Class X Session 2023-24
Subject - Mathematics (Basic)
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The LCM and HCF of two rational numbers are equal, then the numbers must be **[1]**

a) equal b) prime

c) co-prime d) composite

2. If $n = 2^3 \times 3^4 \times 5^4 \times 7$, then the number of consecutive zeros in n, where n is a natural number, is **[1]**

a) 2 b) 3

c) 7 d) 4

3. If 2 is a root of the equation $x^2 + ax + 12 = 0$ and the quadratic equation $x^2 + ax + q = 0$ has equal roots, then q = **[1]**

a) 20 b) 16

c) 12 d) 8

4. The graphic representation of the equations x + 2y = 3 and 2x + 4y + 7 = 0 gives a pair of **[1]**

a) parallel lines b) none of these

c) coincident lines d) intersecting lines

5. The roots of the quadratic equation $x^2 - 11x - 10 = 0$ are **[1]**

a) None of these b) not real roots

c) real and equal d) real and distinct

6. If (a, b) is the mid-point of the line segment joining the points A(10, –6) and B(k, 4) and a – 2b = 18, the value **[1]**

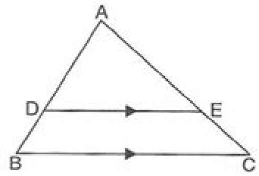
of k is

a) 40
b) 4
c) 30
d) 22

7. In △ABC and △PQR, ∠B = ∠Q, ∠R = ∠C and AB = 2QR, then, the triangles are [1]

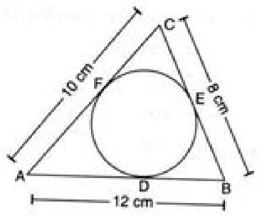
a) Similar but not congruent.
b) Neither congruent nor similar.
c) Congruent as well as similar.
d) Congruent but not similar.

8. In the given figure, $DE||BC$. AB = 15cm, BD = 6cm, AC = 25cm, then AE is equal to [1]



a) 18 cm.
b) 20 cm.
c) 15 cm.
d) 10 cm.

9. A circle is inscribed in $\triangle ABC$ having sides 8 cm, 10 cm and 12 cm as shown in the figure. Then the measure of AD and BE are... [1]



a) AD = 8 cm, BE = 5 cm.
b) AD = 8 cm, BE = 6 cm
c) AD = 5 cm, BE = 7 cm
d) AD = 7 cm, BE = 5 cm

10. $\sin^2A + \sin^2A\tan^2A =$ [1]

a) $\tan^2A$
b) $\cos^2A$
c) None of these
d) $\sin^2A$

11. From the top of a hill, the angles of depression of two consecutive km stones due east are found to be 30° and 45°. The height of the hill is [1]

a) $(\sqrt{3}-1)$ Km
b) $\frac{1}{2}(\sqrt{3}-1)$ Km
c) $(\sqrt{3}+1)$ Km
d) $\frac{1}{2}(\sqrt{3}+1)$ Km

12. If $cosec\,\theta = \sqrt{10}$ then sec θ = ? [1]

a) $\frac{2}{\sqrt{10}}$
b) $\frac{3}{\sqrt{10}}$
c) $\frac{\sqrt{10}}{3}$
d) $\frac{1}{\sqrt{10}}$

13. If the area of a sector of a circle bounded by an arc of length 5π cm is equal to 20π cm^2, then find it's radius [1]

a) 10 cm
b) 16 cm
c) 12 cm
d) 8 cm

14. A piece of wire 20cm long is bent into the form of an arc of a circle subtending an angle of 60^o at its centre. The [1]

radius of the circle is

a) $\frac{20}{6+\pi}$ cm
b) $\frac{30}{6+\pi}$ cm
c) $\frac{60}{\pi}$ cm
d) $\frac{15}{6+\pi}$ cm

15. A card is selected from a deck of 52 cards. The probability of its being a red face card is **[1]**

a) $\frac{3}{13}$
b) $\frac{1}{2}$
c) $\frac{2}{12}$
d) $\frac{3}{26}$

16. For the following distribution: **[1]**

Class:	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The sum of the lower limits of the median class and modal class is:

a) 25
b) 30
c) 35
d) 15

17. A sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder. If the radius of the sphere is r, then the volume of the cylinder is **[1]**

a) $2\pi r^3$
b) $8\pi r^3$
c) $\frac{8}{3}\pi r^3$
d) $4\pi r^3$

18. The mean of 25 observations is 36. If the mean of the first 13 observations is 32 and that of the last 13 observations is 39, then the 13th observation is **[1]**

a) 13
b) 23
c) 32
d) 36

19. **Assertion (A):** Point (0, 3) has image (0, –3). **[1]**
Reason (R): Image of (0, k) is (0, –k) only.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

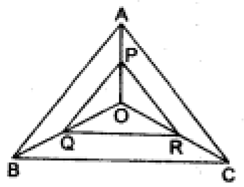
20. **Assertion (A):** For any two positive integers a and b, HCF (a, b) $\times$ LCM (a, b) = a $\times$ b **[1]**
Reason (R): The HCF of the two numbers is 8 and their product is 280. Then their LCM is 40.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

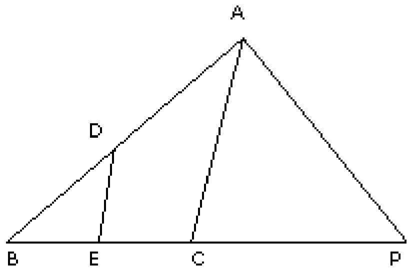
21. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the pair of linear equations are consistent, or inconsistent: $\frac{4}{3}x$ + 2y = 8; 2x + 3y = 12. **[2]**

22. In the given figure $PQ \| AB$ and $PR \| AC$. Prove that $QR \| BC$. **[2]**

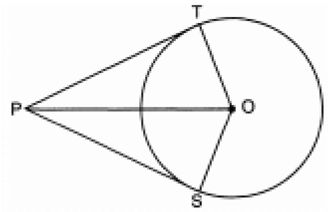


OR

In the figure, DE || AC and, $\frac{BE}{EC} = \frac{BC}{CP}$ prove that

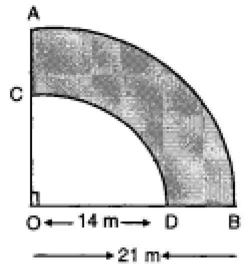


23. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle$SPT = 120^{o}, Prove that OP = 2PS **[2]**



24. If 4 cosθ = 11 sinθ, find the value $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$. **[2]**

25. ABCD is a flower bed. If OA =21 m and OC = 14 m, find the area of the bed. **[2]**



OR

The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 2 days. [Take π = 3.14.]

