ત્રિકોણમિતિય વિધેયો

ખૂણાનું માપન

(i) 1 રેડિયન = $\frac{180^\circ}{\pi}$ = 57° 16′ 22″ (લગભગ)

(ii) $1^\circ = \frac{\pi}{180}$ રેડિયન, 1° = (60′ મિ), 1′ = 60″ (60 સે)

(iii) r ત્રિજ્યાવાળા વર્તુળના ચાપની લંબાઈ

(i) $l = \frac{\pi r \alpha}{180}$, જ્યાં r = વર્તુળની ત્રિજ્યા

α = ચાપે કેન્દ્ર આગળ આંતરેલા ખૂણાનું અંશ માપ

(iii) $l = r\theta$, જ્યાં θ = ચાપે કેન્દ્ર આગળ આંતરેલા ખૂણાનું રેડિયન માપ

(iv) કલાકકાંટા (x) અને મિનિટકાંટા (y) વચ્ચે બનતા ખૂણાનું અંશ માપ $\alpha = \frac{|60x - 11y|}{2}$

(v) n બાજુવાળા નિયમિત બહુકોણ દ્વારા બનતા અંતર્ગત ખૂણાના માપનો સરવાળો = $(n - 2)\pi$ છે.

(1) ત્રિકોણમિતિય વિધેયોના પ્રદેશ વિસ્તાર, શૂન્યોનો ગણ અને મુખ્ય આવર્તમાન :

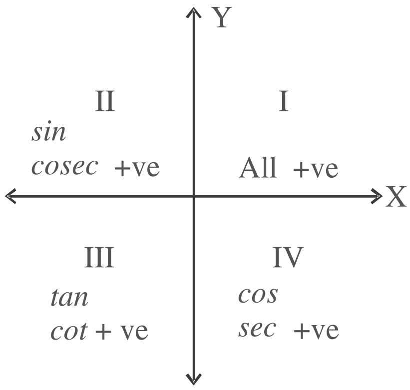
વિધેય	પ્રદેશ	વિસ્તાર	શૂન્યોનો ગણ	મુખ્ય આવર્તમાન
sin	R	[–1, 1]	$\{k\pi \mid k \in Z\}$	2π
cos	R	[–1, 1]	$\{(2k+1)\frac{\pi}{2} \mid k \in Z\}$	2π
tan	$R - \{(2k+1)\frac{\pi}{2} \mid k \in Z\}$	R	$\{k\pi \mid k \in Z\}$	π
cot	$R - \{k\pi \mid k \in Z\}$	R	$\{(2k+1)\frac{\pi}{2} \mid k \in Z\}$	π
sec	$R - \{(2k+1)\frac{\pi}{2} \mid k \in Z\}$	R – (–1, 1)	$\emptyset$	2π
cosec	$R - \{k\pi \mid k \in Z\}$	R – (–1, 1)	$\emptyset$	2π

(2) વધતું-ઘટતું વિધેય :

વિધેય/ચરણ	I	II	III	IV
cos	↓	↓	↑	↑
sin	↑	↓	↓	↑
tan	↑	↑	↑	↑
cot	↓	↓	↓	↓
sec	↑	↑	↓	↓
cosec	↓	↑	↑	↓

અહીં,
↑ = વધતું વિધેય
↓ = ઘટતું વિધેય દર્શાવે છે.

(3) જુદા-જુદા ચરણમાં ત્રિકોણમિતિય વિધેયનાં મૂલ્યનાં ચિહ્ન :



આકૃતિ 12.1

(4) ત્રિકોણમિતિય વિધેયનાં મૂલ્યો :

θ	**0**	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0	–1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	–1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	અવ્યાખ્યાયિત	$-\sqrt{3}$	0	અવ્યાખ્યાયિત	0

(5) વિશિષ્ટ માપના ખૂણા માટે ત્રિવિધેયોનાં મૂલ્યો :

ત્રિ-વિધેયો → ખૂણા ↓	*sin*	*cos*	*tan*	*cot*
$15° = \frac{\pi}{12}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$2 - \sqrt{3}$	$2 + \sqrt{3}$
$75° = \frac{5\pi}{12}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$2 + \sqrt{3}$	$2 - \sqrt{3}$
$18° = \frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{5+2\sqrt{5}}$
$72° = \frac{2\pi}{5}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$
$36° = \frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{5-2\sqrt{5}}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$
$54° = \frac{3\pi}{10}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$	$\sqrt{5-2\sqrt{5}}$
$22\frac{1°}{2} = \frac{\pi}{8}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2} - 1$	$\sqrt{2} + 1$
$67\frac{1°}{2} = \frac{3\pi}{8}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\sqrt{2} + 1$	$\sqrt{2} - 1$

સરવાળાં સૂત્રો :

(1) $cos(\alpha + \beta) = cos\alpha\, cos\beta - sin\alpha\, sin\beta$

(2) $cos(\alpha - \beta) = cos\alpha\, cos\beta + sin\alpha\, sin\beta$

(3) $sin(\alpha + \beta) = sin\alpha\, cos\beta + cos\alpha\, sin\beta$

(4) $sin(\alpha - \beta) = sin\alpha\, cos\beta - cos\alpha\, sin\beta$

(5) $tan(\alpha + \beta) = \dfrac{tan\alpha + tan\beta}{1 - tan\alpha\, tan\beta}$

(6) $tan(\alpha - \beta) = \dfrac{tan\alpha - tan\beta}{1 + tan\alpha\, tan\beta}$

(7) $sin(\alpha + \beta)\ sin(\alpha - \beta) = sin^2\alpha - sin^2\beta$

(8) $cos(\alpha + \beta)\ cos(\alpha - \beta) = cos^2\alpha - sin^2\beta$

(9) $cot(\alpha + \beta) = \dfrac{cot\alpha\, cot\beta - 1}{cot\beta + cot\alpha}$

(10) $cot(\alpha - \beta) = \dfrac{cot\alpha\, cot\beta + 1}{cot\beta - cot\alpha}$

(11) $sin(-\theta) = -sin\theta$ (અયુગ્મ વિધેય)

$cos(-\theta) = cos\theta$ (યુગ્મ વિધેય)

નોંધ : *cos* તથા *sec* સિવાયના તમામ ત્રિકોણમિતીય વિધેયો અયુગ્મ વિધેયો છે.

(12) $sin\left(\dfrac{\pi}{2} + \theta\right) = cos\theta$ તથા $cos\left(\dfrac{\pi}{2} + \theta\right) = -sin\theta$

અવયવ સૂત્રો :

(1) $2sin\alpha\, cos\beta = sin(\alpha + \beta) + sin(\alpha - \beta)$

(2) $2cos\alpha\, sin\beta = sin(\alpha + \beta) - sin(\alpha - \beta)$

(3) $2cos\alpha\, cos\beta = cos(\alpha + \beta) + cos(\alpha - \beta)$

(4) $2sin\alpha\, sin\beta = -cos(\alpha + \beta) + cos(\alpha - \beta)$

(5) $sin\alpha + sin\beta = 2sin\left(\dfrac{\alpha + \beta}{2}\right)\ cos\left(\dfrac{\alpha - \beta}{2}\right)$

(6) $sin\alpha - sin\beta = 2cos\left(\dfrac{\alpha + \beta}{2}\right)\ sin\left(\dfrac{\alpha - \beta}{2}\right)$

(7) $cos\alpha + cos\beta = 2cos\left(\dfrac{\alpha + \beta}{2}\right)\ cos\left(\dfrac{\alpha - \beta}{2}\right)$

(8) $cos\alpha - cos\beta = -2sin\left(\dfrac{\alpha + \beta}{2}\right)\ sin\left(\dfrac{\alpha - \beta}{2}\right)$

ગુણિત-ઉપગુણિત સંખ્યાઓ માટે ત્રિકોણમિતિય વિધેયનાં મૂલ્યો :

(1) $sin2\theta = 2sin\theta\, cos\theta = \dfrac{2tan\theta}{1 + tan^2\theta}$

(2) $cos2\theta = cos^2\theta - sin^2\theta = 1 - 2sin^2\theta = 2cos^2\theta - 1 = \dfrac{1 - tan^2\theta}{1 + tan^2\theta}$

(3) $tan2\theta = \dfrac{2tan\theta}{1 - tan^2\theta}$

(4) $cos^2\theta = \dfrac{1 + cos2\theta}{2}$, $sin^2\theta = \dfrac{1 - cos2\theta}{2}$, $tan^2\theta = \dfrac{1 - cos2\theta}{1 + cos2\theta}$

(5) $sin3\theta = 3sin\theta - 4sin^3\theta$

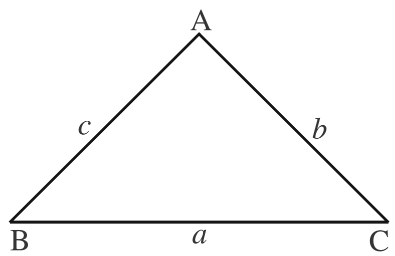
(6) $cos3\theta = 4cos^3\theta - 3cos\theta$

(7) $tan3\theta = \dfrac{3tan\theta - tan^3\theta}{1 - 3tan^2\theta}$

(8) $sin^3\theta = \dfrac{3sin\theta - sin3\theta}{4}$

(9) $cos^3\theta = \dfrac{3cos\theta + cos3\theta}{4}$

ત્રિકોણના ગુણધર્મો



આકૃતિ 12.2

(1) sine સૂત્ર

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} = 2R$$

(2) cosine સૂત્ર

$$cosA = \frac{b^2 + c^2 - a^2}{2bc}, \quad cosB = \frac{a^2 + c^2 - b^2}{2ac}, \quad cosC = \frac{a^2 + b^2 - c^2}{2ab}$$

(3) પ્રક્ષેપ સૂત્ર

$a = b\,cosC + c\,cosB$, $b = a\,cosC + c\,cosA$, $c = a\,cosB + b\,cosA$

(4) અર્ધકોણવાળા સૂત્ર

(i) $sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$, $sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

(ii) $cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$, $cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

(iii) $tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, $tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$, $tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

(5) ત્રિકોણનું ક્ષેત્રફળ

$$\Delta = \frac{1}{2}ab\,sinC = \frac{1}{2}bc\,sinA = \frac{1}{2}ac\,sinB$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \frac{abc}{4R}$$

(6) અંતઃત્રિજ્યાના સૂત્રો

(i) $r = \frac{\Delta}{s}$

(ii) $r = (s - a)tan\frac{A}{2}$	$cot\,A = \frac{b^2 + c^2 - a^2}{4\Delta}$
$r = (s - b)tan\frac{B}{2}$	$cot\,B = \frac{a^2 + c^2 - b^2}{4\Delta}$
$r = (s - c)tan\frac{C}{2}$	$cot\,C = \frac{a^2 + b^2 - c^2}{4\Delta}$

(iii) $r = 4R\,sin\frac{A}{2}\,sin\frac{B}{2}\,sin\frac{C}{2}$

ત્રિકોણમિતિય સમીકરણ :

(i) $cos\theta = a,\ |a| \le 1$ નો ઉકેલ

$\theta = 2k\pi \pm cos^{-1}a,\ k \in Z$

(ii) $sin\theta = a,\ |a| \le 1$ નો ઉકેલ

$\theta = k\pi + (-1)^k sin^{-1}a,\ k \in Z$

(iii) $tan\theta = a,\ a \in R$ નો ઉકેલ

$\theta = k\pi + tan^{-1}a,\ k \in Z$

(iv) $a\,cos\theta + b sin\theta = c$ ના ઉકેલ માટેની શરત $\left|\frac{c}{r}\right| \le 1$, જ્યાં $r = \sqrt{a^2 + b^2}$

$\theta = 2k\pi \pm cos^{-1}\frac{c}{r} - \beta,\ k \in Z$, જ્યાં $cos\beta = \frac{a}{r},\ sin\beta = \frac{b}{r}$.

નોંધ : જો $c^2 > a^2 + b^2$ એટલે કે $c^2 > r^2$ તો ઉકેલગણ $\emptyset$ થાય.

ત્રિકોણમીતિય પ્રતિવિધેયનાં સૂત્રો

1. (i) $sin^{-1}(-x) = -sin^{-1}x,\ |x| \le 1$

(ii) $cos^{-1}(-x) = \pi - cos^{-1}x,\ |x| \le 1$

(iii) $tan^{-1}(-x) = -tan^{-1}x,\ x \in R$

(iv) $cot^{-1}(-x) = \pi - cot^{-1}x,\ x \in R$

(v) $sec^{-1}(-x) = \pi - sec^{-1}x,\ |x| \ge 1$

(iv) $cosec^{-1}(-x) = -cosec^{-1}x,\ |x| \ge 1$

2. (i) $sin^{-1}\frac{1}{x} = cosec^{-1}x;\ |x| \ge 1$

(ii) $cos^{-1}\frac{1}{x} = sec^{-1}x,\ |x| \ge 1$

(iii) $tan^{-1}\frac{1}{x} = \begin{cases} cot^{-1}x, & x > 0 \\ cot^{-1}x - \pi, & x < 0 \end{cases}$

3. (i) $sin^{-1}x + cos^{-1}x = \frac{\pi}{2} \quad |x| \le 1$

(ii) $tan^{-1}x + cot^{-1}x = \frac{\pi}{2} \quad x \in R$

(iii) $sec^{-1}x + cosec^{-1}x = \frac{\pi}{2} \quad |x| \ge 1$

4. જો $x > 0,\ y > 0$, તો

(i) $$tan^{-1}x + tan^{-1}y = \begin{cases} tan^{-1}\left(\frac{x+y}{1-xy}\right) & xy < 1 \\ \pi + tan^{-1}\left(\frac{x+y}{1-xy}\right) & xy > 1 \\ \frac{\pi}{2} & xy = 1 \end{cases}$$

(ii) $tan^{-1}x - tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(iii) $tan^{-1}x + tan^{-1}y + tan^{-1}z = tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

નોંધ : (i) જો $xy + yz + zx = 1$ તો $tan^{-1}x + tan^{-1}y + tan^{-1}z = \frac{\pi}{2}$

(ii) જો $x + y + z = xyz$ તો $tan^{-1}x + tan^{-1}y + tan^{-1}z = \pi$

5. **આંતરસંબંધ :** $0 < x < 1$

(a) $sin^{-1}x = cos^{-1}\sqrt{1-x^2} = tan^{-1}\frac{x}{\sqrt{1-x^2}}$

(b) $cos^{-1}x = sin^{-1}\sqrt{1-x^2} = tan^{-1}\frac{\sqrt{1-x^2}}{x}$

(c) $tan^{-1}x = sin^{-1}\frac{x}{\sqrt{1+x^2}} = cos^{-1}\frac{1}{\sqrt{1+x^2}}$

વિશિષ્ટ સૂત્રો :

1. **ત્રણ ખૂણાના સરવાળાનાં સૂત્રો :**

(1) $sin(A + B + C) = sinA\,cosB\,cosC + cosA\,sinB\,cosC + cosA\,cosB\,sinC - sinA\,sinB\,sinC$

(2) $cos(A + B + C) = cosA\,cosB\,cosC - sinA\,sinB\,cosC - sinA\,cosB\,sinC - cosA\,sinB\,sinC$
$= cosA\,cosB\,cosC\ (1 - tanA\,tanB - tanB\,tanC - tanA\,tanC)$

(3) $tan(A + B + C) = \frac{tanA + tanB + tanC - tanA\,tanB\,tanC}{1 - tanA\,tanB - tanB\,tanC - tanA\,tanC}$

2. (i) $\sqrt{\frac{1+sin\theta}{1-sin\theta}} = tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = sec\theta + tan\theta$

(ii) $\sqrt{\frac{1-sin\theta}{1+sin\theta}} = tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right) = sec\theta - tan\theta$

(iii) $\sqrt{\frac{1+cos\theta}{1-cos\theta}} = cot\,\frac{\theta}{2} = cosec\theta + cot\theta$

(iv) $\sqrt{\frac{1-cos\theta}{1+cos\theta}} = tan\,\frac{\theta}{2} = cosec\theta - cot\theta$

(v) $cos\theta\ cos2\theta\ cos2^2\theta\ ...\ cos2^{n-1}\theta = \frac{sin2^n\theta}{2^n sin\theta},\ n \in Z$ $(\theta \neq n\pi)$

(vi) $cosA + cos(A + B) + cos(A + 2B) + ... + cos(A + (n - 1)B)$

$$= \frac{sin\frac{nB}{2}}{sin\frac{B}{2}} cos\left\{A + (n-1)\frac{B}{2}\right\}$$

(vii) $tan\theta \, tan\left(\frac{\pi}{3} - \theta\right) tan\left(\frac{\pi}{3} + \theta\right) = \tan 3\theta$

(viii) $A + B + C = \pi \Rightarrow tanA + tanB + tanC = tanA\, tanB\, tanC$

બહુવિકલ્પી પ્રશ્નો

(1) બે નિયમિત બહુકોણની બાજુઓની સંખ્યાનો ગુણોત્તર 5 : 4 છે અને તેમની બાજુઓના અંતર્ગત ખૂણાઓના રેડિયન માપ વચ્ચે તફાવત $\frac{\pi}{30}$ છે, તો બંને બહુકોણની બાજુઓની સંખ્યા ...... થાય.

(A) 15, 12 (B) 20, 16 (C) 10, 8 (D) 25, 20

ઉકેલ : N બાજુવાળા નિયમિત બહુકોણના અંતર્ગત ખૂણાના (ધારો $N > n$) માપનો સરવાળો $= (N - 2)\pi$

$\therefore$ અંતર્ગત એક ખૂણાનું માપ $= \frac{(N-2)\pi}{N}$

n બાજુવાળા નિયમિત બહુકોણના અંતર્ગત એક ખૂણાનું માપ $= \frac{(n-2)\pi}{n}$

$\therefore$ તેમની વચ્ચે તફાવત $= \frac{\pi}{30}$

$\therefore \frac{(N-2)\pi}{N} - \frac{(n-2)\pi}{n} = \frac{\pi}{30}$

$\Rightarrow \frac{2}{n} - \frac{2}{N} = \frac{1}{30}$

તથા $\frac{N}{n} = \frac{5}{4}$ આથી $N = \frac{5}{4}n$

$\therefore \frac{2}{n} - \frac{2}{\left(\frac{5}{4}\right)n} = \frac{1}{30}$

$\therefore \frac{2}{n} - \frac{8}{5n} = \frac{1}{30}$

$\therefore \frac{2}{5n} = \frac{1}{30}$

$\therefore n = 12$ તથા $N = \frac{5}{4}n = 15$ **જવાબ : (A)**

(2) એક સમબાજુ ત્રિકોણની બાજુનું માપ 4 સેમી છે. તેનું એક શિરોબિંદુ વર્તુળનું કેન્દ્ર છે. એ વર્તુળની એક ચાપ ત્રિકોણને બે એકરૂપ ભાગમાં વિભાજિત કરે છે. આ વર્તુળની ત્રિજ્યા ...... હોય.

