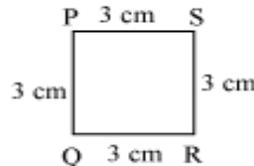
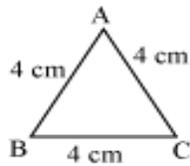


Perimeter and Area

Perimeter of Regular Shapes

Look at the following figures.



What do we observe in these figures?

It can be observed that all the sides of the given figures are equal. These types of figures are known as **regular polygons**.

The first, second, and third figures are the figures of an equilateral triangle, a square, and a regular pentagon respectively.

How can we find the perimeter of a regular polygon?

Let us now discuss some examples based on the perimeter of a regular polygon to understand this concept better.

Example 1:

What is the perimeter of a regular hexagon with side 15 cm?

Solution:

Number of sides of a regular hexagon = 6

Length of each side of the regular hexagon = 15 cm

Perimeter of a regular hexagon = Number of sides of the hexagon \times Length of a side of the hexagon

\therefore Perimeter of the regular hexagon = (6×15) cm = 90 cm

Example 2:

A piece of string, which is 120 cm long, is used to form a regular pentagon. What will be the length of each side of this regular pentagon?

Solution:

Number of sides of a regular pentagon = 5

A piece of string 120 cm long is used to form a regular pentagon

i.e., perimeter of the regular pentagon = 120 cm

However, perimeter of a regular polygon = Number of sides of the polygon \times Length of a side of the polygon

$$\therefore \text{Length of a side of a pentagon} = \frac{\text{Perimeter of the pentagon}}{\text{Number of sides of the pentagon}}$$

$$= \left(\frac{120}{5}\right) \text{ cm}$$
$$= 24 \text{ cm}$$

Thus, the length of each side of the regular pentagon is 24 cm.

Example 3:

Find the cost of fencing a square park of side 375 m at the rate of Rs 16 per metre.

Solution:

Length of one side of square = 375 m

To find the cost of fencing around the square park, we have to find the perimeter of the square park.

Perimeter of the square park = 4 \times Length of a side

$$= (4 \times 375) \text{ m}$$

$$= 1500 \text{ m}$$

It is given that the cost of fencing 1 m of the park is Rs 16.

Therefore, cost of fencing 1500 m = Rs (1500 \times 16) = Rs 24000

Thus, the cost of fencing around the square park is Rs 24000.

Perimeter of Rectangle

Rectangle is a quadrilateral with opposite sides equal. Let us try to find the general formula for perimeter of any rectangle with given length and breadth with the help of an example.

Let us discuss some more examples based on the perimeter of a rectangle.

Example 1:

Find the cost of fencing a rectangular park of length 217 m and breadth 183 m at the rate of Rs 12.50 per metre.

Solution:

Length of the rectangle = 217 m

Breadth of the rectangle = 183 m

To find the cost of fencing the rectangular park, we have to find out the perimeter of the rectangular park.

Now, perimeter of the rectangular park = $2 \times (\text{length} + \text{breadth})$

$$= 2 \times (217 \text{ m} + 183 \text{ m})$$

$$= 2 \times (400 \text{ m})$$

$$= 800 \text{ m}$$

Cost of fencing = Rs 12.50 per metre

$$\therefore \text{Cost of fencing } 800 \text{ m} = \text{Rs } (12.50 \times 800) = \text{Rs } 10000$$

Thus, the cost of fencing the whole rectangular park is Rs 10000.

Example 2:

The lid of a rectangular box of size 60 cm by 20 cm is sealed all around with tape. Find the length of the tape required.

Solution:

Length of the rectangular box = 60 cm

Breadth of the rectangular box = 20 cm

The rectangular box is sealed all around with tape i.e., the tape covers the boundary of the rectangular box.

∴ Length of tape required = Perimeter of the rectangular box

$$= 2 \times (\text{Length} + \text{Breadth})$$

$$= 2 \times (60 \text{ cm} + 20 \text{ cm})$$

$$= 2 \times (80 \text{ cm})$$

$$= 160 \text{ cm}$$

Example 3:

A rectangular piece of land measures 0.75 km by 0.5 km. Each side is to be fenced with 6 rows of wires. Find the length of the wire required.

Solution:

Length of the rectangular land = 0.75 km

Breadth of the rectangular land = 0.5 km

It is given that each side of the land is to be fenced with 6 rows of wires.

Therefore, the total length of wire required for fencing is 6 times the perimeter of the land.

$$\text{Perimeter of the rectangular land} = 2 \times (\text{Length} + \text{Breadth})$$

$$= 2 \times (0.75 \text{ km} + 0.5 \text{ km})$$

$$= 2 \times (1.25 \text{ km})$$

$$= 2.5 \text{ km}$$

$$\therefore \text{Length of wire required} = (6 \times 2.5 \text{ km}) = 15 \text{ km}$$

Example 4:

Chulbul takes 10 rounds of a rectangular park, which is 65 m long and 35 m wide. Find the total distance covered by her.

Solution:

Length of the rectangular park = 65 m

Breadth of the rectangular park = 35 m

Total distance covered by Chulbul in one round is the perimeter of the rectangular park.

Perimeter of the rectangular park = $2 \times (\text{Length} + \text{Breadth})$

$$= 2 \times (65 \text{ m} + 35 \text{ m})$$

$$= 2 \times (100 \text{ m})$$

$$= 200 \text{ m}$$

Therefore, total distance covered by Chulbul in 1 round = 200 m

\therefore Distance covered by Chulbul in 10 rounds = $(10 \times 200 \text{ m}) = 2000 \text{ m}$

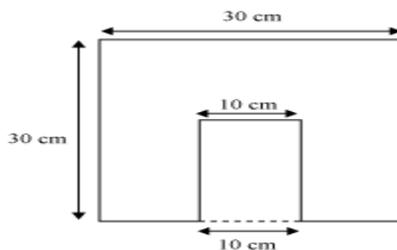
However, $1 \text{ m} = \left(\frac{1}{1000}\right) \text{ km}$

$$\therefore 2000 \text{ m} = \left(2000 \times \frac{1}{1000}\right) \text{ km} = 2 \text{ km}$$

Thus, the distance covered by Chulbul is 2 km.

Example 5:

A rectangular portion is cut off from one side of a square sheet of paper as shown in the figure. If the length of the rectangular portion is 15 cm, then what will be the difference between the perimeters of the sheet of paper before and after cutting off the rectangular portion?



Solution:

Original perimeter of the sheet of paper = $4 \times 30 \text{ cm} = 120 \text{ cm}$

Length of the rectangular portion that is cut off = 15 cm

Breadth of the rectangular portion that is cut off = 10 cm

After cutting off the rectangular portion, three sides of the sheet of paper remain the same, but one side gets changed.