Section C

26. Three sets of English, Hindi and Mathematics books have to be stacked in such a way that all the books are stored topic wise and the height of each stack is the same. The number of English books is 96, the number of Hindi books is 240 and the number of Mathematics books is 336. Assuming that the books are of the same thickness, determine the number of stacks of English, Hindi and Mathematics books. **[3]**

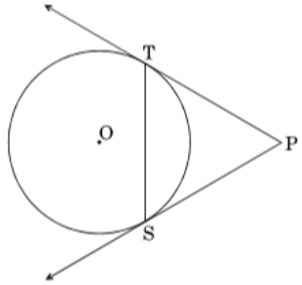
27. If α and β are the zeros of the polynomial f(x) = $6x^2$ + x - 2, find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$ **[3]**

28. A taken 3 hours more than B to walk a distance of 30 km. But if A doubles his speed, he is ahead of B by $1\frac{1}{2}$hours. Find their original speed. **[3]**

OR

A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

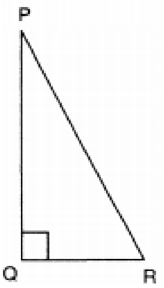
29. In the given figure, PT and PS are tangents to a circle with centre O, from a point P, such that PT = 4 cm and $\angle$TPS = 60^{o}. Find the length of the chord TS. Also, find the radius of the circle. **[3]**



30. Evaluate: $\frac{\sin 30^\circ + \tan 45^\circ - cosec 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$ **[3]**

OR

In the given $\triangle$PQR, right-angled at Q, QR = 9 cm and PR - PQ = 1 cm. Determine the value of sin R + cos R.



31. A piggy bank contains hundred 50-p coins, seventy Rs.1 coin, fifty Rs.2 coins and thirty Rs.5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin **[3]**

 i. will be a Rs. 1 coin?
 ii. will not be a Rs.5 coin
 iii. will be 50-p or a Rs. 2 coin?

Section D

32. If x = - 2 is a root of the equation $3x^2 + 7x + p = 0$, find the value of k so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$ are equal. **[5]**

OR

Solve for x: $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$

33. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio. **[5]**

34. A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them is being 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid. **[5]**

OR

An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar, if one cubic cm of iron weighs 10 g.

35. Calculate the mode of the following frequency distribution table : **[5]**

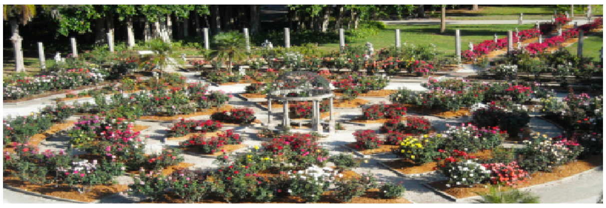
Marks	Number of students
25 or more than 25	52
35 or more than 35	47
45 or more than 45	37
55 or more than 55	17

65 or more than 65	8
75 or more than 75	2
85 or more than 85	0

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.



In their garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row.

(i) How many rows are there of rose plants?

(ii) Also, find the total number of rose plants in the garden.

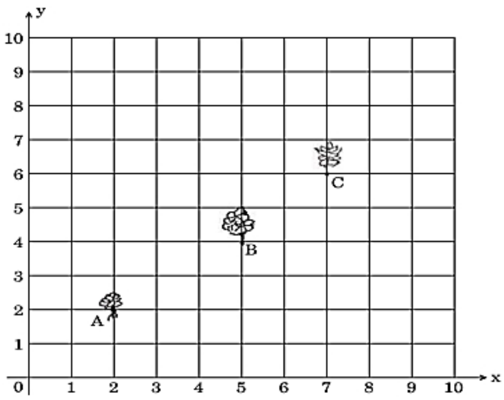
OR

If total number of plants are 80 in the garden, then find number of rows?

(iii) How many plants are there in 6th row.

37. **Read the text carefully and answer the questions:** **[4]**

Reena has a 10 m $\times$ 10 m kitchen garden attached to her kitchen. She divides it into a 10 $\times$ 10 grid and wants to grow some vegetables and herbs used in the kitchen. She puts some soil and manure in that and sow a green chilly plant at A, a coriander plant at B and a tomato plant at C. Her friend Kavita visited the garden and praised the plants grown there. She pointed out that they seem to be in a straight line. See the below diagram carefully:



(i) Find the distance between A and B?

(ii) Find the mid-point of the distance AB?

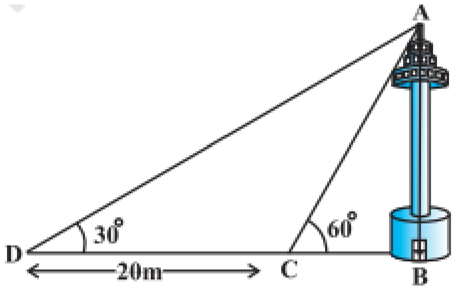
OR

Find the mid point of BC.

(iii) Find the distance between B and C?

38. **Read the text carefully and answer the questions:** **[4]**

A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60^{o} from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30^{o}.



(i) Find the width of the canal.