(A) $\sqrt{12\sqrt{3}}$ સેમી (B) $\frac{\sqrt{12\sqrt{3}}}{\pi}$ સેમી (C) $\sqrt{\frac{\pi}{12\sqrt{3}}}$ સેમી (D) $\sqrt{\frac{12\sqrt{3}}{\pi}}$ સેમી

ઉકેલ : સમબાજુ ત્રિકોણનું ક્ષેત્રફળ $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(4)^2 = 4\sqrt{3}$

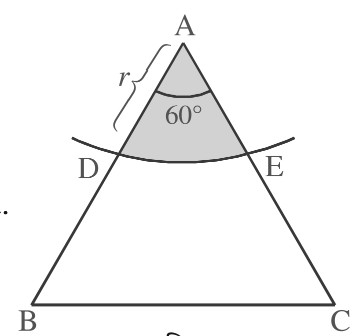
$\therefore \frac{1}{2}$ સમબાજુ ત્રિકોણનું ક્ષેત્રફળ $= 2\sqrt{3}$

સમબાજુ ΔABC માં A ને કેન્દ્ર તરીકે લઈ દોરેલ વર્તુળની કોઈ ચાપ $\widehat{DE}$ એ ΔABC ને બે સમાન ભાગમાં વિભાજિત કરે છે. જેથી વૃત્તાંશ ADE શક્ય બને.

હવે, વૃત્તાંશ ADE નું ક્ષેત્રફળ $= \frac{1}{2}r^2\theta = 2\sqrt{3}$, જ્યાં $\theta = \frac{\pi}{3}$

$\therefore r^2 = \frac{2\sqrt{3} \times 6}{\pi}$

$\therefore r = \sqrt{\frac{12\sqrt{3}}{\pi}}$ સેમી



આકૃતિ 12.3

જવાબ : (D)

(3) એક ચતુષ્કોણના ચાર ખૂણાનાં માપ સમાંતર શ્રેણીમાં છે. સૌથી મોટા તથા નાના ખૂણાના માપનો ગુણોત્તર 2 : 1 છે, તો સૌથી મોટા ખૂણાનું રેડિયન માપ ...... છે.

(A) $\frac{5\pi}{6}$ (B) $\frac{2\pi}{3}$ શ્ર (C) $\frac{5\pi}{9}$ (D) આમાંથી એક પણ નહિ.

ઉકેલ : ધારો કે ચતુષ્કોણના ચાર ખૂણાઓનાં માપ $a - 3d$, $a - d$, $a + d$, $a + 3d$ છે.

$\therefore$ ચારેયનો સરવાળો 2π છે.

$a - 3d + a - d + a + d + a + 3d = 2\pi$

$\therefore$ $4a = 2\pi$

$\therefore$ $a = \frac{\pi}{2}$

$\frac{\text{સૌથી મોટા ખૂણાનું માપ}}{\text{સૌથી નાના ખૂણાનું માપ}} = \frac{2}{1}$

$\therefore$ $\frac{a + 3d}{a - 3d} = 2$

$\therefore$ $\frac{\pi}{2} + 3d = 2\left(\frac{\pi}{2} - 3d\right)$

$\therefore$ $9d = \frac{\pi}{2}$

$\therefore$ $d = \frac{\pi}{18}$

$\therefore$ સૌથી મોટા ખૂણાનું રેડિયન માપ $= a + 3d = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$ **જવાબ : (B)**

(4) r ત્રિજ્યાવાળા વર્તુળના કોઈ એક ચાપની લંબાઈ, તે જ વર્તુળના બંધ $\frac{1}{8}$(વર્તુળ) પરિમિતિ જેટલી છે, તો આ ચાપે કેન્દ્ર આગળ આંતરેલા ખૂણાનું રેડિયન માપ ...... છે.

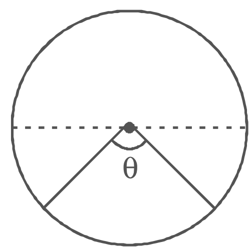
(A) π (B) $\pi - 2$ (C) $\pi + 2$ (D) $\pi + 3$

ઉકેલ : r ત્રિજ્યાવાળા વર્તુળના ચાપની લંબાઈ l

$= $ બંધ $\left(\frac{1}{8}\text{વર્તુળ}\right)$ની પરિમિતિ

$= \frac{\pi r}{4} + 2r$

$= \left(\frac{\pi}{4} + 2\right)r$



આકૃતિ 12.4

પરંતુ જો r ત્રિજ્યાવાળા વર્તુળ દ્વારા કેન્દ્ર આગળ આંતરેલા, ખૂણાનું રેડિયન માપ θ હોય, તો

ચાપની લંબાઈ $l = r\theta$

$\therefore$ $r\theta = \left(\frac{\pi}{4} + 2\right)r$

$\therefore$ $\theta = \frac{\pi}{4} + 2$ રેડિયન **જવાબ : (C)**

(5) $\frac{sin^4 x}{2} + \frac{cos^4 x}{3} = \frac{1}{5}$ તો,

(A) $tan^2 x = \frac{1}{5}$ (B) $tan^2 x = \frac{1}{3}$

(C) $\frac{sin^8 x}{8} + \frac{cos^8 x}{27} = \frac{1}{125}$ (D) $\frac{sin^8 x}{8} + \frac{cos^8 x}{27} = \frac{2}{125}$

ઉકેલ : $\frac{sin^4 x}{2} + \frac{cos^4 x}{3} = \frac{1}{5}$

$\therefore$ $\frac{sin^4 x}{2} + \frac{(1 - sin^2 x)^2}{3} = \frac{1}{5}$

ધારો કે $sin^2x = m$

$\therefore \frac{m^2}{2} + \frac{(1-m)^2}{3} = \frac{1}{5}$

$\therefore \frac{3m^2 + 2(1-m)^2}{6} = \frac{1}{5}$

$\therefore 5[3m^2 + 2(1 - 2m + m^2)] = 6$

$\therefore 25m^2 - 20m + 4 = 0$

$\therefore (5m - 2)^2 = 0$

$\therefore m = \frac{2}{5} = sin^2x$

$\therefore sin^2x = \frac{2}{5} \Rightarrow cos^2x = \frac{3}{5}$

$\therefore tan^2x = \frac{2}{3}$

$\therefore$ (A), (B) શક્ય નથી.

હવે, $\frac{sin^8x}{8} + \frac{cos^8x}{27} = \frac{(sin^2x)^4}{8} + \frac{(cos^2x)^4}{27} = \frac{\left(\frac{2}{5}\right)^4}{8} + \frac{\left(\frac{3}{5}\right)^4}{27} = \frac{2+3}{5^4} = \frac{1}{125}$ **જવાબ : (C)**

(6) $x = sec\theta - tan\theta$, $y = cosec\theta + cot\theta$ તો $xy + 1 = ...$

(A) $x + y$ (B) $x - y$ (C) $y - x$ (D) $-x - y$

ઉકેલ : $xy + 1 = (sec\theta - tan\theta)(cosec\theta + cot\theta), + 1$

$= \left(\frac{1 - sin\theta}{cos\theta}\right)\left(\frac{1 + cos\theta}{sin\theta}\right) + 1$

$= \frac{1 + cos\theta - sin\theta - sin\theta cos\theta + sin\theta cos\theta}{sin\theta cos\theta}$

$= \frac{1 + cos\theta - sin\theta}{sin\theta\, cos\theta}$

$= \frac{sin^2\theta + cos^2\theta + cos\theta - sin\theta}{sin\theta\, cos\theta}$

$= tan\theta + cot\theta + cosec\theta - sec\theta$

$= (cot\theta + cosec\theta) - (sec\theta - tan\theta)$

$= y - x$ **જવાબ : (C)**

(7) θ અને ϕ બંને લઘુકોણ છે. જો $sin\theta = \frac{1}{2}$ તથા $cos\phi = \frac{1}{3}$ હોય, તો $\theta + \phi \in$ ......[IIT : 2004]

(A) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ (C) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{5\pi}{6}, \pi\right)$

ઉકેલ : θ અને ϕ બંને લઘુકોણ છે.

$sin\theta = \frac{1}{2}$. તેથી $\theta = \frac{\pi}{6}$ (1)

cos વિધેય પ્રથમ ચરણમાં ઘટતું વિધેય હોવાથી,

$cos\,\phi = \frac{1}{3} < \frac{1}{2} = cos\frac{\pi}{3}$

હવે, $cos\,\phi = \frac{1}{3}$

સ્પષ્ટ છે કે, $\frac{\pi}{3} < \phi < \frac{\pi}{2}$

$\therefore \frac{\pi}{3} + \theta < \theta + \phi < \frac{\pi}{2} + \theta$

$\therefore \frac{\pi}{3} + \frac{\pi}{6} < \theta + \phi < \frac{\pi}{2} + \frac{\pi}{6}$ ((1) પરથી)

$\therefore \frac{\pi}{2} < \theta + \phi < \frac{2\pi}{3}$

$\therefore \theta + \phi \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ **જવાબ : (B)**

(8) $sin^2\theta + 3cos\theta - 2 = 0$ તો $cos^3\theta + sec^3\theta = ...,\ \theta \neq (2k+1)\frac{\pi}{2},\ k \in Z$

(A) 18 (B) 9 (C) 4 (D) $\frac{1}{4}$

ઉકેલ : $sin^2\theta + 3cos\theta - 2 = 0$

$\therefore\ 1 - cos^2\theta + 3cos\theta - 2 = 0$

$\therefore\ cos^2\theta - 3cos\theta + 1 = 0$

$\therefore\ cos\theta - 3 + sec\theta = 0$ $(cos\theta \neq 0)$

$\therefore\ cos\theta + sec\theta = 3$

$\therefore\ (cos\theta + sec\theta)^3 = cos^3\theta + sec^3\theta + 3cos\theta\, sec\theta(cos\theta + sec\theta)$

$\therefore\ 27 = cos^3\theta + sec^3\theta + 3(3)$

$\therefore\ cos^3\theta + sec^3\theta = 18$ **જવાબ : (A)**

(9) જો $x = sin^2\theta\, cos\theta$ અને $y = cos^2\theta\, sin\theta$ હોય તો, નીચેનામાંથી કયું સત્ય છે ? $0 < \theta < \frac{\pi}{2},\ x > y > 0$

(A) $(x^2y)^{\frac{2}{3}} + (xy^2)^{\frac{2}{3}} = 1$

(B) $\left(\frac{x^2}{y}\right)^{\frac{2}{3}} + \left(\frac{y^2}{x}\right)^{\frac{2}{3}} = 1$

(C) $x^2 + y^2 = x^2y^2$

(D) આમાંથી એક પણ નહિ.

ઉકેલ : અહીં, $\frac{x}{y} = \frac{sin^2\theta cos\theta}{sin\theta cos^2\theta} = tan\theta$ મળે.

$$\therefore\ \frac{x^2}{y^2} + 1 = sec^2\theta$$

$$\therefore\ cos^2\theta = \frac{y^2}{x^2+y^2},\ sin^2\theta = \frac{x^2}{x^2+y^2}$$

આથી, $x = sin^2\theta\, cos\theta$ પરથી, $x = \frac{x^2}{x^2+y^2} \cdot \frac{y}{\sqrt{x^2+y^2}}$

$\therefore\ (x^2 + y^2)^{\frac{3}{2}} = xy$. આથી, $x^2 + y^2 = (xy)^{\frac{2}{3}}$

$$\therefore\ \frac{x^2}{(xy)^{\frac{2}{3}}} + \frac{y^2}{(xy)^{\frac{2}{3}}} = 1.\ \text{આથી,}\ \frac{x^{\frac{4}{3}}}{y^{\frac{2}{3}}} + \frac{y^{\frac{4}{3}}}{x^{\frac{2}{3}}} = 1$$

$$\therefore\ \left(\frac{x^2}{y}\right)^{\frac{2}{3}} + \left(\frac{y^2}{x}\right)^{\frac{2}{3}} = 1$$ **જવાબ : (B)**

(10) $sin\theta\, cos\theta$ ની ન્યૂનતમ કિંમત ...... છે.

(A) 1 (B) 0 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

ઉકેલ : $(sin\theta + cos\theta)^2 \geq 0$

$\therefore\ sin^2\theta + 2sin\theta cos\theta + cos^2\theta \geq 0$

$\therefore\ 1 + 2sin\theta cos\theta \geq 0$

$\therefore \; sin\theta cos\theta \; \geq \; -\frac{1}{2}$

ન્યૂનતમ કિંમત $= -\frac{1}{2}$ છે.

બીજી રીત :

$sin\theta cos\theta = \frac{1}{2} \cdot 2sin\theta cos\theta = \frac{1}{2} \cdot sin2\theta$

હવે $sin2\theta \geq -1$. આથી $\frac{1}{2}sin2\theta \geq -\frac{1}{2}$

ન્યૂનતમ કિંમત $= -\frac{1}{2}$ **જવાબ : (C)**

નોંધ : $\theta = \frac{3\pi}{4}$ માટે $sin\theta \, cos\theta = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}}\right) = -\frac{1}{2}$ છે.

(11) $sinx + sin^2x = 1$ તો $cos^{12}x + 3cos^{10}x + 3cos^8x + cos^6x + 2cos^4x + cos^2x - 2 =$ ......

(A) 0 (B) 1 (C) 2 (D) sin^2x

ઉકેલ : $sinx + sin^2x = 1 \Rightarrow sinx = 1 - sin^2x = cos^2x$

$cos^{12}x + 3cos^{10}x + 3cos^8x + cos^6x + 2cos^4x + cos^2x - 2 =$ ......

$= sin^6x + 3sin^5x + 3sin^4x + sin^3x + 2sin^2x + sinx - 2$

$= \left[(sin^2x)^3 + \binom{3}{1}(sin^2x)^2\,(sinx) + \binom{3}{2}(sin^2x)\,(sinx)^2 + \binom{3}{3}(sinx)^3\right] + (sin^2x + sinx) - 2 + sin^2x$

$= [sin^2x + sinx]^3 + [sin^2x + sinx] - 2 + sin^2x$

$= 1 + 1 - 2 + sin^2x$

$= sin^2x$ **જવાબ : (D)**

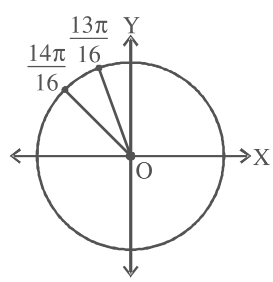
(12) $\frac{sin3\alpha}{cos2\alpha}$ નું મૂલ્ય અંતરાલો (p) તથા (q)માંથી કયા અંતરાલમાં ધન છે ? કયા અંતરાલ માટે ઋણ છે ? તે નક્કી કરો.

(p) $\left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$ (q) $\left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$

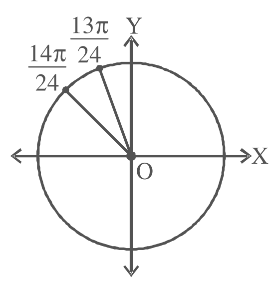
(A) (p) ધન (q) ઋણ (B) (p) ઋણ (q) ધન (C) બંને ધન (D) બંને ઋણ

ઉકેલ : (p) $\frac{13\pi}{48} < \alpha < \frac{14\pi}{48}$

$\Rightarrow \frac{13\pi}{16} < 3\alpha < \frac{14\pi}{16}$



આકૃતિ 12.5



આકૃતિ 12.6

$\therefore$ P(3α) બીજા ચરણમાં છે.

$\therefore \; sin3\alpha > 0$

હવે, $\frac{13\pi}{48} < \alpha < \frac{14\pi}{48}$

$\frac{13\pi}{24} < 2\alpha < \frac{14\pi}{24}$

$\therefore$ P(2α) બીજા ચરણમાં છે.

$cos 2\alpha < 0$

$\therefore \quad \dfrac{sin 3\alpha}{cos 2\alpha} < 0$

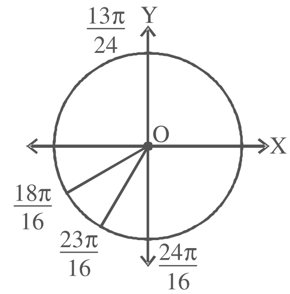
∴ અંતરાલ (p) માટે ઋણ છે.

(q) $\dfrac{18\pi}{48} < \alpha < \dfrac{23\pi}{48}$

$\Rightarrow \quad \dfrac{18\pi}{16} < 3\alpha < \dfrac{23\pi}{16}$

∴ P(3α) ત્રીજા ચરણમાં છે.

$\therefore \quad sin 3\alpha < 0$



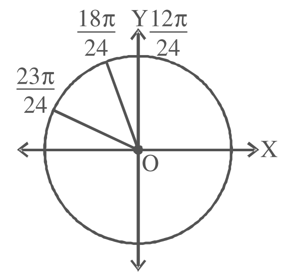
આકૃતિ 12.7

$\dfrac{18\pi}{48} < \alpha < \dfrac{23\pi}{48}$

$\Rightarrow \quad \dfrac{18\pi}{24} < 2\alpha < \dfrac{23\pi}{24}$

∴ P(2α) બીજા ચરણમાં છે.

$\therefore \quad cos 2\alpha < 0$



આકૃતિ 12.8

$\therefore \quad \dfrac{sin 3\alpha}{cos 2\alpha} > 0$

∴ (q) માટે ધન છે. **જવાબ : (B)**

(13) નીચેનામાંથી કોનું મૂલ્ય સૌથી મોટું છે ?

