CBSE Test Paper 05
Chapter 9 Mechanical Properties of Solids

1. In a materials testing laboratory, a metal wire made from a new alloy is found to break when a tensile force of 90.8 N is applied perpendicular to each end. If the diameter of the wire is 1.84 mm, what is the breaking stress of the alloy? **1**
   a. 3.41 $\times 10^7$ Pa
   b. 3.61 $\times 10^7$ Pa
   c. 3.31 $\times 10^7$ Pa
   d. 3.51 $\times 10^7$ Pa
2. Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013 $\times 10^5$ Pa), Final volume = 100.5 litre **1**
   a. 2.226 $\times 10^9$ Pa
   b. 2.126 $\times 10^9$ Pa
   c. 2.326 $\times 10^9$ Pa
   d. 2.026 $\times 10^9$ Pa
3. The edge of an aluminum cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminum is 25 GPa. What is the vertical deflection of this face? **1**
   a. 4.0 $\times 10^{-7}$ m
   b. 6.0 $\times 10^{-6}$ m
   c. 8.0 $\times 10^{-6}$ m
   d. 2.0 $\times 10^{-6}$ m
4. A perfectly rigid body is one **1**
   a. which does not move on application of force
   b. whose shape and size do not change on application of force
   c. which starts flowing like water on application of force
   d. whose shape and size change on application of force
5. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03 $\times 10^3$ kg m^{-3}? **1**
   a. 1.054 $\times 10^3$ kg/ m^3

b. $1.074 \times 10^3\ kg/m^3$
c. $1.094 \times 10^3\ kg/m^3$
d. $1.034 \times 10^3\ kg/m^3$

6. Is stress a vector quantity? **1**
7. Is it possible to double the length of a metallic wire by applying a force over it? **1**
8. How are we able to break a wire by repeated bending? **1**
9. The Young's modulus for steel is much more than that for rubber. For the same longitudinal strain, which one will have greater tensile stress? **2**
10. The length of a metal is l_1, when the tension in it is T_1 and is l_2 when tension is T_2. Find the original length of wire? **2**
11. Which is more elastic rubber or steel? Explain. **2**
12. Explain the following: **3**
    i. Concrete beams used in large buildings have greater depth than breadths.
    ii. Load bearing bars are generally made in I-section.
    iii. Pillar with distributed ends is preferred over a pillar with rounded ends.
13. A force of 5×10^3 N is applied tangentially to the upper face of a cubical block of steel of side 30 cm. Find the displacement of the upper face relative to the lower one, and the angle of shear. The shear modulus of steel is 8.3×10^{10} pa. **3**
14. The Marina trench is located in the Pacific Ocean, and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume 0.32 m^3 bis dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom? (Bulk modulus of steel, k = 1.6×10^{11} N/m^2) **3**
15. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path. **5**

Answer

1. a. 3.41×10^7 Pa

   Explanation: given tensile force F = 90.8 N

   diameter d = 1.84 mm

   $radius\ r = \frac{1.84}{2} = 0.92\ mm\ = 0.92 \times\ 10^{-3}\ m$

   $stress = \frac{tensile\ force}{area} = \frac{90.8}{\pi r^2} = \frac{90.8}{3.14 \times (0.92 \times 10^{-3})^2}$

   $stress = 3.41 \times 10^7\ N/m^2$

2. d. 2.026×10^9 Pa

   Explanation: bulk modulus is given by B= $\frac{\Delta P}{\Delta V/V}$

   $given\ \Delta P = 100\ atm = 100 \times 1.013 \times 10^5\ pa$

   $V_{INITIAL} = 100.0\ lit \qquad V_{FINAL} = 100.5\ lit$

   $\Delta V = V_{FINAL} - V_{INITIAL} = 100.5 - 100.0 = 0.5 B = \frac{100\ \times 1.013 \times 10^5}{0.5/100.0}$

