Three Dimensional Geometry

Question1

If the shortest distance between the lines $\frac{x-4}{1}=\frac{y+1}{2}=\frac{z}{-3}$ and $\frac{x-\lambda}{2}=\frac{y+1}{4}=\frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is : [27-Jan-2024 Shift 1]

Options:

A. 5

B. 8

C. 7

D. 10

Answer: B

Solution:

$$\frac{x-4}{1}=\frac{y+1}{2}=\frac{z}{-3}$$

$$\frac{x-\lambda}{2}=\frac{y+1}{4}=\frac{z-2}{-5}$$

the shortest distance between the lines

$$=\left|\frac{(\vec{a}-\vec{b})\cdot(\vec{d}_1\times\vec{d}_2)}{|\vec{d}_1\times\vec{d}_2|}\right|$$

$$=\left|\frac{\begin{vmatrix}\lambda-4 & 0 & 2\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}\right|$$

$$=\left|\frac{(\lambda-4)(-10+12)-0+2(4-4)}{|2\hat{i}-1\hat{j}+0\hat{k}}\right|$$

$$\frac{6}{\sqrt{5}}=\left|\frac{2(\lambda-4)}{\sqrt{5}}\right|$$

$3=|\lambda-4|$

$\lambda-4=\pm3$

$\lambda=7, 1$

Sum of all possible values of λ is $=8$

Question2

The distance, of the point (7, −2, 11) from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$, is :
[27-Jan-2024 Shift 1]

Options:

A. 12

B. 14

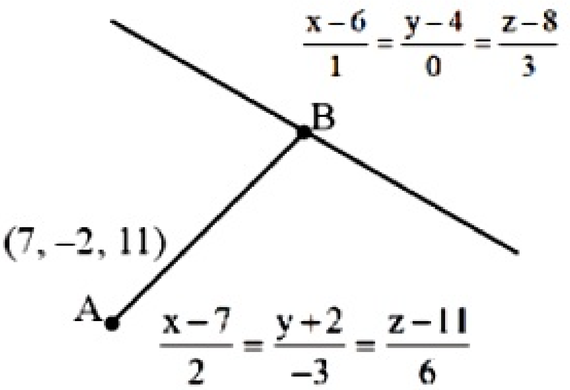
C. 18

D. 21

Answer: B

Solution:

$B = (2\lambda + 7, -3\lambda - 2, 6\lambda + 11)$



Point B lies on $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$

$$\frac{2\lambda + 7 - 6}{1} = \frac{-3\lambda - 2 - 4}{0} = \frac{6\lambda + 11 - 8}{3}$$

$-3\lambda - 6 = 0$

$\lambda = -2$

$B \Rightarrow (3, 4, -1)$

$AB = \sqrt{(7-3)^2 + (4+2)^2 + (11+1)^2}$

$= \sqrt{16 + 36 + 144}$

$= \sqrt{196} = 14$

Question3

The position vectors of the vertices A, B and C of a triangle are $2\hat{i} - 3\hat{j} + 3\hat{k}$, $2\hat{i} + 2\hat{j} + 3\hat{k}$ and $-\hat{i} + \hat{j} + 3\hat{k}$ respectively. Let l denotes the length of the angle bisector AD of ∠BAC where D is on the line segment BC, then $2l^2$ equals :
[27-Jan-2024 Shift 2]

Options:

A. 49

B. 42

C. 50

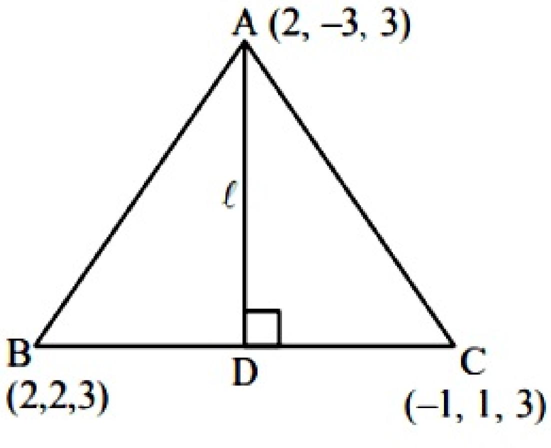
D. 45

Answer: D

Solution:

$AB = 5$

$AC = 5$



$\therefore$D is midpoint of BC

$D\left(\frac{1}{2}, \frac{3}{2}, 3\right)$

$$\therefore l = \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(-3 - \frac{3}{2}\right)^2 + (3-3)^2}$$

$$l = \sqrt{\frac{45}{2}}$$

$\therefore 2l^2 = 45$

Question4

Let the position vectors of the vertices A, B and C of a triangle be $2\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}+2\hat{k}$ respectively. Let l_1, l_2 and l_3 be the lengths of perpendiculars drawn from the ortho center of the triangle on the sides AB, BC and CA respectively, then $l_1^2+l_2^2+l_3^2$ equals :
[27-Jan-2024 Shift 2]