(A) $tan\ 1$ (B) $tan\ 2$ (C) $tan\ 3$ (D) $tan\ 4$

ઉકેલ : $tan 2 < 0$, કારણ કે $\dfrac{\pi}{2} < 2 < \pi$

$tan 3 < 0$, કારણ કે $\dfrac{\pi}{2} < 3 < \pi$

$tan 4 = tan(4 - \pi) > 0$ કારણ કે $0 < 4 - \pi < \dfrac{\pi}{2}$. વળી $0 < 4 - \pi < 1 < \dfrac{\pi}{2}$

$\therefore \quad tan(4 - \pi) = tan 4 < tan 1$ (tan પ્રથમ ચરણમાં વધતું વિધેય છે.)

∴ $tan\ 1$ સૌથી મોટી સંખ્યા છે. **જવાબ : (A)**

(14) જો $y = \dfrac{sec^2\theta - tan\theta}{sec^2\theta + tan\theta}$ તો $y \in$ ......

(A) $\left(\frac{1}{3}, 3\right)$ (B) $\left(-3, -\frac{1}{3}\right)$ (C) $\left[\frac{1}{3}, 3\right]$ (D) $\left(\frac{1}{3}, 4\right)$

ઉકેલ : ધારો કે $y = \dfrac{sec^2\theta - tan\theta}{sec^2\theta + tan\theta}$

$\therefore \quad y = \dfrac{1 + tan^2\theta - tan\theta}{1 + tan^2\theta + tan\theta}$

$\therefore \quad y + y tan^2\theta + y tan\theta = 1 + tan^2\theta - tan\theta$

$\therefore \quad (y - 1) tan^2\theta + (y + 1)\ tan\theta + (y - 1) = 0$

$tan\theta$ માં દ્વિઘાત સમીકરણનાં વાસ્તવિક બીજ માટે $\Delta \geq 0$ જરૂરી છે.

$\therefore \; (y + 1)^2 - 4(y - 1)^2 \ge 0$

$\therefore \; y^2 + 2y + 1 - 4(y^2 - 2y + 1) \ge 0$

$\therefore \; -3y^2 + 10y - 3 \ge 0$

$\therefore \; 3y^2 - 10y + 3 \le 0$

$\therefore \; (3y - 1)(y - 3) \le 0 \Rightarrow y \in \left[\frac{1}{3}, 3\right]$

$\therefore \; y \in \left[\frac{1}{3}, 3\right]$ **જવાબ : (C)**

(15) કોઈ વાસ્તવિક $\theta \in R$ માટે $cos\theta = x + \frac{1}{x}, x \ne 0$ હોય તો નીચેનામાંથી કયું સત્ય બને ?

(A) θ લઘુકોણ છે. (B) θ ગુરુકોણ છે. (C) θ કાટકોણ છે. (D) θ શક્ય નથી.

ઉકેલ : $cos\theta = x + \frac{1}{x}$

$\therefore \; cos^2\theta = x^2 + 2 + \frac{1}{x^2} > 2$ શક્ય નથી. **જવાબ : (D)**

(16) $\sqrt{cos^2x - 10cosx + 25} + 2$ નો વિસ્તાર ...... છે.

(A) [4, 6] (B) (6, 8) (C) [6, 8] (D) [8, 10]

ઉકેલ : $\sqrt{cos^2x - 10cosx + 25} + 2$

$= \sqrt{(cosx - 5)^2} + 2$

$= |cosx - 5| + 2$

$-1 \le cosx \le 1 \Leftrightarrow -6 \le cosx - 5 \le -4$

$\Leftrightarrow 4 \le |cosx - 5| \le 6$

$\Leftrightarrow 6 \le |cosx - 5| + 2 \le 8$

$\Leftrightarrow 6 \le f(x) \le 8$

$\therefore$ વિસ્તાર [6, 8] **જવાબ : (C)**

(17) $\sum_{i=1}^{3} sin^2\theta_i = 0$ તો $\sum_{i=1}^{3} cos\,\theta_i$ ની કિંમત નીચેનામાંથી કઈ શક્ય ન બને ?

(A) 3 (B) –3 (C) –1 (D) –2

ઉકેલ : $\sum_{i=1}^{3} sin^2\theta_i = 0$

$\therefore \; sin^2\theta_1 + sin^2\theta_2 + sin^2\theta_3 = 0$

$\therefore \; sin\theta_1 = sin\theta_2 = sin\theta_3 = 0$

$\therefore \; cos\theta_1 = cos\theta_2 = cos\theta_3 = \pm 1$ (1)

$cos\theta_1$	$cos\theta_2$	$cos\theta_3$	સરવાળો
1	1	1	3
–1	–1	–1	–3
1	1	–1	1
–1	–1	1	–1

આમ, કોષ્ટક પરથી સ્પષ્ટ છે કે સરવાળો –2 શક્ય નથી. **જવાબ : (D)**

(18) જો $sec^2\theta = \dfrac{4xy}{(x+y)^2}$ સત્ય હોય, તો નીચેનામાંથી કયું સત્ય છે ? $(x \neq -y)$ [IIT : 1996]

(A) $x + y = 0$ (B) $x = y,\ x \neq 0$ (C) $x = y$ (D) $x \neq y$

ઉકેલ : $sec^2\theta \geq 1$

$$\frac{4xy}{(x+y)^2} \geq 1 \iff 4xy \geq (x+y)^2 \qquad ((x+y)^2 > 0)$$

$$\iff x^2 - 2xy + y^2 \leq 0$$

$$\iff (x-y)^2 \leq 0$$

$(x-y)^2 < 0$ શક્ય નથી.

$\therefore\ x - y = 0$

$\therefore\ x = y$ અને $x \neq 0$ **જવાબ : (B)**

(19) $\sqrt{\dfrac{1-sin A}{1+sin A}} = \ldots\ldots \quad -\dfrac{\pi}{2} < A < \dfrac{\pi}{2}$

(A) $secA + tanA$ (B) $secA - tanA$ (C) $-secA + tanA$ (D) $-(secA + tanA)$

ઉકેલ : $\sqrt{\dfrac{1-sinA}{1+sinA}} \times \sqrt{\dfrac{1-sinA}{1-sinA}}$

$$= \frac{1-sinA}{\sqrt{1-sin^2A}}$$

$$= \frac{1-sinA}{|cosA|}$$

$$= \frac{1-sinA}{cosA} \qquad \left[\left(\frac{-\pi}{2} < A < \frac{\pi}{2}\right) \text{ હોવાથી } cosA > 0\right]$$

$= secA - tanA$ **જવાબ : (B)**

(20) જો $A = sin^8\theta + cos^{14}\theta,\ \forall\theta \in R$ હોય તો, નીચેનામાંથી કયું સત્ય બને ?

(A) $A \geq 1$ (B) $0 < A \leq 1$ (C) $\frac{1}{2} \leq A \leq \frac{3}{2}$ (D) આમાંથી એક પણ નહિ.

ઉકેલ : આપણે જાણીએ છીએ કે,

$0 \leq sin^2A \leq 1$ તથા $0 \leq cos^2A \leq 1$

$\Rightarrow sin^4A \leq sin^2A$	$cos^4A \leq cos^2A$
$\Rightarrow sin^6A \leq sin^4A \leq sin^2A$	$\Rightarrow cos^6A \leq cos^4A \leq cos^2A$
$\Rightarrow sin^8A \leq sin^2A$	$\Rightarrow cos^{14}A \leq cos^2A$

$0 < sin^8A + cos^{14}A \leq sin^2A + cos^2A$ ($\because$ $sin^8A + cos^{14}A$ ની કિંમત શૂન્ય શક્ય નથી.)

$\Rightarrow 0 < sin^8A + cos^{14}A \leq 1$ **જવાબ : (B)**

નોંધ : $sinA$ તથા $cosA$ બંને એક સાથે શૂન્ય હોય, તો જ $sin^8A + cos^{14}A$ ની કિંમત શૂન્ય થાય અને $sin^2A + cos^2A = 1$ હોવાથી, આ શક્ય નથી.

(21) $\cos\theta - \sin\theta = \frac{1}{5}$, $0 < \theta < \frac{\pi}{2}$

I	II	
(i) $\frac{(\cos\theta + \sin\theta)}{2}$	(*a*) $\frac{4}{5}$	(*b*) $\frac{7}{10}$
(ii) $\cos\theta$	(*c*) $\frac{24}{25}$	(*d*) $\frac{7}{25}$

(A) (i) (*a*) (ii) (*c*) (B) (i) (*b*) (ii) (*d*) (C) (i) (*a*) (ii) (*d*) (D) (i) (*b*) (ii) (*a*)

ઉકેલ : $\cos\theta = \frac{1}{5} + \sin\theta$

$\therefore\ 5\cos\theta = 1 + 5\sin\theta$

$\therefore\ 25\cos^2\theta = 1 + 10\sin\theta + 25\sin^2\theta$

$\therefore\ 25(1 - \sin^2\theta) = 1 + 10\sin\theta + 25\sin^2\theta$

$\therefore\ 50\sin^2\theta + 10\sin\theta - 24 = 0$

$\therefore\ 25\sin^2\theta + 5\sin\theta - 12 = 0$

$\therefore\ (5\sin\theta - 3)(5\sin\theta + 4) = 0$

$\therefore\ \sin\theta = \frac{3}{5}$ કારણ કે $0 < \theta < \frac{\pi}{2}$

$0 < \theta < \frac{\pi}{2}$. આથી, $\cos\theta = \frac{4}{5}$

$\therefore\ \frac{\sin\theta + \cos\theta}{2} = \frac{7}{10}$

$\therefore$ (i) $\rightarrow$ (*b*) (ii) $\rightarrow$ (*a*) **જવાબ : (D)**

(22) $2 - \cos x + \sin^2 x$ ની મહત્તમ તથા ન્યૂનતમ કિંમતનો ગુણોત્તર ...... છે.

(A) $\frac{7}{4}$ (B) $\frac{11}{4}$ (C) $\frac{13}{4}$ (D) આમાંથી એક પણ નહિ.

ઉકેલ : $f(x) = 2 - \cos x + \sin^2 x$

$= 2 - \cos x + 1 - \cos^2 x$

$= 3 - \cos x - \cos^2 x$

$= 3 + \frac{1}{4} - \frac{1}{4} - \cos x - \cos^2 x$

$= \frac{13}{4} - \left(\cos x + \frac{1}{2}\right)^2$

$f(x)$ મહત્તમ મેળવવા $\left(\cos x + \frac{1}{2}\right)^2$ ન્યૂનતમ થવું જોઈએ.

$\therefore\ \left(\cos x + \frac{1}{2}\right) = 0$ થવું જોઈએ. આથી, મહત્તમ કિંમત $= \frac{13}{4}$ મળે, તે માટે $\cos x = -\frac{1}{2}$ જરૂરી છે.

$f(x)_{max} = \frac{13}{4}$

$f(x)$ ન્યૂનતમ મેળવવા $\left(\cos x + \frac{1}{2}\right)^2$ મહત્તમ થવું જોઈએ. આથી, $\cos x = 1$ હોવું જોઈએ.

$\therefore\ f(x) = \frac{13}{4} - \left(1 + \frac{1}{2}\right)^2 = \frac{13}{4} - \frac{9}{4} = 1$

$\therefore\ f(x)_{min} = 1$

$\therefore\ \frac{f(x)_{max}}{f(x)_{min}} = \frac{13}{4}$ **જવાબ : (C)**

(23) $sec^2(a + 2)x + a^2 - 1 = 0, -\pi < x < \pi$ સમીકરણના ઉકેલ (a, x) હોય, તો આ ઉકેલની સંખ્યા ...... છે.

(A) 2 (B) 1 (C) 3 (D) અનંત

ઉકેલ : $sec^2(a + 2)x + a^2 - 1 = 0$

$\therefore$ $sec^2(a + 2)x = 1 - a^2$

$\therefore$ $a = 0$ માટે જ શક્ય બને. આથી, $a = 0$

$a = 0 \Rightarrow sec^2 2x = 1 \Rightarrow sec2x = \pm 1$

$-\pi < x < \pi$. આથી, $-2\pi < 2x < 2\pi$

$sec2x = 1 \Rightarrow x = 0$ તથા $sec2x = -1 \Rightarrow 2x = \pi, -\pi \Rightarrow x = \frac{\pi}{2}, \frac{-\pi}{2}$

$\therefore$ ઉકેલની સંખ્યા 3 છે. **જવાબ : (C)**

(24) $tanx - tan^2x = 1$ તો $tan^4x - 2tan^3x - tan^2x + 2tanx + 1$ ની કિંમત ...... છે.

(A) 1 (B) 2 (C) 3 (D) 4

ઉકેલ : $tan^4x - 2tan^3x + tan^2x - 2tan^2x + 2tanx + 1$

$= (tan^2x - tanx)^2 - 2(tan^2x - tanx) + 1 = (-1)^2 - 2(-1) + 1 = 1 + 2 + 1 = 4$ **જવાબ : (D)**

(25) $2cos^2\frac{x}{2}sin^2x = x^2 + x^{-2}, 0 < x \le \frac{\pi}{2}$ ને ...... [IIT : 1984]

(A) એક પણ વાસ્તવિક ઉકેલ નથી. (B) એક જ વાસ્તવિક ઉકેલ મળે.

(C) એક કરતાં વધુ ઉકેલ મળે. (D) આમાંથી એક પણ નહિ.

ઉકેલ : અહીં, $\left(x - \frac{1}{x}\right)^2 \ge 0$ લેતાં,

$\therefore$ $x^2 - 2 + \frac{1}{x^2} \ge 0$

$\therefore$ $x^2 + \frac{1}{x^2} \ge 2$

$\therefore$ $2cos^2\frac{x}{2}sin^2x \ge 2$

$\therefore$ $cos^2\frac{x}{2}sin^2x \ge 1$

$cos^2\frac{x}{2}sin^2x > 1$ શક્ય નથી.

વળી, $cos^2\frac{x}{2}sin^2x = 1$ પણ ના હોઈ શકે, કારણ કે, $cos^2\frac{x}{2} \le 1, sin^2x \le 1$

$\therefore$ $sin^2x = cos^2\frac{x}{2} = 1$ થવા જોઈએ.

પરંતુ $cos^2\frac{x}{2} = 1$ તો $sin^2x = 4sin^2\frac{x}{2}\ cos^2\frac{x}{2} = 4(0)(1) = 0$ થાય.

$\therefore$ વાસ્તવિક ઉકેલની સંખ્યા '0' છે. **જવાબ : (A)**

(26) જો $tan\alpha = \frac{m}{m+1}$ અને $tan\beta = \frac{1}{2m+1}$ હોય, તો $(\alpha + \beta)$ ની શક્ય કિંમત ...... છે.

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

ઉકેલ : $tan(\alpha + \beta) = \dfrac{tan\alpha + tan\beta}{1 - tan\alpha\, tan\beta}$

$$= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)}$$

$$= \frac{m(2m+1) + (m+1)}{(m+1)(2m+1) - m} = \frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} = 1$$

$\therefore \ (\alpha + \beta) = \frac{\pi}{4}$ **જવાબ : (B)**

(27) $\dfrac{2(sin1^\circ + sin2^\circ + sin3^\circ + ... + sin89^\circ)}{2(cos1^\circ + cos2^\circ + ... + cos44^\circ) + 1} = ...$

(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) 1

ઉકેલ : $2(sin1^\circ + sin2^\circ + sin3^\circ + ... + sin89^\circ)$

$= 2[(sin1^\circ + sin89^\circ) + (sin2^\circ + sin88^\circ) + ... + (sin44^\circ + sin46^\circ) + sin45^\circ]$

$= 2\left[2sin45^\circ cos44^\circ + 2sin45^\circ cos43^\circ + ... + 2sin45^\circ cos1^\circ + \frac{1}{\sqrt{2}}\right]$

$= 2\sqrt{2}(cos44^\circ + cos43^\circ + ... + cos1^\circ) + \sqrt{2}$ (1)

હવે, $\dfrac{2(sin1^\circ + sin2^\circ + ... + sin89^\circ)}{2(cos1^\circ + cos2^\circ + ... + cos44^\circ) + 1}$

$= \dfrac{\left[2\sqrt{2}(cos44^\circ + cos43^\circ + ... + cos1^\circ) + \sqrt{2}\right]}{2(cos1^\circ + cos2^\circ + ... + cos44^\circ) + 1}$ ((1) પરથી)

$= \dfrac{\sqrt{2}\left[2(cos1^\circ + cos2^\circ + ... + cos44^\circ) + 1\right]}{2(cos1^\circ + cos2^\circ + ... + cos44^\circ) + 1}$

$= \sqrt{2}$ **જવાબ : (A)**

(28) $sin(y + z - x)$, $sin(z + x - y)$, $sin(x + y - z)$ સમાંતર શ્રેણીમાં છે. તો $tanx$, $tany$, $tanz$ એ ......

(A) સમાંતર શ્રેણીમાં છે. (B) સમગુણોત્તર શ્રેણીમાં છે.

(C) સ્વરિત શ્રેણીમાં છે. (D) આમાંથી એક પણ નહિ.

ઉકેલ : $sin(y + z - x)$, $sin(z + x - y)$, $sin(x + y - z)$ સમાંતર શ્રેણીમાં છે.

$\therefore \ sin(y + z - x) - sin(z + x - y) = sin(z + x - y) - sin(x + y - z)$

$\therefore \ 2cosz\, sin(y - x) = 2cosx\, sin(z - y)$

$\therefore \ cosz[siny\, cosx - cosy\, sinx] = cosx(sinz\, cosy - cosz\, siny)$

$\therefore \ cosz\, siny\, cosx - cosz\, cosy\, sinx = cosx\, sinz\, cosy - cosx\, cosz\, siny$

$\therefore \ tany - tanx = tanz - tany$ $(cosx\, cosy\, cosz \neq 0)$

$\therefore \ 2tany = tanx + tanz$

$\therefore \ tanx, tany, tanz$ સમાંતર શ્રેણીમાં છે. **જવાબ : (A)**

(29) $cos(\alpha + \beta) = \frac{4}{5}$ અને $sin(\alpha - \beta) = \frac{5}{13}$; $0 < \alpha, \beta < \frac{\pi}{4}$ હોય, તો $tan2\alpha$ = ...

