PRODUCT OF THREE VECTORS (XII, R. S. AGGARWAL)

EXERCISE 25 A [Pg. No.: 1071]

1. Prove that

(i) $\left[\hat{i}\ \ \hat{j}\ \ \hat{k}\right]=\left[\hat{j}\ \ \hat{k}\ \ \hat{i}\right]=\left[\hat{k}\ \ \hat{j}\ \ \hat{i}\right]=1$ (ii) $\left[\hat{i}\ \ \hat{k}\ \ \hat{j}\right]=\left[\hat{k}\ \ \hat{j}\ \ \hat{i}\right]=\left[\hat{j}\ \ \hat{i}\ \ \hat{k}\right]=1$

Sol. (i). $[\hat{i}\ \ \hat{j}\ \ \hat{k}]=[\hat{i}\times\hat{j}]\cdot\hat{k}$

$=\hat{k}\cdot\hat{k}=1$ ……….(i)

$[\hat{j}\ \ \hat{k}\ \ \hat{i}]=[\hat{j}\times\hat{k}]\cdot\hat{i}$

$=\hat{i}\cdot\hat{i}=1$ ……….(ii)

$[\hat{k}\ \ \hat{j}\ \ \hat{i}]=[\hat{k}\times\hat{j}]\cdot\hat{i}$

$=\hat{i}\cdot\hat{i}=1$ ……….(iii)

from (i), (ii) and (iii), we have. $[\hat{i}\ \ \hat{j}\ \ \hat{k}]=[\hat{j}\ \ \hat{k}\ \ \hat{i}]=[\hat{k}\ \ \hat{i}\ \ \hat{j}]=1$

(ii) $[\hat{i}\ \ \hat{k}\ \ \hat{j}]=\left(\hat{i}\times\hat{k}\right)\cdot\hat{j}$

$=-\hat{j}\cdot\hat{j}=-1$…..(i)

$[\hat{k}\ \ \hat{j}\ \ \hat{i}]=[\hat{k}\times\hat{j}]\cdot\hat{i}=-\hat{i}\cdot\hat{i}=-1$ ……..(ii) $[\hat{j}\ \ \hat{i}\ \ \hat{k}]=[\hat{j}\times\hat{i}]\cdot\hat{k}=-\hat{k}\cdot\hat{k}=-1$ ………..(iii)

from (i), (ii) and (iii), we have, $[\hat{i}\ \ \hat{k}\ \ \hat{j}]=[\hat{k}\ \ \hat{j}\ \ \hat{i}]=[\hat{j}\ \ \hat{i}\ \ \hat{k}]=-1$ Proved

2. Find $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]$, when

(i) $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$

(ii) $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}, \vec{b}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}-\hat{j}+2\hat{k}$

(iii) $\vec{a}=2\hat{i}-3\hat{j}$, $\vec{b}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=3\hat{i}-\hat{k}$

Sol. (i) Given: - $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$, $\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$

Now, $[\vec{a}\ \vec{b}\ \vec{c}]=\begin{vmatrix}2 & 1 & 3\\ -1 & 2 & 2\\ 3 & 1 & 2\end{vmatrix}$

$=\begin{vmatrix}0 & 1 & 0\\ -5 & 2 & -5\\ 1 & 1 & -1\end{vmatrix}\begin{cases}c_1\to c_1-2c_2\\ c_3\to c_3-3c_2\end{cases}==\begin{vmatrix}-5 & -5\\ 1 & -1\end{vmatrix}\begin{cases}\text{extanding}\\ \text{by } c_2\end{cases}$

$=-1(5+5)=-10$

(ii) Given :→ $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$, $\vec{c}=3\hat{i}-\hat{j}+\hat{k}$

Now, $[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

$$=\begin{vmatrix}2 & -3 & 4\\ 1 & 2 & -1\\ 3 & -1 & 2\end{vmatrix}=-\begin{vmatrix}1 & 2 & -1\\ 2 & -3 & 4\\ 3 & -1 & 2\end{vmatrix}\{R_1\leftrightarrow R_2=-\begin{vmatrix}1 & 2 & -1\\ 0 & -7 & 6\\ 0 & -7 & 5\end{vmatrix}\begin{cases}R_1\to R_2-2R_1\\ R_3\to R_3-3R_1\end{cases}$$

$$=-\begin{vmatrix}-7 & 2\\ -7 & 5\end{vmatrix}\{\text{expanding by } c_1 \quad = -(-35+42)= -7$$

(iii) $\vec{a}=2\hat{i}-3\hat{j}, \vec{b}=\hat{i}+\hat{j}-\hat{k}, \vec{c}=3\hat{i}-\hat{k}$

Now $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

$$=\begin{vmatrix}2 & -3 & 0\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix}=2\begin{vmatrix}1 & -1\\ 0 & -1\end{vmatrix}+\begin{vmatrix}1 & -1\\ 3 & -1\end{vmatrix}\{\text{by } R_1 \quad =2(-1-0)+(-1+3)=-2+6=4$$

3. Find the volume of the parallelepiped whose coterminous edges are represented by vectors

(i) $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+2\hat{j}-\hat{k}$

(ii) $\vec{a}=-3\hat{i}+7\hat{j}+5\hat{k}\ \vec{b}=-5\hat{i}+7\hat{j}-3\hat{k}, \vec{c}=7\hat{i}-5\hat{j}-3\hat{k}$

(iii) $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}, \vec{b}=2\hat{i}+\hat{j}-\hat{k}, \vec{c}=\hat{j}+\hat{k}$ (iv) $\vec{a}=6\hat{i}, \vec{b}=2\hat{j}, \vec{c}=5\hat{i}$

Sol. (i) Volume of parallelepiped $=\left|\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right|$

Now $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$$=\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 2 & -1\end{vmatrix}=\begin{vmatrix}0 & 1 & 1\\ 0 & -1 & 1\\ 2 & 2 & -1\end{vmatrix}=2\begin{vmatrix}1 & 1\\ -1 & 1\end{vmatrix}\ \{\text{expanding by } c_1$$

$=2(1+1)=4$

Hence required value = 4 cubic units

(ii) volume of parallelepiped $=\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]$

$$\text{Now}\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]=\begin{vmatrix}-3 & 7 & 5\\ -5 & 7 & -3\\ 7 & -5 & -3\end{vmatrix}=-3\begin{vmatrix}7 & -3\\ -5 & -3\end{vmatrix}+5\begin{vmatrix}7 & 5\\ -5 & -3\end{vmatrix}+7\begin{vmatrix}7 & 5\\ 7 & -3\end{vmatrix}\{\text{ by } c_1$$

$$=-3\{-21-15\}+5\{-21+25\}+7\{-21-35\}\ =-3\times(-36)+5\times4+7\times(-56)$$

$=108+20+(-392)= -392+128= -264$ $\therefore |[\vec{a}\ \vec{b}\ \vec{c}]|=|-234|$

Here, required value = 264 cubic units.