   $B = 2.026 \times 10^9\ pa$

3. a. 4.0×10^{-7} m

   Explanation: Edge of the aluminium cube, *L* = 10 cm = 0.1 m

   The mass attached to the cube, *m* = 100 kg

   Shear modulus (η) of aluminium = 25 GPa = 25×10^9 Pa

   $shear\ modulus\ (\eta) = \frac{shear\ stress}{shear\ strain} = \frac{T \times L}{A \times \Delta L}$

   Where,

   T = restoring fprce=Applied force(F) = *m*g = 100 × 9.8 = 980 N

   A = Area of one of the faces of the cube = 0.1 × 0.1 = 0.01 m^2

   Δ*L* = Vertical deflection of the cube

   $\therefore shear\ modulus\ (\eta) = \frac{shear\ stress}{shear\ strain} = \frac{T \times L}{A \times \Delta L}$

   $\Delta L = \frac{T \times L}{\eta \times A} = \frac{980 \times 0.1}{10^{-2} \times 25 \times 10^9}$

   $\Delta L = 3.92 \times 10^{-7}\ m\ \approx 4.0 \times 10^{-7}\ m$

4. b. whose shape and size do not change on application of force

   Explanation: A perfectly rigid body is hypothetical in nature but for some phenomena (in rotational bodies) we assume bodies are perfectly rigid i.e. the intermolecular forces are always in equilibrium irrespective of the external forces due to which their shape and size are constant.

5. d. $1.034 \times 10^3\ \mathrm{kg/m^3}$

Explanation: Let the given depth be h.

Pressure at the given depth, p = 80.0 atm = $80 \times 1.01 \times 10^5$ Pa

Density of water at the surface, $\rho_1 = 1.03 \times 10^3 \mathrm{kg\ m^{-3}}$

Let ρ_2 be the density of water at the depth h.

Let V_1 be the volume of water of mass m at the surface.

Let V_2 be the volume of water of mass m at the depth h.

Let ΔV be the change in volume.

$$\Delta V = V_1 - V_2 \quad = m\left[\frac{1}{\rho_1} - \frac{1}{\rho_2}\right]$$

$$volumetric\ strain \ = \frac{\Delta V}{V_1} = m\left[\frac{1}{\rho_1} - \frac{1}{\rho_2}\right] \times \frac{\rho_1}{m}\,\frac{\Delta V}{V_1} = 1 - \frac{\rho_1}{\rho_2} \quad \rightarrow (1)$$

$$bulk\ modulus\ B = \frac{P}{\frac{\Delta V}{V_1}}$$

$$\frac{\Delta V}{V_1} = \frac{P}{B}\ \ but\ compressibility\ \frac{1}{B}\ of\ water\ is\ 45.8 \times 10^{-11}\ pa^{-1}$$

$$\frac{\Delta V}{V_1} = 80 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} \ = 3.71 \times 10^{-3} \quad \rightarrow (2)$$

from equation 1 and 2

$$1 - \frac{\rho_1}{\rho_2} = 3.71 \times 10^{-3}$$

$$\rho_2 = \frac{1.03 \times 10^3}{1-(3.71 \times 10^{-3})} \rho_2 = 1.034 \times 10^3\ kg/m^3$$

6. Stress = $\frac{\text{Magnitude of restoring force by solid}}{\text{Area of cross - section}}$

as deforming and restoring force are equal and opposite so no net direction is involved.

Hence, the stress is not a vector quantity just like pressure.

7. No, it is not possible because wires actually break much before it is stretched to double the length as within elastic limit, strain is only order of 10“3,

8. If we bend wires repeatedly elastic fatigue occurs in wires due to which wires loses their elastic behavior and become plastic in nature.Thus they can be broken easily.

9. $Y=\frac{\text{stress}}{\text{str ain}}$ As per question, longitudinal strain for rubber and steel are equal.

$\therefore Y \propto$ stress

$\therefore \frac{Y_{steel}}{Y_{Rubber}}=\frac{(Stress)_{Steel}}{(Stress)_{Rubber}} As\, the Y_{steel}>Y_{Rubber}$

$\therefore \frac{Y_{steel}}{Y_{Rubber}}>1$

$\therefore$ (Stress) $_{\text{steel}}$ is larger than $(\text{Stress})_{\text{Rubber}}$.