(ii) Find the height of tower.

OR

Find the distance between top of tower and point C.

(iii) Find the distance between top of the tower and point D.

Solution

Section A

1. **(a)** equal

   Explanation: If we assume that a and b are equal and consider a = b = k

   Then,

   HCF (a, b)= k

   LCM (a, b) = k

2. **(b)** 3

   Explanation: Since, it is given that

   $n = 2^3 \times 3^4 \times 5^4 \times 7$

   $= 2^3 \times 5^4 \times 3^4 \times 7$

   $= 2^3 \times 5^3 \times 5 \times 3^4 \times 7$

   $= (2 \times 5)^3 \times 5 \times 3^4 \times 7$

   $= 5 \times 3^4 \times 7 \times (10)^3$

   So, this means the given number n will end with 3 consecutive zeroes.

3. **(b)** 16

   Explanation: 2 is root equation $x^2 + ax + 12 = 0$

   $\therefore (2)^2 + a \times 2 + 12 = 0 \Rightarrow 4 + 2a + 12 = 0$

   $\Rightarrow 2a + 16 = 0$

   $\Rightarrow a = \frac{-16}{2} = -8$

   and given that roots of $x^2 + ax + q = 0$ are equal.

   $\therefore\ b^2 - 4ac = 0$

   $\Rightarrow a^2 - 4q = 0 \Rightarrow (-8)^2 - 4q = 0$

   $\Rightarrow 64 - 4q = 0 \Rightarrow 4q = 64$

   $\Rightarrow q = \frac{64}{4} = 16$

   $\therefore q = 16$

4. **(a)** parallel lines

   Explanation: Given: Two equations, x + 2y = 3

   ⇒ x + 2y – 3 = 0 .... (i)

   2x + 4y + 7 = 0 .... (ii)

   We know that the general form for a pair of linear equations in 2 variables x and y is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

   Comparing with above equations,

   we have $a_1 = 1$, $b_1 = 2$, $c_1 = -3$; $a_2 = 2$, $b_2 = 4$, $c_2 = 7$

   $\frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}; \frac{c_1}{c_2} = \frac{-3}{7}$

   Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

   $\therefore$Both lines are parallel to each other.

5. **(d)** real and distinct

   Explanation: Here, a = 1, b = -11, c = -10

   Then, $b^2 - 4ac = (-11)^2 - 4 \times 1 \times (-10)$

   $\Rightarrow 121 + 40 = 161$

   Since, $b^2 - 4ac > 0$

   Therefore, The roots of the quadratic equation $x^2 - 11x - 10 = 0$ is real and distinct.

6.

(d) 22

Explanation: 

∵ P is the midpoint of AB

a = $\frac{10+k}{2}$

2a = 10 + k ...(i)

b = $\frac{-6+4}{2}$

2b = -6 + 4

2b = -2

b = -1

Now, a - 2b = 18 (given)

a - 2(-1) = 18

a = 18 - 2

a = 16

From equation (i)

2 × 16 = 10 + k

32 - 10 = k

∴ k = 22

7. **(a)** Similar but not congruent.

Explanation: In △ABC and △PQR ∠B = ∠Q, ∠R = ∠C and AB = 2QR

Then, the triangles are similar, by AA similarity rule, but not congruent because, for congruency, sides should also be equal.

8.

(c) 15 cm.

Explanation: Since $DE \parallel BC, \frac{AD}{DB} = \frac{AE}{EC} (by\ BPT) \Rightarrow \frac{9}{6} = \frac{AE}{25-AE} \Rightarrow AE = 15cm$

9.

(d) AD = 7 cm, BE = 5 cm

Explanation: Let AD = x and BE = y

∴ BD = 12 - x ⇒ BE = y

But BD = BE (Tangents to a circle from an external point B)

⇒y = 12 - x ⇒ x + y = 12 ……….(i)

Also, AF = x

and CF = 10 - x

and CE = 8 - y

∴ 10 - x = 8 - y

x - y = 2……..(ii)

On solving eq. (i) and (ii), we get

x = 7 and y = 5

Therefore AD = 7 cm and BE = 5 cm

10. **(a)** $\tan^2$A

Explanation: Given: $\sin^2$A + $\sin^2$A $\tan^2$A

= $\sin^2$A(1 + $\tan^2$A)

= $\sin^2$A($\sec^2$A)

= $\sin^2$A × $\frac{1}{\cos^2 A}$

= $\frac{\sin^2 A}{\cos^2 A}$

= $\tan^2$A

11.

(d) $\frac{1}{2}(\sqrt{3}+1)$ Km

Explanation: Let AB be the hill.

From right △BAD, we have

$\frac{AD}{AB} = \cot 45^o \Rightarrow \frac{x}{h} = 1 \Rightarrow$ x = h

From right △BAC, we have

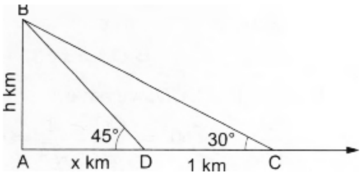
$\frac{AC}{AB}$ = cot 30^o $\Rightarrow \frac{x+1}{h} = \sqrt{3} \Rightarrow x = (h\sqrt{3} - 1)$

From (i) and (ii),we have

h(h$\sqrt{3}$ - 1) ⇒ h($\sqrt{3}$ - 1) = 1

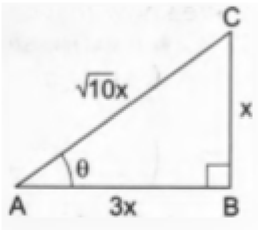
$\Rightarrow \quad h = \frac{1}{(\sqrt{3}-1)}$

$\Rightarrow h = \left\{\frac{1}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}\right\} = \frac{1}{2}(\sqrt{3}+1)$



12.