(iii) Given:-

$\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$

$\vec{b}=2\hat{i}+\hat{j}-\hat{k}$

$\vec{c}=\hat{j}+\hat{k}$

Now, $[\vec{a}\ \vec{b}\ \vec{c}]$

$$=\begin{vmatrix}1 & -2 & 3\\ 2 & 1 & -1\\ 0 & 1 & 1\end{vmatrix}=\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}-2\begin{vmatrix}-2 & 3\\ 1 & 1\end{vmatrix}\{\text{by } c_1=(1+1)-2((-2-3)=2+10=12$$

Here, volume of parallelepiped = 12 which units

(iv) Value of parallelepiped $= |[\vec{a}\ \vec{b}\ \vec{c}]|$ cubic units

$= |[6\hat{i}\ 2\hat{j}\ 5\hat{i}]|$ cubic units $= |(6\hat{i} \times 2\hat{j}) \cdot 5\hat{i}|$ cubic units $= |12\hat{k} \cdot 5\hat{i}|$ cubic units

= 0 cubic units Here, $\vec{a}$, $\vec{b}$ & $\vec{c}$ are co-planar.

4. Show that the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, when

(i) $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

(ii) $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = 7\hat{j} + 3\hat{k}$

(iii) $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$

Sol. (i) Given:-

$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$

$\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

Now, $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$

$= \begin{vmatrix} 1 & -1 & 1 \\ -2 & 1 & -1 \\ 1 & -2 & -2 \end{vmatrix} \begin{cases} R_2 \to R_2 + R_1 \\ R_3 + R_3 - 2R_1 \end{cases} = \begin{vmatrix} 1 & -1 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 0 \end{vmatrix} \{R_2 \to R_3 + R_2$

$= 0$

$\because [\vec{a}\ \vec{b}\ \vec{c}] = 0$

Hence, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are Co-planar.

(ii) Given:- $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{c} = 7\hat{j} + 3\hat{k}$

Now, $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 3 & -1 \\ 0 & -7 & -3 \\ 0 & 7 & 3 \end{vmatrix} \{R_2 + R_2 - 2R_1$

Hence, R_2 Proportional R_3 $\therefore [\vec{a}\ \vec{b}\ \vec{c}] = 0$

Hence, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are Co-planar.

(iii) Given:-

$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$

Now, $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 2 & -1 & 2 \\ 1 & 2 & -3 \\ 3 & -4 & 7 \end{vmatrix} = \begin{vmatrix} 2 & -1 & -2 \\ -3 & 4 & -7 \\ 3 & -4 & 7 \end{vmatrix} \{R_2 \to R_2 - 2R_1 = \begin{vmatrix} 2 & -1 & 2 \\ -3 & 4 & -7 \\ 0 & 0 & 0 \end{vmatrix} \{R_3 + R_3 + R_2 = 0$

Hence, $\vec{a}$, $\vec{b}$ & $\vec{c}$ are coplanar.

5. Find the value of λ for which the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, where

(i) $\vec{a} = \left(2\hat{i} - \hat{j} + \hat{k}\right), \vec{b} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\vec{c} = \left(3\hat{i} + \lambda\hat{j} + 5\hat{k}\right)$

(ii) $\vec{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}, \vec{b} = -7\hat{i} - 5\hat{j}$ and $\vec{c} = \hat{i} - 4\hat{j} - 3\hat{k}$

(iii) $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

Sol. (i) $\therefore \vec{a}, \vec{b}$ & $\vec{c}$ are coplanar.

$\therefore [\vec{a}, \vec{b}\ \vec{c}] = 0$

$$= \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 0 & \lambda & 5 \end{vmatrix} = 0 = \begin{vmatrix} 0 & -5 & -5 \\ 1 & 2 & 3 \\ 0 & \lambda-6 & 5 \end{vmatrix} = 0 \begin{cases} R_1 + R_1 - 2R_2 \\ R_3 + R_3 - 3R_3 \end{cases}$$

$\Rightarrow -1\begin{vmatrix} -5 & -5 \\ \lambda-6 & -4 \end{vmatrix} = 0$ {expandiy by $c_1 \Rightarrow 20 + 5(\lambda - 6) = 0$

$\Rightarrow 20 + 5\lambda - 30 = 0 \Rightarrow 5\lambda - 10 = 0 \Rightarrow \lambda = 2$ Ans.

(ii) $\because \vec{a}. \vec{b}$ and $\vec{c}$ are co-planar

$\therefore [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} \lambda & -10 & -5 \\ -7 & -5 & 0 \\ 1 & -4 & -3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} -5 & -0 \\ -4 & -3 \end{vmatrix} + 10\begin{vmatrix} -7 & -0 \\ 1 & -3 \end{vmatrix} - 5\begin{vmatrix} -7 & -5 \\ 1 & -4 \end{vmatrix} = 0$$

$\Rightarrow 15\lambda + 210 - 5(28 + 5) = 0 \Rightarrow 15\lambda + 210 - 165 = 0 \Rightarrow 15\lambda + 45 = 0 \Rightarrow \lambda = -3$ Ans.