10. Let l $\Rightarrow$ original Length of material - wire

A $\Rightarrow$ area of metal - wire

Change in length in the first case $=(l_1-l)$

Change in length is second case $=(l_2-l)$

Now, Young Modulus Y = $\frac{\text{Longitudinal stress}}{\text{longitudinal strain}}=\frac{\frac{T}{A}}{\frac{\Delta l}{l}}$

for first case : $Y_1=\frac{T_1 l}{A(l_1-l)}$

for second case : $Y_2=\frac{T_2 l}{A(l_2-l)}$

as young's modulus remains same $\Rightarrow Y_1=Y_2$

$\frac{T_1 l}{A(l_1-l)}=\frac{T_2 l}{A(l_2-l)}$

$\Rightarrow T_1(l_2-l)=T_2(l_1-l)$

$T_1 l_2-T_1 l=T_2 l_1-T_2 l$

$l(T_2-T_1)=T_2 l_1-T_1 l_2$

$l=\frac{T_2 l_1-T_1 l_2}{T_2-T_1}$

11. Let length and area of rubber and steel rod = l and a respectively

Let Yr = Young’s modulus of elasticity for rubber

Ys = Young’s modulus of elasticity for steel when Stretching force F is applied, Let

Δlr Extension in rubber

Δ ls = Extension in steel

Now, Δlr will be greater that Δls.

Now $Y=\frac{Fl}{a\Delta l}=\frac{\text{Normal stress}}{\text{Longitudinal strain}}$

So, $\text{Yr}=\frac{Fl}{a\Delta lr}$; $\text{Ys}=\frac{Fl}{a\Delta ls}$

Since $\Delta lr>\Delta ls$

So, $Y_r < Y_s$

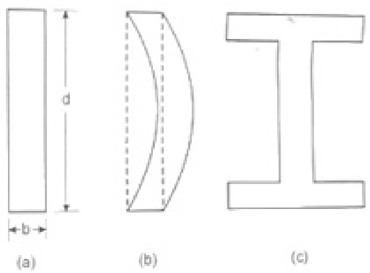
Hence more the modulus of elasticity more elastic is the material, so, steel is more elastic than rubber.

12. i. The major function of beam is to resist moment generated by applied load or to resist the deflection which generates again high Moment on beam. So this applied Moment is resisted through the special property of beam "EI" which includes type of material (Modulus of Elasticity) and dimensions of beam (width & depth). Concrete beams are commonly used in large buildings to support the weight of roof. It is found that depression of a beam of length l, breadth b and depth d when loaded at the centre by a load W and supported at the ends on walls in the middle is given by

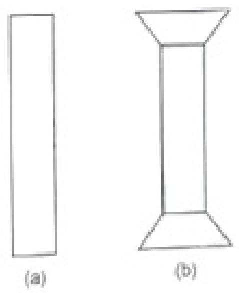
$$\delta = \frac{Wl^3}{4bd^3\gamma}$$

Thus, to have less depression we prefer a beam of greater depth (because $\delta \propto \frac{1}{d^3}$).

ii. A bar of rectangular section with large depth may buckle as shown in Fig. (b), when the load W is not at the right place. To avoid this, we use a I-section beam. It provides a large load bearing surface, enough depth to prevent bending. The shape reduces the weight of the beam without sacrificing the strength and hence reduces the cost.



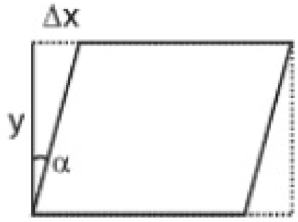
iii. A pillar with distributed ends, as shown in Fig. (b) is preferred over a pillar with rounded ends because it supports much more load than a pillar with rounded ends.