(c) $\frac{\sqrt{10}}{3}$

Explanation: 

$cosec\theta = \frac{AC}{BC} = \frac{\sqrt{10}}{1} = \frac{\sqrt{10}x}{x} \Rightarrow AC = \sqrt{10x}$ and BC = x.

∴ AB^2 = AC^2 - BC^2 $= (\sqrt{10}x)^2 - \left(x^2\right)$ = 10x^2 - x^2 = 9x^2

$\Rightarrow AB = \sqrt{9x^2} = 3x$

$\therefore \sec\theta = \frac{AC}{AB} = \frac{\sqrt{10}x}{3x} = \frac{\sqrt{10}}{3}$

13.

(d) 8 cm

Explanation: We have given length of the arc and area of the sector bounded by that arc and we are asked to find the radius of the circle.

We know that area of the sector $= \frac{\theta}{360} \times \pi r^2$.

Length of the arc $= \frac{\theta}{360} \times 2\pi r$

Now we will substitute the values.

Area of the sector $= \frac{\theta}{360} \times \pi r^2$

$20\pi = \frac{\theta}{360} \times \pi r^2$ ……..(1)

Length of the arc $= \frac{\theta}{360} \times 2\pi r$

$5\pi = \frac{\theta}{360} \times 2\pi r$ ……..(2)

$\frac{20\pi}{5\pi} = \frac{\frac{\theta}{360} \times \pi r^2}{\frac{\theta}{360} \times 2\pi r}$

$\frac{20}{5} = \frac{r^2}{2r}$

$\therefore 4 = \frac{r}{2}$

∴ r = 8

Therefore, radius of the circle is 8 cm.

14.

(c) $\frac{60}{\pi}$ cm

Explanation: Given: Length of arc = 20 cm

$\Rightarrow \frac{\theta}{360^\circ} \times 2\pi$r= 20

$\Rightarrow \frac{60^\circ}{360^\circ} \times 2\pi$r= 20

$\Rightarrow \frac{\pi r}{3}$ = 20

$\Rightarrow r\left(\frac{\pi}{3}\right)$ = 20

$\Rightarrow r\left(\frac{\pi}{3}\right) = 20$

$\Rightarrow$ r = $\frac{60}{\pi}$ cm

15.

(d) $\frac{3}{26}$

Explanation: In a deck of 52 cards, there are 12 face cards i.e. 6 red (3 hearts and 3 diamonds) and 6 black cards (3 spade and 3 clubs)

So, probability of getting a red face card = 6/52 = 3/26

16. **(a)** 25

Explanation:

Class	Frequency	Cumulative frequency
0-5	10	10
5-10	15	25
10-15	12	37
15-20	20	57
20-25	9	66

Here, $\frac{N}{2} = \frac{66}{2}$ = 33, which lies in the interval 10-15.

Therefore, lower limit of the median class is 10.

The highest frequency is 20, which lies in the interval = 15-20.

Therefore, lower limit of modal class is 15.

Hence, required sum is = 10 + 15 = 25

17. **(a)** $2\pi r^3$

Explanation: Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder and the radius of the sphere is *r*.

Thus, height of the cylinder = diameter = 2r and base radius = r

Volume of the cylinder = $\pi \times r^2 \times 2r = 2\pi r^3$

18.

(b) 23

Explanation: Let terms be $x_1, x_2, x_3, ..., x_{25}$

According to the question,

$\frac{x_1+x_2+x_3+.......+x_{25}}{25} = 36$

$\Rightarrow x_1 + x_2 + x_3 + ... x_{25} = 900$ ... (i)

And $\frac{x_1+x_2+x_3+......+x_{13}}{13} = 32$

$\Rightarrow x_1 + x_2 + x_3 + ... x_{13} = 416$ ... (ii)

Also, $\frac{x_{13}+x_{14}+x_{15}+.......+x_{25}}{13} = 39$

$\Rightarrow x_{13} + x_{14} + x_{15} + ... x_{25} = 507$ ... (iii)

Adding eq. (ii) and (iii), we get,

$x_1 + x_2 + x_3 + ... x_{13} + x_{14} + x_{15} + ... x_{25} = 416 + 507 = 923$

$\Rightarrow x_1 + x_2 + x_3 + ... x_{25} + x_{13} = 923$

$\Rightarrow 900 + x_{13} = 923$

$\Rightarrow x_{13} = 23$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Image of points of type (0, k) is (0, –k) only.

20.

(c) A is true but R is false.

Explanation: HCF (a, b) × LCM (a, b) = a × b

⇒ 8 × LCM = 280

⇒ LCM = $\frac{280}{8}$ = 35

A is true but R is false.

Section B

21. Given equations are:

$\frac{4}{3}x$ + 2y = 8; 2x + 3y = 12

Compare equation $\frac{4}{3}x + 2y = 8$ with $a_1x + b_1y + c_1 = 0$ and 2x + 3y = 12

with $a_2x + b_2y + c_2 = 0$, We get, $a_1 = \frac{4}{3}$, $a_1 = \frac{4}{3}$, $b_1 = 2$, $c_1 = -8$, $a_2 = 2$, $b_2 = 3$, $c_2 = -12$

$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3}$ and $\frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, the lines have infinitely many solutions.

Hence, they are consistent.

22. In$\triangle OAB, PQ \| AB$

By using Thales theorem, we get

$\therefore \quad \frac{OP}{PA} = \frac{OQ}{QB}$ ....(i)

In$\triangle AOC, PR \| AC$

By using Thales theorem, we have

$\therefore \quad \frac{OP}{PA} = \frac{OR}{RC}$ ....(ii)

From (i) and (ii) we get

$\frac{\text{OQ}}{QB} = \frac{OR}{RC}$ in $\triangle OBC$.