(iii) $\because \vec{a}, \vec{b}$ and $\vec{c}$ are co-planar

$\therefore [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & -1 & 1 \\ 3 & 1 & -1 \\ 0 & -1 & \lambda \end{vmatrix} = 0 \quad \{c_1 + c_1 - c_3 \Rightarrow -3\begin{vmatrix} -1 & 1 \\ -1 & \lambda \end{vmatrix} = 0 \Rightarrow -3(-\lambda + 1) = 0 \Rightarrow \lambda = 1 \text{ Ans}$$

6. If $\vec{a} = \left(2\hat{i} - \hat{j} + \hat{k}\right), \vec{b} = \left(\hat{i} - 3\hat{j} - 5\hat{k}\right)$ and $\vec{c} = \left(3\hat{i} - 4\hat{j} - \hat{k}\right)$, find $\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]$ and interpret the result

Sol. $[\vec{a}\ \vec{b}\ \vec{c}]$

$$= \begin{vmatrix} 2 & -1 & 1 \\ 1 & -3 & -5 \\ 3 & -4 & -1 \end{vmatrix} = \begin{vmatrix} -3 & -5 \\ -4 & -1 \end{vmatrix} - 1\begin{vmatrix} -1 & 1 \\ -4 & -1 \end{vmatrix} + 3\begin{vmatrix} -1 & -1 \\ -3 & -5 \end{vmatrix} \{\text{expanding by } c_1$$

$= 2(3 - 20) - (1 + 4) + 3((5 + 3) = -34 - 5 + 24 = -15$

7. the volume of the parallelepiped whose edges are $\left(-12\hat{i} + \lambda\hat{k}\right), \left(3\hat{j} - \hat{k}\right)$ and $\left(2\hat{i} + \hat{j} - 15\hat{k}\right)$ is 546 cubic units. Find the value of λ

Sol. Volume of parallelepiped = 546 cube units.

$$\Rightarrow \begin{vmatrix} -12 & 0 & \lambda \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix} = \pm 546 \Rightarrow -12\begin{vmatrix} 3 & -1 \\ 1 & -15 \end{vmatrix} + \lambda\begin{vmatrix} 0 & 3 \\ 2 & 1 \end{vmatrix} = \pm 546$$

$\Rightarrow -12(-45 + 1) + \lambda(0 - 6) = \pm 546 \Rightarrow 528 - 6\lambda = \pm 546 \Rightarrow 528 - 6\lambda = 546$

or, $528 - 6\lambda = -546$

$\Rightarrow -6\lambda = 18$ or $-6\lambda = -1074 \Rightarrow \lambda = -3$ or $\lambda = 179$ Ans.

8. Show that the vectors $\vec{a} = (\hat{i} + 3\hat{j} + \hat{k}), \vec{b} = (2\hat{i} - \hat{j} - \hat{k})$ and $\vec{c} = (7\hat{j} + 3\hat{k})$ are parallel to the same plane

Hints show that $[\vec{a} \quad \vec{b} \quad \vec{c}] = 0$

Sol. Given:- $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = 7\hat{j} + 3\hat{k}$

Now, $[\vec{a} \ \vec{b} \ \vec{c}]$

$$= \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 1 \\ 0 & -7 & -3 \\ 0 & 7 & 3 \end{vmatrix} \{R_2 \to R_2 - 2R_1 = \begin{vmatrix} -7 & -3 \\ 7 & 3 \end{vmatrix} = -21 + 21 = 0$$

Hence, $\vec{a}$, $\vec{b}$ & $\vec{c}$ ar ll to the same plane

9. If the vectors $(a\hat{i} + a\hat{j} + c\hat{k}), (\hat{i} + \hat{k})$ and $(c\hat{i} + x\hat{j} + b\hat{k})$ be coplanar, show that $c^2 = ab$

Sol. Since, the three given vectors are co-planar,

$$\Rightarrow \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & a & c \\ 1 & 0 & 1 \\ 0 & c & b \end{vmatrix} = 0 \ \{R_1 \to R_1 - R_2$$

Expanding by c_1, we have

$$\Rightarrow -1\begin{vmatrix} a & c \\ c & b \end{vmatrix} = 0 \Rightarrow ab - c^2 = 0 \Rightarrow c^2 = ab \text{ proved}$$

10. show that the four points position vectors $(4\hat{i} + 8\hat{j} + 12\hat{k})$, $(2\hat{i} + 4\hat{j} + 6\hat{k}), (3\hat{i}, 5\hat{j}, 4\hat{k})$ and $(5\hat{i} + 8\hat{j} + 5\hat{k})$ are coplanar

Sol. Let, position vector of A = $4\hat{i} + 8\hat{j} + 12\hat{k}$ Let, position vector of B = $2\hat{i} + 4\hat{j} + 6\hat{k}$

Let, position vector of C = $3\hat{i} + 5\hat{j} + 4\hat{k}$ Let, position vector of D = $5\hat{i} + 8\hat{j} + 5\hat{k}$

Now, $\vec{AB}$ = P.V. of B – P.V of A

$= (2\hat{i} + 4\hat{j} + 6\hat{k}) - (4\hat{i} + 8\hat{j} + 12\hat{k} = -2\hat{i} - 4\hat{j} - 6\hat{k}$

$\vec{BC}$ = P.V of C – P.V of B

$= (3\hat{i} + 5\hat{j} + 4\hat{k}) - (2\hat{i} + 4\hat{j} + 6\hat{k} = \hat{i} + \hat{j} - 2\hat{k}$

$\vec{AD}$ = P.V of D – P.V of A $= (5\hat{i} + 8\hat{j} + 5\hat{k}) - (4\hat{i} + 8\hat{j} + 12\hat{k}) = \hat{i} - 7\hat{k}$

Now, $[\vec{AB} \ \vec{BC} \ \vec{AD}]$

$$= \begin{vmatrix} -2 & -4 & -6 \\ 1 & 1 & -2 \\ 1 & 0 & -7 \end{vmatrix} = \begin{vmatrix} 0 & -4 & -20 \\ 0 & 1 & 5 \\ 1 & 0 & -7 \end{vmatrix} \begin{cases} R_1 \to R_1 + 2R_3 \\ R_2 \to R_1 - R_3 \end{cases} = \begin{vmatrix} -4 & -20 \\ 1 & 5 \end{vmatrix} = -20 + 20 = 0$$

Hence, $\vec{AB}$, $\vec{BC}$ and $\vec{AD}$ are coplanar. i.e. A, B, C & D are co-planar.