13. applied tangential force $F = 5 \times 10^3 N$

Side of cube $l = 30\text{cm} = 30 \times 10^{-2}\text{m} = 0.30\text{m}$

Area A of the upper face, $A = (0.30)^2 m^2$
Shear modulus of steel, $G = 8.3 \times 10^{10} Pa$
using equation G = $\frac{\text{shearing stress}}{\text{shearing strain}}$ = $\frac{Fl}{A \triangle x}$
where Δx
$\Rightarrow$ displacement of upper face relatives to lower
$\Delta x = \frac{Fl}{AG} = \frac{5 \times 10^3 \times 0.30}{(0.30)^2 \times 8.3 \times 10^{10}}$
$\Delta x = 2.008 \times 10^{-7} m$



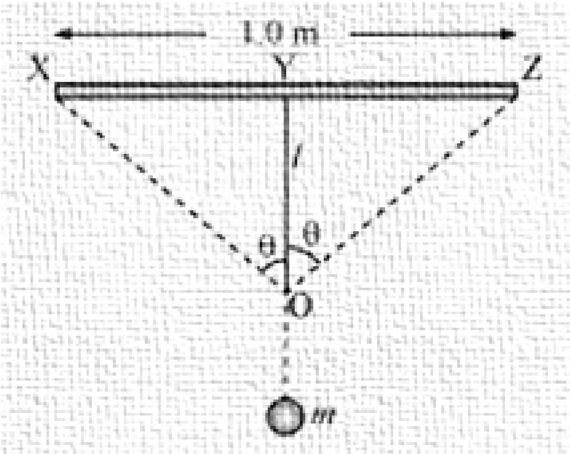
$\therefore$ Angle of shear α is given by tan $\alpha = \frac{\Delta x}{y}$
$\alpha = \tan^{-1}\left(\frac{\Delta x}{y}\right)$
$= \tan^{-1}\left(\frac{2 \times 10^{-7}}{0.30}\right) = \tan^{-1}\left(0.67 \times 10^{-6}\right)$

14. Water pressure at the bottom of the trench, p = 1.1×10^8 Pa

    Initial volume of the steel ball when it remains in air, V_1 = 0.32 m^3

    Now we know that, bulk modulus of steel, k = 1.6 $\times$ 10^{11} Nm^{-2}
    Now the ball falls at the bottom of the Pacific Ocean, which is 11 km beneath the surface of water. Due to the pressure of water there, the volume will of course decrease by some amount.
    Let the change in the volume of the ball on reaching the bottom of the trench be ΔV.
    Now from the equation of bulk modulus, $k = \frac{p}{\frac{\Delta V}{V_1}}$
    $\Rightarrow \Delta V = \frac{pV_1}{B}$
    $\Rightarrow \Delta V = \frac{1.1 \times 10^8 \times 0.32}{1.6 \times 10^{11}}$ (putting all the values of p, V_1 and ΔV)
    $\Rightarrow \Delta V = 2.2 \times 10^{-4} \text{m}^3$
    Therefore, the change in volume of the ball on reaching the bottom of the trench is $= 2.2 \times 10^{-4} \text{m}^3$.

15. Given: Mass, m = 14.5 kg, Length of the steel wire, l = 1.0 m



Angular velocity, $\omega = 2 rev/s = 2 \times 2\pi rad/s = 12.56 rad/s$

Cross - section area of the wire, A = 0.065 cm$^2 = 0.065 \times 10^{-4}$m^2

Let ΔL be the elongation of the wire when the mass is at the lowest point of its path.

At lowest point of vertical circle,

Net Force = Centripetal Force

$F - mg = mr\omega^2$ (where r is radius of vertical circle)

$F - mg = ml\omega^2$ (since r = l, l is length of wire)

$F = (14.5 \times 9.8) + \left(14.5 \times 1 \times 12.56^2\right)$

F = 2429.53 N

Young's modulus of steel, $Y = 2 \times 10^{11} Pa$

Now, Young modulus, $Y = \frac{Fl}{A \triangle l} \Rightarrow \triangle l = \frac{Fl}{YA}$

$\triangle l = \frac{2429.53 \times 1}{2 \times 10^{11} \times 0.065 \times 10^{-4}}$

$\triangle l = 1.87 \times 10^{-3}$m