(A) $\frac{33}{56}$ (B) $\frac{56}{33}$ (C) $\frac{55}{32}$ (D) $\frac{32}{55}$

ઉકેલ : $0 < \alpha < \frac{\pi}{4}$

$0 < \beta < \frac{\pi}{4}$

$0 < \alpha + \beta < \frac{\pi}{2}$

$0 < \alpha < \frac{\pi}{4}$

$-\frac{\pi}{4} < -\beta < 0$

$-\frac{\pi}{4} < \alpha - \beta < \frac{\pi}{4}$ અને $sin(\alpha - \beta) = \frac{5}{13} > 0$

$\therefore$ $P(\alpha - \beta)$ પ્રથમ ચરણમાં જ હોય.

$\therefore$ $tan(\alpha + \beta) = \frac{3}{4}$ તથા $tan(\alpha - \beta) = \frac{5}{12}$

$2\alpha = (\alpha + \beta) + (\alpha - \beta)$

$\therefore$ $tan2\alpha = tan[(\alpha + \beta) + (\alpha - \beta)]$

$$= \frac{tan(\alpha + \beta) + tan(\alpha - \beta)}{1 - tan(\alpha + \beta)\, tan(\alpha - \beta)}$$

$$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \left(\frac{3}{4}\right)\left(\frac{5}{12}\right)} = \frac{56}{33}$$

જવાબ : (B)

(30) જો $sin^{-1}x \le cos^{-1}x$ હોય તો, $x \in$ ......

(A) $\left(-1, \frac{1}{\sqrt{2}}\right)$ (B) $\left[-1, \frac{1}{\sqrt{2}}\right]$ (C) $\left[\frac{1}{\sqrt{2}}, 1\right]$ (D) આમાંથી એકપણ નહિ

ઉકેલ : $sin^{-1}x \le cos^{-1}x$

$sin^{-1}x \le \frac{\pi}{2} - sin^{-1}x$

$\therefore$ $2sin^{-1}x \le \frac{\pi}{2}$

$\therefore$ $sin^{-1}x \le \frac{\pi}{4}$

વળી, $-\frac{\pi}{2} \le sin^{-1}x \le \frac{\pi}{4}$

$\therefore$ $sin\left(\frac{-\pi}{2}\right) \le x \le sin\frac{\pi}{4}$

$\therefore$ $-1 \le x \le \frac{1}{\sqrt{2}}$

જવાબ : (B)

(31) $2cos^{-1}x = sin^{-1}\left(2x\sqrt{1 - x^2}\right)$ નો ઉકેલ ......

(A) $\left[\frac{1}{\sqrt{2}}, 1\right]$ (B) $\left[\frac{-1}{\sqrt{2}}, 1\right]$ (C) $\left[-1, \frac{-1}{\sqrt{2}}\right]$ (D) $\left(\frac{1}{\sqrt{2}}, 1\right)$

ઉકેલ : $2cos^{-1}x = sin^{-1}\left(2x\sqrt{1 - x^2}\right)$

ધારો કે, $cos^{-1}x = \theta$, $\theta \in [0, \pi]$

$\therefore$ $x = cos\theta$

$\therefore$ $2\theta = sin^{-1}\left(2cos\theta\sqrt{1 - cos^2\theta}\right)$

$\therefore \quad 2\theta = sin^{-1}(2cos\theta sin\theta)$ $\qquad$ ($sin\theta \geq 0$ કારણ કે $0 \leq \theta \leq \pi$)

$\therefore \quad 2\theta = sin^{-1}(sin2\theta)$

હવે, $-\frac{\pi}{2} \leq 2\theta \leq \frac{\pi}{2}$

$\therefore \quad -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ તથા $\theta \in [0, \pi]$ હોવાથી, $\theta \in \left[0, \frac{\pi}{4}\right]$ થાય.

વળી, પ્રથમ ચરણમાં *cos* ઘટતું વિધેય હોવાથી,

$0 \leq \theta \leq \frac{\pi}{4} \Rightarrow cos0 \geq cos\theta \geq cos\frac{\pi}{4}$

$\therefore \quad \frac{1}{\sqrt{2}} \leq cos\theta \leq 1$

$\therefore \quad x \in \left[\frac{1}{\sqrt{2}}, 1\right]$ $\qquad$ **જવાબ : (A)**

(32) $f(x) = sin^{-1}\left(\frac{1+x^2}{2|x|}\right)$ હોય, તો $x \in$ ...... શક્ય છે.

(A) $\{-1, 1\}$ $\qquad$ (B) $[-1, 1]$ $\qquad$ (C) $[0, 1]$ $\qquad$ (D) $[-1, 0]$

ઉકેલ : આપણે જાણીએ છીએ કે, $(1 \pm x)^2 \geq 0$

$\therefore \quad 1 \pm 2x + x^2 \geq 0$

$\therefore \quad 1 + x^2 \geq 2|x|$

$\therefore \quad \frac{1+x^2}{2|x|} \geq 1$

$f(x) = sin^{-1}\left(\frac{1+x^2}{2|x|}\right)$ હોવાથી $\frac{1+x^2}{2|x|} > 1$ શક્ય નથી.

$\therefore \quad \frac{1+x^2}{2|x|} = 1$

$\therefore \quad x = \pm 1$

$\therefore \quad x \in \{-1, 1\}$ $\qquad$ **જવાબ : (A)**

(33) $sin^{-1}\left(cot\left(sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}} + cos^{-1}\frac{\sqrt{12}}{4} + sec^{-1}\sqrt{2}\right)\right) = ...$

(A) 1 $\qquad$ (B) 0 $\qquad$ (C) -1 $\qquad$ (D) 2

ઉકેલ : $sin^{-1}\left(cot\left(sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}} + cos^{-1}\frac{\sqrt{3}}{2} + sec^{-1}\sqrt{2}\right)\right)$

$= sin^{-1}\left(cot\left(sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) + cos^{-1}\frac{\sqrt{3}}{2} + sec^{-1}\sqrt{2}\right)\right)$ $\qquad \left(\because \sqrt{\frac{2-\sqrt{3}}{4}} = \sqrt{\frac{4-2\sqrt{3}}{8}} = \frac{\sqrt{3}-1}{2\sqrt{2}}\right)$

$= sin^{-1}\left(cot\left(\frac{\pi}{12} + \frac{\pi}{6} + \frac{\pi}{4}\right)\right) = sin^{-1}\left(cot\frac{\pi}{2}\right) = sin^{-1}(0) = 0$ $\qquad$ **જવાબ : (B)**

(34) $f(x) = x^{11} + x^9 + x^7 + x^3 + 1$ તથા

$f(sin^{-1}(sin8)) = \alpha$ જ્યાં α = અચળ હોય, તો $f(tan^{-1}(tan8)) =$ ......

(A) α (B) $\alpha - 2$ (C) $\alpha + 2$ (D) $2 - \alpha$

ઉકેલ : $\frac{5\pi}{2} < 8 < 3\pi$

$\therefore -\frac{\pi}{2} < 8 - 3\pi < 0$

$sin(8 - 3\pi) = -sin8$

$\therefore \alpha = f(sin^{-1}(sin8))$

$= f(sin^{-1}(-sin(8 - 3\pi)))$

$= f(-sin^{-1}(sin(8 - 3\pi))$

$= f(-8 + 3\pi)$ (1)

હવે, $tan(8 - 3\pi) = tan8$

$f(tan^{-1}(tan8)) = f(tan^{-1}(tan(8 - 3\pi))$

$= f(8 - 3\pi)$ (2)

$f(x) = x^{11} + x^9 + x^7 + x^3 + 1$

$f(-x) = -x^{11} - x^9 - x^7 - x^3 + 1$

$f(x) + f(-x) = 2$

$\therefore f(-8 + 3\pi) + f(8 - 3\pi) = 2$

$\therefore \alpha + f(8 - 3\pi) = 2$ ((1) પરથી)

$\therefore f(8 - 3\pi) = 2 - \alpha$

$\therefore f(tan^{-1}(tan8)) = 2 - \alpha$ ((2) પરથી)

જવાબ : **(D)**

(35) $sinx - 3sin2x + sin3x = cosx - 3cos2x + cos3x$ નો ઉકેલ ...... હોય. [IIT : 1989]

(A) $n\pi + \frac{\pi}{8}, n \in Z$ (B) $\frac{n\pi}{2} + \frac{\pi}{8}, n \in Z$

(C) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}, n \in Z$ (D) $2n\pi + cos^{-1}\frac{2}{3}, n \in Z$

ઉકેલ : $sinx - 3sin2x + sin3x = cosx - 3cos2x + cos3x$

$\therefore (sin3x + sinx) - 3sin2x = (cos3x + cosx) - 3cos2x$

$\therefore 2sin2x\, cosx - 3sin2x = 2cos2x\, cosx - 3cos2x$

$\therefore sin2x(2cosx - 3) = cos2x\ (2cosx - 3)$

$\therefore (sin2x - cos2x)\ (2cosx - 3) = 0$

પરંતુ, $cosx = \frac{3}{2}$ શક્ય નથી.

$\therefore sin2x - cos2x = 0$

$\therefore tan2x = 1$ (જો $cos2x = 0 \Rightarrow sin2x = 0$ જે શક્ય નથી. આથી, $cos2x \neq 0$)

$\therefore tan2x = tan\frac{\pi}{4}$

$\therefore 2x = n\pi + \frac{\pi}{4}, n \in Z$

$\therefore x = \frac{n\pi}{2} + \frac{\pi}{8}, n \in Z$

જવાબ : **(B)**

(36) $tanx + secx = 2cosx$ ના $[0, 2\pi]$ માં ઉકેલની સંખ્યા ...... છે.

(A) 0 (B) 1 (C) 2 (D) 3

ઉકેલ : $tanx + secx = 2cosx$

$\therefore \frac{sin\,x}{cos\,x} + \frac{1}{cos\,x} = 2cosx$

$\therefore sinx + 1 = 2cos^2x$

$\therefore sinx + 1 = 2(1 - sin^2x)$

$\therefore \quad 2sin^2x + sinx - 1 = 0$

$\therefore \quad (2sinx - 1)(sinx + 1) = 0$

$\therefore \quad sinx = \frac{1}{2}$ અથવા $sinx = -1$

$x \in [0, 2\pi]$ હોવાથી,

$sinx = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$ અથવા $\frac{5\pi}{6}$

પરંતુ $sinx = -1 \Rightarrow cosx = 0$ જે શક્ય નથી. ($tanx$ વ્યાખ્યાયિત છે.)

$\therefore \quad sinx = -1$ શક્ય નથી.

$\therefore$ ઉકેલની સંખ્યા 2 છે. **જવાબ : (C)**

(37) $2sin^2\theta - 3sin\theta - 2 = 0$ નો વ્યાપક ઉકેલ ...... છે.

(A) $n\pi + (-1)^n \frac{\pi}{6};\ n \in Z$ (B) $n\pi + (-1)^n \frac{\pi}{2};\ n \in Z$

(C) $n\pi + (-1)^n \frac{5\pi}{6};\ n \in Z$ (D) $n\pi + (-1)^n \frac{7\pi}{6};\ n \in Z$

ઉકેલ : $2sin^2\theta - 3sin\theta - 2 = 0$

$\therefore \quad (2sin\theta + 1)(sin\theta - 2) = 0$

$\therefore \quad sin\theta = -\frac{1}{2}$ અથવા $sin\theta = 2$

$sin\theta = 2 > 1$ નો ઉકેલગણ $\emptyset$ છે.

$sin\theta = -\frac{1}{2} \Rightarrow sin\theta = sin\left(\frac{7\pi}{6}\right)$

$\Rightarrow \theta = n\pi + (-1)^n\left(\frac{7\pi}{6}\right),\ n \in Z$ **જવાબ : (D)**

(38) $\begin{vmatrix} sinx & cosx & cosx \\ cosx & sinx & cosx \\ cosx & cosx & sinx \end{vmatrix} = 0$ ના $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$ માં ભિન્ન ઉકેલની સંખ્યા ...... છે. [IIT : 2001]

(A) 0 (B) 2 (C) 1 (D) 3

ઉકેલ : $\begin{vmatrix} sinx & cosx & cosx \\ cosx & sinx & cosx \\ cosx & cosx & sinx \end{vmatrix} = 0$

$\therefore \quad \begin{vmatrix} sinx - cosx & 0 & cosx \\ cosx - sinx & sinx - cosx & cosx \\ 0 & cosx - sinx & sinx \end{vmatrix} = 0 \qquad C_1 - C_2,\ C_2 - C_3$

$\therefore \quad (sinx - cosx)^2 \begin{vmatrix} 1 & 0 & cosx \\ -1 & 1 & cosx \\ 0 & -1 & sinx \end{vmatrix} = 0$

$\therefore \quad (sinx - cosx)^2\ [1(sinx + cosx) + cosx(1 - 0)] = 0$

$\therefore \quad (sinx - cosx)^2(sinx + 2cosx) = 0$

$\therefore \quad sinx = cosx$ અથવા $sinx = -2cosx$

$\therefore \quad tanx = 1$ અથવા $tanx = -2$

અહીં $x \in \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$ હોવાથી, $tanx \in [-1, 1]$

$\therefore \quad tanx = -2$ શક્ય નથી.

$\therefore \quad tanx = 1 \Rightarrow x = \frac{\pi}{4}$. આથી, ઉકેલની સંખ્યા 1 છે. **જવાબ : (C)**

(39) સમીકરણ $7cosx + 5sinx = 2k + 1$ નો શક્ય ઉકેલ, k પૂર્ણાંક હોય, તેવા પૂર્ણાંક k ની સંખ્યા ...... છે. [IIT : 2002]

(A) 4 (B) 8 (C) 10 (D) 12

ઉકેલ : $7cosx + 5sinx = 2k + 1$

$a = 7,\ b = 5$

$\therefore\ r^2 = a^2 + b^2 = 49 + 25 = 74$. આથી, $r = \sqrt{74}$

$\therefore\ -\sqrt{74} \le 2k + 1 \le \sqrt{74}$ જરૂરી છે.

$\therefore\ |2k + 1| \le \sqrt{74}$

શક્ય પૂર્ણાંક $k = 0, 1, 2, 3, -1, -2, -3, -4$

$\therefore$ શક્ય પૂર્ણાંકની સંખ્યા 8 છે. **જવાબ : (B)**

(40) $cos(\alpha - \beta) = 1,\ cos(\alpha + \beta) = \frac{1}{e},\ \alpha, \beta \in [-\pi, \pi]$ એ બંને સમીકરણનું સમાધાન કરતી ક્રમયુક્ત જોડ (α, β) ની સંખ્યા ...... છે. [IIT : 2005]

(A) 0 (B) 1 (C) 2 (D) 4

ઉકેલ : $-\pi \le \alpha \le \pi$

$-\pi \le \beta \le \pi \Rightarrow \pi \ge -\beta \ge -\pi$

$\therefore\ -\pi \le \alpha \le \pi,\ -\pi \le -\beta \le \pi$

$-2\pi \le \alpha - \beta \le 2\pi$

$cos(\alpha - \beta) = 1$ તથા $\alpha - \beta \in [-2\pi, 2\pi] \Rightarrow \alpha - \beta = 0, 2\pi, -2\pi$

(i) $\alpha - \beta = 0$ (ii) $\alpha = \beta + 2\pi$ (iii) $\alpha = \beta - 2\pi$ નો વિચાર કરીએ.

$\alpha = \beta + 2\pi$ લઈએ, તો

$-\pi \le \beta \le \pi \Rightarrow \pi \le 2\pi + \beta = \alpha \le 3\pi$ જે શક્ય નથી. કારણ કે $-\pi \le \alpha \le \pi$

$[\alpha = \pi,\ \beta = -\pi$ શક્ય છે, પરંતુ $cos(\alpha + \beta) = \frac{1}{e} \ne 1]$

આ જ રીતે, $\alpha = \beta - 2\pi$ પણ શક્ય નથી.

$\therefore\ \alpha = \beta$ લેતાં,

$cos(\alpha + \beta) = \frac{1}{e} \Rightarrow cos2\alpha = \frac{1}{e}$ જ્યાં $-\pi \le \alpha \le \pi$ એટલે કે $-2\pi \le 2\alpha \le 2\pi$

$\therefore\ -2\pi \le 2\alpha \le 2\pi$ અને $cos2\alpha = \frac{1}{e}$

$[-2\pi, 0]$ માં બે ઉકેલ અને $[0, 2\pi]$ માં બે ઉકેલ મળે.

$\therefore$ ઉકેલની કુલ સંખ્યા 4 છે. **જવાબ : (D)**

(41) $\theta \in (0, 2\pi)$ માટે $2sin^2\theta - 5sin\theta + 2 > 0$ નો ઉકેલ ...... હોઈ શકે. [IIT : 2006]

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$ (C) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{41\pi}{8}, \pi\right)$

ઉકેલ : $2sin^2\theta - 5sin\theta + 2 > 0$

$\therefore\ 2\left(sin\theta - \frac{1}{2}\right)(sin\theta - 2) > 0$

$\therefore\ sin\theta < \frac{1}{2}$ અથવા $sin\theta > 2$

$sin\theta > 2$ શક્ય નથી.

$\therefore\ sin\theta < \frac{1}{2}$

$\therefore$ ઉકેલગણ $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ છે. **જવાબ : (A)**

(42) સમીકરણ $2sin^2\theta - cos2\theta = 0$ અને $2cos^2\theta - 3sin\theta = 0$ નાં $[0, 2\pi]$ માં ઉકેલની સંખ્યા ...... છે.

(A) 0 (B) 1 (C) 2 (D) 4

ઉકેલ : $2sin^2\theta - cos2\theta = 0$

$\therefore\ 2sin^2\theta - (1 - 2sin^2\theta) = 0$

$\therefore\ 4sin^2\theta = 1$

$\therefore\ sin\theta = \pm\frac{1}{2}$

હવે, $2cos^2\theta - 3sin\theta = 0$

$\therefore\ 2(1 - sin^2\theta) - 3sin\theta = 0$

$\therefore\ 2sin^2\theta + 3sin\theta - 2 = 0$

$\therefore\ (2sin\theta - 1)(sin\theta + 2) = 0$

$\therefore\ sin\theta = \frac{1}{2}$ અથવા $sin\theta = -2$

$sin\theta = -2$ શક્ય નથી.

આથી સમીકરણનો સામાન્ય ઉકેલ $sin\theta = \frac{1}{2}$ છે. $\theta \in [0, 2\pi] \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore$ ઉકેલની સંખ્યા 2 છે. **જવાબ : (C)**

(43) $3sin^2x - 7sinx + 2 = 0$ ના $[0, 5\pi]$ માં ઉકેલની સંખ્યા ...... છે.