Thus, perimeter of the sheet of paper after the rectangular portion is cut off

$$= 30 \text{ cm} + 30 \text{ cm} + 30 \text{ cm} + (30 - 10) \text{ cm} + 15 \text{ cm} + 10 \text{ cm} + 15 \text{ cm}$$

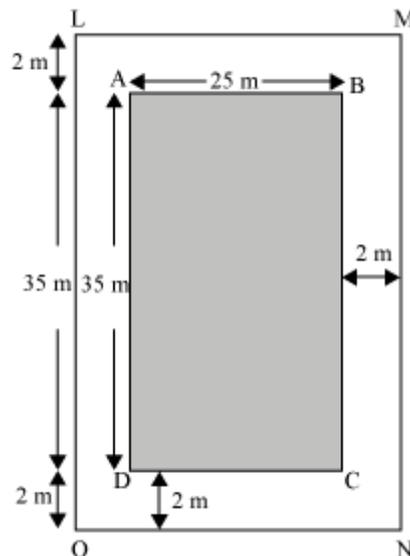
$$= 30 \text{ cm} + 30 \text{ cm} + 30 \text{ cm} + 20 \text{ cm} + 15 \text{ cm} + 10 \text{ cm} + 15 \text{ cm}$$

$$= 150 \text{ cm}$$

Thus, difference between the two perimeters = $150 \text{ cm} - 120 \text{ cm} = 30 \text{ cm}$

Example 6:

What is the difference between the perimeters of rectangles ABCD and LMNO?



Solution:

Length of rectangle ABCD = 35 m

Breadth of rectangle ABCD = 25 m

∴ Perimeter of rectangle ABCD = $2(35\text{ m} + 25\text{ m}) = 2 \times 60\text{ m} = 120\text{ m}$

Length of rectangle LMNO = $2\text{ m} + 35\text{ m} + 2\text{ m} = 39\text{ m}$

Breadth of rectangle LMNO = $2\text{ m} + 25\text{ m} + 2\text{ m} = 29\text{ m}$

∴ Perimeter of rectangle LMNO = $2(39\text{ m} + 29\text{ m}) = 2 \times 68\text{ m} = 136\text{ m}$

∴ Difference between the perimeters of the two rectangles = $136\text{ m} - 120\text{ m} = 16\text{ m}$

Example 7:

The perimeters of a square and a rectangle are equal. If the length of the rectangle is 9 cm and its breadth is 3 cm less than its length, then what is the length of each side of the square?

Solution:

Length (l) of the rectangle = 9 cm

Thus, breadth (b) of the rectangle = $(9 - 3)\text{ cm} = 6\text{ cm}$

∴ Perimeter of the rectangle = $2 \times (l + b) = 2 \times (9 + 6)\text{ cm} = 2 \times 15\text{ cm} = 30\text{ cm}$

It is given that the perimeters of the square and the rectangle are equal.

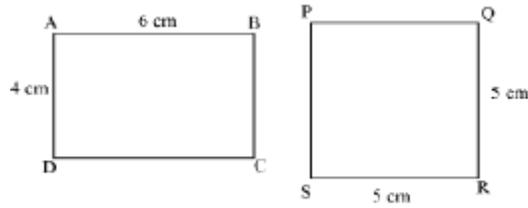
Therefore, perimeter of the square = $4 \times \text{length of the square} = 30\text{ cm}$

Thus, length of each side of the square = $\frac{30\text{ cm}}{4} = 7.5\text{ cm}$

Area of Rectangle and Square

We usually come across the situations in our life when we need to find the area of various types of things such as area of a piece of land, area of wall to be painted, area of cloth required etc. The most common shapes that we see in our life are square and rectangle and thus, it becomes necessary for us to learn how to find their area.

Look at the figures given below.



Here, the first figure, i.e. ABCD, is a rectangle of length 6 cm and breadth 4 cm whereas the second figure, i.e. PQRS, is a square of side 5 cm.

Can we find which of the two shapes has the greater area?

It is difficult to answer this question by merely looking at the figures. To find the area of a rectangle or a square, we have to know the formula for each of them.

If the measure of the diagonal of the square is known, then its area can be calculated using the following formula.

$$\text{Area of square} = \frac{(\text{Diagonal})^2}{2}$$

Now, let us consider a real life situation to understand the concept better.

The owner of a paddy field decides to construct a 3 m wide path outside the field along its boundary. What will be the cost of constructing the path at the rate of Rs 500 per m²?

Now, let us discuss how to convert units of area.

As, we have already studied about the conversion of units for length and they are as follows:

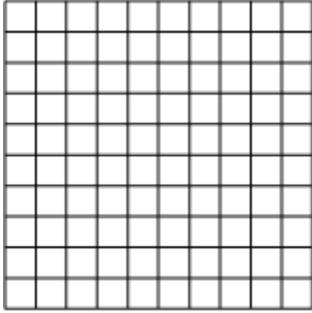
1 centimetre = 10 millimetres

1 metre = 100 centimetres

1 kilometre = 1000 metres

In the same manner, we can convert the unites of areas as well.

Let us consider a square of side 1 cm and divide that square into 100 small squares, each of side 1 mm.



It is evident from the figure that area of a square of side 1 cm will be equal to the areas of 100 small squares of side 1 mm.

$$\Rightarrow 1 \text{ cm}^2 = 100 \times 1 \text{ mm}^2$$

$$\Rightarrow 1 \text{ cm}^2 = 100 \text{ mm}^2$$

Similarly, we can say that $1 \text{ m}^2 = 1 \text{ m} \times 1 \text{ m} = 100 \text{ cm} \times 100 \text{ cm} = 10000$

Now, to convert 1 km^2 into m^2 , we will proceed as follows:

$$1 \text{ km}^2 = 1 \text{ km} \times 1 \text{ km} = 1000 \text{ m} \times 1000 \text{ m} = 1000000 \text{ m}^2.$$

It is quite significant to note that when we convert a unit of area to a smaller unit, the resulting number of units will be bigger.

The areas of land are usually measured in hectares where 1 hectare = Area of a square of side 100 m.

$$\text{i.e. } 1 \text{ hectare} = 100 \text{ m} \times 100 \text{ m} = 10000 \text{ m}^2.$$

Let us have a look at some more examples to be clear with the concept.

Example 1:

A paddy field is in the form of a square with length 100 m. Find the area of the field.

Solution:

It is given that the length of a side of the square paddy field is 100 m.

$$\therefore \text{Area of the paddy field} = (\text{Side})^2$$

$$= (100 \text{ m})^2$$

$$= 10000 \text{ m}^2$$

Example 2:

A square park is 300 m long diagonally. What is the area of the park?

Solution:

Length of diagonal of square park = 300 m

$$\text{Area of square park} = \frac{(\text{Diagonal})^2}{2}$$

$$\Rightarrow \text{Area of square park} = \frac{(300)^2}{2} \text{ m}^2$$

$$\Rightarrow \text{Area of square park} = \frac{90000}{2} \text{ m}^2$$

$$\Rightarrow \text{Area of square park} = 45000 \text{ m}^2$$

Example 3:

All the four walls of a room have to be painted. If all the four walls have the equal length of 4 m and equal breadth of 3 m, then find the total cost of painting the walls at the rate of Rs 7 per m².

Solution:

Length of a wall = 4 m

Breadth of a wall = 3 m

Now, area of one wall = length × breadth

$$= 4\text{m} \times 3\text{m}$$

$$= 12 \text{ m}^2$$

$$\therefore \text{Total area of the four walls} = 4 \times 12 \text{ m}^2$$

$$= 48 \text{ m}^2$$

Cost of painting the walls = Rs 7 per m².

$$\therefore \text{Total cost of painting the four walls} = \text{Rs } (7 \times 48)$$

$$= \text{Rs } 336$$

Example 4:

A 90 cm long wire is bent into a rectangle of length 30 cm and an 80 cm long wire is bent in the form of a square. Which encloses more area – the rectangle or the square, and by how much?

