CBSE Test Paper 01

Chapter 2 Inverse Trigonometric Functions

1. The period of the function f(x) = cos4x + tan3x is
   a. $\frac{\pi}{3}$
   b. π
   c. None of these
   d. $\frac{\pi}{2}$
2. If $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$. Then, x=.
   a. $\frac{1}{\sqrt{3}}$
   b. $\frac{1}{\sqrt{2}}$
   c. 2
   d. 1
3. The value of $\tan 15^0 + \cot 15^0$ is
   a. 4
   b. Not defined
   c. $\sqrt{3}$
   d. $2\sqrt{3}$
4. The values of x which satisfy the trigonometric equation $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ are:
   a. ± 2
   b. $\pm\frac{1}{2}$
   c. $\pm\frac{1}{\sqrt{2}}$
   d. $\pm\sqrt{2}$
5. The minimum value of sinx - cosx is
   a. $-\sqrt{2}$

b. -1

c. 0

d. 1

6. The principle value of $\tan^{-1}\sqrt{3}$ is ________.
7. If y = 2 $\tan^{-1}$x + $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ for all x, then ________ < y < ________.
8. The value of cos ($\sin^{-1}$x + $\cos^{-1}$x), $|x| \leq 1$ is ________.
9. Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.
10. Write the principal value of $\cos^{-1}1$ [cos(680)°].
11. Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$. **(1)**
12. Find the value of the expression $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.
13. Solve the equation: 2$\tan^{-1}$(cosx) = $\tan^{-1}$(2cosec x).
14. Find the value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$. **(2)**
15. Prove that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$.
16. Solve for x, $\tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{3} = \frac{\pi}{4}, \sqrt{6} > x > 0$.
17. Find the value of the following: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.
18. Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$.

Solution

1. b. π, **Explanation:** $f(\pi) = (cos\ 4\pi + tan 3\pi\ )$ gives the same value as f (0). Therefore, the period of the function is π.

2. a. $\frac{1}{\sqrt{3}}$, **Explanation:** $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$
Put x = tanθ
$3\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - 4\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + 2\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \frac{\pi}{3}$
$3\sin^{-1}(\sin 2\theta) - 4\cos^{-1}(\cos 2\theta) + 2\tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$
$3.2\theta - 4.2\theta + 2.2\theta = \frac{\pi}{3} \Rightarrow 2\theta = \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$
$\therefore \tan^{-1}x = \frac{\pi}{6} \Rightarrow x = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

3. a. 4, **Explanation:** $\tan 15^0 + \cot 15^0 = \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$
$= \frac{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2}{2}$ = $\frac{8}{2} = 4$

4. c. $\pm\frac{1}{\sqrt{2}}$, **Explanation:** $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$
$\tan^{-1}\left[\frac{\left(\frac{x-1}{x-2}\right)+\left(\frac{x+1}{x+2}\right)}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\Pi}{4}$
$\tan^{-1}\left[\frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)-(x+1)(x-1)}\right] = \frac{\Pi}{4}$
$\left(\frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}\right) = \tan^{-1}\left(\frac{\Pi}{4}\right)$
$\left(\frac{2x^2-4}{-3}\right) = 1$
$\therefore 2x^2 - 4 = -3$
$\Rightarrow 2x^2 = 1$
$x = \pm\frac{1}{\sqrt{2}}$

5. a. $-\sqrt{2}$, **Explanation:** Since, range of sine function and cosine function is [-1,1]. But, sine is increasing function and cosine is decreasing function. Therefore, the lowest that both together can attain is -45^0.
$\left(-\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{\sqrt{2}}\right) = -\sqrt{2}$

6. $\frac{\pi}{3}$

7. -2π, 2π
8. 0
9. Let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \theta$
   $\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$
   We know that $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
   $\Rightarrow \sin\theta = \sin\frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$
   Therefore, principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$
10. We know that, principal value branch of $\cos^{-1}$ x is [0, 180°].
   Since, 680° $\in$ [0,180°], so write 680° as 2 $\times$ 360°-40°
   Now, $\cos^{-1}$[cos (680)°] = $\cos^{-1}$ [cos(2 ;$\times$ 360°-40°)]
   = $\cos^{-1}$(cos40°) $[\because \cos(4\pi - \theta) = \cos\theta]$
   Since, 40°$\in$ [0,180°]
   $\therefore$ $\cos^{-1}$[cos(680°)] = 40°
   $\left[\because \cos^{-1}(\cos\theta) = \theta; \forall\theta \in [0, 180^\circ]\right]$
   which is the required principal value.
11. **LHS** = $\tan^{-1}\sqrt{x}$
   Let $tan\theta = \sqrt{x}$
   $\tan^2\theta = x$
   R.H.S. $= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$
   $= \frac{1}{2}\cos^{-1}(\cos 2\theta) = \frac{1}{2} \times 2\theta = \theta$
   $= \tan^{-1}\sqrt{x}$
12. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$
   $= \tan^{-1}\left(\tan\frac{4\pi-\pi}{4}\right)$
   $= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$
   $= \tan^{-1}\left[-\tan\frac{\pi}{4}\right]$
   $= \tan^{-1}\tan\left(-\frac{\pi}{4}\right) = -\frac{\pi}{4}$
13. 2 $\tan^{-1}$(cos x) = $\tan^{-1}$(2 cosec x)
   $\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2}{\sin x}\right)$
   $\Rightarrow \frac{2\cos x}{1-\cos^2 x} = \frac{2}{\sin x}$
   $\Rightarrow \frac{\cos x}{\sin x} = 1$
   $\Rightarrow$ cot x = 1 $\Rightarrow x = \frac{\pi}{4}$

14. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$=\sin^{-1}\left(\sin\frac{3\pi-\pi}{3}\right)$

$=\sin^{-1}\left[\sin\left(\pi-\frac{\pi}{3}\right)\right]$

$=\sin^{-1}\sin\frac{\pi}{3}=\frac{\pi}{3}$

15. **To prove,** $tan^{-1}(1)+tan^{-1}(2)+tan^{-1}(3)=\pi$

LHS = $tan^{-1}(1)+tan^{-1}(2)+tan^{-1}(3)$

$=\tan^{-1}(\tan\frac{\pi}{4})+\frac{\pi}{2}-\cot^{-1}(2)+\frac{\pi}{2}-\cot^{-1}(3)\ [\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}]$

$=\frac{\pi}{4}+\pi-[\cot^{-1}(2)+\cot^{-1}(3)]\ [\because \tan^{-1}(\tan\theta)=\theta;\forall\theta\in(-\frac{\pi}{2},\frac{\pi}{2})]$

$=\frac{5\pi}{4}-\left[\tan^{-1}(\frac{1}{2})+\tan^{-1}\left(\frac{1}{3}\right)\right]\ [\because \cot^{-1}x=\tan^{-1}\frac{1}{x},x>0]$

$=\frac{5\pi}{4}-\left[\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right)\right]\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right),\text{ if } xy<1\right]$

$=\frac{5\pi}{4}-\tan^{-1}\left(\frac{5/6}{5/6}\right)$

$=\frac{5\pi}{4}-\tan^{-1}(1)=\frac{5\pi}{4}-\frac{\pi}{4}=\frac{4\pi}{4}=\pi$= **RHS (Hence Proved)**

16. Here, we have to find the value of x .Now, we are given that

$\tan^{-1}\frac{x}{2}+\tan^{-1}\frac{x}{3}=\frac{\pi}{4},\sqrt{6}>x>0$

$\Rightarrow \tan^{-1}\left(\frac{\frac{x}{2}+\frac{x}{3}}{1-\frac{x^2}{6}}\right)=\frac{\pi}{4}\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right);xy<1\right]$

$\Rightarrow \frac{\frac{3x+2x}{6}}{\frac{6-x^2}{6}}=\tan\frac{\pi}{4}$ { taking tan on both sides}

$\Rightarrow \frac{5x}{6-x^2}=1\ [\because \tan\frac{\pi}{4}=1]$

$\Rightarrow$ 5x = 6-x^2

$\Rightarrow$ x^2 + 5x - 6 = 0

$\Rightarrow$ x^2 + 6x - x - 6 = 0

$\Rightarrow$ x (x + 6) - 1 (x + 6) = 0

$\Rightarrow$ (x-1) (x + 6) = 0

$\therefore$ x = 1 or - 6

But it is given that, $\sqrt{6}>\text{x}>0\ \Rightarrow \text{x}>0$

$\therefore$ x = - 6 is rejected.

Hence, x = 1 is the only solution of the given equation.

17. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$= \tan^{-1}\left[2\cos\left(2\sin^{-1}\sin\frac{\pi}{6}\right)\right]$
$= \tan^{-1}\left[2\cos\left(2 \times \frac{\pi}{6}\right)\right]$
$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$
$= \tan^{-1}\left[2 \times \frac{1}{2}\right]$ = $\tan^{-1}1$
$= \tan^{-1}\tan\frac{\pi}{4} = \frac{\pi}{4}$

18. Let θ = $\sin^{-1}(\frac{12}{13})$
$\Rightarrow \sin\theta$ = $\frac{12}{13}$
$\Rightarrow \sqrt{1 - cos^2\theta} = \frac{12}{13}$
$\Rightarrow 1 - cos^2\theta = \frac{(12)^2}{(13)^2}$
$\Rightarrow cos^2\theta = \frac{(5)^2}{(13)^2}$
$\Rightarrow cos\theta = \frac{5}{13}$
Since, $\tan\theta$ = $\frac{sin\theta}{cos\theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$
$\Rightarrow \theta = tan^{-1}(\frac{12}{5})$
Thus, $\theta = sin^{-1}(\frac{12}{13}) = tan^{-1}(\frac{12}{5})$
Similarly, $cos^{-1}(\frac{5}{13}) = tan^{-1}(\frac{12}{5})$
We have, LHS = $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right)$
$= \tan^{-1}\left(\frac{12}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{63}{16}\right)$
$= \left[\tan^{-1}\left(\frac{12}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right)\right] + \tan^{-1}\left(\frac{63}{16}\right)$
{since $\frac{12}{5} \times \frac{3}{4} = \frac{9}{5} > 1$, therefore , $\tan^{-1}A + \tan^{-1}B = \pi + \tan^{-1}\frac{A+\_B}{1-AB}$)
$=\pi + \tan^{-1}\left(\frac{\frac{12}{5}+\frac{3}{4}}{1-\left(\frac{12}{5}\right)\left(\frac{3}{4}\right)}\right) + \tan^{-1}\left(\frac{63}{16}\right)$
$= \pi + \tan^{-1}\left(-\frac{63}{16}\right) + \tan^{-1}\left(\frac{63}{16}\right)$
$= \pi - \tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\left(\frac{63}{16}\right)$ = π Hence Proved.