CBSE SAMPLE PAPER - 06

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If a line with direction ratio 2 : 1 : 1 intersects the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z+2}{3}$ at A and B then $|\overrightarrow{\mathrm{AB}}|$ is: **[1]**

   a) $2\sqrt{3}$ b) $2\sqrt{5}$

   c) $2\sqrt{7}$ d) $2\sqrt{6}$

2. $\int \frac{(x^2+1)}{(x^4+1)} dx = ?$ **[1]**

   a) $\frac{1}{\sqrt{2}}\tan^{-1}\left\{\frac{1}{\sqrt{2}}\left(\mathrm{x}-\frac{1}{\mathrm{x}}\right)\right\}+\mathrm{C}$ b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(x-\frac{1}{x}\right)+C$

   c) $\frac{1}{\sqrt{2}}\cot^{-1}\left\{\left(x-\frac{1}{x}\right)\right\}+C$ d) None of these

3. A unit vector perpendicular to both $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is **[1]**

   a) $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$ b) $\hat{i}+\hat{j}+\hat{k}$

   c) $\hat{i}-\hat{j}+\hat{k}$ d) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

4. The area of the region bounded by the circle $x^2 + y^2 = 1$ is **[1]**

   a) 2π sq units b) π sq units

   c) 3π sq units d) 4π sq units

5. Area of the region $\{(x,y)\} \in R^2 : y \geq \sqrt{|x+3|}, 5y \leq (x+9) \leq 15\}$ is equal to **[1]**

   a) $\frac{5}{3}$ b) $\frac{3}{2}$

   c) $\frac{1}{6}$ d) $\frac{4}{3}$

6. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the probability of **[1]**

getting exactly one red ball is

a) $\frac{45}{196}$
b) $\frac{15}{56}$
c) $\frac{15}{29}$
d) $\frac{135}{392}$

7. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident involving a scooter driver, car driver, and truck driver are 0.01, 0.03, and 0.15, respectively. One of the insured persons meets with an accident. The probability that the person is a scooter driver, is **[1]**

a) $\frac{1}{52}$
b) $\frac{19}{52}$
c) $\frac{15}{52}$
d) $\frac{3}{52}$

8. The point at which the maximum value of x + y, subject to the constraints x + 2y $\leq$ 70, 2x + y $\leq$ 95, x, y $\geq$ 0 is obtained, is **[1]**

a) (20, 35)
b) (30, 25)
c) (35, 20)
d) (40,15)

9. If A (1, 2, -3) and B (-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$ then the direction cosines of $\overrightarrow{AB}$ **[1]**

a) $\frac{-1}{2}, -1, 1$
b) -2, - 4, 4
c) $\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$
d) none of these

10. Integrating factor of the differential equation $\frac{dy}{dx}$ + y tan x - sec x = 0 is: **[1]**

a) sec x
b) $e^{\cos x}$
c) $e^{\sec x}$
d) cos x

11. The solution of $\frac{dy}{dx} = \sqrt{1 - x^2 - y^2 + x^2y^2}$ is, where, C is an arbitrary constant. **[1]**

a) $2\sin^{-1} y = x\sqrt{1-x^2} + \cos^{-1} x + C$
b) $2\sin^{-1} y = \sqrt{1-x^2} + \sin^{-1} x + C$
c) $2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C$
d) $\sin^{-1} y = \sin^{-1} x + C$

12. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x + \cot^5 x)}$ equals **[1]**

a) $\frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$
b) $\frac{\pi}{40}$
c) $\frac{1}{5}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$
d) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

13. The general solution of the differential equation $\frac{ydx - xdy}{y} = 0$ is **[1]**

a) $y = Cx^2$
b) $x = Cy^2$
c) y = Cx
d) xy = C

14. If y = ax^2 + bx + c, then $y^3 \frac{d^2y}{dx^2}$ is **[1]**

a) a constant
b) a function of x only
c) a function of y only
d) a function of x and y

15. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ then x = ?. **[1]**

a) 1
b) -2
c) $\frac{1}{2}$
d) 2

16. The solution of the differential equation $\frac{dy}{dx} - \frac{y(x+1)}{x} = 0$ is given by **[1]**

a) $xy = e^x + C$

b) $y = xe^{x + c}$

c) $x = ye^x$

d) $y = x + c$

17. The domain of the function $y = \sin^{-1}(-x^2)$ is **[1]**

a) [0, 1]

b) ϕ

c) (0, 1)

d) [–1, 1]

18. Find the shortest distance between the lines $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda\left(\hat{i} - \hat{j} + \hat{k}.\right)$ $and\ \vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + \widehat{2k}.\right)$, $\lambda, \mu \in R$ **[1]**

a) $\frac{4\sqrt{2}}{2}$

b) $\frac{3\sqrt{2}}{5}$

c) $\frac{3\sqrt{2}}{2}$

d) $\frac{5\sqrt{2}}{2}$

19. Let the feasible region of the linear programming problem with the objective function Z = ax + by is unbounded and let M and m be the maximum and minimum value of Z, respectively. **[1]**

Now, consider the following statements

Assertion (A): M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.

Reason (R): m is the minimum value of Z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, Z has no minimum value.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If y = sin x, then $\frac{d^3y}{dx^3}$ = -1 at x = 0. **[1]**

Reason (R): If y = f(x)·g(x), then $\frac{dy}{dx}$ = f(x)· $\frac{d}{dx}$g(x) + g(x)$\frac{d}{dx}$f(x).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the general solution of the differential equation: $(x + y)\frac{dy}{dx} = 1$ **[2]**

22. If y = $\sqrt{x^2+1}$ - log $\left\{\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}\right\}$, find $\frac{dy}{dx}$. **[2]**

23. Find the angle between the pair of lines: **[2]**

$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$ and $\vec{r} = \hat{i} - \hat{j} + 2\hat{k} - \mu(2\hat{i} + 4\hat{j} - 4\hat{k})$

OR

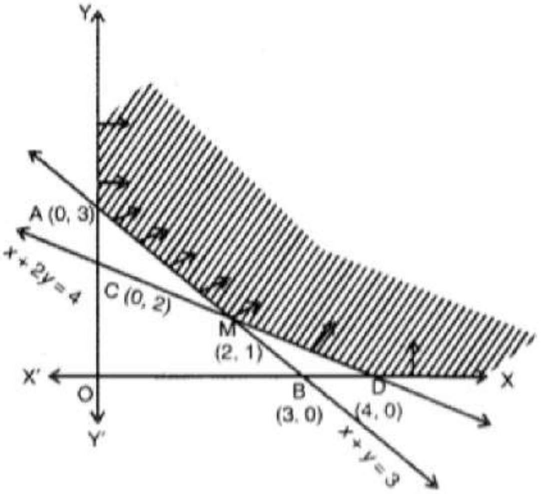
Find the vector and Cartesian equations of the line passing through the points A(2, -3, 0) and B(-2, 4, 3).

24. Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferred from bag I to bag II and then two balls are drawn at random (without replacement) from bag II. The balls so drawn are both found to be black. Find the probability that the transferred ball is black. **[2]**

25. Evaluate $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$ **[2]**

Section C

26. The feasible region for an LPP is shown in fig. Evaluate Z = 4x + y at each of the corner points of this region. Find the minimum value of Z, if it exists. **[3]**



27. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$ **[3]**

OR

Find the area of the region bounded by $x^2 = 16y$, y = l, y = 4 and the y-axis in the first quadrant.

