frac{1}{2\sqrt{4a^2 - x^2}}(-2x)\right)$$

$$\frac{d^2P}{dx^2} = -2\left(\frac{\sqrt{4a^2-x^2} - (x)\left(\frac{-2x}{2\sqrt{4^2-x^2}}\right)}{\left(4a^2-x^2\right)}\right)$$

$$= -2\left(\frac{\left(4a^2-x^2\right)+x^2}{\left(4a^2-x^2\right)^{3/2}}\right)$$

$$\Rightarrow \frac{d^2P}{dx^2}\bigg]_{x=a\sqrt{2}} = -2\left(\frac{4a^2}{\left(4a^2-2a^2\right)^{3/2}}\right) = \frac{-2}{(2\sqrt{2})a} < 0$$

Perimeter is maximum at a critical point.

OR

From the above results know that x = y = $\sqrt{2}a$

a = radius

Here, x = y = $10\sqrt{2}$

Perimeter = P = 4 $\times$ side = $40\sqrt{2}$ cm

37. **Read the text carefully and answer the questions:**

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.

Article	School	A	B	C
Fans		40	25	35
Mats		50	40	50
Plates		20	30	40

(i)

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} \textit{Fans} & \textit{Mats} & \textit{Plates} \end{array} \\ \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \end{array}$$

$$Q = \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{array}{l} \textit{Fans} \\ \textit{Mats} \\ \textit{Plates} \end{array}$$

(ii) Clearly, total funds collected by each school is given by the matrix

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 1000 + 5000 + 1000 \\ 625 + 4000 + 1500 \\ 875 + 5000 + 2000 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

$\therefore$ Funds collected by school A is ₹7000.
Funds collected by school B is ₹6125.
Funds collected by school C is ₹7875.

(iii)

$$\textit{New price matrix } Q = 20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{array}{l} \textit{Fans} \\ \textit{Mats} \\ \textit{Plates} \end{array}$$

$$\Rightarrow Q = \begin{bmatrix} 25 + 25 \times 0.20 \\ 100 + 100 \times 0.20 \\ 50 + 50 \times 0.20 \end{bmatrix} \begin{array}{l} \textit{Fans} \\ \textit{Mats} \\ \textit{Plates} \end{array}$$

$$Q = \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{array}{l} \textit{Fans} \\ \textit{Mats} \\ \textit{Plates} \end{array}$$

$$\text{New price matrix Q} = 20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{array}{l} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{array}$$

$$\Rightarrow Q = \begin{bmatrix} 25 + 25 \times 0.20 \\ 100 + 100 \times 0.20 \\ 50 + 50 \times 0.20 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

$$Q = \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

OR

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix}$$

$$PQ = \begin{bmatrix} 1200 + 6000 + 1200 \\ 750 + 4800 + 1800 \\ 1050 + 6000 + 2400 \end{bmatrix} = \begin{bmatrix} 8400 \\ 7350 \\ 9450 \end{bmatrix}$$

Total fund collected = 8400 + 7350 + 9450 = ₹25,200

38. **Read the text carefully and answer the questions:**

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.

(i) Required probability = P(one ticket with prime number and other ticket with a multiple of 4)

$$= 2\left(\frac{15}{50} \times \frac{12}{49}\right) = \frac{36}{245}$$

(ii) P(First ticket shows an even number and second ticket shows an odd number)

$$= \frac{25}{50} \times \frac{25}{49} = \frac{25}{98}$$