Solution:

Perimeter of the rectangle = Length of the wire = 90 cm

Length of the rectangle = 30 cm

Let the breadth of the rectangle be b .

$$\therefore \text{Perimeter of the rectangle} = 90 \text{ cm} = 2(30 \text{ cm} + b)$$

$$\Rightarrow 90 \text{ cm} = 2 \times 30 \text{ cm} + 2 \times b$$

$$\Rightarrow 90 \text{ cm} = 60 \text{ cm} + 2b$$

$$\Rightarrow 2b = 90 \text{ cm} - 60 \text{ cm} = 30 \text{ cm}$$

$$\Rightarrow b = \frac{30 \text{ cm}}{2} = 15 \text{ cm}$$

Thus, area enclosed by the rectangle = length \times breadth = 30 cm \times 15 cm = 450 cm²

Perimeter of the square = Length of the wire = 80 cm

Let the length of the square be l .

$$\therefore \text{Perimeter of the square} = 80 \text{ cm} = 4l$$

$$\Rightarrow l = \frac{80 \text{ cm}}{4} = 20 \text{ cm}$$

Thus, area enclosed by the square = side \times side = 20 cm \times 20 cm = 400 cm²

Therefore, the rectangle encloses 450 cm² – 400 cm² = 50 cm² more area than the square.

Example 5:

From a rectangular sheet of paper of 30 cm length and 600 cm² area, the biggest possible square is cut out. What is the area of the sheet of paper left?

Solution:

Length of the rectangular sheet of paper = 30 cm

Let the width of the rectangular sheet of paper be w .

Area of the sheet of paper = $600 \text{ cm}^2 = 30 \text{ cm} \times w$

$$\Rightarrow w = \frac{600 \text{ cm}^2}{30 \text{ cm}} = 20 \text{ cm}$$

\therefore Width of the sheet = 20 cm

The biggest possible square that can be cut off from this sheet has each side of length equal to the width of the square i.e., 20 cm.

Thus, area of the square that is cut off = $20 \text{ cm} \times 20 \text{ cm} = 400 \text{ cm}^2$

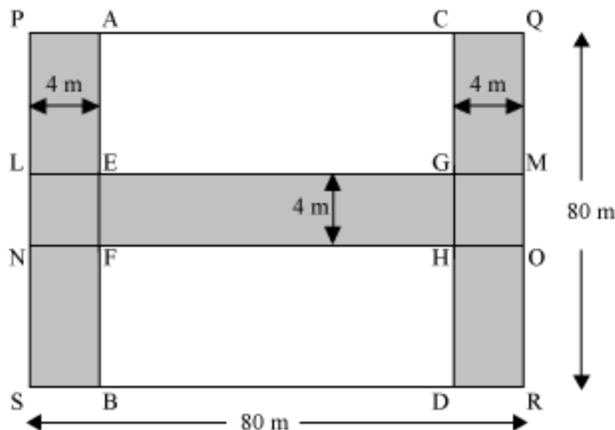
Thus, area of the sheet left = Original area of the sheet – Area of the square that is cut off

$$= 600 \text{ cm}^2 - 400 \text{ cm}^2$$

$$= 200 \text{ cm}^2$$

Example 6:

Two jogging tracks, each of width 4 m, run along the two opposite sides inside a park. Another jogging track runs through the centre of the park and intersects the two tracks perpendicularly. What is the total area of the tracks?



Solution:

In the given figure, the shaded portion represents the jogging tracks.

Now, area of the roads

= area of PABS + area of CQRD + area of LMON – Area of LEFN – area of GMOH

= $(80 \text{ m} \times 4 \text{ m}) + (80 \text{ m} \times 4 \text{ m}) + (80 \text{ m} \times 4 \text{ m}) - (4 \text{ m} \times 4 \text{ m}) - (4 \text{ m} \times 4 \text{ m})$

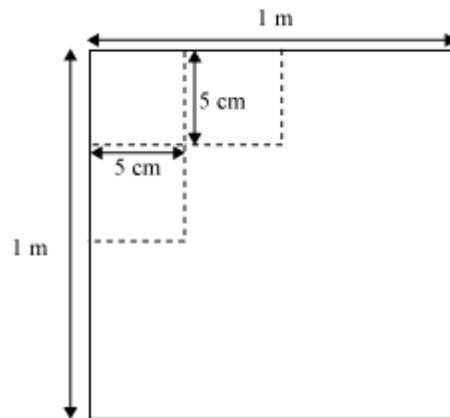
= $320 \text{ m}^2 + 320 \text{ m}^2 + 320 \text{ m}^2 - 16 \text{ m}^2 - 16 \text{ m}^2$

= $960 \text{ m}^2 - 32 \text{ m}^2$

= 928 m^2

Example 7:

Raju was given a sheet of paper having all the sides of length 1 m. He was asked to draw squares with side 5 cm on the given sheet without leaving any space. How many squares can be drawn?



Solution:

Given, length of the sheet = 1 m = 100 cm

The length of all sides of the sheet is the same i.e., the sheet is in the form of a square.

Therefore, area of the sheet = $(\text{side})^2$

= $(100 \text{ cm})^2$

= 10000 cm^2

Now, side of each square to be drawn = 5 cm

$$\therefore \text{Area of one square} = (\text{Side})^2$$

$$= (5 \text{ cm})^2$$

$$= 25 \text{ cm}^2$$

Number of squares that can be drawn on the sheet

$$\begin{aligned} &= \frac{\text{Area of the sheet}}{\text{Area of one square}} \\ &= \frac{10000}{25} = 400 \end{aligned}$$

Thus, 400 squares can be drawn on the sheet.

Example 8:

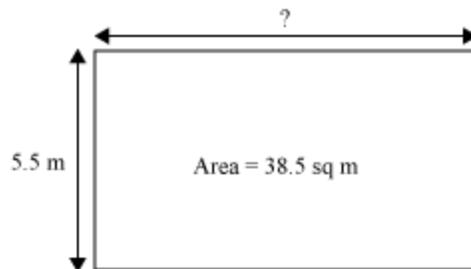
The area of a rectangular pond is 38.5 m^2 . If the breadth of the pond is 5.5 m , then find the length of the pond.

Solution:

Given, area of the pond = 38.5 m^2

Breadth of the pond = 5.5 m

We have to find the length of the pond.



Since the pond is rectangular in shape,

$$\text{Length} \times \text{Breadth} = \text{Area of the pond}$$

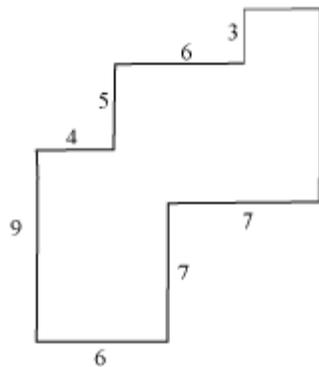
$$\Rightarrow \text{Length} \times 5.5 \text{ m} = 38.5 \text{ m}^2$$

$$\Rightarrow \text{Length} = \left(\frac{38.5}{5.5} \right) \text{m} = 7 \text{ m}$$

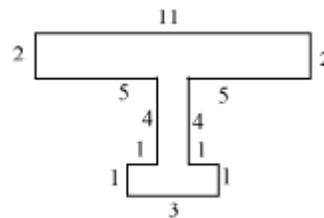
Thus, the length of the pond is 7 m.