28. $\int \frac{2+\sin 2x}{1+\cos 2x} e^x dx$ **[3]**

OR

Evaluate: $\int \frac{x^2}{(a^6-x^6)} dx$

29. A line passing through the point A with position vector $\vec{a}=4\hat{i}+2\hat{j}+2\hat{k}$ is parallel to the vector $\overrightarrow{\mathrm{b}}=2\hat{\mathrm{i}}+3\hat{\mathrm{j}}+6\hat{\mathrm{k}}$. Find the length of the perpendicular drawn on this line from a point P with position vector $\overrightarrow{\mathrm{r}}_1=\hat{\mathrm{i}}+2\hat{\mathrm{j}}+3\hat{\mathrm{k}}$. **[3]**

OR

Find the image of the point having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane $r.\left(2\hat{i}-\hat{j}+\hat{k}\right)+3=0$

30. Find the area of the region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$ **[3]**

31. Find the values of a and b so that the function f(x) defined by **[3]**

$$f(x)=\begin{cases} x+a\sqrt{2}\sin x & \text{, if } 0\leq x<\pi/4 \\ 2x\cot x+b & \text{, if } \pi/4\leq x<\pi/2 \\ a\cos 2x-b\sin x, & \text{if } \pi/2\leq x\leq \pi \end{cases}$$ becomes continuous on $[0,\pi]$

Section D

32. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k},\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{p}$, which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}\cdot\vec{c}=18$. **[5]**

33. Solve the following system of equations by using determinants: **[5]**

x + y + z = 1

ax + by + cz = k

$a^2 x + b^2 y + c^2 z = k^2$

OR

If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}.

34. Show that the function f: $R_0 \rightarrow R_0$, defined as f (x) = $\frac{1}{x}$, is one-one onto, where R_0 is the set non-zero real numbers. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0? **[5]**

OR

Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R.

35. Evaluate $\int_0^{\pi} x \log \sin x dx$ **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Ankit wants to construct a rectangular tank for his house that can hold 80 ft^3 of water. He wants to construct on one corner of terrace so that sufficient space is left after construction of tank. For that he has to keep width of tank constant 5ft, but the length and heights are variables. The top of the tank is open. Building the tank cost ₹20 per sq. foot for the base and ₹10 per sq. foot for the side.

(i) Express cost of tank as a function of height(h).

(ii) Verify by second derivative test that cost is minimum at critical point.

(iii) Find the value of h at which c(h) is minimum.

OR

Find the minimum cost of tank?

37. **Read the text carefully and answer the questions:** **[4]**

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.



(i) Write the matrix summarizing sales data of 2019 and 2020.

(ii) Find the matrix summarizing sales data of 2020.

(iii) Find the total number of cars sold in two given years, by each dealer?

OR

If each dealer receives a profit of ₹ 50000 on sale of a Hatchback, ₹100000 on sale of a Sedan and ₹200000 on sale of an SUV, then find the amount of profit received in the year 2020 by each dealer.

38. **Read the text carefully and answer the questions:** **[4]**

Family photography is all about capturing groups of people that have family ties. These range from the small group, such as parents and their children. New-born photography also falls under this umbrella. Mr Ramesh, His wife Mrs Saroj, their daughter Sonu and son Ashish line up at random for a family photograph, as shown in figure.

(i) Find the probability that daughter is at one end, given that father and mother are in the middle.

(ii) Find the probability that mother is at right end, given that son and daughter are together.

Solution

CBSE SAMPLE PAPER - 06

Class 12 - Mathematics

Section A

1. **(d)** $2\sqrt{6}$

   Explanation: $2\sqrt{6}$

2. **(a)** $\frac{1}{\sqrt{2}}\tan^{-1}\left\{\frac{1}{\sqrt{2}}\left(x - \frac{1}{x}\right)\right\} + C$

   Explanation: Formula:- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

   Therefore ,

   $\Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-2+2}dx$

   $= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}dx$

   Put $x - \frac{1}{x} = t \Rightarrow \left(1 + \frac{1}{x^2}\right)dx = dt$

   $\Rightarrow \int \frac{1}{t^2+2}dt = \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}} + c$

   $= \frac{1}{\sqrt{2}}\tan^{-1}\left[\frac{1}{\sqrt{2}}\left(x - \frac{1}{x}\right)\right] + c$

3. **(a)** $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

   Explanation: $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

   Let:

   $\vec{a} = \hat{i} + \hat{j} + 0\hat{k}$

   $\vec{b} = 0\hat{i} + \hat{j} + \hat{k}$

   $\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$

   $= \hat{i} - \hat{j} + \hat{k}$

   $\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{1+1+1}$

   $= \sqrt{3}$

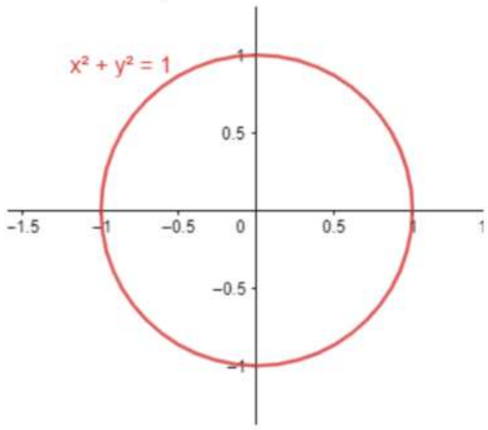
   Unit vector perpendicular to $\vec{a}$ and $\vec{b} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

4. **(b)** π sq units

   Explanation:

   Given;

   The circle $x^2 + y^2 = 1$



   By the symmetry of the circle with x-axis and y-axis.

   Required area

$= 4\int_0^1 (\sqrt{1-x^2})dx$

$= \left[\int \sqrt{a^2-x^2}dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]$

$= 4\int_0^1 (\sqrt{1^2-x^2})dx$

$= 4\left[\frac{x\sqrt{1^2-x^2}}{2} + \frac{1^2}{2}\sin^{-1}\left(\frac{x}{1}\right)\right]_0^1$

$= 4\left(0 - \frac{\pi}{4} - 0 - 0\right)$

$= \pi$ sq.units

5. **(b)** $\frac{3}{2}$

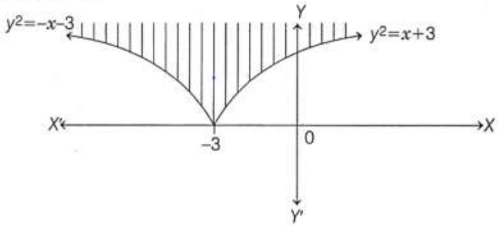
Explanation: Here, $\{(x,y)\} \in R^2 : y \geq \sqrt{|x+3|}\ , 5y \leq (x+9) \leq 15\}$

$\therefore \quad y \geq \sqrt{x+3}$

$\Rightarrow \quad y \geq \begin{cases} \sqrt{x+3}, \text{ when } x \geq -3 \\ \sqrt{-x-3}, \text{ when } x \leq -3 \end{cases}$

or $y^2 \geq \begin{cases} x+3, \text{ when } x \geq -3 \\ -3-x, \text{ when } x \leq -3 \end{cases}$

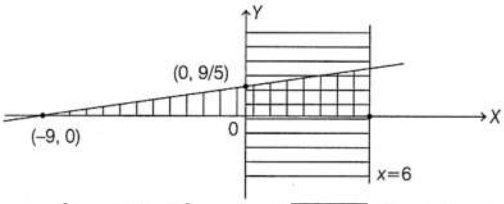
Shown as



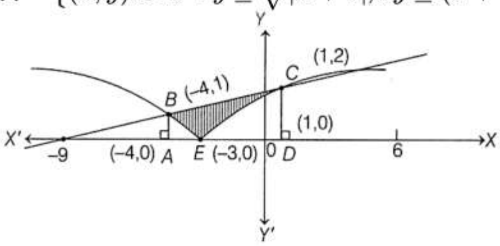
Also, $5y \leq (x+9) \leq 15$

$\Rightarrow \quad (x+9) \geq 5$ y and $x \leq 6$

Shown as



$\therefore \quad \left\{(x,y) \in R^2 : y \geq \sqrt{|x+3|}, 5y \leq (x+9) \leq 15\right\}$



$\therefore$ Required area = Area of trapezium ABCD - Area of ABE under parabola - Area of CDE under parabola

$= \frac{1}{2}(1+2)(5) - \int_{-4}^{-3}\sqrt{-(x+3)}dx - \int_{-3}^{1}\sqrt{(x+3)}dx$

$= \frac{15}{2} - \left[\frac{(-3-x)^{3/2}}{-\frac{3}{2}}\right]_{-4}^{-3} - \left[\frac{(x+3)^{3/2}}{\frac{3}{2}}\right]_{-3}^{1}$

$= \frac{15}{2} + \frac{2}{3}[0-1] - \frac{2}{3}[8-0] = \frac{15}{2} - \frac{2}{3} - \frac{16}{3} = \frac{15}{2} - \frac{18}{3} = \frac{3}{2}$

6. **(b)** $\frac{15}{56}$

Explanation: Probability of getting exactly one red (R) ball $= P_R \cdot P_B \cdot P_B + P_B \cdot P_R \cdot P_R + P_B \cdot P_B \cdot P_R$

$= \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{5}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{5}{6}$

$= \frac{15}{4 \cdot 7 \cdot 6} + \frac{15}{4 \cdot 7 \cdot 6} + \frac{15}{4 \cdot 7 \cdot 6}$

$= \frac{5}{56} + \frac{5}{56} + \frac{5}{56} = \frac{15}{56}$.