(A) 0 (B) 5 (C) 6 (D) 10

ઉકેલ : $3sin^2x - 7sinx + 2 = 0$

$\therefore\ (3sinx - 1)(sinx - 2) = 0$

$\therefore\ sinx = \frac{1}{3}$ અથવા $sinx = 2$. પરંતુ $sinx = 2$ શક્ય નથી.

$sinx = \frac{1}{3} > 0$ હોવાથી, $x \in [0, \pi]$ માં બે ઉકેલ, $x \in [2\pi, 3\pi]$ માં બે ઉકેલ તેમજ $x \in [4\pi, 5\pi]$માં અન્ય બે ઉકેલ મળે. તેથી કુલ 6 ઉકેલ મળે. **જવાબ : (C)**

(44) x એ એવી નાનામાં નાની ધન વાસ્તવિક સંખ્યા છે કે જેથી $\log_{cosx} sinx + \log_{sinx}cosx = 2$ થાય, તો $x =$ ......

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

ઉકેલ : $\log_{cosx} sinx = a$ લેતાં,

$a + \frac{1}{a} = 2$

$\therefore\ a^2 - 2a + 1 = 0$

$\therefore\ a = 1$

$\therefore\ \log_{cosx}sinx = 1$

$\therefore\ sinx = cosx$

$\therefore\ tanx = 1$

$\therefore\ x = \frac{\pi}{4}$, જે નાનામાં નાની ધન સંખ્યા છે. **જવાબ : (C)**

(45) સમીકરણ $sinx\,cosy = 1$ નો ઉકેલ મળે તેવી ક્રમયુક્ત જોડ (x, y)ની સંખ્યા ...... છે, $x, y \in [0, 2\pi]$

(A) 1 (B) 2 (C) 3 (D) 4

ઉકેલ : નીચેના બે વિકલ્પો માટે $sinx\ cosy = 1$ સત્ય થાય.

(i) $sinx = cosy = 1$ અથવા (ii) $sinx = cosy = -1$

(i) જો $sinx = cosy = 1$ હોય તો,

$(x, y) = \left(\frac{\pi}{2}, 0\right)$ અથવા $\left(\frac{\pi}{2}, 2\pi\right)$ $(x, y \in [0, 2\pi])$

(ii) જો $sinx = cosy = -1$ હોય, તો $(x, y) = \left(\frac{3\pi}{2}, \pi\right)$

$\therefore$ ઉકેલની સંખ્યા 3 છે. **જવાબ : (C)**

(46) જો $sin^3x\, sin3x = \sum_{m=0}^{n} c_m\, cosmx$ તો $n = ...$ જ્યાં $c_0, c_1, ... , c_n$ અચળો છે, $c_n \neq 0$ [IIT : 1981]

(A) 4 (B) 5 (C) 6 (D) 7

ઉકેલ : $sin^3x\, sin3x = \sum_{m=0}^{n} c_m\, cosmx$

$$\therefore \sum_{m=0}^{n} c_m\, cosmx = \frac{1}{4}(3sinx - sin3x)sin3x$$

$$= \frac{1}{4}[3sin3x\, sinx - sin^2 3x]$$

$$= \frac{1}{8}[3(2sin3x\, sinx) - 2sin^2 3x]$$

$$= \frac{1}{8}[3(-cos4x + cos2x) - (1 - cos6x)]$$

$$c_0 + c_1 cosx + c_2 cos2x + c_3 cos3x + ... + c_n cosnx = \frac{1}{8}[-3cos4x + 3cos2x - 1 + cos6x]$$

$\therefore n = 6$

જવાબ : **(C)**

(47) $sin\frac{\pi}{14}\, sin\frac{3\pi}{14}\, sin\frac{5\pi}{14}\, sin\frac{7\pi}{14}\, sin\frac{9\pi}{14}\, sin\frac{11\pi}{14}\, sin\frac{13\pi}{14} = ...$

(A) $\frac{1}{32}$ (B) $\frac{1}{128}$ (C) $\frac{1}{16}$ (D) $\frac{1}{64}$

ઉકેલ : $sin\frac{\pi}{14}\, sin\frac{3\pi}{14}\, sin\frac{5\pi}{14}\, sin\frac{7\pi}{14}\, sin\frac{9\pi}{14}\, sin\frac{11\pi}{14}\, sin\frac{13\pi}{14}$

$$= sin\frac{\pi}{14}\, sin\frac{3\pi}{14}\, sin\frac{5\pi}{14}\, sin\frac{\pi}{2}\, sin\left(\pi - \frac{5\pi}{14}\right) \cdot sin\left(\pi - \frac{3\pi}{14}\right) \cdot sin\left(\pi - \frac{\pi}{14}\right)$$

$$= \left(sin\frac{\pi}{14}\, sin\frac{3\pi}{14}\, sin\frac{5\pi}{14}\right)^2$$

$$= \left(cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right) cos\left(\frac{\pi}{2} - \frac{3\pi}{14}\right) cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right)\right)^2$$

$$= \left[cos\frac{\pi}{7}\, cos\frac{2\pi}{7}\, cos\frac{3\pi}{7}\right]^2$$

$$= \left[\frac{1}{2sin\frac{\pi}{7}}\left(2sin\frac{\pi}{7}\, cos\frac{\pi}{7}\right) cos\frac{2\pi}{7}\, cos\frac{3\pi}{7}\right]^2$$

$$= \left[\frac{1}{2sin\frac{\pi}{7}}\left(sin\frac{2\pi}{7}\, cos\frac{2\pi}{7}\right) cos\left(\pi - \frac{4\pi}{7}\right)\right]^2$$

$$= \left[\frac{1}{2^2 sin\frac{\pi}{7}}\left(2sin\frac{2\pi}{7}\, cos\frac{2\pi}{7}\right)\left(-cos\frac{4\pi}{7}\right)\right]^2$$

$$= \left[\frac{1}{2 \times 4\, sin\frac{\pi}{7}}\left(2sin\frac{4\pi}{7}\, cos\frac{4\pi}{7}\right)\right]^2$$

$$= \left[\frac{1}{8\, sin\frac{\pi}{7}}\left(sin\frac{8\pi}{7}\right)\right]^2$$

$$= \frac{1}{64\, sin^2\frac{\pi}{7}}\, sin^2\left(\pi + \frac{\pi}{7}\right)$$

$$= \frac{1}{64\, sin^2\frac{\pi}{7}} \cdot sin^2\frac{\pi}{7}$$

$$= \frac{1}{64}$$

જવાબ : **(D)**

(48) સમીકરણ $x + y = \frac{2\pi}{3}$ તથા $cosx + cosy = \frac{3}{2}$ નો ઉકેલ ...... છે. [IIT : 1987]

(A) R (B) $\emptyset$ (C) [–1, 1] (D) {–1, 1}

ઉકેલ : $cosx + cosy = \frac{3}{2}$

$\therefore \quad 2cos\left(\frac{x+y}{2}\right) cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$

$\therefore \quad 2cos\left(\frac{2\pi}{3(2)}\right) cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$

$\therefore \quad 2\left(\frac{1}{2}\right) cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$

$\therefore \quad cos\left(\frac{x-y}{2}\right) = \frac{3}{2} > 1$ જે શક્ય નથી.

$\therefore$ ઉકેલગણ $\emptyset$ છે. **જવાબ : (B)**

(49) $sin\frac{\pi}{18} \; sin\frac{5\pi}{18} \; sin\frac{7\pi}{18} = ...$

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) આમાંથી એકપણ નહિ

ઉકેલ : $sin\frac{\pi}{18} \; sin\frac{5\pi}{18} \; sin\frac{7\pi}{18} = sin10° \, sin50° \, sin70°$

$= \frac{1}{2}(2sin70° sin10°)sin50°$

$= \frac{1}{2}(-cos80° + cos60°)sin50°$

$= \frac{1}{2}\left(-sin10° + \frac{1}{2}\right)sin50°$

$= -\frac{1}{4}[2sin50° sin10°] + \frac{1}{4}sin50°$

$= \frac{1}{4}[cos60° - cos40°] + \frac{1}{4}sin50°$

$= \frac{1}{4}\left[\frac{1}{2} - cos40° + sin50°\right]$

$= \frac{1}{4}\left[\frac{1}{2} - cos40° + cos40°\right]$

$= \frac{1}{8}$ **જવાબ : (C)**

નોંધ : $= sin10° \, sin(60° - 10°) \, sin(60° + 10°) = \frac{1}{4} sin3(10°) = \frac{1}{8}$

(50) $cos(x - y)$, $cosx$, $cos(x + y)$ સ્વરિત શ્રેણીમાં છે, તો $cosx \cdot sec\frac{y}{2} = ...$ $\left(sin\frac{y}{2} \neq 0\right)$

(A) $\pm\sqrt{2}$ (B) ±2 (C) $\pm\sqrt{3}$ (D) ±4

ઉકેલ : $cos(x - y)$, $cosx$, $cos(x + y)$ સ્વરિત શ્રેણીમાં છે.

$\therefore \quad \frac{1}{cos(x-y)}, \frac{1}{cosx}, \frac{1}{cos(x+y)}$ સમાંતર શ્રેણીમાં છે.

$\therefore \quad \frac{1}{cos(x-y)} + \frac{1}{cos(x+y)} = \frac{2}{cosx}$

$\therefore \quad \frac{cos(x+y) + cos(x-y)}{cos(x+y)cos(x-y)} = \frac{2}{cosx}$

$\therefore \quad \frac{2cosx\,cosy}{cos^2x - sin^2y} = \frac{2}{cosx}$

$\therefore \quad cos^2x \, cosy = cos^2x - sin^2y$

$\therefore \quad cos^2x - cos^2x \, cosy = sin^2y$

$\therefore \quad cos^2x \; (1 - cosy) = sin^2y$

$\therefore \quad cos^2x\left(2sin^2\frac{y}{2}\right) = \left(2sin\frac{y}{2}\, cos\frac{y}{2}\right)^2$

$\therefore \quad cos^2x\left(2sin^2\frac{y}{2}\right) = 4sin^2\frac{y}{2}\, cos^2\frac{y}{2}$

$\therefore \quad cos^2x = 2cos^2\frac{y}{2}$

$\therefore \quad \frac{cos^2x}{cos^2\frac{y}{2}} = 2$

$\therefore \quad cos^2x \, sec^2\frac{y}{2} = 2$

$\therefore \quad cosx \, sec\frac{y}{2} = \pm\sqrt{2}$

જવાબ : (A)

(51) $A > 0, B > 0, A + B = \frac{\pi}{3}$, તો $tanA\ tanB$ નું મહત્તમ મૂલ્ય ...... છે.

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{\sqrt{3}}$

ઉકેલ : ધારો કે, $tanA\ tanB = y$

હવે, $A + B = \frac{\pi}{3}$

$\therefore\ tan(A + B) = tan\frac{\pi}{3}$

$\therefore\ \frac{tanA + tanB}{1 - tanA\,tanB} = \sqrt{3}$

$\therefore\ tanA + tanB = \sqrt{3}(1 - tanA\,tanB)$

$\therefore\ \frac{y}{tanB} + tanB = \sqrt{3}(1 - y)$

$\therefore\ y + tan^2B = \sqrt{3}\,tanB(1 - y)$

$\therefore\ tan^2B - \sqrt{3}\,tanB(1 - y) + y = 0$

આ $tanB$ માં દ્વિઘાત સમીકરણ છે.

વાસ્તવિક ઉકેલ માટે $\Delta \geq 0$ થવું જોઈએ.

$\therefore\ 3(1 - y)^2 - 4y \geq 0$

$\therefore\ 3(1 - 2y + y^2) - 4y \geq 0$

$\therefore\ 3y^2 - 10y + 3 \geq 0$

$\therefore\ (y - 3)(3y - 1) \geq 0$

$A > 0, B > 0$ અને $A + B = \frac{\pi}{3}$

$\therefore\ 0 < A < \frac{\pi}{3}$ અને $0 < B < \frac{\pi}{3}$

$\therefore\ 0 < tanA < \sqrt{3}$ તથા $0 < tanB < \sqrt{3}$

$\therefore\ 0 < tanA\ tanB < 3$

$\therefore\ tanA\ tanB \geq 3$ શક્ય નથી.

$\therefore\ tanA\ tanB \leq \frac{1}{3}$

$\therefore\ tanA\,tanB$ ની મહત્તમ સીમા $\frac{1}{3}$ છે.

એ જોઈ શકાય કે $tanA = tanB = \frac{1}{\sqrt{3}}$ શક્ય છે.

$\therefore$ મહત્તમ મૂલ્ય $\frac{1}{3}$ છે. **જવાબ : (A)**

(52) $tanA = \frac{(1 - cosB)}{sinB}$ તો $tan2A =$ ......

(A) $tanB$ (B) $sinB$ (C) $tan2B$ (D) $tan4B$

ઉકેલ : $tanA = \frac{(1 - cosB)}{sinB} = \frac{2sin^2\frac{B}{2}}{2sin\frac{B}{2}\,cos\frac{B}{2}} = tan\frac{B}{2}$

$\therefore\ tanA = tan\frac{B}{2}$

$\therefore\ A = \frac{B}{2} + n\pi$

$\therefore\ 2A = B + 2n\pi$

$\therefore\ tan2A = tanB$ **જવાબ : (A)**

(53) $sin^4\theta - 2sin^2\theta - 1 = 0$ નાં $[0, 2\pi]$ માં શક્ય ઉકેલની સંખ્યા ...... છે. [IIT : 1984]

(A) 0 (B) 1 (C) 2 (D) 3

ઉકેલ : $sin^4\theta - 2sin^2\theta - 1 = 0$

$\therefore\ sin^4\theta - 2sin^2\theta + 1 - 2 = 0$

$\therefore\ (1 - sin^2\theta)^2 = 2$

$\therefore\ cos^4\theta = 2$, જે શક્ય નથી.

$\therefore$ ઉકેલ સંખ્યા 0 છે. **જવાબ : (A)**

(54) કોઈ ધન પૂર્ણાંક n માટે, $sin\frac{\pi}{2n} + cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$ હોય તો n ની ...... કિંમતો શક્ય બને. [IIT : 1994]

(A) 1 (B) 2 (C) 3 (D) 4

ઉકેલ : $sin\frac{\pi}{2n} + cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$

$\therefore \left(sin\frac{\pi}{2n} + cos\frac{\pi}{2n}\right)^2 = \frac{n}{4}$

$\therefore 1 + sin\frac{2\pi}{2n} = \frac{n}{4}$

$\therefore sin\frac{\pi}{n} = \frac{n}{4} - 1$

$\therefore sin\frac{\pi}{n} = \frac{n-4}{4}$ (1)

હવે, n ધન પૂર્ણાંક છે.

$0 \le sin\frac{\pi}{n} \le 1$

$sin\frac{\pi}{n} = 0$ તો $n = 4$. પરંતુ $sin\frac{\pi}{n} \ne 0$ ((1) પરથી)

$sin\frac{\pi}{n} = 1$ તો $n = 8$ પરંતુ $sin\frac{\pi}{8} \ne 1$

$\therefore 0 < sin\frac{\pi}{n} < 1 \Rightarrow 0 < \frac{n-4}{4} < 1$

$\therefore 0 < n - 4 < 4$

$\therefore 4 < n < 8$

$\therefore n = 5, 6, 7$ શક્ય બને.

$\therefore n$ ની 3 કિંમતો શક્ય બને. **જવાબ : (C)**

(55) સંકર સંખ્યા ω એ 1 નું ઘનમૂળ હોય તો

$sin\left((\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\right)$ = ... $(\omega \ne 1)$

(A) $-\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{\sqrt{2}}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{2}$

ઉકેલ : $sin\left((\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\right)$

$\omega^{10} = \omega^9 \cdot \omega = \omega$

$\omega^{23} = \omega^{21} \cdot \omega^2 = \omega^2$

$sin\left((\omega + \omega^2)\pi - \frac{\pi}{4}\right)$

$= sin\left(-\pi - \frac{\pi}{4}\right)$ $1 + \omega + \omega^2 = 0$. આથી, $\omega + \omega^2 = -1$

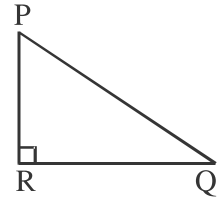
$= -sin\left(\pi + \frac{\pi}{4}\right) = sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ **જવાબ : (C)**

(56) ΔPQR માટે $R = \frac{\pi}{2}$ હોય તથા $tan\frac{P}{2}$ અને $tan\frac{Q}{2}$ એ દ્વિઘાત સમીકરણ $ax^2 + bx + c = 0$ $(a \ne 0)$ નાં બે બીજ હોય, તો ...... સત્ય બને.

(A) $a + b = c$ (B) $a + c = b$ (C) $b + c = a$ (D) $b = c$

ઉકેલ : સમીકરણ $ax^2 + bx + c = 0$ નાં બે બીજ $tan\frac{P}{2}$ અને $tan\frac{Q}{2}$ છે.

$\therefore tan\frac{P}{2} + tan\frac{Q}{2} = -\frac{b}{a}$ અને $tan\frac{P}{2}\ tan\frac{Q}{2} = \frac{c}{a}$



આકૃતિ 12.9

વળી, ΔPQR માટે, $R = \frac{\pi}{2}$ હોવાથી, $P + Q = \frac{\pi}{2}$

$\therefore \frac{P}{2} + \frac{Q}{2} = \frac{\pi}{4}$

$\therefore tan\left(\frac{P}{2} + \frac{Q}{2}\right) = 1$

$\therefore \frac{tan\frac{P}{2} + tan\frac{Q}{2}}{1 - tan\frac{P}{2}\, tan\frac{Q}{2}} = 1$

$\therefore \frac{\frac{-b}{a}}{1 - \frac{c}{a}} = 1$

$\therefore \frac{-b}{a - c} = 1$

$\therefore -b = a - c$

$\therefore c = a + b$ **જવાબ : (A)**

(57) $f(\theta) = sin\theta(sin\theta + sin3\theta)$ માટે, નીચેનામાંથી કયું સત્ય બને ?