Example 9:

By splitting the following figures into rectangles, find their areas. (The measures are given in centimetres)



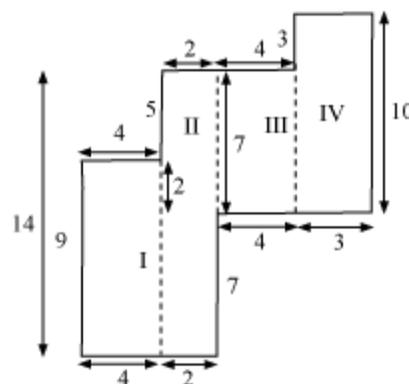
(a)



(b)

Solution:

(a) By splitting the given figure into rectangles, we will obtain the following figure.



For rectangle I,

Length = 4 cm and breadth = 9 cm

$$\therefore \text{Area of rectangle I} = \text{Length} \times \text{Breadth} = (4 \times 9) \text{ sq cm} = 36 \text{ cm}^2$$

For rectangle II,

Length = 14 cm and breadth = 2 cm

$$\therefore \text{Area of rectangle II} = \text{Length} \times \text{Breadth} = (14 \times 2) \text{ sq cm} = 28 \text{ cm}^2$$

For rectangle III,

Length = 7 cm and breadth = 4 cm

$$\therefore \text{Area of rectangle III} = \text{Length} \times \text{Breadth} = (7 \times 4) \text{ sq cm} = 28 \text{ cm}^2$$

For rectangle IV,

Length = 10 cm and breadth = 3 cm

$$\therefore \text{Area of rectangle IV} = \text{Length} \times \text{Breadth} = (10 \times 3) \text{ sq cm} = 30 \text{ cm}^2$$

$$\therefore \text{Area of the given figure} = \text{Area of rectangle I} + \text{Area of rectangle II} +$$

Area of rectangle III + Area of rectangle IV

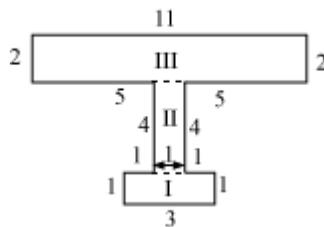
$$= 36 \text{ cm}^2 + 28 \text{ cm}^2 + 28 \text{ cm}^2 + 30 \text{ cm}^2$$

$$= (36 + 28 + 28 + 30) \text{ cm}^2$$

$$= 122 \text{ cm}^2$$

Thus, the area of the given figure is 122 cm².

(b) By splitting the given figure into rectangles, we will obtain the following figure.



For rectangle I,

Length = 3 cm and breadth = 1 cm

$$\therefore \text{Area of rectangle I} = \text{Length} \times \text{Breadth} = (3 \times 1) \text{ cm}^2 = 3 \text{ cm}^2$$

For rectangle II,

Length = 4 cm and breadth = 1 cm

$$\therefore \text{Area of rectangle II} = \text{Length} \times \text{Breadth} = (4 \times 1) \text{ cm}^2 = 4 \text{ cm}^2$$

For rectangle III,

Length = 11 cm and breadth = 2 cm

$$\therefore \text{Area of rectangle III} = \text{Length} \times \text{Breadth} = (11 \times 2) \text{ cm}^2 = 22 \text{ cm}^2$$

$$\therefore \text{Area of the given figure} = \text{Area of rectangle I} + \text{Area of rectangle II} +$$

Area of rectangle III

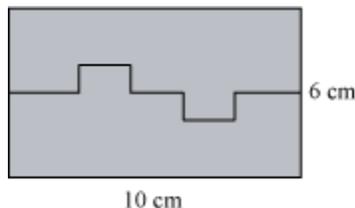
$$= (3 + 4 + 22) \text{ cm}^2$$

$$= 29 \text{ cm}^2$$

Thus, the area of the given figure is 29 cm².

Areas of Congruent Parts of Rectangles

Consider the following figure.



In this figure, we can observe that the rectangle is composed of two congruent parts.

Can we find the areas of these congruent parts?

Each part is a polygon and we do not know the formula to find the areas of such polygons. However, if we notice then, we can see that the area of each part is equal to the half of the area of rectangle.

We know that the area of rectangle is length \times breadth.

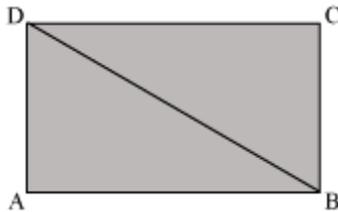
In the given figure, two such parts form a rectangle.

$$\therefore \text{Area of each part} = \frac{1}{2} \times \text{Area of rectangle}$$

$$\begin{aligned} &= \frac{1}{2} \times l \times b \\ &= \frac{1}{2} \times 10 \times 6 \\ &= 30 \text{ sq. cm} \end{aligned}$$

Thus, the area of each part is 30 sq. cm.

In the same way, each diagonal of a rectangle divides the rectangle into two congruent triangles.



We can find the area of each triangle as follows.

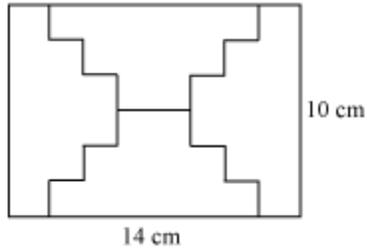
$$\text{Area of } \triangle ABD \text{ (or } \triangle BCD) = \frac{1}{2} \times \text{Area of rectangle} = \frac{1}{2} \times \text{length} \times \text{breadth}$$

It is also the formula to find the area of triangle.

Let us solve some more examples to understand the concept better.

Example 1:

The following rectangle is composed of congruent polygons. Find the area of each polygon.



Solution:

The given rectangle is composed of four congruent polygons.

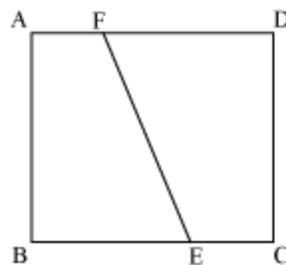
Now, the area of each polygon $= \frac{1}{4} \times \text{Area of rectangle}$

$$\begin{aligned}
 &= \frac{1}{4} \times l \times b \\
 &= \frac{1}{4} \times 14 \times 10 \\
 &= 35 \text{ sq. cm}
 \end{aligned}$$

Thus, the area of each polygon is 35 sq. cm.

Example 2:

In the given figure, the square ABCD is composed of two congruent trapeziums. Find the area of trapezium ABEF, if the side of the square is 8 cm.



Solution:

In the given figure, the square ABCD is composed of two congruent trapeziums ABEF and CDFE.

$$\therefore \text{Area of trapezium ABEF} = \frac{1}{2} \times \text{Area of square ABCD}$$

$$\begin{aligned}
&= \frac{1}{2} \times (8)^2 \\
&= \frac{1}{2} \times 64 \\
&= 32 \text{ sq. cm}
\end{aligned}$$

Thus, the area of trapezium ABEF is 32 sq. cm.