11. Show that the four points with position vectors $(6\hat{i} - 7\hat{j}), (16\hat{i} - 19\hat{j} - 4\hat{k}), (3\hat{j} - 6\hat{k})$ and $(2\hat{i} - 5\hat{j} + 10\hat{k})$ are coplanar

Sol. Let, position vector of A = $6\hat{i} - 7\hat{j}$

position vector of B = $16\hat{i}-19\hat{j}-4\hat{k}$ position vector of C = $3\hat{i}-6\hat{k}$

position vector of D = $2\hat{i}-5\hat{j}+10\hat{k}$

Now, $\overrightarrow{AB}$ = P.V. of B – P.V of A

$\Rightarrow \overrightarrow{AB} = (16\hat{i}-19\hat{j}-4\hat{k})-(16\hat{i}-7\hat{j}) \Rightarrow \overrightarrow{AB} = 10\hat{i}-12\hat{j}-4\hat{k}$

$\overrightarrow{BC}$ = P.V of C – P.V of B $= (3\hat{j}-6\hat{k})-(16\hat{i}-19\hat{j}-4\hat{k}) = -16\hat{i}+22\hat{j}-2\hat{k}$

An $\overrightarrow{AD}$ = P.V of D – P.V of A

$= (2\hat{i}-5\hat{j}+10\hat{k})-(6\hat{i}-7\hat{j}) = -4\hat{i}+2\hat{j}+10\hat{k}$

Now, $[\overrightarrow{AB}\ \overrightarrow{BC}\ \overrightarrow{AD}]$

$$= \begin{vmatrix} 10 & -12 & -4 \\ -16 & 22 & -2 \\ -4 & 2 & 10 \end{vmatrix} = 10\begin{vmatrix} 22 & -2 \\ 2 & 10 \end{vmatrix} + 16\begin{vmatrix} -12 & -4 \\ 2 & 10 \end{vmatrix} - 4\begin{vmatrix} -12 & -4 \\ 22 & -2 \end{vmatrix}$$

$= 10\,(220 + 4) + 16\,(-120 + 8) - 4\,(24 + 88) = 2240 - 1792 - 448 = 0$

Hence, $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AD}$ are co-planar. i.e. A, B, C & D are co-planar.

12. Find the value of λ for which the four points with position vectors $\left(\hat{i}+2\hat{j}+3\hat{k}\right), \left(3\hat{i}-\hat{j}+2\hat{k}\right), \left(-2\hat{i}+\lambda\hat{j}+\hat{k}\right)$ and $\left(6\hat{i}-4\hat{j}+2\hat{k}\right)$ are coplanar

Sol. Let, P.V of A = $\hat{i}+2\hat{j}+3\hat{k}$

P.V of B = $3\hat{i}-\hat{j}+2\hat{k}$

P.V of C = $-2\hat{i}+\lambda\hat{j}+\hat{k}$

Let, P.V of D = $6\hat{i}-4\hat{j}+12\hat{k}$

Now, $\overrightarrow{AB} = (3-1)\hat{i}+(-1-2)\hat{j}+(2-3)\hat{k} = 2\hat{i}-3\hat{j}-3\hat{k}$

$\overrightarrow{BC} = (-2-3)\hat{i}+(\lambda+1)\hat{j}+(1-2)\hat{k} = -5\hat{i}+(\lambda+1)\hat{j}-\hat{k}$

$\overrightarrow{AD} = (6-1)\hat{i}+(-4-2)\hat{j}+(2-3)\hat{k} = 5\hat{i}-6\hat{j}-\hat{k}$

Since, A, B, C and D are coplanar,

$\Rightarrow [\overrightarrow{AB}\ \overrightarrow{BC}\ \overrightarrow{AD}] = 0$

$$\Rightarrow \begin{vmatrix} 2 & -3 & -1 \\ -5 & \lambda+1 & -1 \\ 5 & -6 & -1 \end{vmatrix} = 0 \quad \Rightarrow \begin{vmatrix} -3 & 3 & 0 \\ -10 & \lambda+7 & 0 \\ 5 & -6 & -1 \end{vmatrix} = 0 \begin{cases} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{cases}$$

$\Rightarrow -1\begin{vmatrix} -3 & 3 \\ -10 & \lambda+7 \end{vmatrix} = 0 \Rightarrow -3\lambda - 21 + 30 = 0 \Rightarrow -3\lambda + 9 = 0 \Rightarrow \lambda = 3$ Ans

13. Find the value of λ for which the four points with position vectors $\left(-\hat{j}+\hat{k}\right), \left(2\hat{i}-\hat{j}-\hat{k}\right), \left(\hat{i}+\lambda\hat{j}+\hat{k}\right)$ and $\left(3\hat{j}+3\hat{k}\right)$ are coplanar

Sol. Let, Position vector of A = $-\hat{j}+\hat{k}$ Let, Position vector of B = $2\hat{i}-\hat{j}-\hat{k}$

Let, Position vector of C = $\hat{i}+\lambda\hat{j}+\hat{k}$ Let, Position vector of D = $3\hat{j}+3\hat{k}$

Now, $\overrightarrow{AB}$ = P.V of B – P.V of A $= 2\hat{i}-2\hat{k}$

$\overrightarrow{BC}$ = P.V of C – P.V of B $= -\hat{i} + (\lambda+1)\hat{j} + 2\hat{k}$

$\overrightarrow{AD}$ = P.V of D – P.V of A $= 4\hat{i} + 2\hat{k}$ $\quad\quad \because$ A, B, C and D are Co-planer

$\Rightarrow [\overrightarrow{AB}\ \overrightarrow{BC}\ \overrightarrow{AD}] = 0$

$$\Rightarrow \begin{vmatrix} 2 & 0 & -2 \\ -1 & \lambda+1 & 2 \\ 4 & 0 & 2 \end{vmatrix} = 0 \quad \Rightarrow \begin{vmatrix} 2 & 0 & 0 \\ -1 & \lambda+1 & 3 \\ 4 & 0 & 6 \end{vmatrix} = 0 \ \{C_3 \to C_3 + C_1 \Rightarrow 2\begin{vmatrix} \lambda+1 & 3 \\ 0 & 6 \end{vmatrix} = 0$$