Therefore, in $\triangle OBC$, Q and R are points on OB and OC respectively such that

$\frac{OQ}{QB} = \frac{OR}{RC}$

Hence, by the converse of Thales' theorem, $QR \| BC$

OR

In △ABC, DE ⊥ AC

$\therefore \frac{BD}{DA} = \frac{BE}{CP}$ ....(1)[By Thales's Theorem]

Also

$\frac{BE}{EC} = \frac{BC}{CP}$ (given) ....(ii)

∴ From (i) and (ii), we get

$\frac{BD}{DA} = \frac{BC}{CP} \therefore DC \perp AP$ [By the converse of Thales Theorem]

23. Given that $\angle SPT = 120^\circ$

or, $\angle OPS = \frac{120^\circ}{2} = 60^\circ$ (as OP bisect $\angle SPT$)

Also, $\angle PTO = 90^\circ$ (as radius ⊥ tangent)

∴ In right triangle POS.

$\cos \angle OPS = \frac{PS}{OP}$

or, $\frac{1}{2} = \frac{PS}{OP}$

or, OP = 2 PS

24. Given :$4cos\theta = 11sin\theta$

or, $cos\theta$ = $\frac{11}{4}\sin\theta$

Now, $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$= $\frac{11 \times \frac{11}{4}\sin\theta - 7\sin\theta}{11 \times \frac{11}{4}\sin\theta + 7\sin\theta}$

= $\frac{\sin\theta\left(\frac{121}{4} - 7\right)}{\sin\theta\left(\frac{121}{4} + 7\right)}$

= $\frac{121-28}{121+28} = \frac{93}{149}$

25. We have, OA = R = 21 m and OC = r = 14 m

∴Area of the flower bed = Area of a quadrant of a circle of radius R - Area of a quadrant of a circle of radius r

$= \frac{1}{4}\pi R^2 - \frac{1}{4}\pi r^2$

$= \frac{\pi}{4}\left(R^2 - r^2\right)$

$= \frac{1}{4} \times \frac{22}{7}\left(21^2 - 14^2\right) \text{cm}^2$

$= \left\{\frac{1}{4} \times \frac{22}{7} \times (21+14)(21-14)\right\} m^2$

$= \left\{\frac{1}{4} \times \frac{22}{7} \times 35 \times 7\right\} \text{m}^2$

= 192.5 m^2

OR

The hour hand covers 4 complete circles in 2 days (48 hours)

Distance = $2 \times \frac{22}{7} \times 4 \times 4$

= 100.57 cm

The minute hand covers = 48 Circles in 2 days (Each hour = 1 circle)

Distance =$2 \times \frac{22}{7} \times 6 \times 48$

= 1810.23 cm

Total distance = 100.57 + 1810.23

= 1910.8 cm

Section C

26. In order to arrange the books as required, we have to find the largest number that divides 96, 240 and 336 exactly.

Clearly, such a number is their HCF.

We have,

$96 = 2^5 \times 3, 240 = 2^4 \times 3 \times 5$ and $336 = 2^4 \times 3 \times 7$

$\therefore$ HCF of 96, 240 and 336 is $2^4 \times 3 = 48$

So, there must be 48 books in each stack.

$\therefore$ Number of stacks of English books $= \frac{96}{48} = 2$

Number of stacks of Hindi books $= \frac{240}{48} = 5$

Number of stacks of Mathematics books $= \frac{336}{48} = 7$

27. Let f(x) = $6x^2 + x - 2$

a = 6, b = 1 and c = -2

And α and β are the zeros of polynomial,

$\alpha + \beta = -\frac{b}{a} = -\frac{1}{6}$

$\alpha\beta = \frac{c}{a} = \frac{-2}{6} = \frac{-1}{3}$

$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(-\frac{1}{6}\right)^2 - 2\left(-\frac{1}{3}\right)}{\left(-\frac{1}{3}\right)}$

$= -\frac{\frac{1}{36}+\frac{2}{3}}{\frac{1}{3}}$

$= -\frac{\frac{25}{36}}{\frac{1}{3}}$

$= -\frac{25}{36} \times \frac{3}{1}$

$= -\frac{25}{12}$

28. Let the original speed of A and B are x km/h and y km/h respectively.

According to the question,

$\frac{30}{x} - \frac{30}{y} = 3$

or $\frac{1}{x} - \frac{1}{y} = \frac{1}{10}$

or $u + v = \frac{1}{10}$ ....(i)

and $\frac{30}{y} - \frac{30}{2x} = \frac{3}{2}$

$\frac{1}{y} - \frac{1}{2x} = \frac{1}{20}$

$v - \frac{u}{2} = \frac{1}{20}$ ...(ii)

On adding (i) and (ii), we get,

u $- \frac{u}{2} = \frac{1}{10} + \frac{1}{20}$

$\frac{u}{2} = \frac{3}{20} \Rightarrow \frac{1}{u} = \frac{10}{3}$

or $x = \frac{10}{3}$
and y = 5
$\therefore$ speed of A is 3.3 km/h

OR

Let the usual speed of the plane = x km/hr.
Distance to the destination = 1500 km
Case (i):
we know that, $Speed = \frac{Distance}{Time}$
$\Rightarrow Time = \frac{Distance}{speed}$
So, in case(i) Time = $\frac{1500}{x}$ Hrs
Case (iI)
Distance to the destination = 1500 km
Increased speed = 100 km/hr
So, speed = x+100
So, in case(ii) Time = $\frac{1500}{x+100}$ Hrs
So, according to the question
$\therefore \frac{1500}{x} - \frac{1500}{x+100} = \frac{30}{60}$
$\Rightarrow x^2 + 100x - 300000 = 0$
$\Rightarrow x^2 + 600x - 500x - 300000 = 0$
$\Rightarrow (x + 600)(x - 500) = 0$
$\Rightarrow x = 500$ or $x = -600$
Since, speed can not be negative, x = 500
Therefore, Speed of plane = 500 km/hr.