(A) $f(\theta) < 0, \forall\theta \ge 0$ (B) $f(\theta) < 0, \forall\theta < 0$

(C) $f(\theta) \ge 0, \forall\theta \in R$ (D) $f(\theta) < 0, \forall\theta \in R$

ઉકેલ : $f(\theta) = sin\theta(sin\theta + sin3\theta)$

$= sin\theta(2sin2\theta\ \ cos\theta)$

$= (2sin\theta cos\theta)sin2\theta$

$= sin^2 2\theta \ge 0, \forall\theta \in R$ **જવાબ : (C)**

(58) $\alpha + \beta = \frac{\pi}{2}$ અને $\beta + \gamma = \alpha$ તો $tan\alpha = ...$

(A) $2(tan\beta + tan\gamma)$ (B) $tan\beta + 2tan\gamma$ (C) $tan\beta + tan\gamma$ (D) $2tan\beta + tan\gamma$

ઉકેલ : અહીં, $\alpha + \beta = \frac{\pi}{2}$ છે.

$\therefore \alpha = \frac{\pi}{2} - \beta$

$tan\alpha = tan\left(\frac{\pi}{2} - \beta\right) = cot\beta = \frac{1}{tan\beta}$

$\therefore tan\alpha\, tan\beta = 1$ (1)

હવે, $\beta + \gamma = \alpha$

$\therefore \gamma = \alpha - \beta$

$\therefore tan\gamma = tan(\alpha - \beta) = \frac{tan\alpha - tan\beta}{1 + tan\alpha\, tan\beta}$ ((1) પરથી)

$\therefore tan\gamma = \frac{tan\alpha - tan\beta}{2}$

$\therefore tan\alpha = tan\beta + 2tan\gamma$ **જવાબ : (B)**

(59) $(cos\alpha_1)(cos\alpha_2)\ ...\ (cos\alpha_n)$ ની મહત્તમ કિંમત ...... છે,

જ્યાં $0 \le \alpha_1, \alpha_2, \alpha_3, ..., \alpha_n \le \frac{\pi}{2}$ તથા $cot\alpha_1 \cdot cot\alpha_2 \ ...\cdot cot\alpha_n = 1$

(A) $\frac{1}{2^{\frac{n}{2}}}$ (B) $\frac{1}{2^n}$ (C) $2^{\frac{1}{n}}$ (D) 1

ઉકેલ : ધારો કે $y = (cos\alpha_1)(cos\alpha_2)\ ...\ (cos\alpha_n)$

હવે, $\cot\alpha_1 \cot\alpha_2 \ldots \cot\alpha_n = 1$

$$\therefore \quad \frac{\cos\alpha_1}{\sin\alpha_1}\cdot\frac{\cos\alpha_2}{\sin\alpha_2}\ldots\frac{\cos\alpha_n}{\sin\alpha_n} = 1$$

$\therefore \quad \cos\alpha_1 \cdot \cos\alpha_2 \ldots \cos\alpha_n = \sin\alpha_1 \cdot \sin\alpha_2 \ldots \sin\alpha_n = y$

$y^2 = (\cos\alpha_1 \cos\alpha_2 \cos\alpha_3 \ldots \cos\alpha_n)(\sin\alpha_1 \sin\alpha_2 \ldots \sin\alpha_n)$

$$\therefore \quad y^2 = \frac{(2\sin\alpha_1 \cos\alpha_1)(2\sin a_2 \cos\alpha_2)\ldots(2\sin a_n \cos\alpha_n)}{2^n}$$

$$\therefore \quad y^2 = \frac{\sin2\alpha_1 \sin2\alpha_2 \ldots \sin2\alpha_n}{2^n}$$

$0 \le \alpha_1, \alpha_2, \ldots \alpha_n \le \frac{\pi}{2}$

$\therefore \quad 0 \le 2\alpha_1, 2\alpha_2, \ldots, 2\alpha_n \le \pi$

$\therefore \quad 0 \le \sin2\alpha_1 \sin2\alpha_2, \ldots, \sin2\alpha_n \le 1$

$$\therefore \quad 0 \le \frac{\sin2\alpha_1 \sin2\alpha_2 \ldots \sin2\alpha_n}{2^n} \le \frac{1}{2^n}$$

$\therefore \quad 0 \le y^2 \le \frac{1}{2^n}$

$\therefore \quad 0 \le y \le \frac{1}{2^{\frac{n}{2}}}$

$\therefore$ y ની મહત્તમ કિંમત $\frac{1}{2^{\frac{n}{2}}}$ છે.

વળી $\alpha_1, \alpha_2 \ldots \alpha_n = \frac{\pi}{4}$ લેતાં,

$\cot\alpha_1 \cot\alpha_2 \ldots \cot\alpha_n = 1$

તથા $\cos\alpha_1 \cos\alpha_2 \ldots \cot\alpha_n = \left(\frac{1}{\sqrt{2}}\right)^n = \frac{1}{2^{\frac{n}{2}}}$

જવાબ : (A)

(60) α, β, γ એવી વાસ્તવિક સંખ્યાઓ છે કે જેથી $\alpha + \beta + \gamma = \pi$ હોય, તો $\sin\alpha + \sin\beta + \sin\gamma$ ની ન્યૂનતમ કિંમત ......

(A) હંમેશા ધન હોય. (B) ઋણ હોઈ શકે. (C) 0 થાય. (D) –3 થાય.

ઉકેલ : $\alpha + \beta + \gamma = \pi$

$\alpha = -\frac{\pi}{2}, \beta = -\frac{\pi}{2}, \gamma = 2\pi$ માટે આ સત્ય છે.

$$\sin\alpha + \sin\beta + \sin\gamma = \sin\left(\frac{-\pi}{2}\right) + \sin\left(\frac{-\pi}{2}\right) + \sin2\pi$$
$$= (-1) + (-1) + 0 = -2$$

$\therefore$ $\sin\alpha + \sin\beta + \sin\gamma$ ની કિંમત –2 છે.

(A), (C), (D) શક્ય નથી.

$\therefore$ કિંમત ઋણ હોઈ શકે. **જવાબ : (B)**

[(A) તો સ્પષ્ટ અસત્ય છે. (D) સત્ય માટે $\alpha = \beta = \gamma = \frac{3\pi}{2}, \frac{7\pi}{2}$ વગેરે આ શક્ય નથી, કારણ કે $\alpha + \beta + \gamma = \pi$. (C) પણ સ્પષ્ટ રીતે અસત્ય છે.]

(61) $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ નો વિસ્તાર ...... છે.

(A) [4, 10] (B) [–4, 10] (C) [3, 10] (D) [–3, 10]

ઉકેલ : $f(\theta) = 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right)$

$$= 5\cos\theta + 3\left(\cos\theta \cos\frac{\pi}{3} - \sin\theta \sin\frac{\pi}{3}\right)$$

$= 5cos\theta + \left(\frac{3}{2}cos\theta - \frac{3\sqrt{3}}{2}sin\theta\right)$

$= \frac{13}{2}\ cos\theta - \frac{3\sqrt{3}}{2}sin\theta$. આથી, $a = \frac{13}{2}$, $b = -\frac{3\sqrt{3}}{2}$

$r^2 = a^2 + b^2 = \frac{169+27}{4} = 49$. આથી, $r = 7$

$\therefore$ f નો વિસ્તાર $[-7, 7]$ છે.

$\therefore\ -7 \le 5cos\theta + 3cos\left(\theta + \frac{\pi}{3}\right) \le 7$

$\Rightarrow -4 \le 5cos\theta + 3cos\left(\theta + \frac{\pi}{3}\right) + 3 \le 10$

માંગેલ વિસ્તાર $[-4, 10]$ છે. **જવાબ : (B)**

(62) $f(\theta) = sin^2\theta$ નું મુખ્ય આવર્તમાન ...... છે. [AIEEE : 2002]

(A) π^2 (B) π (C) 2π (D) $\frac{\pi}{2}$

ઉકેલ : $f(\theta) = sin^2\theta = sin^2(\pi + \theta) = f(\pi + \theta)$

આવી નાનામાં નાની ધન વાસ્તવિક સંખ્યા π છે. આથી, મુખ્ય આવર્તમાન $= \pi$ **જવાબ : (B)**

(63) α, β એવી સંખ્યા છે કે જેથી $\pi < \alpha - \beta < 3\pi$

$sin\alpha + sin\beta = \frac{-21}{65}$ અને $cos\alpha + cos\beta = \frac{-27}{65}$, તો $cos\left(\frac{\alpha-\beta}{2}\right) = ...$ [AIEEE : 2004]

(A) $\frac{-6}{65}$ (B) $\frac{3}{\sqrt{130}}$ (C) $\frac{6}{65}$ (D) $\frac{-3}{\sqrt{130}}$

ઉકેલ : $sin\alpha + sin\beta = \frac{-21}{65}$. આથી, $2sin\left(\frac{\alpha+\beta}{2}\right)cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-21}{65}$ (1)

$cos\alpha + cos\beta = \frac{-27}{65}$. આથી $2cos\left(\frac{\alpha+\beta}{2}\right)cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-27}{65}$ (2)

(1) અને (2) વર્ગોનો સરવાળો કરતાં,

$\therefore\ 4sin^2\left(\frac{\alpha+\beta}{2}\right)cos^2\left(\frac{\alpha-\beta}{2}\right) + 4cos^2\left(\frac{\alpha+\beta}{2}\right)cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{441+729}{65^2}$

$\therefore\ 4cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{18}{65}$

$\therefore\ cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{9}{130}$

$\therefore\ cos\left(\frac{\alpha-\beta}{2}\right) = \pm\frac{3}{\sqrt{130}}$

$\therefore\ cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}}$ કારણ કે $\frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2}$ **જવાબ : (D)**

(64) સમીકરણ $2sin^2x + 5sinx - 3 = 0$ ના $[0, 3\pi]$ માં શક્ય ઉકેલોની સંખ્યા ...... છે. [AIEEE : 2006]

(A) 4 (B) 6 (C) 1 (D) 2

ઉકેલ : $2sin^2x + 5sinx - 3 = 0$

$\therefore \; sinx = \frac{1}{2}$ અથવા $sinx = -3$

હવે, $sinx = -3$ શક્ય નથી.

$\therefore \; sinx = \frac{1}{2}$ અને $x \in [0, 3\pi]$

$\therefore$ $[0, \pi]$ માં બે ઉકેલ અને $[2\pi, 3\pi]$ માં અન્ય બે ઉકેલ મળે.

$\therefore \; \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ ચાર ઉકેલો શક્ય બને.

ઉકેલોની કુલ સંખ્યા 4 છે. **જવાબ : (A)**

(65) $0 < x < \pi$ અને $cosx + sinx = \frac{1}{2}$ હોય, તો $tanx = ...$

(A) $\frac{1-\sqrt{7}}{4}$ (B) $\frac{4-\sqrt{7}}{3}$ (C) $\frac{-4 \pm \sqrt{7}}{3}$ (D) $\frac{1+\sqrt{7}}{4}$

ઉકેલ : $cosx + sinx = \frac{1}{2}$ આપેલ છે.

$\therefore \; 1 + tanx = \frac{1}{2}secx$ $(cosx \neq 0)$

$\therefore \; 2 + 2tanx = secx$

$\therefore \; 4 + 8tanx + 4tan^2x = tan^2x + 1$

$\therefore \; 3tan^2x + 8tanx + 3 = 0$

$\therefore \; tanx = \frac{-8 \pm \sqrt{28}}{6} = \frac{-8 \pm 2\sqrt{7}}{6} = \frac{-4 \pm \sqrt{7}}{3}$ **જવાબ : (C)**

(66) જો $cos(\beta - \gamma) + cos(\gamma - \alpha) + cos(\alpha - \beta) = -\frac{3}{2}$ હોય, તો P : $cos\alpha + cos\beta + cos\gamma = 0$ અને Q : $sin\alpha + sin\beta + sin\gamma = 0$ માટે ...... [AIEEE 2009]

(A) P અસત્ય છે અને Q સત્ય છે. (B) P અને Q બંને સત્ય છે.

(C) P અને Q બંને અસત્ય છે. (D) P સત્ય છે અને Q અસત્ય છે.

ઉકેલ : $(cos\alpha + cos\beta + cos\gamma)^2 + (sin\alpha + sin\beta + sin\gamma)^2$

$= 3 + 2[(cos\alpha\, cos\beta + sin\alpha\, sin\beta) + (cos\beta\, cos\gamma + sin\beta\, sin\gamma) + (cos\alpha\, cos\gamma + sin\alpha\, sin\gamma)]$

$= 3 + 2[cos(\alpha - \beta) + cos(\beta - \gamma) + cos(\alpha - \gamma)]$

$= 3 + 2\left(-\frac{3}{2}\right) = 0$

$\therefore \; cos\alpha + cos\beta + cos\gamma = 0$ તથા $sin\alpha + sin\beta + sin\gamma = 0$ બંને સત્ય છે. **જવાબ : (B)**

(67) $a_1, a_2 \ldots a_{n+1}$ પદો સમાંતર શ્રેણીમાં હોય, તો $tan^{-1}\frac{d}{1+a_1a_2} + tan^{-1}\frac{d}{1+a_2a_3} + \ldots + n$ પદ સુધી = ...

(જ્યાં $d > 0,\ a_i > 0,\ \forall i$) (સામાન્ય તફાવત છે.)

(A) $tan^{-1}\frac{nd}{1+a_1a_{n+1}}$ (B) $tan^{-1}\left(\frac{nd}{1+a_1a_n}\right)$

(C) $tan^{-1}\left(\frac{(n-1)d}{1+a_1a_n}\right)$ (D) આમાંથી એકપણ નહિ

ઉકેલ : $a_1, a_2, a_3, \ldots a_{n+1}$ સમાંતર શ્રેણીમાં છે.

$\therefore \quad a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots a_{n+1} - a_n = d$ (1)

$tan^{-1}\dfrac{d}{1+a_1a_2} + tan^{-1}\dfrac{d}{1+a_1a_3} + \ldots$ (n પદ સુધી)

$= tan^{-1}\left(\dfrac{a_2-a_1}{1+a_1a_2}\right) + tan^{-1}\left(\dfrac{a_3-a_2}{1+a_2a_3}\right) + \ldots +$ (n પદ સુધી)

$= (tan^{-1}a_2 - tan^{-1}a_1) + (tan^{-1}a_3 - tan^{-1}a_2) + \ldots + [tan^{-1}(a_{n+1}) - tan^{-1}(a_n)]$

$= tan^{-1}(a_{n+1}) - tan^{-1}a_1$

$= tan^{-1}\left(\dfrac{a_{n+1}-a_1}{1+a_{n+1}a_1}\right)$ (2)

હવે, (1) પરથી

$a_2 - a_1 + a_3 - a_2 + a_4 - a_3 + \ldots + a_{n+1} - a_n = nd$

$\therefore \quad a_{n+1} - a_1 = nd$

$\therefore \quad tan^{-1}\left(\dfrac{a_{n+1}-a_1}{1+a_{n+1}a_1}\right) = tan^{-1}\left(\dfrac{nd}{1+a_{n+1}a_1}\right)$ **જવાબ : (A)**

(68) $sin^2x + cos^4x, \ \forall x \in$ R નો વિસ્તાર ...... છે. [AIEEE : 2011]

(A) $\left[\frac{13}{16}, 1\right]$ (B) $[1, 2]$ (C) $\left[\frac{3}{4}, \frac{13}{16}\right]$ (D) $\left[\frac{3}{4}, 1\right]$

ઉકેલ : $sin^2x + cos^4x = \left(\dfrac{1-cos2x}{2}\right) + \left(\dfrac{1+cos2x}{2}\right)^2$

$= \dfrac{1-cos2x}{2} + \dfrac{1+2cos2x+cos^2 2x}{4}$

$= \dfrac{2-2cos2x+1+2cos2x+cos^2 2x}{4}$

$= \dfrac{3+cos^2 2x}{4}$

$= \frac{3}{4} + \frac{1}{4}\left(\dfrac{1+cos4x}{2}\right)$

$= \frac{7}{8} + \frac{1}{8}cos4x$

$cos4x$ નો વિસ્તાર $[-1, 1]$ છે.

$\therefore \quad \frac{7}{8} + \frac{1}{8}\ cos4x$ નો વિસ્તાર $\left[\frac{7}{8} - \frac{1}{8}, \frac{7}{8} + \frac{1}{8}\right] = \left[\frac{3}{4}, 1\right]$ છે. **જવાબ : (D)**

(69) $sin\theta + sin4\theta + sin7\theta = 0$ નું $(0, \pi)$ માં સમાધાન કરતી θ ની કિંમતો ...... છે. [AIEEE : 2011]

(A) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$ (B) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

(C) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$ (D) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

ઉકેલ : $sin\theta + sin4\theta + sin7\theta = 0$

$\therefore \quad (sin7\theta + sin\theta) + sin4\theta = 0$

$\therefore \quad 2sin4\theta \, cos3\theta + sin4\theta = 0$

$\therefore \quad sin4\theta \; (2cos3\theta + 1) = 0$

$\therefore \quad sin4\theta = 0$ અથવા $cos3\theta = -\frac{1}{2}$

(i) $sin4\theta = 0$ લેતાં,

$0 < \theta < \pi \Rightarrow 0 < 4\theta < 4\pi$

$\therefore \quad 4\theta = \pi, 2\pi, 3\pi$. આથી, $\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ (1)

(ii) $cos3\theta = -\frac{1}{2}$ લેતાં,

$0 < \theta < \pi \Rightarrow 0 < 3\theta < 3\pi$

$\therefore \quad 3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$. આથી, $\theta = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$ (2)

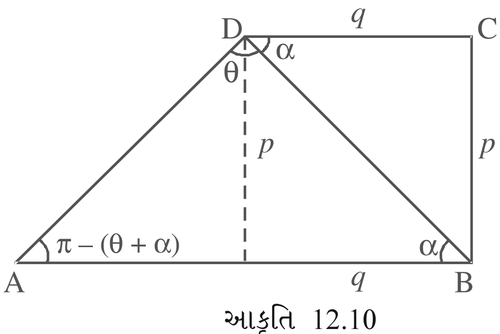
$\therefore \quad \theta = \frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$ **જવાબ : (D)**

(70) કોઈ એક સમલંબ ચતુષ્કોણ ABCD માટે $\overline{AB} \parallel \overline{CD}$ તથા $\overline{BC} \perp \overline{CD}$ હોય તથા જો $m\angle ADB = \theta$, $BC = p$ તથા $CD = q$ હોય, તો AB = ... [JEE Main : 2013]

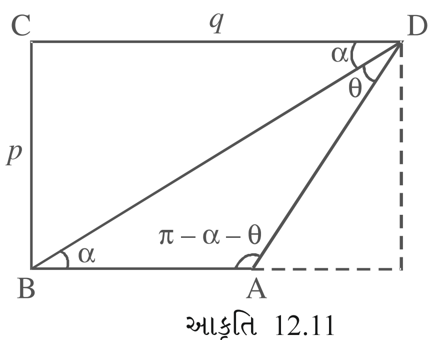
(A) $\dfrac{(p^2 + q^2)\,sin\theta}{p\,cos\theta + q\,sin\theta}$ (B) $\dfrac{p^2 + q^2\,cos\theta}{p\,cos\theta + q\,sin\theta}$

(C) $\dfrac{p^2 + q^2}{p^2\,cos\theta + q^2\,sin\theta}$ (D) $\dfrac{(p^2 + q^2)\,sin\theta}{(p\,cos\theta + q\,sin\theta)^2}$

ઉકેલ :



આકૃતિ 12.10



આકૃતિ 12.11

સમલંબ ચતુષ્કોણ ABCD માટે, $\overline{AB} \parallel \overline{CD}$ તેમજ $\overline{BC} \perp \overline{CD}$ આપેલ હોવાથી, આકૃતિમાં દર્શાવ્યા મુજબ કાટકોણ ΔBDC રચાશે. જ્યાં $m\angle BDC = \alpha$

$\therefore \quad BD^2 = BC^2 + CD^2 = p^2 + q^2$ (1)

અને $cos\alpha = \dfrac{CD}{BD} = \dfrac{q}{\sqrt{p^2+q^2}}$ તેમજ $sin\alpha = \dfrac{BC}{BD} = \dfrac{p}{\sqrt{p^2+q^2}}$ (2), (3)

વળી, ΔABD માટે, *sine* સૂત્રનો ઉપયોગ કરતાં,

$\dfrac{AB}{sin\theta} = \dfrac{BD}{sin\,A}$ મળે.