Which is the required solution

7. **(a)** $\frac{1}{52}$

Explanation: Let P(A) = P(scooter) = $\frac{2000}{12000} = \frac{1}{6}$

$P(B) = P(car) = \frac{4000}{12000} = \frac{1}{3}$

and $P(C) = P$ (truck) $= \frac{6000}{12000} = \frac{1}{2}$

Let E = Event that person meets with accident.

Then, $P\left(\frac{E}{A}\right) = \frac{1}{100}, P\left(\frac{E}{B}\right) = \frac{3}{100}, P\left(\frac{E}{C}\right) = \frac{15}{100}$

∴ Required probability

$$= \frac{P(A)\cdot P\left(\frac{E}{A}\right)}{P(A)\cdot P\left(\frac{E}{A}\right)+P(B)\cdot P\left(\frac{E}{B}\right)+P(C)\cdot P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100}+\frac{1}{3}\times\frac{3}{100}+\frac{1}{2}\times\frac{15}{100}} = \frac{\frac{1}{6}}{\frac{1}{6}+1+\frac{15}{2}}$$

$$\frac{\frac{1}{6}}{\frac{1+6+45}{6}} = \frac{1}{52}$$

8. **(d)** (40,15)

Explanation: We need to maximize the function z = x + y Converting the given inequations into equations, we obtain

x + 2y = 70, 2x + y = 95, x = 0 and y = 0

Region represented by $x + 2y \leq 70$:

The line x + 2y = 70 meets the coordinate axes at A(70, 0) and B(0, 35) respectively. By joining these points we obtain the line x + 2y = 70. Clearly (0, 0) satisfies the inequation $x + 2y \leq 70$. So, the region containing the origin represents the solution set of the inequation $x + 2y \leq 70$.

Region represented by $2x + y \leq 95$:

The line 2x + y = 95 meets the coordinate axes at $C\left(\frac{95}{2}, 0\right)$ respectively. By joining these points we

obtain the line 2x + y = 95

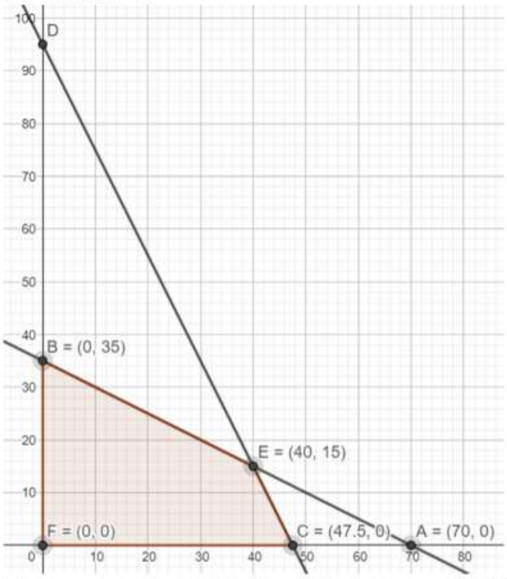
Clearly (0, 0) satisfies the inequation $2x + y \leq 95$. So, the region containing the origin represents the solution set of the inequation $2x + y \leq 95$

Region represented by $x \geq 0$ and $y \geq 0$:

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$, and $y \geq 0$

The feasible region determined by the system of constraints $x + 2y \leq 70$, $2x + y \leq 95$, $x \geq 0$, and $y \geq 0$

are as follows.



The corner points of the feasible region are O(0, 0), C($\frac{95}{2}$, 0) E(40, 15) and B(0, 35).

The value fo Z at these corner points are as follows.

Corner point : z = x + y

O(0, 0) : 0 + 0 = 0

$C\left(\frac{95}{2}, 0\right) : \frac{95}{2} + 0 = \frac{95}{2}$

E(40, 15) : 40 + 15 = 55

B(0, 35) : 0 + 35 = 35

We see that maximum value of the objective function Z is 55 which is at (40, 15).

9. **(c)** $\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$

Explanation: Given points are A(1, 2, -3) and B(-1, -2, 1)

$\Rightarrow$ Direction ratio of $\overrightarrow{AB}$ are (-2, -4, 4)

$\Rightarrow$ Direction cosines of $\overrightarrow{AB}$ are $(\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3})$

10. **(a)** sec x

Explanation: Given that $\frac{dy}{dx}$ + y tan x - sec x = 0

Here, P = tan x, Q = sec x

IF = $e^{\int Pdx} = e^{\int \tan x dx}$

= $e^{\log \sec x}$

= sec x

11. **(c)** $2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C$

Explanation: $2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C$

12. **(a)** $\frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

Explanation: Let $I = \int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x + \cot^5 x)}$

$= \int_{\pi/6}^{\pi/4} \frac{(1+\tan^2 x)\tan^5 x}{2\tan x(\tan^{10} x+1)} dx$ $[\because \sin 2x = \frac{2tanx}{1+\tan^2 x}]$

$= \frac{1}{2}\int_{\pi/6}^{\pi/4} \frac{\tan^4 x sec^2 x}{(\tan^{10} x+1)} dx$

Put $\tan^5 x = t$ $[\because \sec^2 x = 1 + \tan^2 x]$

$\Rightarrow 5 \tan^4 x \sec^2 x\, dx = dt$

x	$\frac{\pi}{6}$	$\frac{\pi}{4}$
t	$\left(\frac{1}{\sqrt{3}}\right)^5$	1

$\therefore I = \frac{1}{2} \cdot \frac{1}{5}\int_{(1/\sqrt{3})^5} \frac{dt}{t^2+1} = \frac{1}{10}\left(\tan^{-1}(t)\right)_{(1/\sqrt{3})^5}^{1}$

$= \frac{1}{10}\left(\tan^{-1}(1) - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

$= \frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

13. **(c)** y = Cx

Explanation: It is given that $\frac{\text{ydx}-\text{xdy}}{\text{y}} = 0$

$\Rightarrow \frac{\text{ydx}-\text{xdy}}{\text{xy}} = 0$

$\Rightarrow \frac{1}{x}dx - \frac{1}{y}dy = 0$

Integrating both sides, we get,

log|x| - log|y| = log k

$\Rightarrow \log\left|\frac{x}{y}\right| = \log k$

$\Rightarrow \frac{x}{y} = k$

$\Rightarrow \text{y} = \frac{1}{\text{k}}\text{x}$

$\Rightarrow$ y = Cx where C = $\frac{1}{k}$

14. **(c)** a function of y only

Explanation: y = ax^2 + bx + c

$\frac{dy}{dx} = 2ax + b$

$\frac{d^2y}{dx^2} = 2a$

$y^3\frac{d^2y}{dx^2} = 2ay^3$ = A function of y only

15. **(c)** $\frac{1}{2}$

Explanation: We know that A × A^{-1} = I

$$\begin{pmatrix} 2x & 0 \\ x & x \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2x \times 1 + 0 \times (-1) & 2x \times 0 + 0 \times 2 \\ x \times 1 + x \times (-1) & x \times 0 + x \times 2x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2x & 0 \\ 0 & 2x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