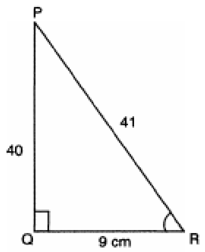
29. $\because$ PT = PS (tangents from an external point P)
$\therefore \angle PTS = \angle PST$
Using Angle Sum Property in $\triangle PTS$
$\angle PTS + \angle PST + \angle TPS = 180^o$
$2\angle PTS = 180 - 60 = 120^o$
$\angle PTS = 60^o$
$\Rightarrow$ PTS Is a equilateral triangle
So, TS = 4 cm
Now, In $\triangle PTO$
As PO is angle bisector of $\angle TPS$, $\angle OTP = 90^o$
$\tan 30^o = \frac{OT}{TP}$
$\frac{1}{\sqrt{3}} = \frac{OT}{4}$
OT = $\frac{4}{\sqrt{3}}$
OT = $\frac{4\sqrt{3}}{3}$ cm
$\therefore$ radius of circle = $\frac{4\sqrt{3}}{3}$ cm

30. We have $\frac{\sin 30^\circ + \tan 45^\circ - cosec 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$
after putting values,we get
$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$
$= \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}$
$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$ Rationalise it, we get
$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$
$= \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2 - (4)^2}$

$= \frac{27+16-24\sqrt{3}}{27-16}$

$= \frac{43-24\sqrt{3}}{11}$

OR



$P^2 + B^2 = H^2$ (By Pythagoras theorem)

$PQ^2 + QR^2 = PR^2$

$PQ^2 + 9^2 = PR^2$

$PQ^2 + 81 = PR^2$

$PQ^2 + 81 = (PQ+1)^2 \ (\because PR - PQ = 1)$

$PQ^2 + 81 = PQ^2 + 1 + 2PQ$

$PQ^2 - PQ^2 + 81 - 1 = 2PQ$

$80 = 2PQ$

or, $PQ = 40$

$PR - PQ = 1$ (Given)

or, $PR = 1 + 40$

or, $PR = 41$

Now, sinR $= \frac{P}{H} = \frac{PQ}{PR} = \frac{40}{41}$

cosR = $\frac{B}{H} = \frac{9}{41}$

$\therefore$ sinR + cosR = $\frac{40}{41} + \frac{9}{41} = \frac{49}{41}$

31. Number of 50-p coins = 100.

Number of Rs. 1 coins = 70.

Number of Rs. 2 coins = 50.

Number of Rs. 5 coins = 30.

Therefore, the total number of outcomes = 100+70+50+30=250

i. Suppose E1 be the event of getting a Rs. 1 coin.

The number of favorable outcomes = 70.

Therefore, P(getting a Rs. 1 coin) = $P(E_1) = \frac{Number \text{ of outcomes favorable to } E_1}{\text{Number of all possible outcomes}} = \frac{70}{250}$

Thus, the probability that the coin will be a Rs. 1 coin is $\frac{7}{25}$.

ii. Suppose E_2 be the event of not getting a Rs. 5.

Number of favorable outcomes = 250 - 30 = 220

Therefore, P(not getting a Rs. 5 coin)

$= P(E_2) = \frac{\text{Number of outcomes favorable to } E_2}{\text{Number of all possible outcomes}} = \frac{220}{250} = \frac{22}{25}$

Thus, probability that the coin will not be a Rs. 5 coin is $\frac{22}{25}$.

iii. Suppose E_3 be the event of getting a 50-p or a Rs. 2 coins.

Number of favorable outcomes = 100 + 50 = 150

Therefore, P(getting a 50-p or a Rs. 2 coin) = P(E_3) = $\frac{\text{Number of outcomes favorable to} E_3}{\text{Number of all possible outcomes}} = \frac{150}{250} = \frac{3}{5}$

Thus, probability that the coin will be a 50-p or a Rs. 2 coin is $\frac{3}{5}$.

Section D

32. Here x = - 2 is the root of the equation $3x^2 + 7x + p = 0$

then, $3(-2)^2 + 7(-2) + p = 0$

or, p=2

Roots of the equation $x^2 + 4kx + k^2 - k + 2 = 0$ are equal,then,

$16k^2 - 4\left(k^2 - k + 2\right) = 0$

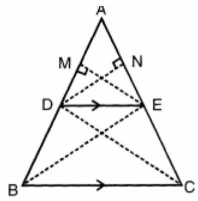
or, $16k^2 - 4k^2 + 4k - 8 = 0$

or, $12k^2 + 4k - 8 = 0$
or, $3k^2 + k - 2 = 0$
or, (3k-2)(k + 1) = 0
or, $k = \frac{2}{3}, -1$
Hence, roots = $\frac{2}{3}, -1$

OR

We have given,
$\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$
Let $\frac{2x}{(x-5)}$ be y
$\therefore \quad y^2 + 5y - 24 = 0$
Now factorise,
$y^2 + 8y - 3y - 24 = 0$
$y(y+8) - 3(y+8) = 0$
$(y+8)(y-3) = 0$
$y = 3, -8$
Putting y=3
$\frac{2x}{x-5} = 3$
2x = 3x - 15
x = 15
Putting y = -8
$\frac{2x}{x-5} = -8$
2x = -8x + 40
10x = 40
x = 4
Hence, x is 15 , 4

33. Given: ABC is a triangle in which DE $\parallel$ BC.
To prove: $\frac{AD}{BD} = \frac{AE}{CE}$
Construction: Draw $DN \perp AE$ and $EM \perp AD$., Join BE and CD.
Proof :



In $\triangle ADE$,
Area of $\Delta ADE = \frac{1}{2} \times AE \times DN$ ...(i)
In ΔDEC,
Area of $\Delta DCE = \frac{1}{2} \times CE \times DN$ ...(ii)
Dividing equation (l) by equation (ii),
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEC)} = \frac{AE}{CE}$ ...(iii)
Similarly, In ΔADE,
Area of $\Delta ADE = \frac{1}{2} \times AD \times EM$ ...(iv)
In ΔDEB,
Area of $\Delta DEB = \frac{1}{2} \times EM \times BD$ ...(v)
Dividing equation (iv) by equation (v),
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{AD}{BD}$ ...(vi)
ΔDEB and ΔDEC lie on the same base DE and between two parallel lines DE and BC.