$\Rightarrow 6(\lambda+1) - 0 = 0 \Rightarrow 6\lambda + 6 = 0 \Rightarrow \lambda = -1$

14. Using vector method show that the points $A(4,5,1), B(0,-1,-1), C(3,9,4)$ and $D(-4,4,4)$ are coplanar

Sol. $\overrightarrow{AB} = (0-4)\hat{i} + (-1-5)\hat{j} + (-1-1)\hat{k} = -4\hat{i} - 6\hat{j} - 2\hat{k}$

$\overrightarrow{BC} = (3-0)\hat{i} + (9+1)\hat{j} + (4+1)\hat{k} = 3\hat{i} + 10\hat{j} + 5\hat{k}$

$\overrightarrow{AD} = (-4-4)\hat{i} + (4-5)\hat{j} + (4-1)\hat{k} = -8\hat{i} - \hat{j} + 3\hat{k}$

Now, $[\overrightarrow{AB}\ \overrightarrow{BC}\ \overrightarrow{AD}]$

$$= \begin{vmatrix} -4 & -6 & -2 \\ 3 & 10 & 5 \\ -8 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 0 & 0 & -2 \\ -7 & -5 & 5 \\ -14 & -10 & 3 \end{vmatrix} \quad \begin{cases} C_1 \to C_1 - 2C_3 \\ C_2 \to C_2 - 3C_3 \end{cases}$$

$$= \begin{vmatrix} -7 & -5 \\ -14 & -10 \end{vmatrix} = -2(70-70) = 0$$

$\Rightarrow \overrightarrow{AB}, \overrightarrow{BC}$ and $\overrightarrow{AD}$ are co-planar $\Rightarrow$ A, B, C and D are co-planar

15. Find the value of λ for which the points $A(3,2,1), B(4,\lambda,5), C(4,2,-2)$ and $D(6,5,-1)$ are coplanar

Sol. Given points A (3, 2, 1)

B (4, λ, 5), C (4, 2, -2) and D (6, 5, -1) are co-planar.

Now, $\overrightarrow{AB} = (4-3)\hat{i} + (\lambda-2)\hat{j} + (5-1)\hat{k} = \hat{i} + (\lambda-2)\hat{j} + 4\hat{k}$

$\overrightarrow{BC} = (4-4)\hat{i} + (2-\lambda)\hat{j} + (-2-5)\hat{k} = (2-\lambda)\hat{j} - 7\hat{k}$

$\overrightarrow{AD} = (6-3)\hat{i} + (5-2)\hat{j} + (-1-1)\hat{k} = 3\hat{i} + 3\hat{j} - 2\hat{k}$

Now, $[\overrightarrow{AB}\ \overrightarrow{BC}\ \overrightarrow{AD}] = 0$

$$= \begin{vmatrix} 1 & \lambda-2 & 4 \\ 0 & 2-\lambda & -7 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

Expanding by C_1

$$= \begin{vmatrix} 2-\lambda & -7 \\ 3 & -2 \end{vmatrix} + 3\begin{vmatrix} \lambda-2 & 4 \\ 2-\lambda & -7 \end{vmatrix} = 0$$

$\Rightarrow -4 + 2\lambda + 21 + 3(-7\lambda + 14 - 8 + 4\lambda) = 0 \Rightarrow 2\lambda + 17 - 9\lambda + 18 = 0$

$\Rightarrow -7\lambda + 35 = 0 \Rightarrow \lambda = 5$ Ans.

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EXERCISE 25 B [Pg. No.: 1073]

1. If $\vec{d}=\left(\sqrt{2}\hat{i}+\sqrt{3}\hat{j}-\sqrt{5}\hat{k}\right)\vec{a}=x\hat{i}+2\hat{j}-z\hat{k}$ and $\vec{b}=3\hat{i}-y\hat{j}+\hat{k}$ are two equal vectors then $x+y+z=?$

Sol. $\because \vec{a}=\vec{b}$

$\Rightarrow x\hat{i}+2\hat{j}-z\hat{k}=3\hat{i}-y\hat{j}+\hat{k} \Rightarrow x = 3,\ y = -2 \text{ and } z = 1 \Rightarrow x + y + z = 3 + (-2) + (-1)$

$\Rightarrow x\ y + z = 0$

2. Write a unit vector in the direction of the sum of the vectors $\vec{a}=\left(2\hat{i}+2\hat{j}-5\hat{k}\right)$ and $\vec{b}=\left(2\hat{i}+\hat{j}-7\hat{k}\right)$

Sol. Given:-

$\vec{a}=2\hat{i}+2\hat{j}-5\hat{k}$

$\vec{b}=2\hat{i}+2\hat{j}-7\hat{k}$

Now, $\vec{a}+\vec{b}=4\hat{i}+3\hat{j}-12\hat{k}$

unit vector along $\vec{a}+\vec{b}$ is,

$\hat{r}=\dfrac{\vec{a}+\vec{b}}{|\vec{a}+\vec{a}|}=\dfrac{4\hat{i}+3\hat{j}-12\hat{k}}{\sqrt{4^2+3^2+(-12)^2}}=\dfrac{1}{13}\left(4\hat{i}+3\hat{j}-12\hat{k}\right)$

3. Write the value of λ so that the vectors $\vec{a}=\left(2\hat{i}+\lambda\hat{j}+\hat{k}\right)$ and $\vec{b}=\left(\hat{i}-2\hat{j}+3\hat{k}\right)$ are perpendicular to each other

Sol. $\because \vec{a}\perp r\vec{b}$

$\Rightarrow \vec{a}\cdot\vec{b}=0 \Rightarrow \left(2\hat{i}+\lambda\hat{j}+\hat{k}\right)\cdot\left(\hat{i}-2\hat{j}+3\hat{k}\right)=0 \Rightarrow 2-2\lambda+3=0 \Rightarrow -2\lambda=-5 \Rightarrow \lambda=\dfrac{5}{2}$

4. Find the value of p for which the vectors $\vec{a}=\left(3\hat{i}+2\hat{j}+9\hat{k}\right)$ and $\vec{b}=\left(\hat{i}-2p\hat{j}+3\hat{k}\right)$ are parallel