To satisfy the above condition 2x = 1

$x = \frac{1}{2}$

16. **(b)** $y = xe^{x + c}$

Explanation: We have,

$\frac{dy}{dx} - \frac{y(x+1)}{x} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y(x+1)}{x}$

$\Rightarrow \frac{dy}{y} = \frac{x+1}{x} dx$

$\Rightarrow \int \frac{dy}{y} = \int \frac{x+1}{x} dx$

$\Rightarrow \int \frac{dy}{y} = \int \left(1 + \frac{1}{x}\right) dx$

$\Rightarrow \log y = x + \log x + c$

$\Rightarrow \log\left(\frac{y}{x}\right) = x + c$

$\Rightarrow \frac{y}{x} = e^{x+c}$

$\Rightarrow y = xe^{x+c}$

17. **(d)** [–1, 1]

Explanation: $y = \sin^{-1}(-x^2) \Rightarrow \sin y = -x^2$

i.e. $-1 \leq -x^2 \leq 1$ (since $-1 \leq \sin y \leq 1$)

$\Rightarrow 1 \geq x^2 \geq -1$

$\Rightarrow 0 \leq x^2 \leq 1$

$\Rightarrow |x| \leq 1$ i.e. $-1 \leq x \leq 1$

18. **(c)** $\frac{3\sqrt{2}}{2}$

Explanation: On comparing the given equations with: $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$, and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$, we get:

$\vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b_1} = \hat{i} - \hat{j} + \hat{k}$, and $\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}, \vec{b_2} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\therefore S.D. = \left| \frac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right| = \left| \frac{(-3\hat{i} + 3\hat{k}).(\hat{i} - 3\hat{j} - 2\hat{k})}{3\sqrt{2}} \right|$

$= \left| \frac{-3 + -6}{3\sqrt{2}} \right| = \left| \frac{-9}{3\sqrt{2}} \right| = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

19. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: In case, the feasible region is unbounded, we have

Assertion: M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.

Reason: Similarly, m is the minimum value of Z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, Z has no minimum value. Hence, Assertion is true and Reason is true but Reason is not the correct explanation of Assertion.

20. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Both A and R are true but R is not the correct explanation of A.

Section B

21. The given differential equation is: $(x+1)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x+y} \Rightarrow \frac{dx}{dy} = x + y \Rightarrow \frac{dx}{dy} - x = y$

This is of the form $\frac{dy}{dx} + Px = Q$

Where P = -1 and Q = y

Now, $I.F = e^{\int P(dy)} = e^{-\int dy} = e^{-y}$

Solution is $(IF)x = \int (IF)Q\, dy + c$

$\Rightarrow e^{-y} \cdot x = \int e^{-y} \cdot y \cdot dy + C$

$x \cdot e^{-y} = y\int e^{-y}dy - \int\left\{\frac{dy}{dy}\int e^{-y}dy\right\}dy + C$

$\Rightarrow x \cdot e^{-y} = -y \cdot e^{-y} + \int e^{-y}dy + C$

$\Rightarrow x \cdot e^{-y} = -y \cdot e^{-y} - e^{-y} + C$

$\Rightarrow$ x = -y - 1 + C.e^{-y}

$\Rightarrow$ x + y + 1 = C.e^{y}

22. We have,

y = $\sqrt{x^2+1}$ - log $\left\{\frac{1+\sqrt{x^2+1}}{x}\right\}$

$\Rightarrow$ y = $\sqrt{x^2+1}$ - log {1 + $\sqrt{x^2+1}$ } + log x

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(x^2+1\right)^{-1/2}$ 2x - $\frac{1}{(1+\sqrt{x^2+1})} \cdot \left\{\frac{1}{2}\left(x^2+1\right)^{-1/2} \cdot 2x\right\}$ + $\frac{1}{x}$

= $\frac{x}{\sqrt{x^2+1}}$ - $\frac{1}{(1+\sqrt{x^2+1})} \cdot \frac{x}{\sqrt{x^2+1}}$ + $\frac{1}{x}$

= $\frac{x\{1+\sqrt{x^2+1}\}-x}{(\sqrt{x^2+1})(1+\sqrt{x^2+1})}$ + $\frac{1}{x}$ = $\frac{x\sqrt{x^2+1}}{(\sqrt{x^2+1})(1+\sqrt{x^2+1})}$ + $\frac{1}{x}$

= $\frac{x}{\{1+\sqrt{x^2+1}\}} + \frac{1}{x}$ = $\frac{(x^2+1)+\sqrt{x^2+1}}{x(1+\sqrt{x^2+1})}$

= $\frac{(\sqrt{x^2+1})(\sqrt{x^2+1})+1}{x(1+\sqrt{x^2+1})}$ = $\frac{\sqrt{x^2+1}}{x}$

23. We know that the angle between two lines given with their vector equation is the angle between their parallel vectors always.

we know that

$\cos\theta = \frac{\overrightarrow{b_1}\cdot\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$

Given,

$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$

$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) - \mu(2\hat{i} + 4\hat{j} - 4\hat{k})$

Where, $\overrightarrow{a_1} = 4\hat{i} - \hat{j}$, $\overrightarrow{b_1} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\overrightarrow{a_2} = \hat{i} - \hat{j} + 2\hat{k}$, $\overrightarrow{b_2} = 2\hat{i} + 4\hat{j} - 4\hat{k}$

$|\overrightarrow{b_1}| = \sqrt{(1)^2 + (2)^2 + (-2)^2} = 3$

$|\overrightarrow{b_2}| = \sqrt{(2)^2 + (4)^2 + (-4)^2} = 6$

Let θ be the angle between the given lines. So using dot product we can find the angle as,

$\cos\theta = \frac{\overrightarrow{b_1}\cdot\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$

= $\frac{(\hat{i}+2\hat{j}-2\hat{k})(2\hat{i}+4\hat{j}-4\hat{k})}{3\times6}$

= $\frac{2+8+8}{18}$ = 1

$\Rightarrow$ cos θ = 1

$\Rightarrow \theta = 0^\circ$

OR

Here $\vec{a} = 2\hat{i} - 3\hat{j}$

The direction ratios of the line are (2 + 2) : (-3 - 4) : (0 - 3)

$\Rightarrow$ 4 : 7 : 3

$\Rightarrow$ -4 : 7 : 3

Therefore, $\overrightarrow{b} = -4\hat{i} + 7\hat{j} + 3\hat{k}$

Therefore,we have

Vector form:

$\overrightarrow{r} = 2\hat{i} - 3\hat{j} + \lambda(-4\hat{i} + 7\hat{j} + 3\hat{k})$

Cartesian form:

$\frac{x-2}{-4} = \frac{y+3}{7} = \frac{z}{3}$

24. Let E_1, E_2, E_3 and A be events such that

E_1 = Both transferred ball from Bag I to bag II are red.

E_2 = Both transferred ball from Bag I to bag II are black.

E_3 = Out of two transferred ball one is red and other is black.

A = drawing a red ball from Bag II.

Here, $P\left(\frac{E_2}{A}\right)$ is required.