$\therefore$ Area (ΔDEB) = Area (ΔDEC)

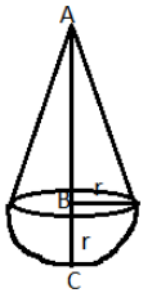
From equation (iii),

$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{AE}{CE}$. ......(vii)

From equation (vi) and equation (vii),

$\frac{AE}{CE} = \frac{AD}{BD}$

$\therefore$ If a line is drawn parallel to one side of a triangle to intersect the other two sides in two points, then the other two sides are divided in the same ratio.

34. 

From the given figure,

Height (AB) of the cone = AC - BC (Radius of the hemisphere)

Thus, height of the cone = Total height - Radius of the hemisphere

= 9.5 - 3.5

= 6 cm

Volume of the solid = Volume of the cone + Volume of the hemisphere

$= \left(\frac{1}{3}\pi r^2 h\right) + \left(\frac{2}{3}\pi r^3\right)$

$= \frac{1}{3}\pi r^2(h + 2r)$

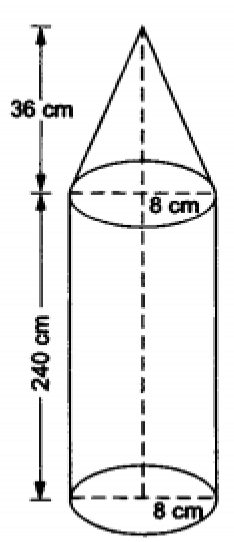
$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5(6 + 2 \times 3.5)$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 13$

= 166.83 cm^3

Thus, total volume of the solid is 166.83 cm^3.

OR



Let us suppose that r denotes the radius of the cylinder = 8 cm.

Suppose R denotes the radius of the cone = 8 cm.

Let h be the height of the cylinder = 240cm.

Suppose H is the height of the cone = 36 cm.

Total volume of the iron = volume of the cylinder + volume of the cone

$= \pi r^2 h + \frac{1}{3}\pi R^2 H = \pi r^2\left(h + \frac{1}{3}H\right)$ [as r=R= 8cm each]

$= \left[\frac{22}{7} \times 8 \times 8 \times \left(240 + \frac{1}{3} \times 36\right)\right] \text{cm}^3$

= 50688 cm^3

$\therefore$ Weight of the pilllar = volume in $cm^3 \times$ weight per cm^3

$= \left(\frac{50688 \times 10}{1000}\right)$ kg $= 506.88$kg

Therefore, the weight of the pillar is 506.88 kg.

35. The given data are:

Marks	**Number of students**
25 or more than 25	52
35 or more than 35	47
45 or more than 45	37
55 or more than 55	17
65 or more than 65	8
75 or more than 75	2
85 or more than 85	0

From above data we can calculate range data as following:

Marks	**Number of students(f)**
25 - 35	52 - 47 = 5
35 - 45	47 - 37 = 10
45 - 55	37 - 17 = 20
55 - 65	17 - 8 = 9
65 - 75	8 - 2 = 6
75 - 85	2 - 0 = 2
85 - 95	0

From table it is clear that maximum class frequency is 20 belonging to class interval 45 - 55

Modal class = 45 - 55

Lower limit (l) of modal class = 45

Class size (h) = 10

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 10

Frequency (f_2) of class succeeding the modal class = 9

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 45 + \left(\frac{20-10}{2 \times 20 - 10 - 9}\right) \times 10$

$= 45 + \frac{10}{21} \times 10$

= 45 + 4.76

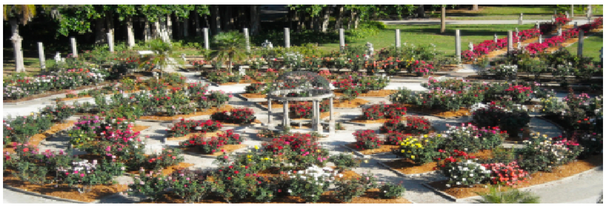
= 49.76

Therefore mode of data is 49.76

Section E

36. **Read the text carefully and answer the questions:**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.



In their garden bed, there are 23 rose plants in the first row, 21 are in the 2nd, 19 in 3rd row and so on. There are 5 plants in the last row.

(i) The number of rose plants in the 1st, 2nd, ... are 23, 21, 19, ... 5

$a = 23$, $d = 21 - 23 = -2$, $a_n = 5$

$\therefore a_n = a + (n - 1)d$

or, $5 = 23 + (n - 1)(-2)$

or, $5 = 23 - 2n + 2$

or, $5 = 25 - 2n$

or, $2n = 20$

or, $n = 10$

(ii) Total number of rose plants in the flower bed,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{10} = \frac{10}{2}[2(23) + (10 - 1)(-2)]$

$S_{10} = 5[46 - 20 + 2]$

$S_{10} = 5(46 - 18)$

$S_{10} = 5(28)$

$S_{10} = 140$

OR

$S_n = 80$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow 80 = \frac{n}{2}[2 \times 23 + (n - 1) \times -2]$

$\Rightarrow 80 = 23n - n^2 + n$

$\Rightarrow n^2 - 24n + 80 = 0$

$\Rightarrow (n - 4)(n - 20) = 0$

$\Rightarrow n = 4$ or $n = 20$

$n = 20$ not possible

$a_{20} = 23 + 19 \times (-2) = -15$

Number of plants cannot be negative.