Sol. $\because \vec{a}\,||\,\vec{b}$

$\Rightarrow \dfrac{3}{1}=\dfrac{2}{-2P}=\dfrac{9}{3}\Rightarrow 3=-\dfrac{1}{P}\Rightarrow P=-\dfrac{1}{3}$

5. Find the value of λ when the projection of $\vec{a}=\left(\lambda\hat{i}+\hat{j}+4\hat{k}\right)$ on $\vec{b}=\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$ is 4 units

Sol. Projection of $\vec{a}$ on $\vec{b}=4$

$\Rightarrow \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=4 \Rightarrow \dfrac{2\lambda+6+12}{\sqrt{2^2+6^2+3^2}}=4 \Rightarrow 2\lambda+18=4\times 7 \Rightarrow 2\lambda=28-18 \Rightarrow \lambda=\dfrac{10}{2}=5$

6. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors such that $\left|\vec{a}+\vec{b}\right|=13$ and $\left|\vec{a}\right|=5,$ find the value of $\left|\vec{b}\right|$

Sol. We have, $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}$

$\Rightarrow 13^2=5^2+|\vec{b}|^2+2\times 0 \quad \left\{\because \vec{a}\perp r\vec{b} \quad \therefore \vec{a}\cdot\vec{b}=0\right.$

$\Rightarrow 169 = 25 + |\vec{b}|^2 \Rightarrow |\vec{b}|^2 = 144 \Rightarrow |\vec{b}| = 12$

7. If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=15$, find $|\vec{x}|$

Sol. $\because \left(\vec{x}-\vec{a}\right)\cdot\left(\vec{x}+\vec{a}\right)=15$

$\Rightarrow |\vec{x}|^2-|\vec{a}|^2=15$

$\Rightarrow |\vec{x}|^2-1=15 \left\{\because \vec{a} \text{ is a unit vector.}\right.$

$\Rightarrow |\vec{x}|^2=16$

$\Rightarrow |\vec{x}|=4.$

8. Find the sum of the vectors $\vec{a}=(\hat{i}-3\hat{k}), \vec{b}=(2\hat{j}-\hat{k})$ and $\vec{c}=(2\hat{i}-3\hat{j}+2\hat{k})$

Sol. $\vec{a}+\vec{b}+\vec{c}$

$=\hat{i}-3\hat{j}+2\hat{j}-\hat{k}+2\hat{i}-3\hat{j}+2\hat{k} \Rightarrow 3\hat{i}-\hat{j}-2\hat{k}$

9. Find the sum of the vectors $\vec{a}=(\hat{i}-2\hat{j}), \vec{b}=(2\hat{i}-3\hat{j})$ and $\vec{c}=(2\hat{i}+3\hat{k})$.

Sol. $\vec{a}+\vec{b}+\vec{c}$

$=\hat{i}-2\hat{j}+2\hat{i}-3\hat{j}+2\hat{i}+3\hat{k} \Rightarrow 5\hat{i}-5\hat{j}+3\hat{k}$

10. Write the projection of the vector $(\hat{i}+\hat{j}+\hat{k})$ along the vector $\hat{j}$

Sol. Proj. of $\left(\hat{i}+\hat{j}+\hat{k}\right)$ on $\hat{j}$

$$=\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\hat{j}}{|\hat{j}|}=\frac{0+1+0}{1}=1$$

11. Write the projection of the vector $(7\hat{i}+\hat{j}-4\hat{k})$ on the vector $(2\hat{i}+6\hat{j}+3\hat{k})$

Sol. let, $\vec{a}=7\hat{i}+\hat{j}-4\hat{k}$

$\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$

$$=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=\frac{14+6-12}{\sqrt{2^2+6^2+3^2}}=\frac{6}{7}\text{ Ans}$$

12. Find $\vec{a}\cdot(\vec{b}\times\vec{c})$ when $\vec{a}=(2\hat{i}+\hat{j}+3\hat{k}), \vec{b}=(-\hat{i}+2\hat{j}+\hat{k})$ and $\vec{c}=(3\hat{i}+\hat{j}+2\hat{k})$

Sol. $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)=[\vec{a}\ \vec{b}\ \vec{c}]$

$$=\begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}=2\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}+1\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}+3\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}$$

$= 2(4-1)+(2-3)+3(1-6)= 2\times 3-1+3\times(-5)=6-1-15=-10$

13. Find a vector in the direction of $\left(2\hat{i}-3\hat{j}+6\hat{k}\right)$ which has magnitude 21 units

Sol. Let, $\vec{a}=2\hat{i}-3\hat{j}+6\hat{k}$

A vector of magnitude 21 in the direction of $\vec{a}$ is given by,

$$\vec{b}=21\cdot\frac{\vec{a}}{|\vec{a}|}=$$

$$\Rightarrow \vec{b}=21\frac{2\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{2^2+(-3)^2+6^2}}=\Rightarrow \vec{b}=\frac{21}{7}\left(2\hat{i}-3\hat{j}+6\hat{k}\right)$$

$$\Rightarrow \vec{b}=3\left(2\hat{i}-3\hat{j}+6\hat{k}\right)\Rightarrow \vec{b}=6\hat{i}-9\hat{j}+18\hat{k}$$

14. If $\vec{a}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right),\vec{b}=\left(-\hat{i}+2\hat{j}+\hat{k}\right)$ and $\vec{c}=\left(3\hat{i}+j\right)$ are such that $\left(\vec{a}+\lambda\vec{b}\right)$ is perpendicular to $\vec{c}$ then find the value of λ

Sol. Given:$\rightarrow$ $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$

$\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$

& $\vec{c}=3\hat{i}+\hat{j}$

$\because \vec{a}+\lambda\vec{b}\perp\vec{c}$

$$\Rightarrow\left(\vec{a}+\lambda\vec{b}\right)\cdot\vec{c}=0\Rightarrow\{2\hat{i}+2\hat{j}+3\hat{k}+\lambda(-\hat{i}+2\hat{j}+\hat{k})\}\cdot(3\hat{i}+\hat{j})=0$$

$$\Rightarrow 6-3\lambda+2+2\lambda=0\ \Rightarrow 8-\lambda=0\Rightarrow\lambda=8$$