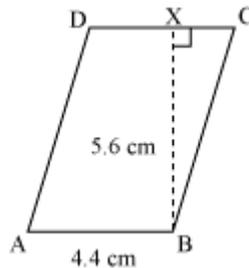
Areas Of Parallelograms

Ramesh and his brother Suresh once got into an argument. Suresh said that the area of a parallelogram is the product of its base and height while Ramesh said that this formula is for the area of a rectangle. Who do you think is right?

Ramesh, of course, is right as the area of a rectangle is the product of its base and height. However, this formula also represents the formula for the area of a parallelogram. Let us consider the following parallelogram. Can we convert it into a rectangle?

Thus, area of a parallelogram = base \times height

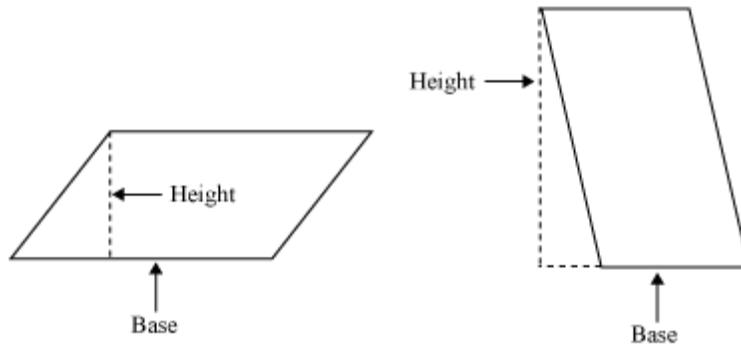
Using this formula, let us calculate the area of parallelogram ABCD drawn below.



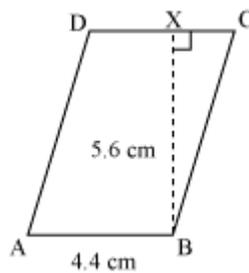
Here, AB is the base of the parallelogram and its corresponding height is BX.

Thus, area of the parallelogram = $AB \times BX = 4.4 \text{ cm} \times 5.6 \text{ cm} = 24.64 \text{ cm}^2$

A point to note is that any side of a parallelogram can be taken as a base and the perpendicular drawn to the base from the opposite vertex is the height of the parallelogram.



In the example cited above, let us draw a perpendicular from vertex B to side CD.



Now, we can take the base of the parallelogram as CD and its corresponding height as BX.

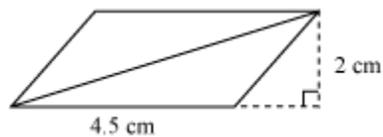
Thus, the area of the parallelogram is also equal to the product of base CD and its corresponding height BX.

Let us solve some examples to understand the concept better.

Solved examples

Example 1:

What is the area of the following parallelogram?



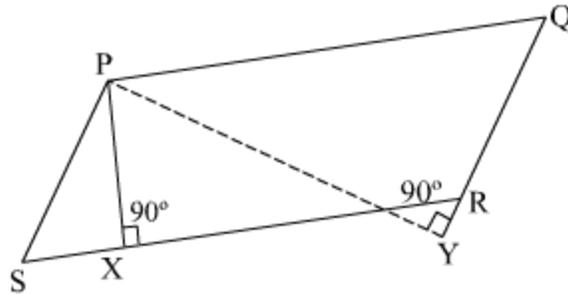
Solution:

Base of the parallelogram = 4.5 cm

Height of the parallelogram = 2 cm

Therefore, area of PQRS = height \times base = 4.5 cm \times 2 cm = 9 cm²

Example 2:



PQRS is a parallelogram.

The lengths of \overline{SR} and \overline{PX} are 7 cm and 2.5 cm respectively. If the length of \overline{PY} is 5 cm, then what will be the length of \overline{RQ} ?

Solution:

It is given that $\overline{SR} = 7$ cm and $\overline{PX} = 2.5$ cm.

Therefore, area of parallelogram PQRS = height \times base = $\overline{SR} \times \overline{PX} = 7 \times 2.5$ cm² = 17.5 cm²

If we choose \overline{RQ} as the base, then its corresponding height will be \overline{PY} .

Now, height \times base = area of parallelogram PQRS

$$\Rightarrow \overline{PY} \times \overline{RQ} = \text{area of PQRS}$$

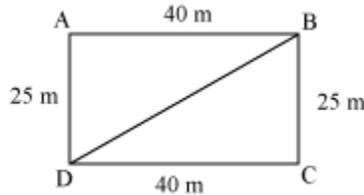
$$\Rightarrow 5 \text{ cm} \times \overline{RQ} = 17.5 \text{ cm}^2$$

$$\Rightarrow \overline{RQ} = \frac{17.5}{5} \text{ cm} = 3.5 \text{ cm}$$

Areas Of Triangles

A gardener wanted to grow roses and tulips in a garden. The garden was rectangular in shape and had a length of 40 m and breadth of 25 m. The gardener wanted to dedicate equal areas to grow these two types of flowers. He did not know anything about geometry, but even then, he thought that if he divided the garden diagonally, then it would be divided into two equal parts. Now, he could grow roses in one part of the garden and tulips in the other part. **Was he correct?**

Let ABCD be the garden.



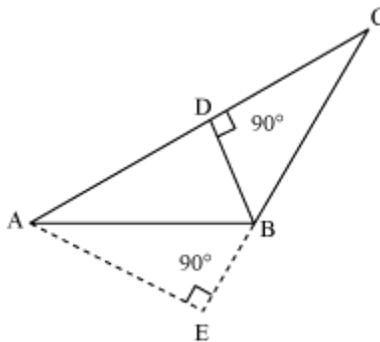
If it is divided along the diagonal BD, then two triangles ABD and BCD are formed.

Thus, the formula for the area of a triangle is:

$$\text{Area of a triangle} = \frac{1}{2} \times (\text{Base} \times \text{Height})$$

To use the formula, any side of a triangle can be taken as the base and the perpendicular drawn to the base from the opposite vertex is the corresponding height of the triangle.

Let us look at $\triangle ABC$ drawn below. Side AC measures 10 cm and side BC measures 4 cm. AE is the perpendicular from vertex A to side BC and measures 8 cm.



Now, how do we find the area of $\triangle ABC$? Let us choose side BC as the base of $\triangle ABC$. Then, segment AE will be its corresponding height.

Therefore, area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height} = \left(\frac{1}{2} \times 4 \times 8\right) \text{ cm}^2 = 16 \text{ cm}^2$

That was easy! Now, can we also find the length of the perpendicular BD?

Note that if we choose side AC as the base of ΔABC , then segment BD will be its corresponding height.

Thus, the area of ΔABC can also be expressed as $\frac{1}{2} \times AC \times BD$.

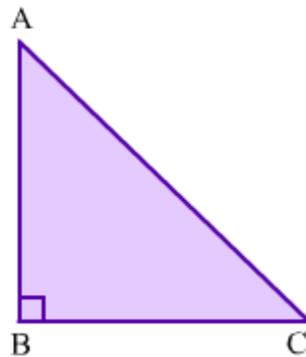
$$\Rightarrow \frac{1}{2} \times 10 \text{ cm} \times BD = 16 \text{ cm}^2$$

$$\Rightarrow BD = \frac{16 \times 2}{10} \text{ cm}$$

$$\Rightarrow BD = 3.2 \text{ cm}$$

Area of the right angled triangle:

Observe the right angled triangle given below:



It can be seen that ΔABC is right angled at B. So, side AB is perpendicular to side BC.