$n = 4$

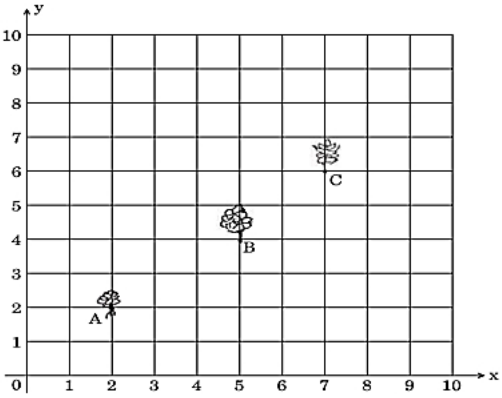
(iii) $a_n = a + (n - 1)d$

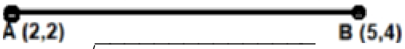
$\Rightarrow a_6 = 23 + 5 \times (-2)$

$\Rightarrow a_6 = 13$

37. **Read the text carefully and answer the questions:**

Reena has a 10 m × 10 m kitchen garden attached to her kitchen. She divides it into a 10 × 10 grid and wants to grow some vegetables and herbs used in the kitchen. She puts some soil and manure in that and sow a green chilly plant at A, a coriander plant at B and a tomato plant at C. Her friend Kavita visited the garden and praised the plants grown there. She pointed out that they seem to be in a straight line. See the below diagram carefully:



(i)


AB = $\sqrt{(5-2)^2+(4-2)^2}$

$=\sqrt{9+4}$

$=\sqrt{13}$

(ii) Middle point AB = $\left(\frac{2+5}{2}, \frac{2+4}{2}\right)$

= (3.5, 3)

OR

Middle point of BC = $\left(\frac{5+7}{2}, \frac{4+6}{2}\right)$

= (6, 5)

(iii)

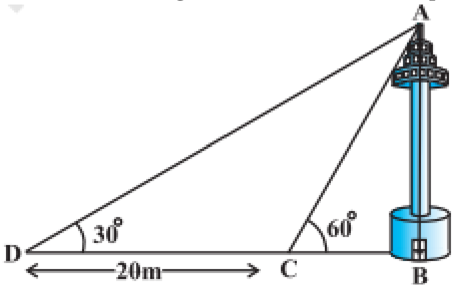

BC = $\sqrt{(7-5)^2+(6-4)^2}$

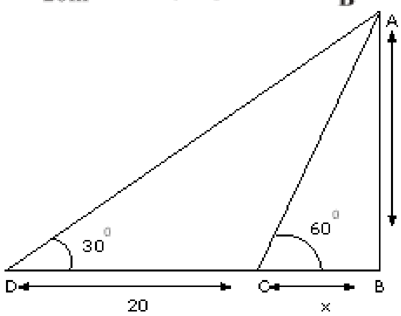
$=\sqrt{4+4}$

$=2\sqrt{2}$

38. **Read the text carefully and answer the questions:**

A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60^o from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30^o.



(i)


Let 'h' (AB) be the height of tower and x be the width of the river.

In $\triangle ABC, \frac{h}{x}=\tan 60^{\circ}$

$\Rightarrow h=\sqrt{3}x$ ...(i)

In $\Delta ABD, \frac{h}{x+20}=\tan 30^{\circ}$

$\Rightarrow h=\frac{x+20}{\sqrt{3}}$ ...(ii)

Equating (i) and (ii),

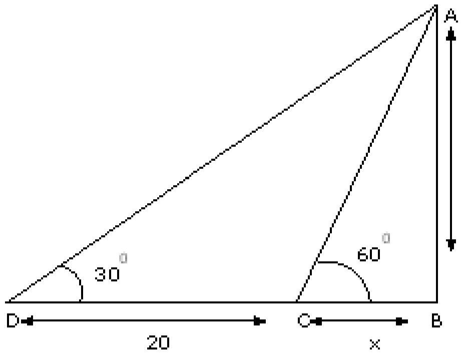
$\sqrt{3}x = \frac{x+20}{\sqrt{3}}$

$\Rightarrow$ 3x = x + 20

$\Rightarrow$ 2x = 20

$\Rightarrow$ x = 10 m

(ii)



Let 'h' (AB) be the height of tower and x be the width of the river.

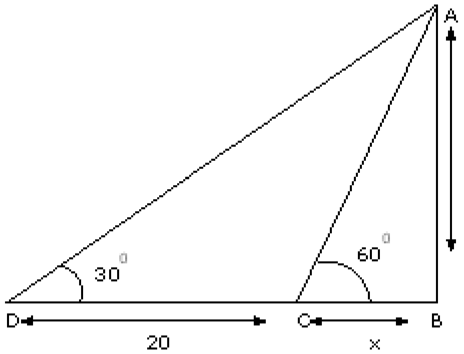
In $\triangle ABC, \frac{h}{x} = \tan 60^\circ$

$\Rightarrow h = \sqrt{3}x$ ...(i)

Put x = 10 in (i), $h = \sqrt{3}x$

$\Rightarrow h = 10\sqrt{3}$m

OR



In $\triangle$ABC

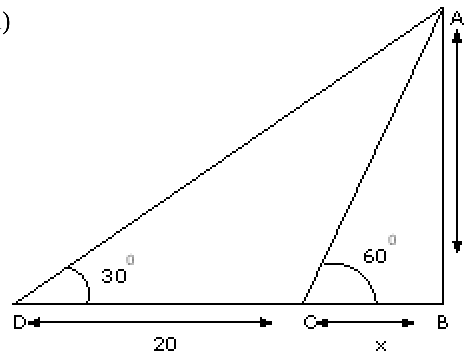
sin 60^o = $\frac{AB}{AC}$

$\Rightarrow$ AC = $\frac{AB}{\sin 30^0}$

$\Rightarrow$ AC = $\frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AC = 20 m

(iii)



In $\triangle$ABD

sin 30^o = $\frac{AB}{AD}$

$\Rightarrow$ AD = $\frac{AB}{\sin 30^\circ}$

$\Rightarrow$ AD = $\frac{10\sqrt{3}}{\frac{1}{2}}$

$\Rightarrow$ AD = $20\sqrt{3}$ m