15. Write a vector of magnitude 15 units in the direction of vector $\left(\hat{i}-2\hat{j}+2\hat{k}\right)$

Sol. Let, $\vec{a}=(\hat{i}-2\hat{j}+2\hat{k})$

A vector of magnitude 15 units in the direction of $\vec{a}=\left(\hat{i}-2\hat{j}+2\hat{k}\right)$ is given by,

$$\vec{b}=15\cdot\frac{\vec{a}}{|\vec{a}|}$$

$$\Rightarrow \vec{b}=15\frac{\left(\hat{i}-2\hat{j}+2\hat{k}\right)}{\sqrt{1+4+4}}\quad\Rightarrow \vec{b}=5\left(\hat{i}-2\hat{j}+2\hat{k}\right)\text{ Ans.}$$

16. If $\vec{a}=\left(\hat{i}+\hat{j}+\hat{k}\right),\vec{b}=\left(4\hat{i}-2\hat{j}+3\hat{k}\right)$ and $\vec{c}=\left(\hat{i}-2\hat{j}+\hat{k}\right)$, find a vector of magnitude 6 units which is parallel to the vector $\left(2\vec{a}-\vec{b}+3\vec{c}\right)$

Sol. $\because \vec{a}=\hat{i}+\hat{j}+\hat{k}$

$$\Rightarrow 2\vec{a}=2\hat{i}+2\hat{j}+2\hat{k}\Rightarrow\vec{b}=4\hat{i}-2\hat{j}+3\hat{k}\Rightarrow-\vec{b}=-4\hat{i}+2\hat{j}-3\hat{k}$$

and, $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$

$$\Rightarrow 3\vec{c}=3\hat{i}-6\hat{j}+3\hat{k}$$

Now, $2\vec{a}-\vec{b}+3\vec{c}=\hat{i}-2\hat{j}+2\hat{k} \Rightarrow |\vec{a}-\vec{b}+3\vec{c}|=\sqrt{1^2+(-2)^2+2^2}=3$

A vector of mag. 6 units parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$ is,

$$\vec{r}=6\cdot\frac{2\vec{a}-\vec{b}+3\vec{c}}{|2\vec{a}-\vec{b}+3\vec{c}|} \quad \Rightarrow \vec{r}=6\cdot\frac{\left(\hat{i}-2\hat{j}+2\hat{k}\right)}{3} \quad \Rightarrow \vec{r}=2\hat{i}-4\hat{j}+4\hat{k}$$

17. Write the projection of the vector $\left(\hat{i}-\hat{j}\right)$ on the vector $\left(\hat{i}+\hat{j}\right)$

Sol. Projection of $\hat{i}-\hat{j}$ on $\hat{i}+\hat{j}$

$$=\frac{\left(\hat{i}-\hat{j}\right)\cdot\left(\hat{i}+\hat{j}\right)}{|\hat{i}+\hat{j}|}=\frac{1-1}{\sqrt{2}}=0.$$

18. Write the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a}\cdot\vec{b}=\sqrt{6}$

Sol. Let θ be the angle b/ω $\vec{a}$ and $\vec{b}$

$$\theta=\cos^{-1}\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\Rightarrow \theta=\cos^{-1}\frac{\sqrt{6}}{\sqrt{3}\cdot 2} \Rightarrow \theta=\cos^{-1}\frac{1}{\sqrt{2}} \Rightarrow \theta=\frac{\pi}{4}$$

19. If $\vec{a}=\left(\hat{i}-7\hat{j}+7\hat{k}\right)$ and $\vec{b}=\left(3\hat{i}-2\hat{j}+2\hat{k}\right)$ then find $\left|\vec{a}\times\vec{b}\right|$

Sol. Given:$\rightarrow$

$\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$

$\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -7 & 7\\ 3 & -2 & 2\end{vmatrix}$

$=\begin{vmatrix}-7 & 7\\ -2 & 2\end{vmatrix}\hat{i}-\begin{vmatrix}1 & 7\\ 3 & 2\end{vmatrix}\hat{j}+\begin{vmatrix}1 & -7\\ 3 & -2\end{vmatrix}\hat{k}=19\hat{i}+19\hat{k} \Rightarrow |\vec{a}\times\vec{b}|=\sqrt{19^2+19^2}=19\sqrt{2}$ Ans

20. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively when $\left|\vec{a}\times\vec{b}\right|=\sqrt{3}$

Sol. Let, θ is the angle between

$\vec{a}$ and $\vec{b}$

Now, $\sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} \Rightarrow \sin\theta=\frac{\sqrt{3}}{1\times 2} \Rightarrow \sin\theta=\frac{\sqrt{3}}{2} \Rightarrow \theta=\frac{\pi}{3}$ or, $2\frac{\pi}{3}$.

21. What conclusion can you draw about vectors $\vec{a}$ and $\vec{b}$ when $\vec{a}\times\vec{b}=\vec{0}$ and $\vec{a}\cdot\vec{b}=0$?

Sol. If, $\vec{a}\times\vec{b}=\vec{0}$ and $\vec{a}\cdot\vec{b}-0$

then, $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$

$\because \vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow |\vec{a}| = 0$ or, $|\vec{b}| = 0$ or, $\vec{a} \parallel \vec{b}$ ....(i)

and, $\vec{a} \cdot \vec{b} - 0$

$\Rightarrow |\vec{a}| = 0$ or, $|\vec{b}| = 0$ or $\vec{a} \perp \vec{b}$ ....(ii)

from (i) and (ii)

$\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$

22. Find the value of λ when the vectors $\vec{a} = \left(\hat{i} + \lambda\hat{j} + 3\hat{k}\right)$ and $\vec{b} = \left(3\hat{i} + 2\hat{j} + 9\hat{k}\right)$ are parallel

Sol. $\because \vec{a} \parallel \vec{b}$

$$\Rightarrow \frac{1}{3} = \frac{\lambda}{2} = \frac{3}{9} \Rightarrow \lambda = \frac{2}{3}.$$

23. Write the value of $\hat{i} \cdot \left(\hat{j} \times \hat{k}\right) + \hat{j} \cdot \left(\hat{i} \times \hat{k}\right) + \hat{k} \cdot \left(\hat{i} \times \hat{j}\right)$

Sol. $\hat{i} \cdot \left(\hat{j} \times \hat{k}\right) + \hat{j} \cdot \left(\hat{i} \times \hat{k}\right) + \hat{k} \cdot \left(\hat{i} \times \hat{j}\right)$

$= \hat{i} \cdot \hat{i} + \hat{j} \cdot \left(-\hat{j}\right) + \hat{k} \cdot \hat{k} = 1 - 1 + 1 = 1$ Ans.