Now, $P(E_1) = \frac{{}^3C_2}{{}^7C_2} = \frac{3!}{2!1!} \times \frac{2! \times 5!}{7!} = \frac{1}{7}$

$P(E_2) = \frac{{}^4C_2}{{}^7C_2} = \frac{4!}{2!2!} \times \frac{2! \times 5!}{7!} = \frac{2}{7}$

$P(E_3) = \frac{{}^3C_1 \times {}^4C_1}{{}^7C_2} = \frac{3! \times 4!}{7!} \times \frac{2! \times 5!}{1!} = \frac{4}{7}$

$P\left(\frac{A}{E_1}\right) = \frac{6}{11}, P\left(\frac{A}{E_2}\right) = \frac{4}{11}, P\left(\frac{A}{E_3}\right) = \frac{5}{11}$

Therefore, $P\left(\frac{E_2}{A}\right) = \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{P(E_1) \cdot \left(\frac{A}{E_1}\right) + P(E_2) \cdot \left(\frac{A}{E_2}\right) + P(E_3) \cdot \left(\frac{A}{E_3}\right)}$

after solving

$= \frac{\frac{8}{77}}{\frac{6}{77} + \frac{8}{77} + \frac{20}{77}} = \frac{8}{77} \times \frac{77}{34} = \frac{4}{17}$

Therefore, the probability that the transferred balls were both black $= \frac{4}{17}$

25. We have, $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

$\cos\left[\cos^{-1}\left(-cos\frac{\pi}{6}\right) + \frac{\pi}{6}\right]$

$= \cos\left[\cos^{-1}\left(\cos\frac{5\pi}{6}\right) + \frac{\pi}{6}\right]$

$= \cos\left(\frac{5\pi}{6} + \frac{\pi}{6}\right)\{\because \cos^{-1}\cos x = x,\ x \in [0, \pi]\}$

$= \cos\left(\frac{6\pi}{6}\right)$

$= \cos(\pi) = -1$

Section C

26. Consider $x + y = 3$

When x = 0, then y = 3 and

when y = 0, then x = 3

So $A(0,3)\ and\ B(3,0)$ are the points on the line $x + y = 3$

Consider $x + 2y = 4$

When x = 0, then y = 2 and when y = 0, then x = 4.

So C (0, 2) and D (4, 0) are the points on the line x + 2y = 4

The two lines $x + y = 3$ and$x + 2y = 4$, intersect each other at $M(2, 1)$.

So the feasible region is unbounded. Therefore, minimum value may or may not occur. If it occurs, it will be on the corner point.

The comer points are $(4, 0), (2, 1)$ and $(0, 3)$ $Z = 4x + y$

At $(4, 0),\ Z = 4(4) + 0 = 16$

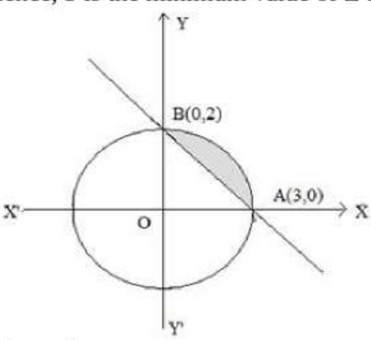
At$(2, 1),\ Z = 4(2) + 1 = 9$

At $(0, 3),\ Z = 4(0) + 3 = 3$ (minimum)

If we draw the graph of $4x + y < 3$, we see that open half plane determined by $4x + y < 3$ and feasible region do not have a point in common other than (0, 3).

Hence, 3 is the minimum value of Z at (0, 3).

27.



$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\frac{x}{3} + \frac{y}{2} = 1$

$\Rightarrow \frac{x^2}{(3)^2}+\frac{y^2}{(2)^2}=1$ is the equation of ellipse and

$\frac{x}{3}+\frac{y}{2}=1$ is the equation of intercept form of line

Area $=\frac{2}{3}\int_0^3\sqrt{9-x^2}dx-\frac{2}{3}\int_0^3(3-x)dx$

$=\frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}sin^{-1}\frac{x}{3}-3x+\frac{x^2}{3}\right]_0^3$

$=\frac{2}{3}\left[(0+\frac{9}{2}(sin^{-1}(-1)-3(3)+\frac{9}{2})-0\right]$

$=\frac{2}{3}[\frac{9\pi}{4}-\frac{9}{2}]$

$=\frac{3}{2}(\pi-2)$sq. unit

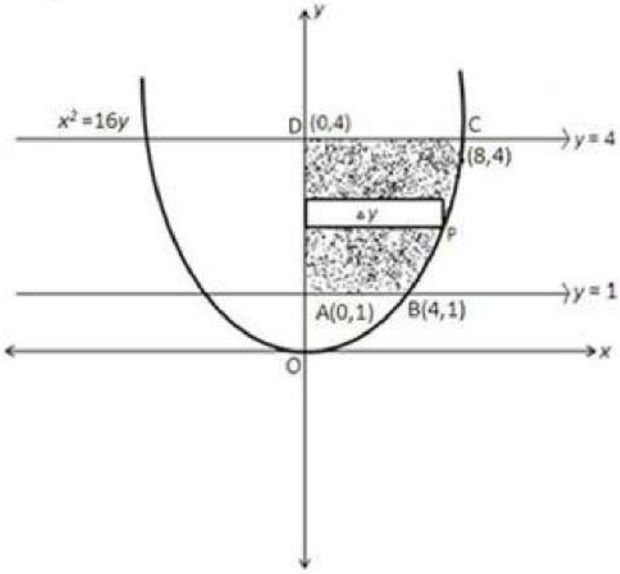
OR

To find region in first quadrant bounded by y = 1, y = 4 and y-axis and

x^2 = 16y.........(i)

Equation (i) represents a parabola with vertex (0, 0) and axes as y=axis.

A rough sketch of the curves is as under:-



Shaded region is required area it is sliced in rectangles of area $x\triangle y$ which slides from y = 1 to y = 4, so

Required area = Area of the Region ABCDA which is given as

$A=\int_1^4 xdy$

$=\int_1^4 4\sqrt{y}dy$

$=4\cdot\left[\frac{2}{3}y\sqrt{y}\right]_1^4$

$=4\cdot\left[\left(\frac{2}{3}\cdot 4\sqrt{4}\right)-\left(\frac{2}{3}\cdot 1\cdot\sqrt{1}\right)\right]$

$=4\left[\frac{16}{3}-\frac{2}{3}\right]$

$A=\frac{56}{3}$ sq. units

28. $I=\int\left(\frac{2+2\sin x.\cos x}{2\cos^2 x}\right)e^x dx$

$\int\left(\frac{2}{2\cos^2 x}+\frac{2\sin x.\cos x}{2\cos^2 x}\right)e^x dx$

$=\int(\sec^2 x+\tan x)e^x dx$

Let f(x) = tanx

$f'(x)=\sec^2 x$

$\because$ We know that $\int e^x\left[f(x)+f'(x)\right]dx=e^x f(x)+c$

$\therefore\int(\sec^2 x+\tan x)e^x dx$

$=e^x.\tan x+c$

OR

To find: $\int\frac{x^2dx}{(a^6-x^6)}$

Formula to be Used in this: $\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+C$

Let y = x^3 ...(i)

Differentiating both sides, we get

dy = $3x^2$ dx

Substituting in given equations,

$\Rightarrow \int \frac{\frac{1}{3}dy}{a^6-y^2}$

$\Rightarrow \frac{1}{3} \int \frac{1}{(a^3)^2-y^2} dy$

$\Rightarrow \frac{1}{3} \times \frac{1}{2a^3} \times \log\left|\frac{a^3+y}{a^3-y}\right| + c$

$\Rightarrow \frac{1}{6a^3}\log\left|\frac{a^3+y}{a^3-y}\right| + C$

From (1),

$\Rightarrow \frac{1}{6a^3}\log\left|\frac{a^3+x^3}{a^3-x^3}\right| + C$

Therefore, we have ..

$\int \frac{x^2 dx}{(a^6-x^6)} = \frac{1}{6a^3}\log\left|\frac{a^3+x^3}{a^3-x^3}\right| + C$

29. Let equation of the line through $\vec{a} = 4\hat{i} + 2\hat{j} + 2\hat{k}$ and parallel to $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ is L.

Now, $\vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

Let M be the foot of perpendicular on the line L drawn from point P with position vector $\vec{r}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$.

So, position vector of M which lies on L is $(2\lambda + 4)i$ + $(3\lambda + 2)\hat{j}$ + $(6\lambda + 2)\hat{k}$.

$\therefore \overrightarrow{PL}$ = $(2\lambda + 3)i$ + $3\lambda\hat{j}$ + $(6\lambda - 1)\hat{k}$

Since $\overrightarrow{PL}$ shall be perpendicular to line L

So, $\overrightarrow{PL}. \vec{b}$ = 0.

