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Principle of Mathematical Induction

TOPIC 1 Problems Based on Sum of Series, Problems Based on Inequality and Divisibility

1. Consider the statement: "P(n) : $n^2 - n + 41$ is prime." Then which one of the following is true? **[Jan. 10, 2019 (II)]**
(a) Both P(3) and P(5) are true.
(b) P(3) is false but P(5) is true.
(c) Both P(3) and P(5) are false.
(d) P(5) is false but P(3) is true.

2. Let $S(K) = 1 + 3 + 5 ... + (2K - 1) = 3 + K^2$. Then which of the following is true **[2004]**
(a) Principle of mathematical induction can be used to prove the formula
(b) $S(K) \Rightarrow S(K+1)$
(c) $S(K) \not\Rightarrow S(K+1)$
(d) $S(1)$ is correct

3. If $a_n = \sqrt{7+\sqrt{7+\sqrt{7+......}}}$ having n radical signs then by methods of mathematical induction which is true **[2002]**
(a) $a_n > 7 \;\forall\; n \geq 1$
(b) $a_n < 7 \;\forall\; n \geq 1$
(c) $a_n < 4 \;\forall\; n \geq 1$
(d) $a_n < 3 \;\forall\; n \geq 1$

Hints & Solutions

1. **(a)** $P(n) = n^2 - n + 41$
   $\Rightarrow$ $P(3) = 9 - 3 + 41 = 47$ (prime)
   & $P(5) = 25 - 5 + 41 = 61$ (prime)
   $\therefore$ $P(3)$ and $P(5)$ are both prime i.e., true.

2. **(b)** $S(K) = 1+3+5+...+(2K-1) = 3 + K^2$

   $S(1): 1 = 3 + 1,$ which is not true

   $\because$ $S(1)$ is not true.

   $\therefore$ P.M.I cannot be applied

   Let $S(K)$ is true, i.e.

   $S(K): 1 + 3 + 5.... + (2K - 1) = 3 + K^2$

   Adding $2K + 1$ on both sides

   $\Rightarrow 1 + 3 + 5.... + (2K - 1) + 2K + 1$

   $= 3 + K^2 + 2K + 1 = 3 + (K + 1)^2 = S(K + 1)$

   $\therefore S(K) \Rightarrow S(K + 1)$

3. **(b)** For $n = 1$, $a_1 = \sqrt{7} < 7$. Let $a_m < 7$.

   Then $a_{m+1} = \sqrt{7 + a_m}$

   $\Rightarrow$ $a^2_{m+1} = 7 + a_m < 7 + 7 < 14.$

   $\Rightarrow$ $a_{m+1} < \sqrt{14} < 7$; So, by the principle of mathematical induction $a_n < 7, \ \forall\ n.$