Scalar Triple Product

Q.1. If $\vec{a},\vec{b},\vec{c}$ are three vectors, show that : $(\vec{a} + \vec{b}).(\vec{b} + \vec{c})\times(\vec{c} + \vec{a}) = 2[\vec{a}\vec{b}\vec{c}]$

Solution : 1

L.H.S. = $(\vec{a} + \vec{b}).(\vec{b} + \vec{c})\times(\vec{c} + \vec{a})$

$= (\vec{a} + \vec{b}).(\vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{c} + \vec{c}\times\vec{a})$

$= (\vec{a} + \vec{b}).(\vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{a})$

$= \vec{a}.\vec{b}\times\vec{c} + \vec{a}.\vec{b}\times\vec{a} + \vec{a}.\vec{c}\times\vec{a} + \vec{b}.\vec{b}\times\vec{c}$

$+ \vec{b}.\vec{b}\times\vec{a} + \vec{b}.\vec{c}\times\vec{a}$

$= [\vec{a}\vec{b}\vec{c}] + [\vec{a}\vec{b}\vec{c}]$ [As, $\vec{a}.\vec{b}\times\vec{a} = 0$, $\vec{b}.\vec{b}\times\vec{a} = 0$,

$= 2[\vec{a}\vec{b}\vec{c}]$ **[Proved.]** & $\vec{b}.\vec{b}\times\vec{c} = 0$& $\vec{a}.\vec{c}\times\vec{a} = 0$.]

Q.2. The vectors i + 3j , 5k and λi – j are coplanar. Find the value of λ.

Solution : 2

Vectors i + 3j , 5k and λi – j are coplanar, [When $\vec{a}$, $\vec{b}$and $\vec{c}$ are coplanar ,

$\vec{a} . (\vec{b}\times\vec{c}) = 0$]

Therefore, | 1 3 0 |

| 0 0 5 |= 0

| λ –1 0 |

Or, 1{(0 – (–5) }+ λ(15) = 0

Or, 15λ = – 5

Or, λ = – 1/3 .

Q.3. Prove that $\vec{a}.(\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}]$

Solution : 3

L.H.S. $= \vec{a}.(\vec{b} + \vec{c}) \times(\vec{a} + 2\vec{b} + 3\vec{c})$

$= \vec{a}.[\ \vec{b} \times\vec{a} + 2\vec{b} \times\vec{b} + 3\vec{b}\times\vec{c} + \vec{c} \times\vec{a} + 2\vec{c}\times\vec{b} + 3\vec{c}\times\vec{c}]$

$= \vec{a}[\ - \vec{a}\times \vec{b} + 0 + 3\vec{b}\times\vec{c} + \vec{c}\times \vec{a} - 2\vec{b}\times\vec{c}]$

$[\text{As, } \vec{b}\times \vec{b} = \vec{c}\times \vec{c} = 0\ ]$

$= \vec{a}.[- \vec{a}\times \vec{b} + \vec{b}\times \vec{c} + \vec{c}\times \vec{a}]$

$= - \vec{a}.(\vec{a}\times \vec{b}) + \vec{a}.(\vec{b}\times \vec{c}) + \vec{a}.(\vec{b}\times \vec{a})$

$= 0 + \vec{a}.(\vec{b}\times \vec{c}) + 0$ $[\text{As, } \vec{a}.(\vec{a}\times \vec{b}) = (\vec{a}\times \vec{a}).\vec{b} = 0\ ]$

$= \vec{a}.(\vec{b}\times \vec{c})$ = R.H.S. **[Proved.]**

Q.4. Find the volume of the parallelepiped whose three co-terminus edges are represented by the vectors : $i + j + k$, $i - j + k$ and $i + 2j - k$.

Solution : 4

$$\text{Volume of parallelepiped} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$= 1(1 - 2) - 1 (- 1 - 1) + 1(2 + 1)$

$= - 1 + 2 + 3$

$= 4$ cubic units.

Q.5. Find the volume of the parallelepiped whose edges (co-turminus) are represented by the vectors : $\vec{a} = 2 i - 3j + 4k$, $\vec{b} = i + 2j - k$, $\vec{c} = 3 i - j + 2k$.

Solution : 5

We have, $\vec{a} = 2i - 3j + 4k$, $\vec{b} = i + 2j - k$ and $\vec{c} = 3i - j + 2k$.

Volume, $V = (\vec{a} \times \vec{b}).\vec{c} = [(2i - 3j + 4k)\times(i + 2j - k)].(3i - j + 2k)$

$$= \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$= 3(3 - 8) - (-1)(-2 - 4) + 2(4 + 3)$

$= -15 - 6 + 14 = -7.$

Hence, $V = |-7| = 7$ cubic units.