That is, $((2\lambda + 3)\hat{i} + 3\lambda\hat{j} + (6\lambda - 1)\hat{k}).(2\hat{i} + 3\hat{j} + 6\hat{k})$ = 0

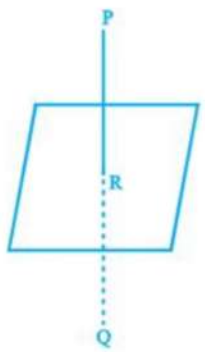
$\Rightarrow \lambda$ = 0.

$\therefore \overrightarrow{PL}$ = $3\hat{i} - \hat{k}$.

$\therefore$ the length of the perpendicular is, $|\overrightarrow{PL}|$ = $\sqrt{9+1}$ = 10 units.

OR

Let the given point be $P\left(\hat{i} + 3\hat{j} + 4\hat{k}\right)$ and Q be the image of P in the plane $\hat{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) + 3 = 0$ as shown in the Fig.



Then PQ is the normal to the plane. Since PQ passes through P and is normal to the given plane, so the equation of PQ is given by $\vec{r} = \left(\hat{i} + 3\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$

Since Q lies on the line PQ, the position vector of Q can be expressed as $\left(\hat{i} + 3\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$ i.e.,

$(1 + 2\lambda)\,i + (3 - \lambda)\,\hat{j} + 4\,(4 + \lambda)\,\hat{k}$

Since R is the mid point of PQ, the position vector of R is $\frac{\left[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(4+\lambda)\hat{k}\right]+\left[\hat{i}+3\hat{j}+4\hat{k}\right]}{2}$

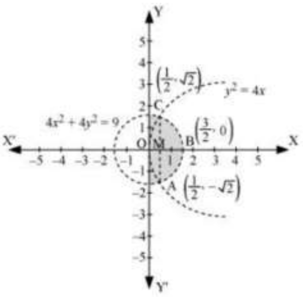
i.e., $(\lambda + 1)\,\hat{i} + \left(3 - \frac{\lambda}{2}\right)\hat{j} + \left(4 + \frac{\lambda}{2}\right)\hat{k}$

Again, since R lies on the plane $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) + 3 = 0$, we have

$\left\{(\lambda - 1)\,\hat{i} + \left(3 - \frac{\lambda}{2}\right)\hat{j} + \left(4 + \frac{\lambda}{2}\right)\hat{k}\right\}.\left(2\hat{i} - \hat{j} - \hat{k}\right) + 3 = 0$

$\Rightarrow \lambda = -2$

Hence, the position vector of Q is $\left(\hat{i} + 3\hat{j} + 4\hat{k}\right) - 2\left(2\hat{i} - \hat{j} + \hat{k}\right)$, i.e., $-3\hat{i} + 5\hat{j} + 2\hat{k}$.

30. 

The area bounded by the curves, $\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$, is shown by shaded region as OABCO

The points of intersection of both the curves are $\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$

We can observed that area OABCO is symmetrical about x-axis

Thus, Area of OABCO = 2 $\times$ Area OBC

Now, Area OBCO = Area OMC + Area MBC

$= \int\limits_0^{\frac{1}{2}} 2\sqrt{x}dx + \int\limits_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2}\sqrt{9-4x^2}dx$

$= \int\limits_0^{\frac{1}{2}} 2\sqrt{x}dx + \int\limits_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2}\sqrt{(3)^2-(2x)^2}dx$

Put 2x = y

$\Rightarrow dx = \frac{dt}{2}$

So, when x = $\frac{3}{2}$, t = 3 and x = $\frac{1}{2}$, t = 1, we get

$= \int\limits_0^{\frac{1}{2}} 2\sqrt{x}dx + \frac{1}{4}\int\limits_1^{3} \sqrt{(3)^2-(t)^2}dt$

$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{1}{2}} + \frac{1}{4}\left[\frac{t}{2}\sqrt{9-t^2} + \frac{9}{2}\sin^{-1}\left(\frac{t}{3}\right)\right]_{-1}^{3}$

$= 2\left[\frac{2}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}}\right] + \frac{1}{4}\left[\left\{\frac{3}{2}\sqrt{9-(3)^2} + \frac{9}{2}\sin^{-1}\left(\frac{3}{3}\right)\right\} - \left\{\frac{1}{2}\sqrt{9-(1)^2} + \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right\}\right]$

$= \frac{2}{3\sqrt{2}} + \frac{1}{4}\left[\left\{0 + \frac{9}{2}\sin^{-1}(1)\right\} - \left\{\frac{1}{2}\sqrt{8} + \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right\}\right]$

$= \frac{\sqrt{2}}{3} + \frac{1}{4}\left[\frac{9\pi}{4} - \sqrt{2} - \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right]$

$= \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right)$

$= \frac{9\pi}{16} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right) + \frac{\sqrt{2}}{12}$

Therefore, the required area is $\left[2 \times \left(\frac{9\pi}{16} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right) + \frac{\sqrt{2}}{12}\right)\right]$

$= \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) + \frac{1}{3\sqrt{2}}$ units

31. Given: f is continuous on $[0, \pi]$

$\therefore$ f is continuous at x = $\frac{\pi}{4}$ and $\frac{\pi}{2}$

At x = $\frac{\pi}{4}$, we have

$\lim\limits_{x \to \frac{\pi}{4}^-} f(x) = \lim\limits_{h \to 0} f\left(\frac{\pi}{4} - h\right)$

$= \lim\limits_{h \to 0}\left[\left(\frac{\pi}{4} - h\right) + a\sqrt{2}\sin\left(\frac{\pi}{4} - h\right)\right]$

$= \left[\frac{\pi}{4} + a\sqrt{2}\sin\left(\frac{\pi}{4}\right)\right] = \left[\frac{\pi}{4} + a\right]$

$\lim\limits_{x \to \frac{\pi}{4}^+} f(x) = \lim\limits_{h \to 0} f\left(\frac{\pi}{4} + h\right)$

$= \lim\limits_{h \to 0}\left[2\left(\frac{\pi}{4} + h\right)\cot\left(\frac{\pi}{4} + h\right) + b\right]$

$= \left[\frac{\pi}{2}\cot\left(\frac{\pi}{4}\right) + b\right] = \left[\frac{\pi}{2} + b\right]$

At x = $\frac{\pi}{2}$, we have,

$\lim\limits_{x \to \frac{\pi}{h}^-} f(x) = \lim\limits_{h \to 0} f\left(\frac{\pi}{2} - h\right) = \lim\limits_{h \to 0}\left[2\left(\frac{\pi}{2} - h\right)\cot\left(\frac{\pi}{2} - h\right) + b\right] = b$

$\lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{2} + h\right) = \lim_{h \to 0} \left[a \cos 2\left(\frac{\pi}{2} + h\right) - b \sin\left(\frac{\pi}{2} + h\right)\right] = -a - b$

Since f is continuous at $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$ we get

$\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^+} f(x)$ and $\lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{x \to \frac{\pi}{4}^+} f(x)$

$\Rightarrow$ - b - a = b and $\frac{\pi}{4} + a = \frac{\pi}{2} + b$

$\Rightarrow b = \frac{-a}{2}$ ...(i) and $\frac{-\pi}{4} = b - a$ ...(ii)

$\Rightarrow \frac{-\pi}{4} = \frac{-3a}{2}$ ...[Substituting the value of b in eq. (ii)]

$\Rightarrow a = \frac{\pi}{6}$

$\Rightarrow b = \frac{-\pi}{12}$ ...[From eq. (i)]

Section D

32. According to the question vectors are

$\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$,

$\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$

$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

Suppose, $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$

We have, $\vec{p}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

$\vec{p} \cdot \vec{a} = 0$

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 4\hat{j} + 2\hat{k}) = 0$

$\Rightarrow \quad x + 4y + 2z = 0$ ...(i)

and $\vec{p} . \vec{b} = 0$

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} - 2\hat{j} + 7\hat{k}) = 0$