Thus, in ΔABC , side AB is the altitude or height and side BC is the base.

Now,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times (\text{Base} \times \text{Height})$$

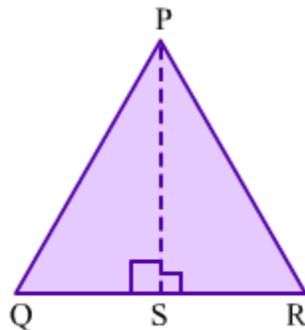
$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2} \times (BC \times AB)$$

So, it can be concluded that:

Area of a right angled triangle = $\frac{1}{2}$ × Product of the lengths of the sides making right angle

Area of the equilateral triangle:

Let us take an equilateral triangle, say ΔPQR , such that $PQ = QR = RP = a$.



If we fold this triangle along side QR such that point Q coincides with point R, we will get two congruent triangles such as ΔPQS and ΔPRS .

In ΔPQS and ΔPRS , we obtain

$$QS = RS = \frac{a}{2} \text{ and}$$

$$\angle PSQ = \angle PSR = 90^\circ$$

By applying Pythagoras theorem on ΔPQS , we obtain

$$(PQ)^2 = (QS)^2 + (SP)^2$$

$$\Rightarrow a^2 = \left(\frac{a}{2}\right)^2 + (SP)^2$$

$$\Rightarrow (SP)^2 = a^2 - \frac{a^2}{4}$$

$$\Rightarrow (SP)^2 = \frac{3}{4}a^2$$

$$\Rightarrow SP = \frac{\sqrt{3}}{2}a$$

Now,

$$\text{Area of } \triangle PQR = \frac{1}{2} \times (\text{Base} \times \text{Height})$$

$$\text{Area of } \triangle PQR = \frac{1}{2} \times QR \times SP$$

$$\Rightarrow \text{Area of } \triangle PQR = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a$$

$$\Rightarrow \text{Area of } \triangle PQR = \frac{\sqrt{3}}{4}a^2$$

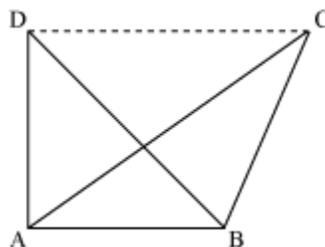
Thus, it can be concluded that

$$\text{Area of equilateral triangle} = \frac{\sqrt{3}}{4} \times (\text{Side})^2$$

One important point to remember about congruent triangles and their area is as follows.

We know that congruent figures are exactly alike. Thus, the areas of two congruent triangles will always be equal. However, its converse is not true. This means that two triangles, which have the same area, need not be congruent.

For example, look at triangles ABC and ABD drawn below. Both share the same base AB. Their height is also the same. Thus, the two triangles have the same area. However, it is clear from the figure that the two triangles are not congruent. Thus, this proves the fact that two triangles having the same area need not be congruent.



Let us solve some examples using the above concepts.

Solved examples

Example 1:

Find the area of the triangle, which has a base of length 9 cm and a height of 11 cm.

Solution:

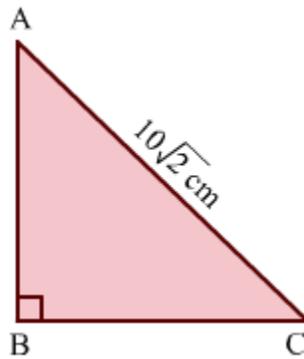
$$\text{Area of triangle} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 9 \text{ cm} \times 11 \text{ cm} = 49.5 \text{ cm}^2$$

Example 2:

Find the area of the isosceles right angled triangle whose hypotenuse measures $10\sqrt{2}$ cm.

Solution:

Let ΔABC be the isosceles right angled triangle right angled at B whose hypotenuse measures $10\sqrt{2}$ cm.



In ΔABC , CA is the hypotenuse and $AB = BC$.

By applying Pythagoras theorem on ΔABC , we obtain

$$\begin{aligned} (CA)^2 &= (AB)^2 + (BC)^2 \\ (10\sqrt{2})^2 &= (AB)^2 + (AB)^2 && (\text{As } AB = BC) \\ \Rightarrow (10\sqrt{2})^2 &= 2(AB)^2 \\ \Rightarrow 2(AB)^2 &= 200 \\ \Rightarrow (AB)^2 &= 100 \\ \Rightarrow AB &= 10 \\ \therefore AB = BC &= 10 \text{ cm} \end{aligned}$$

Now,

Area of a right angled triangle = $\frac{1}{2} \times$ Product of the lengths of the sides making right angle

$$\Rightarrow \text{Area of } \triangle ABC = \frac{1}{2} \times AB \times BC$$

$$\Rightarrow \text{Area of } \triangle ABC = \left(\frac{1}{2} \times 10 \times 10\right) \text{ cm}^2 \quad (\text{AB} = \text{BC} = 10 \text{ cm})$$

$$\Rightarrow \text{Area of } \triangle ABC = 50 \text{ cm}^2$$

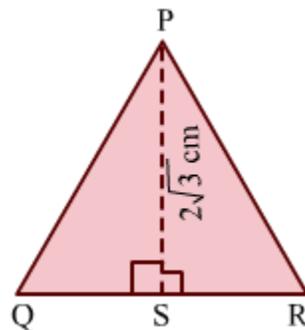
Thus, the area of the required triangle is 50 cm^2 .

Example 3:

Find the area of the equilateral triangle whose height is $2\sqrt{3}$ cm.

Solution:

Let $\triangle PQR$ be the equilateral triangle whose height SP is $2\sqrt{3}$ cm.



In $\triangle PQR$, we have

$$PQ = QR = RP,$$

$$QS = SR = \frac{QR}{2} \text{ and}$$

$$\angle PSQ = \angle PSR = 90^\circ$$

By applying Pythagoras theorem on $\triangle PQS$, we obtain

$$(PQ)^2 = (QS)^2 + (SP)^2$$

$$\Rightarrow (QR)^2 = \left(\frac{QR}{2}\right)^2 + (2\sqrt{3})^2 \quad \left(PQ = QR, QS = \frac{QR}{2} \text{ and } SP = 2\sqrt{3} \text{ cm}\right)$$

$$\Rightarrow (QR)^2 = \frac{(QR)^2}{4} + 12$$

$$\Rightarrow \frac{3}{4}(QR)^2 = 12$$

$$\Rightarrow (QR)^2 = 16$$

$$\Rightarrow QR = 4$$

Thus, $PQ = QR = RP = 4 \text{ cm}$

Now,

Area of equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{Side})^2$

$$\text{Area of } \Delta PQR = \left(\frac{\sqrt{3}}{4} \times 4^2\right) \text{ cm}^2$$

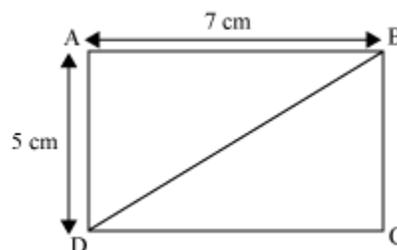
$$\Rightarrow \text{Area of } \Delta PQR = 4\sqrt{3} \text{ cm}^2$$

Thus, the area of the required triangle is $4\sqrt{3} \text{ cm}^2$.