$\therefore \quad \dfrac{AB}{sin\theta} = \dfrac{\sqrt{p^2+q^2}}{sin(\pi - (\theta + \alpha))}$ ((1) પરથી)

$$\frac{AB}{sin\theta} = \frac{\sqrt{p^2 + q^2}}{sin\theta cos\alpha + cos\theta sin\alpha}$$

$$\therefore \quad AB = \frac{\sqrt{p^2 + q^2}\, sin\theta}{sin\theta\left(\frac{q}{\sqrt{p^2 + q^2}}\right) + cos\theta\left(\frac{p}{\sqrt{p^2 + q^2}}\right)}$$ ((2), (3) પરથી)

$$\therefore \quad AB = \frac{(p^2 + q^2)\, sin\theta}{p\, cos\theta + q\, sin\theta}$$

જવાબ : (A)

(71) $tan^{-1}y = tan^{-1}x + tan^{-1}\left(\frac{2x}{1 - x^2}\right),\ 0 < x < \frac{1}{\sqrt{3}}$ હોય, તો $y =$ ......

(A) $\frac{3x - x^3}{1 - 3x^2}$ (B) $\frac{3x + x^3}{1 - 3x^2}$ (C) $\frac{3x - x^3}{1 + 3x^2}$ (D) $\frac{3x + x^3}{1 + 3x^2}$

ઉકેલ : $tan^{-1}y = tan^{-1}\left(\frac{x + \frac{2x}{1 - x^2}}{1 - \frac{2x^2}{1 - x^2}}\right)$ $\left(0 < x < \frac{1}{\sqrt{3}}\right)$

$$= tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

$$\therefore \quad y = \left(\frac{3x - x^3}{1 - 3x^2}\right)$$

જવાબ : (A)

(72) $sin^{-1}(sin 10) =$ ......

(A) 10 (B) $10 - 3\pi$ (C) $3\pi - 10$ (D) આમાંથી એક પણ નહિ.

ઉકેલ : $sin^{-1}(sin 10) = sin^{-1}[sin(3\pi - 10)] = 3\pi - 10$ $\left(-\frac{\pi}{2} < 3\pi - 10 < 0\right)$

જવાબ : (C)

(73) $sin^{-1}\left[x\sqrt{1 - x} - \sqrt{x}\sqrt{1 - x^2}\right] =$ ......

(A) $sin^{-1}\sqrt{x} + sin^{-1}x$ (B) $sin^{-1}x - sin^{-1}\sqrt{x}$ (C) $sin^{-1}\sqrt{x} - sin^{-1}x$ (D) આમાંથી એક પણ નહિ.

ઉકેલ : ધારો કે $x = sin\theta$ અને $\sqrt{x} = sin\alpha$,

અહીં $x \in [0, 1] \Rightarrow \theta, \alpha \in \left[0, \frac{\pi}{2}\right]$

$\therefore \theta - \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$sin^{-1}\left(x\sqrt{1 - x} - \sqrt{x}\sqrt{1 - x^2}\right)$

$= sin^{-1}\left(sin\theta\sqrt{1-sin^2\alpha} - sin\alpha\sqrt{1-sin^2\theta}\right)$

$= sin^{-1}(sin\theta\ cos\alpha - cos\theta\ sin\alpha)$ $(cos\alpha > 0,\ cos\theta > 0)$

$= sin^{-1}(sin(\theta - \alpha))$

$= \theta - \alpha$

$= sin^{-1}x - sin^{-1}\sqrt{x}$ $\left(0 \le \theta, \alpha \le \frac{\pi}{2}\right)$ **જવાબ : (B)**

(74) $cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}$, $n \in N$ તો n ની મહત્તમ કિંમત ......

(A) 6 (B) 7 (C) 5 (D) આમાંથી એક પણ નહિ.

ઉકેલ : $cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}$

$\therefore \frac{n}{\pi} < cot\frac{\pi}{6}$ (cot ઘટતું વિધેય છે.)

$\therefore \frac{n}{\pi} < \sqrt{3}$

$\therefore n < \sqrt{3}\pi \Rightarrow n < 6$

$\therefore$ n ની મહત્તમ કિંમત $= 5$. **જવાબ : (C)**

(75) $\sum_{r=1}^{n} sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right) = ...$

(A) $tan^{-1}\sqrt{n} - \frac{\pi}{4}$ (B) $tan^{-1}\left(\sqrt{n+1}\right) - \frac{\pi}{4}$ (C) $tan^{-1}\sqrt{n}$ (D) $tan^{-1}\left(\sqrt{n+1}\right)$

ઉકેલ : $sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right) = tan^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{1+\sqrt{r}\sqrt{r-1}}\right)$

$= tan^{-1}\sqrt{r} - tan^{-1}\left(\sqrt{r-1}\right)$

$\therefore \sum_{r=1}^{n} sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{1+\sqrt{r}\sqrt{r-1}}\right) = \sum_{r=1}^{n}\left(tan^{-1}\sqrt{r} - tan^{-1}\sqrt{r-1}\right) = tan^{-1}\sqrt{n}$ **જવાબ : (C)**

(76) $\sum_{r=1}^{n} tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right) = ...$

(A) $tan^{-1}2^n$ (B) $tan^{-1}2^n - \frac{\pi}{4}$ (C) $tan^{-1}2^{n+1}$ (D) $tan^{-1}2^{n+1} - \frac{\pi}{4}$

ઉકેલ : $\sum_{r=1}^{n} tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right) = \sum_{r=1}^{n} tan^{-1}\left(\frac{2^r - 2^{r-1}}{1+2^r \cdot 2^{r-1}}\right)$

$= \sum_{r=1}^{n}\left[tan^{-1}(2^r) - tan^{-1}(2^{r-1})\right]$

$= tan^{-1}2^n - tan^{-1}1 = tan^{-1}2^n - \frac{\pi}{4}$ **જવાબ : (B)**

(77) $\sum_{m=1}^{n} tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right) = ...$

(A) $tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$ (B) $tan^{-1}\left(\frac{n^2-n}{n^2-n+2}\right)$ (C) $tan^{-1}\left(\frac{n^2+n+2}{n^2+n}\right)$ (D) આમાંથી એક પણ નહિ.

ઉકેલ : $\sum_{m=1}^{n} tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right) = \sum_{m=1}^{n} tan^{-1}\left(\frac{(m^2+m+1)-(m^2-m+1)}{1+(m^2+m+1)(m^2-m+1)}\right)$

$$= \sum_{m=1}^{n}\left[tan^{-1}(m^2+m+1) - tan^{-1}(m^2-m+1)\right]$$

$$= (tan^{-1}3 - tan^{-1}1) + (tan^{-1}7 - tan^{-1}3) + (tan^{-1}13 - tan^{-1}7) + ... + (tan^{-1}(n^2+n+1) - tan^{-1}(n^2-n+1))$$

$$= tan^{-1}(n^2+n+1) - tan^{-1}1$$

$$= tan^{-1}\left(\frac{n^2+n+1-1}{1+(n^2+n+1)(1)}\right) \qquad (n^2+n+1 > 0)$$

$$= tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$$

જવાબ : (A)

(78) a, b, c ધન વાસ્તવિક સંખ્યા માટે

$\theta = tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$ હોય, તો $tan\theta = ...$ [IIT : 1981]

(A) 0 (B) 2 (C) 1 (D) $\sqrt{3}$

ઉકેલ : ધારો કે $tan\alpha = \sqrt{\frac{a(a+b+c)}{bc}}$, $tan\beta = \sqrt{\frac{b(a+b+c)}{ca}}$, $tan\gamma = \sqrt{\frac{c(a+b+c)}{ab}}$

$0 < \alpha, \beta, \gamma < \frac{\pi}{2}$

$\therefore$ $\theta = \alpha + \beta + \gamma$ મળે.

$$tan\alpha + tan\beta + tan\gamma = \frac{\sqrt{a+b+c}(a+b+c)}{\sqrt{abc}} = \frac{(a+b+c)^{\frac{3}{2}}}{\sqrt{abc}}$$

$$tan\alpha\ tan\beta\ tan\gamma = \frac{(a+b+c)^{\frac{3}{2}}}{\sqrt{abc}}$$

$\therefore$ $tan\alpha + tan\beta + tan\gamma = tan\alpha\ tan\beta\ tan\gamma$

$\therefore$ $\alpha + \beta + \gamma = (2k+1)\pi;\ k \in Z$

પરંતુ $0 < \alpha + \beta + \gamma < \frac{3\pi}{2}$. આથી, $\alpha + \beta + \gamma = \pi$

$\therefore$ $tan\theta = tan(\alpha + \beta + \gamma) = tan\pi = 0$

જવાબ : (A)

બીજી રીત :

ધારો કે $a + b + c = u$

$$\therefore \quad \theta = tan^{-1}\sqrt{\frac{au}{bc}} + tan^{-1}\sqrt{\frac{bu}{ca}} + tan^{-1}\sqrt{\frac{cu}{ab}}$$

હવે, $\sqrt{\frac{au}{bc}} \cdot \sqrt{\frac{bu}{ca}} = \frac{u}{c} = \frac{a+b+c}{c} > 1$

$$\therefore \quad \theta = \pi + tan^{-1}\left(\frac{\sqrt{\frac{au}{bc}} + \sqrt{\frac{bu}{ca}}}{1 - \left(\sqrt{\frac{au}{bc}}\right)\left(\sqrt{\frac{bu}{ca}}\right)}\right) + tan^{-1}\sqrt{\frac{cu}{ab}}$$

$$= \pi + tan^{-1}\left(\frac{\frac{(a+b)\sqrt{u}}{\sqrt{abc}}}{1 - \frac{u}{c}}\right) + tan^{-1}\sqrt{\frac{cu}{ab}}$$

$$= \pi + tan^{-1}\left(\frac{(u-c)\sqrt{u}}{(c-u)} \cdot \frac{c}{\sqrt{abc}}\right) + tan^{-1}\sqrt{\frac{cu}{ab}}$$

$$= \pi - tan^{-1}\sqrt{\frac{cu}{ab}} + tan^{-1}\sqrt{\frac{cu}{ab}}$$

$$= \pi$$

$\therefore \quad tan\theta = tan\pi = 0$ **જવાબ : (A)**

(79) $tan\left[2tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right] = ...$

(A) $\frac{7}{17}$ (B) $\frac{-7}{17}$ (C) $\frac{17}{7}$ (D) $\frac{15}{7}$

ઉકેલ : $tan\left[2tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right]$

$$= tan\left[tan^{-1}\frac{1}{5} + tan^{-1}\frac{1}{5} - tan^{-1}1\right]$$

$$= tan\left[tan^{-1}\frac{\frac{2}{5}}{1 - \frac{1}{25}} - tan^{-1}1\right]$$

$$= tan\left[\tan^{-1}\frac{5}{12} - \tan^{-1}1\right]$$

$$= tan\left[tan^{-1}\left(\frac{\frac{5}{12} - 1}{1 + \frac{5}{12}}\right)\right]$$

$$= tan\left[tan^{-1}\left(\frac{-7}{17}\right)\right] = \frac{-7}{17}$$ **જવાબ : (B)**

(80) $\lim_{n\to\infty} \sum_{r=1}^{\infty} tan^{-1}\left(\frac{1}{2r^2}\right) = ...$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{3}$

ઉકેલ : $\sum_{r=1}^{\infty} tan^{-1}\left(\frac{1}{2r^2}\right) = \sum_{r=1}^{\infty} tan^{-1}\left(\frac{2}{4r^2}\right)$

$$= \sum_{r=1}^{\infty} tan^{-1}\left[\frac{2}{1+4r^2-1}\right]$$

$$= \sum_{r=1}^{\infty} tan^{-1}\left[\frac{(2r+1)-(2r-1)}{1+(2r-1)(2r+1)}\right]$$

$$= \sum_{r=1}^{\infty} \left[tan^{-1}(2r+1) - tan^{-1}(2r-1)\right]$$

$$\therefore \lim_{n\to\infty} \sum_{r=1}^{\infty} tan^{-1}\left(\frac{1}{2r^2}\right) = \lim_{n\to\infty} \sum_{r=1}^{n} \left[tan^{-1}(2r+1) - tan^{-1}(2r-1)\right]$$

$$= \lim_{n\to\infty} \left[tan^{-1}(2n+1) - \frac{\pi}{4}\right] = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

જવાબ : **(B)**

(81) $cot^{-1}\left(\sqrt{cos\alpha}\right) - tan^{-1}\left(\sqrt{cos\alpha}\right) = x$ તો $sinx = ...$ [AIEEE : 2002]

(A) $tan^2\left(\frac{\alpha}{2}\right)$ (B) $cot^2\left(\frac{\alpha}{2}\right)$ (C) $tan\alpha$ (D) $cot\frac{\alpha}{2}$

ઉકેલ : $x = cot^{-1}\left(\sqrt{cos\alpha}\right) - tan^{-1}\left(\sqrt{cos\alpha}\right)$

$$= tan^{-1}\left(\frac{1}{\sqrt{cos\alpha}}\right) - tan^{-1}\left(\sqrt{cos\alpha}\right)$$

$$= tan^{-1}\left(\frac{\frac{1}{\sqrt{cos\alpha}} - \sqrt{cos\alpha}}{1+1}\right)$$

$$= tan^{-1}\left(\frac{1-cos\alpha}{2\sqrt{cos\alpha}}\right)$$

$$\therefore sinx = sin\left[tan^{-1}\left(\frac{1-cos\alpha}{2\sqrt{cos\alpha}}\right)\right]$$

$$sinx = sin\left[sin^{-1}\left(\frac{1-cos\alpha}{1+cos\alpha}\right)\right] \qquad \left(tan^{-1}x = sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$$

$$= sin\left(sin^{-1}\left(tan^2\frac{\alpha}{2}\right)\right)$$

$$= tan^2\frac{\alpha}{2}$$

જવાબ : **(A)**

(82) $sin^{-1}x = 2sin^{-1}a$ સમીકરણનો ઉકેલ મળે તે માટે નીચેનામાંથી કઈ શરત જરૂરી છે ?[AIEEE : 2003]

(A) $|a| \ge \frac{1}{\sqrt{2}}$ (B) $|a| \le \frac{1}{\sqrt{2}}$ (C) $\forall a \in R$ (D) $|a| < \frac{1}{2}$

ઉકેલ : $sin^{-1}x = 2sin^{-1}a$

આપણે જાણીએ છીએ કે, $\frac{-\pi}{2} \le sin^{-1}x \le \frac{\pi}{2}$

$\therefore \frac{-\pi}{2} \le 2sin^{-1}a \le \frac{\pi}{2}$

$\therefore \frac{-\pi}{4} \le sin^{-1}a \le \frac{\pi}{4}$

$\therefore -\frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$

$\therefore |a| \le \frac{1}{\sqrt{2}}$

જવાબ : (B)

(83) $cos^{-1}x - cos^{-1}\frac{y}{2} = \alpha$, તો $4x^2 - 4xy\,cos\alpha + y^2 = ...$ [AIEEE : 2005]

(A) $2sin2\alpha$ (B) 4 (C) $4sin^2\alpha$ (D) $-4sin^2\alpha$

ઉકેલ : ધારો કે $cos^{-1}x = p$, $cos^{-1}\frac{y}{2} = q$, $p \in [0, \pi]$, $q \in [0, 2\pi]$

$\therefore cos^{-1}x - cos^{-1}\frac{y}{2} = \alpha$

હવે, $p - q = \alpha$

$\therefore cos(p - q) = cos\alpha$

$\therefore cosp\,cosq + sinp\,sinq = cos\alpha$

$\frac{xy}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = cos\alpha$ $(sinp > 0,\ sinq > 0)$

$\left(\frac{xy}{2} - cos\alpha\right)^2 = (1 - x^2)\left(1 - \frac{y^2}{4}\right)$

$\frac{x^2y^2}{4} - xycos\alpha + cos^2\alpha = 1 - x^2 - \frac{y^2}{4} + \frac{x^2y^2}{4}$

$x^2 + \frac{y^2}{4} = 1 - cos^2\alpha + xycos\alpha$

$\therefore 4x^2 - 4xycos\alpha + y^2 = 4sin^2\alpha$

જવાબ : (C)

(84) $sin^{-1}\left(\frac{x}{5}\right) + cosec^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ હોય, તો $x = ...$ [AIEEE : 2007]