$\Rightarrow \quad 3x - 2y + 7z = 0$ ...(ii)

Also, given $\vec{p} \cdot \vec{c} = 18$

$\Rightarrow \quad \left(x\hat{i} + y\hat{j} + z\hat{k}\right) \cdot (2\hat{i} - \hat{j} + 4\hat{k}) = 0$

$\Rightarrow \quad 2x - y + 4z = 18$ ...(iii)

Multiplying Eq. (i) by 3 and subtracting it from Eq. (ii), we get

$-14y + z = 0$

Multiplying Eq. (i) by 2 and subtracting it from Eq. (iii), we get

$-9y = 18$

$\Rightarrow y = -2$

On putting y = -2 and z= -28 in Eq. (i), we get

$x + 4(-2) + 2(-28) = 0$

$\Rightarrow x - 8 - 56 = 0$

$\Rightarrow x = 64$

Hence, the required vector is

$\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$

i.e. $\vec{p} = 64\hat{i} - 2\hat{j} - 28\hat{k}$

33. For the given system of equations, we have

D = $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

$\Rightarrow$ D = $\begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$ [Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$]

$\Rightarrow$ D = (b - a) (c - a) $\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$

$\Rightarrow$ D = (b - a) (c - a) $\begin{vmatrix} 1 & 1 \\ b+a & c+a \end{vmatrix}$

$\Rightarrow$ D = (b - a) (c - a) (c + a - b - a) = (b - c) (c - a) (a - b)

$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ k & b & c \\ k^2 & b^2 & c^2 \end{vmatrix}$ = (b - c) (c - k) (k - b)

$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ a & k & c \\ a^2 & k^2 & c^2 \end{vmatrix}$ = (k - c) (c - a) (a - k)

and, $D_3 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & k \\ a^2 & b^2 & k^2 \end{vmatrix}$ = (a - b) (b - k) (k - a)

$\therefore$ x = $\frac{D_1}{D}$, y = $\frac{D_2}{D}$ and z = $\frac{D_3}{D}$

$\Rightarrow$ x = $\frac{(b-c)(c-k)(k-b)}{(b-c)(c-a)(a-b)}$, y = $\frac{(k-c)(c-a)(a-k)}{(b-c)(c-a)(a-b)}$ and z = $\frac{(a-b)(b-k)(k-a)}{(a-b)(b-c)(c-a)}$

Hence, x = $\frac{(c-k)(k-b)}{(c-a)(a-b)}$, y = $\frac{(k-c)(a-k)}{(b-c)(a-b)}$ and z = $\frac{(b-k)(k-a)}{(b-c)(c-a)}$

is the solution of given system of equations.

OR

Given: $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\therefore A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

L.H.S = A^2 - 5A + 7I = A^2 - 5A + $7I^2$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15 & 5-5 \\ -5+5 & 3-10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -7+7 & 0+0 \\ 0+0 & -7+7 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

= R.H.S.

$\Rightarrow A^2 - 5A + 7I_2 = 0$ ...(i)

To find: A^{-1}, multiplying eq. (i) by A^{-1}.

$\Rightarrow A^2A^{-1} - 5A.A^{-1} + 7I_2A^{-1} = 0.A^{-1}$

$\Rightarrow A - 5I_2 + 7A^{-1} = 0$

$\Rightarrow 7A^{-1} = -A + 5I_2$

$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

34. We observe the following properties of f.

Injectivity: Let x, y $\in R_0$ such that f(x) = f(y). Then,

$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f : $R_0 \rightarrow R_0$ is one-one.

Surjectivity: Let y be an arbitrary element of R_0 (co-domain) such that f(x) = y. Then,

$f(x) = y \Rightarrow \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$

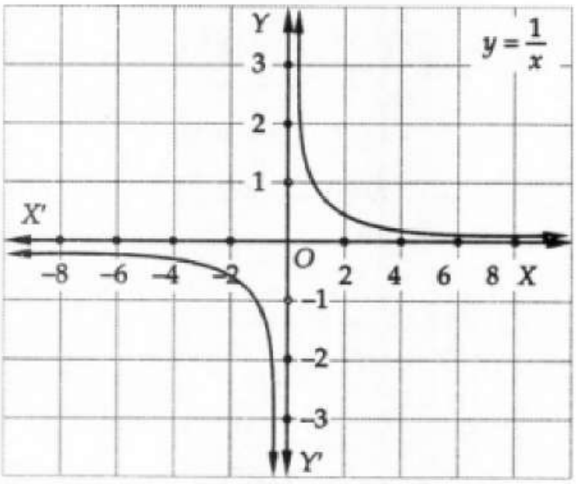
Clearly, $x = \frac{1}{y} \in R_0$ (domain) for all y $\in R_0$ (co-domain).

Thus, for each y $\in R_0$ (co-domain) there exits $x = \frac{1}{y} \in R_0$ (domain) such that $f(x) = \frac{1}{x} = y$

So, f : $R_0 \rightarrow R_0$ is onto.

Hence, f: $R_0 \rightarrow R_0$ is one-one onto.

This is also evident from the graph of f(x) as shown in fig.



Let us now consider f : N → R_0 given by $f(x) = \frac{1}{x}$

For any x, y ∈ N, we find that

f(x) = f(y) ⇒ $\frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f: N → R_0 is one-one.

We find that $\frac{2}{3}, \frac{3}{5}$ etc. in co-domain R_0 do not have their pre-image in domain N. So, f: N →R_0 is not onto.

Thus, f: N →R_0 is one-one but not onto.

OR

R = {(a,b) = |a.b| is divisible by 2.

where $a, b \in A = \{1, 2, 3, 4, 5\}$

reflexivty

For any a∈A,|a−a|=0 Which is divisible by 2.

∴ (a, a) ∈ r for all a ∈ A

So ,R is Reflexive

Symmetric :

Let $(a, b) \in R$ for all a, b ∈ R

|a−b| is divisible by 2

|b−a| is divisible by 2

$(a, b) \in r \Rightarrow (b, a) \in R$

So, R is symmetirc .

Transitive :

Let $(a, b) \in R$ and $(b, c) \in R$ then

$(a, b) \in R$ and $(b, c) \in R$

|a−b| is divisible by 2

|b−c| is divisible by 2

Two cases :

Case 1:

When b is even

$(a, b) \in R$ and $(b, c) \in R$

|a−c| is divisible by 2

|b−c| is divisible by 2

|a−c| is divisible by 2

∴ (a, c) ∈ R

Case 2:

When b is odd

$(a, b) \in R$ and $(b, c) \in R$

|a−c| is divisible by 2

|b−c| is divisible by 2

|a−c| is divisible by 2

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

So R is transitive.

Hence , R is an equivalence relation

35. Given: $\int_0^{\pi} x\ln(\sin x)dx$

Using the property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Let I = $\int_0^{\pi} x\ln(\sin x)dx$

$\Rightarrow \int_0^{\pi}(\pi - x)\ln(\sin(\pi - x))dx = \int_0^{\pi} \pi\ln(\sin x)dx - \int_0^{\pi} x\ln(\sin x)dx$

As sin (π - x) = sin x

$\Rightarrow$ 2I = $\int_0^{\pi} \pi\ln(\sin x)dx = \pi\int_0^{\pi} \ln(\sin x)dx$ ......(i)

Now in $\int_0^{\pi} \ln(\sin x)dx$

Using the property: $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$ (for f(2a - x) = f(x))

$\Rightarrow \int_0^{\pi} \ln(\sin x)dx = 2\int_0^{\frac{\pi}{2}} \ln(\sin x)dx$ ......(ii)

Let Z = $\int_0^{\frac{\pi}{2}} \ln(\sin x)dx$ ......(iii)

Using the property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Z = $\int_0^{\frac{\pi}{2}} \ln\left(\sin\left(\frac{\pi}{2} - x\right)dx = \int_0^{\frac{\pi}{2}} \ln(\cos x)dx\right.$ .....(iv)