24. Find the volume of the parallelepiped whose edges are represented by the vectors $\vec{a} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)$, $\vec{b} = \left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{c} = \left(3\hat{i} - 2\hat{j} + 2\hat{k}\right)$

Sol. Volume of parallelepiped $= |[\vec{a}\ \vec{b}\ \vec{c}]|$

$$\text{Now, } [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -2 & 2 \end{vmatrix}$$

$$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 0 & -7 & 6 \\ 1 & 2 & -1 \\ 0 & -8 & 5 \end{vmatrix} \begin{cases} R_1 + R_1 - 2R_2 \\ R_3 + R_3 - 3R_2 \end{cases}$$

$$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = -1 \begin{vmatrix} -7 & 6 \\ -8 & 5 \end{vmatrix} = -(-35 + 48) = -13 \qquad \Rightarrow |[\vec{a}\ \vec{b}\ \vec{c}]| = |-13| = 13$$

Hence, volume of parallelepiped = 13. cubic units

25. If $\vec{a} = \left(-2\hat{i} - 2\hat{j} + 4\hat{k}\right), \vec{b} = \left(-2\hat{i} + 4\hat{j} - 2\hat{k}\right)$ and $\vec{c} = \left(4\hat{i} - 2\hat{j} - 2\hat{k}\right)$ then prove that $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

Sol. Given:-

$\vec{a} = -2\hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$

$\vec{a} = 4\hat{i} - 2\hat{j} - 2\hat{k}$

Now, $[\vec{a}\ \vec{b}\ \vec{a}]$

$$= \begin{vmatrix} -2 & -2 & 4 \\ -2 & 4 & -2 \\ 4 & -2 & -2 \end{vmatrix} = \begin{vmatrix} -4 & 2 & 2 \\ -2 & 4 & -2 \\ 4 & -2 & -2 \end{vmatrix} \{R_1 \to R_1 + R_2 \quad = \begin{vmatrix} 0 & 0 & 0 \\ -2 & 4 & -2 \\ 4 & -2 & -2 \end{vmatrix} \{R_1 \to R_1 + R_3$$

$= 0$

Hence, $\vec{a}$, $\vec{b}$ & $\vec{c}$ are co-planer.

26. if $\vec{a} = (2\hat{i} + 6\hat{j} + 27\hat{k})$ and $\vec{b} = (\hat{i} + \lambda\hat{j} + \mu\hat{k})$ are such that $\vec{a} \times \vec{b} = \vec{0}$ then find the values of λ and μ

Sol. $\because \vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow \vec{a} \parallel \vec{b} \Rightarrow \frac{2}{1} = \frac{6}{\lambda} = \frac{27}{r} \Rightarrow 2\lambda = 6$ and $2r = 27 \Rightarrow \lambda = 3$ and $r = \frac{27}{2}$ Ans.

27. If θ is the angle between $\vec{a}$ and $\vec{b}$, and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ then what is the value of θ?

Sol. $\because \vec{a} \cdot \vec{b} = |\vec{b} \times \vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

28. When does $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$ holds

Sol. If, $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$

$\Rightarrow |\vec{a} + \vec{b}|^2 = \{|\vec{a}| + |\vec{b}|\}^2 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|$

$\Rightarrow 2|\vec{a}||\vec{b}|\cos\theta = 2|\vec{a}||\vec{b}| \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$

Hence, If, $\vec{a}$ like $\vec{b}$, then, $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$

29. Find the direction cosines of a vector which is equally inclined to the x-axis y-axis and z-axis

Sol. Let, θ = Indignation of vector with axes.

Here direction cosines are, $\ell = \cos\theta$, $m = \cos\theta$ and $n = \cos\theta$

We have, $\ell^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\theta + \cos^2\theta + \cos^2\theta = 1 \Rightarrow 3\cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{3} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{3}}$

Hence, d cosines are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

30. If $P(1,5,4)$ and $Q(4,1,-2)$ be the position vectors of two points P and Q find the direction rations of $\overrightarrow{PQ}$

Sol. $\overrightarrow{PQ} = (4-1)\hat{i} + (1-5)\hat{j} + (-2-4)\hat{k}$

$\Rightarrow \overrightarrow{PQ} = 3\hat{i} - 4\hat{j} - 6\hat{k}$

Here, Direction ratios are 3, -4. -6.

31. Find the direction cosines of the vector $\vec{a} = (\hat{i} + 2\hat{j} + 3\hat{k})$

Sol. Given:- $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Direction of $\vec{a}$ are 1, 2, 3

Direction cosines of $\vec{a}$ are,

$$\ell = \frac{1}{\sqrt{1^2+2^2+3^2}} = \frac{1}{\sqrt{14}}$$

$$m = \frac{2}{\sqrt{1^2+2^2+3^2}} = \frac{2}{\sqrt{14}}$$

and, $n = \frac{3}{\sqrt{1^2+2^2+3^2}} = \frac{3}{\sqrt{14}}$

32. If $\hat{a}$ and $\hat{b}$ are unit vectors such that $\left(\hat{a}+\hat{b}\right)$ is a unit vector, what is the angle between $\hat{a}$ and $\hat{b}$?

Sol. $\because \left(\hat{a}+\hat{b}\right)$ is a unit vector.

$$\Rightarrow |\hat{a}+\hat{b}| = 1 \Rightarrow |\hat{a}+\hat{b}|^2 = 1 \Rightarrow |\hat{a}|^2 + |\hat{b}|^2 + 2|\hat{a}||\hat{b}|\cos\theta = 1$$

$$\Rightarrow 1 + 1 + 2\cos\theta = 1 \quad \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 2\cdot\frac{\pi}{2}$$