Example 4:

ABCD is a rectangle of length 7 cm and breadth 5 cm. What is the area of ΔABD ?

Solution:



If a rectangle is divided diagonally, then the area of each triangle so obtained equals one-half the area of the rectangle.

$$\therefore \text{Area of } \triangle ABD = \frac{1}{2} \times \text{Area of rectangle ABCD}$$

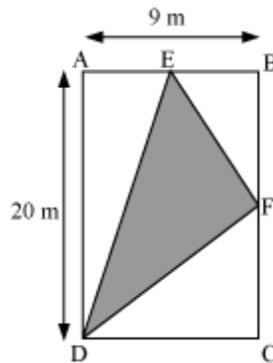
$$= \frac{1}{2} \times \text{Length} \times \text{Width}$$

$$= \frac{1}{2} \times 7 \text{ cm} \times 5 \text{ cm}$$

$$= 17.5 \text{ cm}^2$$

Example 5:

The given figure shows a rectangle ABCD. E and F are the mid-points of sides AB and BC respectively. Find the area of the shaded portion in the given figure.



Solution:

Here, area of the shaded part = area of rectangle ABCD – (area of $\triangle ADE$ + area of $\triangle BEF$ + area of $\triangle CDF$)

Length AD of rectangle ABCD = 20 m

Width AB of rectangle ABCD = 9 m

$$\therefore \text{Area of rectangle ABCD} = 9 \text{ m} \times 20 \text{ m} = 180 \text{ m}^2$$

E is the mid-point of side AB.

$$\therefore AE = EB = \frac{9}{2} \text{ m} = 4.5 \text{ m}$$

Also, F is the mid-point of side BC.

$$\therefore BF = FC = \frac{20}{2} \text{ m} = 10 \text{ m}$$

In $\triangle ADE$, length of base $AD = 20 \text{ m}$

Length of corresponding height $AE = 4.5 \text{ m}$

$$\therefore \text{Area of } \triangle ADE = \frac{1}{2} \times 20 \text{ m} \times 4.5 \text{ m} = 45 \text{ m}^2$$

In $\triangle BEF$, length of base $BF = 10 \text{ m}$

Length of corresponding height $BE = 4.5 \text{ m}$

$$\therefore \text{Area of } \triangle BEF = \frac{1}{2} \times 10 \text{ m} \times 4.5 \text{ m} = 22.5 \text{ m}^2$$

In $\triangle CDF$, length of base $CD = 9 \text{ m}$

Length of corresponding height $CF = 10 \text{ m}$

$$\therefore \text{Area of } \triangle CDF = \frac{1}{2} \times 9 \text{ m} \times 10 \text{ m} = 45 \text{ m}^2$$

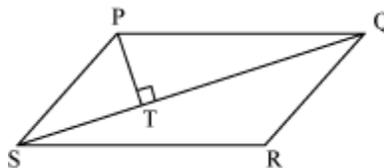
Thus, area of the shaded portion = $[180 - (45 + 22.5 + 45)] \text{ m}^2$

$$= (180 - 112.5) \text{ m}^2$$

$$= 67.5 \text{ m}^2$$

Example 6:

The given figure shows a parallelogram PQRS that has an area of 16 cm^2 . If the length of diagonal QS is 8 cm , then what is the length of perpendicular PT?



Solution:

Diagonal QS divides parallelogram PQRS into two triangles, ΔPQS and ΔQRS .

Thus, the area of ΔPQS is one-half the area of parallelogram PQRS.

$$\therefore \text{Area of } \Delta PQS = \frac{1}{2} \times 16 \text{ cm}^2 = 8 \text{ cm}^2$$

If we choose QS as the base of ΔPQS , then PT is its corresponding height.

Thus, the area of ΔPQS can also be expressed as $\frac{1}{2} \times QS \times PT$.

$$\therefore \frac{1}{2} \times QS \times PT = 8 \text{ cm}^2$$

$$\Rightarrow \frac{1}{2} \times 8 \text{ cm} \times PT = 8 \text{ cm}^2$$

$$\Rightarrow PT = \left(\frac{8 \times 2}{8} \right) \text{ cm} = 2 \text{ cm}$$

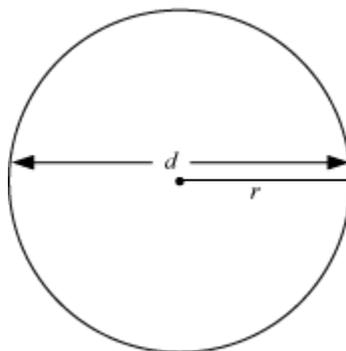
Thus, the length of perpendicular PT is 2 cm.

Circumference of Circle

A car moved certain distance. To cover this distance, its wheel of radius 21 cm made 50 revolutions. Can we calculate this distance covered by the car?

We can find the distance covered by the car, if we can find the circumference of the circular wheel. For this, we have to know the formula for the circumference of a circle from its radius. Let us learn how to find the circumference of the circle from its radius.

For this, let us consider a circle of radius r and diameter d .



The circumference C of the circle is given by the formula

$$C = \pi d \text{ or } C = 2\pi r$$

Here, $d = 2r$ and the approximate value of π (read as 'pi') is 3.14 or $\frac{22}{7}$.

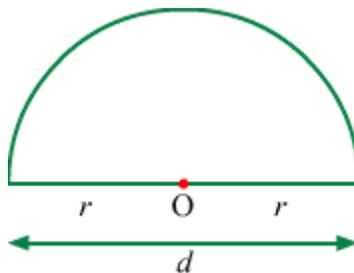
Using this formula for the circumference of a circle, let us find the distance covered by the car.

It is given that the radius of the wheel, $r = 21$ cm

We have, distance covered = $50 \times$ circumference of the circle

$$\begin{aligned} &= 50 \times 2\pi r \\ &= \left(50 \times 2 \times \frac{22}{7} \times 21 \right) \text{ cm} \quad \text{(Putting values of } r \text{ and } \pi) \\ &= 6600 \text{ cm} \\ &= \left(\frac{6600}{100} \right) \text{ m} \quad \left[1 \text{ cm} = \left(\frac{1}{100} \right) \text{ m} \right] \\ &= 66 \text{ m} \end{aligned}$$

In case of semicircle, we find the perimeter P as follows:



$$P = \pi r + 2r$$

or

$$P = \pi r + d$$

Now, let us discuss some examples based on circumference of a circle. In each of the examples, assume the value of π as $\frac{22}{7}$, unless mentioned otherwise.

Example 1:

What is the difference between the circumferences of two circles if the diameter of one is 21 cm and radius of the other is 7 cm?

Solution:

$$\text{Circumference of the circle of diameter 21 cm} = \pi d = \frac{22}{7} \times 21 \text{ cm} = 66 \text{ cm}$$

$$\text{Circumference of the circle of radius 7 cm} = 2\pi r = 2 \times \frac{22}{7} \times 7 \text{ cm} = 44 \text{ cm}$$

Thus, difference between the circumference of the two circles = $(66 - 44) \text{ cm} = 22 \text{ cm}$

Example 2:

The wheel of a vehicle made 150 revolutions to cover a distance of 132 m. Find the diameter of the wheel.