Adding equations (iii) and (iv),

2Z = $\int_0^{\frac{\pi}{2}} \ln(\sin x)dx + \int_0^{\frac{\pi}{2}} \ln(\cos x)dx = \int_0^{\frac{\pi}{2}} \ln(\sin x\cos x)dx$ .....(v)

$\Rightarrow \int_0^{\frac{\pi}{2}} \ln(\sin x\cos x)dx = \int_0^{\frac{\pi}{2}} \ln\left(\frac{2\sin x\cos x}{2}\right)dx$

$\Rightarrow \int_0^{\frac{\pi}{2}} \ln\left(\frac{2\sin x\cos x}{2}\right)dx = \int_0^{\frac{\pi}{2}} (\ln(\sin 2x) - \ln 2)dx$

$\Rightarrow \int_0^{\frac{\pi}{2}} (\ln(\sin 2x))dx - \int_0^{\frac{\pi}{2}} (\ln 2)dx = \int_0^{\frac{\pi}{2}} \ln(\sin 2x)dx - \frac{\pi\ln 2}{2}$ ....(vi)

Now in $\int_0^{\frac{\pi}{2}} \ln(\sin 2x)dx$ put 2x = t

$\Rightarrow$ 2dx = dt and limits changes from 0 to π

2Z = $\frac{1}{2}\int_0^{\pi} \ln(\sin t)dt - \frac{\pi\ln 2}{2}$

From equation (ii) $\frac{1}{2}\int_0^{\pi} \ln(\sin t)dt$ again becomes,

2Z = $\frac{2}{2}\int_0^{\frac{\pi}{2}} \ln(\sin t)dt - \frac{\pi\ln 2}{2}$

From equation (iii)

2Z = Z - $\frac{\pi\ln 2}{2}$

Z = $\int_0^{\frac{\pi}{2}} \ln(\sin x)dx = -\frac{\pi\ln 2}{2}$ ......(vii)

On putting (vii) in (ii) and the obtained result in(i)

2I = $-\pi^2\ln 2$

$\Rightarrow \mathrm{I} = \int_0^{\pi} \mathrm{x}\ln(\sin\mathrm{x})\mathrm{dx} = -\frac{\pi^2}{2}\ln 2$

Section E

36. **Read the text carefully and answer the questions:**

Ankit wants to construct a rectangular tank for his house that can hold 80 ft^3 of water. He wants to construct on one corner of terrace so that sufficient space is left after construction of tank. For that he has to keep width of tank constant 5ft, but the length and heights are variables. The top of the tank is open. Building the tank cost ₹20 per sq. foot for the base and ₹10 per sq. foot for the side.

(i) c(h) = 100 h + 320 + $\frac{1600}{h}$

Let l ft be the length and h ft be the height of the tank. Since breadth is equal to 5 ft. (Given)

∴ Two sides will be 5h sq. feet and two sides will be lh sq. feet. So, the total area of the sides is (10 h + 2lh)ft^2

Cost of the sides is ₹10 per sq. foot. So, the cost to build the sides is (10h + 2Ih) × 10 = ₹(100h + 20Ih)

Also, cost of base = (5 l) × 20 = ₹100 l

∴ Total cost of the tank in ₹ is given by c = 100 h + 20 lh + 100l

Since, volume of tank = 80 ft^3

∴ 5lh = 80 ft^3 $\therefore l = \frac{80}{5h} = \frac{16}{h}$

∴ c(h) = 100h + 20 $\left(\frac{16}{h}\right) h + 100\left(\frac{16}{h}\right)$

= 100h + 320 + $\frac{1600}{h}$

(ii) C(h) = 100h + 320 + $\frac{1600}{h}$

$\frac{dC(h)}{dh} = 100 - \frac{1600}{h^2}$

$\frac{d^2C(h)}{dh^2} = -\left(\frac{-2}{h^3}\right)1600$

at h = 4

$\frac{d^2C(h)}{dh^2}$ = 50 > 0

Hence cost is minimum when h = 4 ft

(iii)To minimize cost, $\frac{dc}{dh}$ = 0

⇒ 100 - $\frac{1600}{h^2}$ = 0

⇒ $100h^2 = 1600 \Rightarrow h^2 = 16 \Rightarrow h = \pm 4$

⇒ h = 4 [∵ height can not be negative]

OR

Minimum cost of tank is given by

c(4) = 400 + 320 + $\frac{1600}{4}$

= 720 + 400 = ₹1120

37. **Read the text carefully and answer the questions:**

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.


(i)
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{matrix} \textit{Hatchback} & \textit{Sedan} & \textit{SUV} \end{matrix} \\ \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix} \end{array}$$

In 2019, dealer A sold 120 Hatchbacks, 50 Sedans and 10 SUV; dealer B sold 100 Hatchbacks, 30 Sedans and 5 SUVs and dealer C sold 90 Hatchbacks, 40 Sedans and 2 SUVs.

∴ Required matrix, say P, is given by

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{matrix} \textit{Hatchback} & \textit{Sedan} & \textit{SUV} \end{matrix} \\ \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix} \end{array}$$

In 2020, dealer A sold 300 Hatchbacks, 150 Sedans, 20 SUVs dealer B sold 200 Hatchbacks, 50 sedans, 6 SUVs dealer C sold 100 Hatchbacks, 60 sedans, 5 SUVs.

∴ Required matrix, say Q, is given by

$$Q = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{matrix} \textit{Hatchback} & \textit{Sedan} & \textit{SUV} \end{matrix} \\ \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix} \end{array}$$

(ii)
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} \textit{Hatchback} & \textit{Sedan} & \textit{SUV} \end{array} \\ \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix} \end{array}$$

In 2020, dealer A sold 300 Hatchback, 150 Sedan, 20 SUV dealer B sold 200 Hatchback, 50 sedan, 6 SUV dealer C sold 100 Hatchback, 60 sedan, 5 SUV.

∴ Required matrix, say Q, is given by

$$Q = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} \textit{Hatchback} & \textit{Sedan} & \textit{SUV} \end{array} \\ \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix} \end{array}$$

(iii) Total number of cars sold in two given years, by each dealer, is given by

$$P + Q = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} \textit{Hatchback} & \textit{Sedan} & \textit{SUV} \end{array} \\ \begin{bmatrix} 120+300 & 50+150 & 10+20 \\ 100+200 & 30+50 & 5+6 \\ 90+100 & 40+60 & 2+5 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} \textit{Hatchback} & \textit{Sedan} & \textit{SUV} \end{array} \\ \begin{bmatrix} 420 & 200 & 30 \\ 300 & 80 & 11 \\ 190 & 100 & 7 \end{bmatrix} \end{array}$$

OR

The amount of profit in 2020 received by each dealer is given by the matrix

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} \textit{Hatchback} & \textit{Sedan} & \textit{SUV} \end{array} \\ \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix} \end{array} \begin{bmatrix} 50000 \\ 100000 \\ 200000 \end{bmatrix}$$

$$= \begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 15000000 + 15000000 + 4000000 \\ 10000000 + 5000000 + 1200000 \\ 5000000 + 6000000 + 1000000 \end{bmatrix}$$

$$= \begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 34000000 \\ 16200000 \\ 12000000 \end{bmatrix}$$

38. **Read the text carefully and answer the questions:**

Family photography is all about capturing groups of people that have family ties. These range from the small group, such as parents and their children. New-born photography also falls under this umbrella. Mr Ramesh, His wife Mrs Saroj, their daughter Sonu and son Ashish line up at random for a family photograph, as shown in figure.

(i) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

Let A denotes the event that daughter is at one end n(A) = 12 and B denotes the event that father, and mother are in the middle n(B) = 4

Also, n(A ∩ B) = 4

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{24}}{\frac{4}{24}} = 1$$

(ii) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

Let A denotes the event that mother is at right end. n(A) = 6 and B denotes the event that son and daughter are together.

n(B) = 12

Also, n (A ∩ B) = 4

$$P\left(A/B\right)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{4}{24}}{\frac{12}{24}}=\frac{1}{3}$$