(A) 4 (B) 5 (C) 1 (D) 3

ઉકેલ : $sin^{-1}\left(\frac{x}{5}\right) + cosec^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$\therefore sin^{-1}\left(\frac{x}{5}\right) + sin^{-1}\frac{4}{5} = \frac{\pi}{2}$

$\therefore sin^{-1}\left(\frac{x}{5}\right) + cos^{-1}\frac{3}{5} = \frac{\pi}{2}$

$\therefore x = 3$

જવાબ : (D)

(85) $cot\left(cosec^{-1}\frac{5}{3} + tan^{-1}\frac{2}{3}\right) = ...$

(A) $\frac{6}{17}$ (B) $\frac{3}{17}$ (C) $\frac{4}{17}$ (D) $\frac{5}{17}$

ઉકેલ : $cosec^{-1}\frac{5}{3} + tan^{-1}\frac{2}{3} = tan^{-1}\frac{3}{4} + tan^{-1}\frac{2}{3} = tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{6}{12}}\right) = tan^{-1}\left(\frac{17}{6}\right)$

$\therefore \quad cot\left(cosec^{-1}\frac{5}{3} + tan^{-1}\frac{2}{3}\right) = cot\left(cot^{-1}\frac{6}{17}\right) = \frac{6}{17}$ જવાબ : **(A)**

(86) $0 < \theta < \frac{\pi}{2}$

$\sum_{m=1}^{6} cosec\left(\theta + \frac{(m-1)\pi}{4}\right)cosec\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$ સમીકરણના ઉકેલ ...... હોય. [IIT : 2009]

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

ઉકેલ : $0 < \theta < \frac{\pi}{2}$

$$\sum_{m=1}^{6} cosec\left(\theta + \frac{(m-1)\pi}{4}\right)cosec\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$$

$$\therefore \quad \sum_{m=1}^{6} \frac{1}{sin\left(\theta + \frac{(m-1)\pi}{4}\right) sin\left(\theta + \frac{m\pi}{4}\right)} = 4\sqrt{2}$$

$$\therefore \quad \sum_{m=1}^{6} \frac{sin\frac{\pi}{4}}{sin\frac{\pi}{4}\, sin\left(\theta + \frac{(m-1)\pi}{4}\right) sin\left(\theta + \frac{m\pi}{4}\right)} = 4\sqrt{2}$$

$$\therefore \quad \sum_{m=1}^{6} \frac{sin\left[\left(\theta + \frac{m\pi}{4}\right) - \left(\theta + \frac{(m-1)\pi}{4}\right)\right]}{sin\frac{\pi}{4}\left[sin\left(\theta + \frac{(m-1)\pi}{4}\right) sin\left(\theta + \frac{m\pi}{4}\right)\right]} = 4\sqrt{2}$$

$$\therefore \quad \sum_{m=1}^{6}\left[cot\left(\theta + \frac{(m-1)\pi}{4}\right) - cot\left(\theta + \frac{m\pi}{4}\right)\right] = 4$$

$$\therefore \quad cot\theta - cot\left(\theta + \frac{\pi}{4}\right) + cot\left(\theta + \frac{\pi}{4}\right) - cot\left(\theta + \frac{2\pi}{4}\right) + ... + cot\left(\theta + \frac{5\pi}{4}\right) - cot\left(\theta + \frac{6\pi}{4}\right) = 4$$

$$\therefore \quad cot\theta - cot\left(\theta + \frac{3\pi}{2}\right) = 4$$

$$\therefore \quad cot\theta + tan\theta = 4$$

$$\therefore \quad \frac{1}{tan\theta} + tan\theta = 4$$

$\therefore \quad 1 + tan^2\theta = 4tan\theta$

$\therefore \quad tan^2\theta - 4tan\theta + 1 = 0$

$\therefore \quad tan\theta = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

$tan\theta = 2 + \sqrt{3} \Rightarrow \theta = \frac{5\pi}{12}$ તથા $tan\theta = 2 - \sqrt{3} \Rightarrow \theta = \frac{\pi}{12}$ **જવાબ : (C), (D)**

(87) x, y, z સમાંતર શ્રેણીમાં છે તથા $tan^{-1}x, tan^{-1}y, tan^{-1}z$ પણ સમાંતર શ્રેણીમાં છે. તો [JEE : 2013] $(0 < x, y, z < 1)$

(A) $x = y = z$ (B) $2x = 3y = 6z$ (C) $6x = 3y = 6z$ (D) $6x = 4y = 3z$

ઉકેલ : x, y, z સમાંતર શ્રેણીમાં છે.

$\therefore \quad x + z = 2y$ (1)

હવે, $tan^{-1}x, tan^{-1}y, tan^{-1}z$ પણ સમાંતર શ્રેણીમાં છે.

$\therefore \quad tan^{-1}x + tan^{-1}z = 2tan^{-1}y$

$\therefore \quad tan^{-1}\left(\frac{x+z}{1-xz}\right) = 2tan^{-1}y = tan^{-1}\left(\frac{y+y}{1-y^2}\right)$

$\therefore \quad tan^{-1}\left(\frac{2y}{1-xz}\right) = tan^{-1}\left(\frac{2y}{1-y^2}\right)$ ((1) પરથી)

$\therefore \quad 1 - xz = 1 - y^2$

$\therefore \quad y^2 = xz$

$\therefore$ x, y, z સમગુણોત્તર શ્રેણીમાં છે તથા x, y, z સમાંતર શ્રેણીમાં છે.

$\therefore \quad x = y = z$ **જવાબ : (A)**

(88) $tan\left[cos^{-1}\left(\frac{4}{5}\right) + tan^{-1}\frac{2}{3}\right] = ...$ [IIT : 1993]

(A) $\frac{6}{17}$ (B) $\frac{7}{16}$ (C) $\frac{16}{7}$ (D) આમાંથી એક પણ નહિ.

ઉકેલ : હવે, $cos^{-1}\frac{4}{5} + tan^{-1}\frac{2}{3} = tan^{-1}\frac{3}{4} + tan^{-1}\frac{2}{3} = tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{6}{12}}\right) = tan^{-1}\left(\frac{17}{6}\right)$

$\therefore \quad tan\left[cos^{-1}\frac{4}{5} + tan^{-1}\frac{2}{3}\right] = tan\left(tan^{-1}\frac{17}{6}\right) = \frac{17}{6}$ **જવાબ : (D)**

(89) $tan\left[cos^{-1}\frac{1}{5\sqrt{2}} - sin^{-1}\frac{4}{\sqrt{17}}\right] = ...$ [IIT : 1994]

(A) $\frac{\sqrt{29}}{3}$ (B) $\frac{29}{3}$ (C) $\frac{\sqrt{3}}{29}$ (D) $\frac{3}{29}$

ઉકેલ : $tan\left[cos^{-1}\frac{1}{5\sqrt{2}} - sin^{-1}\frac{4}{\sqrt{17}}\right]$

$= tan\,[tan^{-1}7 - tan^{-1}4]$

$= tan\left(tan^{-1}\left(\frac{7-4}{1+28}\right)\right) = \frac{3}{29}$ **જવાબ : (D)**

(90) સમીકરણ $tan^{-1}\sqrt{x(x+1)} + sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2}$ નાં વાસ્તવિક બીજની સંખ્યા ...... [IIT : 1999]

(A) 0 (B) 1 (C) 2 (D) અનંત

ઉકેલ : $tan^{-1}\sqrt{x(x+1)} = \frac{\pi}{2} - sin^{-1}\sqrt{x^2 + x + 1}$

$\therefore \quad tan^{-1}\sqrt{x(x+1)} = cos^{-1}\sqrt{x^2 + x + 1}$

$\therefore \quad cos^{-1}\frac{1}{\sqrt{x^2 + x + 1}} = cos^{-1}\sqrt{x^2 + x + 1}$

$\therefore \quad x^2 + x + 1 = 1$

$\therefore \quad x^2 + x = 0$

$\therefore \quad x(x + 1) = 0$

$\therefore \quad x = 0$ અથવા $x = -1$

$\therefore$ વાસ્તવિક બીજની સંખ્યા 2 છે. ચકાસણી કરતા બંને બીજ, આપેલ સમીકરણનું સમાધાન કરે છે.

જવાબ : (C)

(91) $sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - ...\right) + cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^8}{4} - ...\right) = \frac{\pi}{2}$, $0 < |x| < \sqrt{2}$ હોય, તો $x =$ ......

(A) $\frac{1}{2}$ (B) 1 (C) $-\frac{1}{2}$ (D) –1 [IIT : 2015]

ઉકેલ : $sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - ...\infty\right) + cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^8}{4} - ...\infty\right) = \frac{\pi}{2}$

$\therefore \quad \left(x - \frac{x^2}{2} + \frac{x^3}{4} ...\infty\right) = \left(x^2 - \frac{x^4}{2} + \frac{x^8}{4} - ...\infty\right)$

$\therefore \quad \frac{x}{1 - \left(\frac{-x}{2}\right)} = \frac{x^2}{1 - \left(\frac{-x^2}{2}\right)}$ $\qquad (0 < x^2 < 2, \ 0 < |x| < \sqrt{2})$

$\therefore \quad \frac{2x}{2 + x} = \frac{2x^2}{2 + x^2}$

$\therefore \quad x(2 + x^2) = x^2(2 + x)$

$\therefore \quad 2x + x^3 = 2x^2 + x^3$. આથી, $x^2 - x = 0$. આથી, $x(x - 1) = 0$

$\therefore \quad x = 0$ અથવા $x = 1$ પરંતુ $0 < |x| < \sqrt{2}$

$\therefore \quad x = 1$ **જવાબ : (B)**

(92) $sin(cot^{-1}(1 + x)) = cos(tan^{-1}x)$ હોય, તો $x =$ ......

(A) $\frac{1}{2}$ (B) 1 (C) 0 (D) $-\frac{1}{2}$

ઉકેલ : $sin(cot^{-1}(1 + x)) = cos(tan^{-1}x)$

$$\therefore \quad sin\left(sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}\right) = cos\left(cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$

$\therefore \quad \frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{\sqrt{1+x^2}}$. આથી, $1 + x^2 = x^2 + 2x + 2$

$\therefore$ $2x = -1$. આથી, $x = -\frac{1}{2}$ $\left(x = -\frac{1}{2}\right.$ મૂકી ચકાસણી કરો.$\left.\right)$ **જવાબ : (D)**

(93) $\sqrt{1+x^2}\left[\left\{x\,cos(cot^{-1}x) + sin(cot^{-1}x)\right\}^2 - 1\right]^{\frac{1}{2}}$ = ... $(0 < x < 1)$ [IIT : 2008]

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) x (C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

ઉકેલ : $\sqrt{1+x^2}\left[\left\{x\,cos(cot^{-1}x) + sin(cot^{-1}x)\right\}^2 - 1\right]^{\frac{1}{2}}$

$$= \sqrt{1+x^2}\left[\left\{x\left(cos\left(cos^{-1}\frac{x}{\sqrt{1+x^2}}\right)\right) + sin\left(sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right\}^2 - 1\right]^{\frac{1}{2}}$$

$$= \sqrt{1+x^2}\left[\left\{\frac{x^2}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right\}^2 - 1\right]^{\frac{1}{2}}$$

$$= \sqrt{1+x^2}\left[1 + x^2 - 1\right]^{\frac{1}{2}}$$

$$= x\sqrt{1+x^2} \qquad (x > 0)$$

જવાબ : (C)

(94) કોઈ ત્રિકોણની ત્રણ બાજુનાં માપ $3x + 4y$, $4x + 3y$, $5x + 5y$ હોય, તો તે ત્રિકોણ ...... પ્રકારનો હોય $x, y > 0$

(A) કાટકોણ (B) ગુરુકોણ (C) સમબાજુ (D) આમાંથી એક પણ નહિ.

ઉકેલ : ધારો કે ΔABC માં $BC = a = 3x + 4y$, $AC = b = 4x + 3y$, $AB = c = 5x + 5y$

$$cosC = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{(3x+4y)^2 + (4x+3y)^2 - (5x+5y)^2}{2(3x+4y)\,(4x+3y)} = \frac{-2xy}{2(3x+4y)\,(4x+3y)} < 0$$

$\therefore$ $C > \frac{\pi}{2}$. આથી, ΔABC ગુરુકોણ ત્રિકોણ છે. **જવાબ : (B)**

(95) a માપવાળા અને n બાજુવાળા નિયમિત બહુકોણના અંતર્ગત અને બહિર્ગત વર્તુળની ત્રિજ્યાઓનો સરવાળો ......

(A) $\frac{a}{4} cot \frac{\pi}{2n}$ (B) $a\ cot \frac{\pi}{n}$ (C) $\frac{a}{2} cot \frac{\pi}{2n}$ (D) $a\ cot \frac{\pi}{2n}$

ઉકેલ : $tan \frac{\pi}{n} = \frac{a}{2r}$

$sin \frac{\pi}{n} = \frac{a}{2R}$

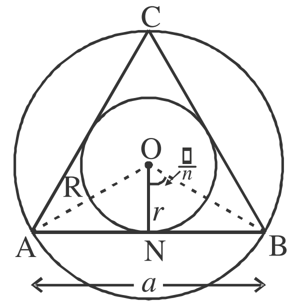
$\therefore\ r = \frac{a}{2} cot \frac{\pi}{n}$

$R = \frac{a}{2} cosec \frac{\pi}{n}$

$\therefore\ r + R = \frac{a}{2}\left[cot \frac{\pi}{n} + cosec \frac{\pi}{n}\right]$

$= \frac{a}{2}\left[\frac{1 + cos \frac{\pi}{n}}{sin \frac{\pi}{n}}\right]$

$= \frac{a}{2}\left[\frac{2cos^2 \frac{\pi}{2n}}{2sin \frac{\pi}{2n} cos \frac{\pi}{2n}}\right] = \frac{a}{2} cot \frac{\pi}{2n}$ જવાબ : **(C)**



આકૃતિ 12.12

(96) $(2cos\theta - 1)(2cos2\theta - 1)(2cos4\theta - 1) =$ ......

(A) $\frac{2cos8\theta + 1}{2cos\theta + 1}$ (B) $\frac{2cos8\theta - 1}{2cos\theta + 1}$ (C) $\frac{2cos8\theta + 1}{2cos\theta - 1}$ (D) આમાંથી એક પણ નહિ.

ઉકેલ : $(2cos\theta - 1)(2cos2\theta - 1)(2cos4\theta - 1)$

$= \frac{(2cos\theta + 1)(2cos\theta - 1)(2cos2\theta - 1)(2cos4\theta - 1)}{(2cos\theta + 1)}$

$= \frac{(4cos^2\theta - 1)(2cos2\theta - 1)(2cos4\theta - 1)}{2cos\theta + 1}$

$= \frac{\left[4\left(\frac{1 + cos2\theta}{2}\right) - 1\right](2cos2\theta - 1)(2cos4\theta - 1)}{2cos\theta + 1}$

$= \frac{(2cos2\theta + 1)(2cos2\theta - 1)(2cos4\theta - 1)}{2cos\theta + 1}$

$= \frac{4cos^2 4\theta - 1}{2cos\theta + 1}$

$= \frac{2(1 + cos8\theta) - 1}{2cos\theta + 1}$

$= \frac{2cos8\theta + 1}{2cos\theta + 1}$ જવાબ : **(A)**

(97) $(tan^{-1}x)^2 + (cot^{-1}x)^2 = \frac{5\pi^2}{8}$ હોય, તો $x =$ ......

(A) -1 (B) 1 (C) 0 (D) આમાંથી એક પણ નહિ.

ઉકેલ : $(tan^{-1}x)^2 + (cot^{-1}x)^2 = \frac{5\pi^2}{8}$

$\therefore\ (tan^{-1}x)^2 + \left(\frac{\pi}{2} - tan^{-1}x\right)^2 = \frac{5\pi^2}{8}$

$\therefore\ (tan^{-1}x)^2 + (tan^{-1}x)^2 - \pi(tan^{-1}x) + \frac{\pi^2}{4} = \frac{5\pi^2}{8}$

$\therefore\ 2(tan^{-1}x)^2 - \pi(tan^{-1}x) - \frac{3\pi^2}{8} = 0$

ધારો કે $y = tan^{-1}x$

$\therefore\ 2y^2 - \pi y - \frac{3\pi^2}{8} = 0$

$\therefore\ 16y^2 - 8\pi y - 3\pi^2 = 0$

$\therefore\ (4y - 3\pi)\ (4y + \pi) = 0$

$\therefore\ y = -\frac{\pi}{4}$ અથવા $y = \frac{3\pi}{4}$

$\therefore\ tan^{-1}x = -\frac{\pi}{4}$ અથવા $tan^{-1}x = \frac{3\pi}{4}$

$\therefore\ tan^{-1}x = -\frac{\pi}{4}$ $\left(tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right)$

$\therefore\ x = tan\left(-\frac{\pi}{4}\right)$

$\therefore\ x = -1$

$x = -1$ની ચકાસણી કરતા તે સમીકરણનું સમાધાન કરે છે. આથી $x = -1$ ઉકેલ છે. **જવાબ : (A)**

(98) $e^{-\frac{\pi}{2}} < \theta < \frac{\pi}{2}$ હોય, તો નીચેનામાંથી કયું સત્ય બને ? $(0 < cos\theta < 1)$

(A) $cos(log\theta) > log(cos\theta)$ (B) $cos(log\theta) < log(cos\theta)$

(C) $cos(log\theta) = log(cos\theta)$ (D) $cos(log\theta) = \frac{2}{3}log(cos\theta)$

ઉકેલ : $e^{-\frac{\pi}{2}} < \theta < \frac{\pi}{2}$

$log e^{-\frac{\pi}{2}} < log\theta < log\frac{\pi}{2}$

$\therefore\ -\frac{\pi}{2} < log\theta < \frac{\pi}{2}$ $\left(\frac{\pi}{2} < e \Rightarrow log\frac{\pi}{2} < 1 < \frac{\pi}{2}\right)$

$\therefore\ cos(log\theta) > 0$ (1)

હવે, $0 < cos\theta < 1$

$\therefore\ log(cos\theta) < log 1$

$\therefore\ log(cos\theta) < 0$ (2)

(1) અને (2) પરથી

$cos(log\theta) > log(cos\theta)$ **જવાબ : (A)**