Solution:

Distance covered by the wheel = 132 m = 13200 cm

Number of revolutions made by the wheel = 150

$$\begin{aligned}\text{Now, distance covered in one revolution} &= \frac{\text{Total distance covered by the vehicle}}{\text{Number of revolutions}} \\ &= \frac{13200 \text{ cm}}{150} \\ &= 88 \text{ cm}\end{aligned}$$

We know that the distance covered by the wheel in one revolution equals its circumference.

\therefore Circumference of the wheel = 88 cm

We know, the circumference of the wheel = πd , where d is the diameter.

$$\Rightarrow \frac{22}{7} \times d = 88 \text{ cm}$$

$$\Rightarrow d = \left(88 \times \frac{7}{22} \right) \text{ cm} = 28 \text{ cm}$$

Thus, the diameter of the wheel is 28 cm.

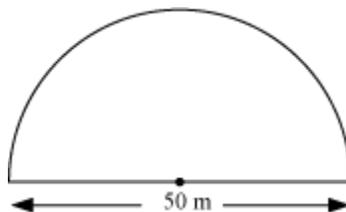
Example 3:

A gardener wants to fence a semi-circular garden of diameter 50 m with three rounds of wire. Find the length of the wire that he needs to purchase. Also find the costs of the wire, if it costs Rs 6 per metre. (Take $\pi = 3.14$)

Solution:

It is given that the diameter of the circle, $d = 50$ m

To find the length of wire required, we have to find the perimeter of the semi-circular field. The perimeter of the semi-circular field includes the circumference of the semi-circle and the diameter.



Now, perimeter of the semi-circular field = Circumference of the semi-circle
+ diameter of the circle

$$= \frac{1}{2} \times \pi d + d$$

$$= \left(\frac{1}{2} \times 3.14 \times 50 + 50 \right) \text{ m}$$

$$= (78.5 + 50) \text{ m}$$

$$= 128.5 \text{ m}$$

1 round of wire requires 128.5 m of wire.

∴ 3 rounds of wire requires $(128.5 \times 3) \text{ m} = 385.5 \text{ m}$ wire

Therefore, the gardener has to purchase 385.5 m of wire.

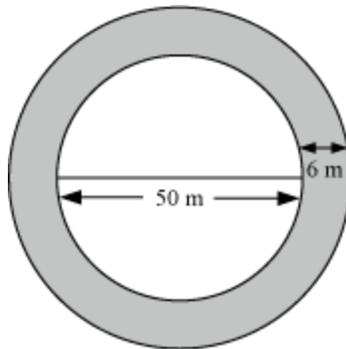
It is given that cost of 1 m of wire = Rs 6

∴ Cost of 385.5 m wire = Rs $(385.5 \times 6) = \text{Rs } 2313$

Therefore, the cost of the wire is Rs 2313.

Area of Circle

The figure drawn below shows a circular park of diameter 50 m. The shaded region represents a 6 m wide jogging track, which runs along the boundary of the park.



This path requires to be gravelled at the rate of Rs 12 per square metre. Now, how do we calculate the required amount?

In many cases, we have to find the area of semicircular shapes. Area of a semicircle is exactly half the area of the corresponding circle.

Mathematically, we can calculate the same using the following formula.

$$\text{Area of semicircle} = \frac{1}{2} \times \pi r^2$$

Let us discuss some examples based on the area of circles and semicircles in order to understand the concept better.

Example 1:

The circumference of a circle is 77 cm. Find its area.

Solution:

Let r be the radius of the circle.

It is given that $2\pi r = 77$ cm.

$$\Rightarrow r = \frac{77}{2 \times \frac{22}{7}}$$

$$\Rightarrow r = \frac{7 \times 7}{2 \times 2}$$

$$\Rightarrow r = \frac{49}{4}$$

$$\text{Area of circle} = \pi r^2$$

$$= \frac{22}{7} \times \left(\frac{49}{4}\right)^2$$

$$= 471.625 \text{ cm}^2$$

Thus, area of the circle is 471.625 cm².

Example 2:

Rajeev wants to paint the semi-circular face of a wooden article. If the diameter of the semi-circle is 3.5 m, then find the cost of painting it at the rate of Rs 100/m²?

$$\left(\text{Use } \pi = \frac{22}{7} \right)$$

Solution:

Diameter of the semi-circular face, $d = 3.5$ m

We know that the diameter of a circle, $d = 2r$, where r is the radius of the circle.

$$\therefore 2r = 3.5 \text{ m}$$

$$r = \left(\frac{3.5}{2}\right) \text{ m} = 1.75 \text{ m}$$

$$\begin{aligned} \text{Area of the semi-circular face} &= \frac{1}{2} \times \pi r^2 = \left(\frac{1}{2} \times \frac{22}{7} \times (1.75)^2\right) m^2 \\ &= \left(\frac{1}{2} \times \frac{22}{7} \times 1.75 \times 1.75\right) m^2 = 4.8125 m^2 \end{aligned}$$

Now, cost of painting 1 m^2 area = Rs 100

Therefore, cost of painting 4.8125 m^2 area = Rs (100×4.8125) = Rs 481.25

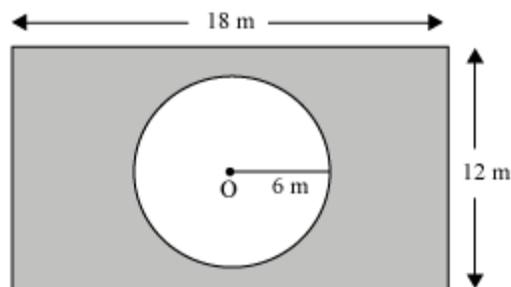
Thus, the cost of painting the top face of the wooden block is Rs 481.25.

Example 3:

A rectangular garden is 18 m long and 12 m wide. A sprinkler is used to water the garden and covers a radius of 6 m. What area of the garden is not watered by the sprinkler? [Use $\pi = 3.14$]

Solution:

The given information can be represented as



Here, the rectangle represents the garden, while point O shows the sprinkler.

The portion of the garden that is not watered by the sprinkler is shown by the shaded part.

It is clear that, area of the garden that is not watered by the sprinkler

= Area of the garden – area of circle with radius 6 m

Area of the garden = length \times breadth = 18 m \times 12 m = 216 m²

Also, we know area of a circle = πr^2 , where r is the radius of the circle.

\therefore Area of the circle with radius 6 m = $\pi \times (6 \text{ m})^2 = (3.14 \times 36) \text{ m}^2 = 113.04 \text{ m}^2$

Thus, area of the garden that is not watered by the sprinkler = (216 – 113.04) m²

= 102.96 m²

Example 4:

The second's hand of a circular clock is 10 cm long. What area does the tip of the second's hand cover in half an hour? (Take $\pi = 3.14$)

Solution:

Radius of the second's hand, $r = 10 \text{ cm}$

The second's hand of a clock takes 1 minute to complete one revolution.

We know half an hour = 30 minutes

Therefore, number of revolution made by the second's hand in half an hour = 30

Area covered by the second's hand in one revolution = $\pi r^2 = (3.14 \times 10 \times 10) \text{ cm}^2$

= 314 cm²

Thus, area covered by the second's hand in 30 revolutions = (30 \times 314) cm² = 9420 cm²