Options:

A. $\frac{1}{5}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

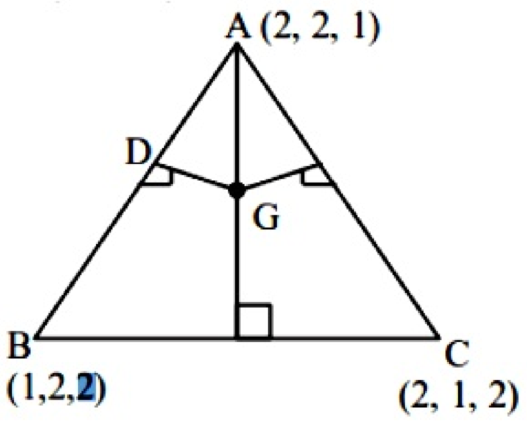
Answer: B

Solution:

$\triangle$ABC is equilateral

Orthocentre and centroid will be same

$G\left(\frac{5}{3}, \frac{5}{3}, \frac{5}{3}\right)$



Mid-point of AB is $D\left(\frac{3}{2}, 2, \frac{3}{2}\right)$

$$\therefore \ell_1 = \sqrt{\frac{1}{36}+\frac{1}{9}+\frac{1}{36}}$$

$$\ell_1 = \sqrt{\frac{1}{6}} = \ell_2 = \ell_3$$

$$\therefore \ell_1^2+\ell_2^2+\ell_3^2 = \frac{1}{2}$$

Question5

Let the image of the point (1, 0, 7) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be the point (α, β, γ). Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y-axis and z-axis respectively and an acute angle with x-axis?
[27-Jan-2024 Shift 2]

Options:

A. $(1, -2, 1 + \sqrt{2})$

B. $(1, 2, 1 - \sqrt{2})$

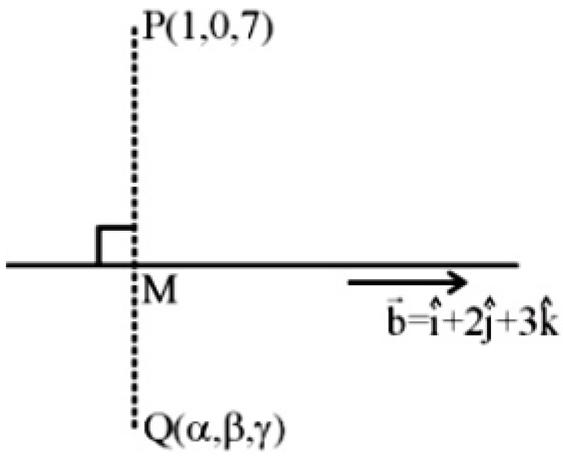
C. $(3, 4, 3 - 2\sqrt{2})$

D. $(3, -4, 3 + 2\sqrt{2})$

Answer: C

Solution:

$$L_1 = \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$



$M(\lambda, 1 + 2\lambda, 2 + 3\lambda)$

$\overrightarrow{PM} = (\lambda - 1)\hat{i} + (1 + 2\lambda)\hat{j} + (3\lambda - 5)\hat{k}$

$\overrightarrow{PM}$ is perpendicular to line L_1

$\overrightarrow{PM} \cdot \overrightarrow{b} = 0 \quad \left(\overrightarrow{b} = \hat{i} + 2\hat{j} + 3\hat{k}\right)$

$\Rightarrow \lambda - 1 + 4\lambda + 2 + 9\lambda - 15 = 0$

$14\lambda = 14 \Rightarrow \lambda = 1$

$\therefore M = (1, 3, 5)$

$\overrightarrow{Q} = 2\overrightarrow{M} - \overrightarrow{P}$ [M is midpoint of $\overrightarrow{P}$ & $\overrightarrow{Q}$]

$\overrightarrow{Q} = 2\hat{i} + 6\hat{j} + 10\hat{k} - \hat{i} - 7\hat{k}$

$\overrightarrow{Q} = \hat{i} + 6\hat{j} + 3\hat{k}$

$\therefore (\alpha, \beta, \gamma) = (1, 6, 3)$

Required line having direction cosine (l, m, n)

$l^2 + m^2 + n^2 = 1$

$\Rightarrow l^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = 1$

$l^2 = \frac{1}{4}$

$\therefore l = \frac{1}{2}$ [Line make acute angle with x-axis]

Equation of line passing through $(1, 6, 3)$ will be

$\overrightarrow{r} = \left(\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu\left(\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}\right)$

Option (3) satisfying for $\mu = 4$

Question6

The lines $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$ and $\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$ is l, then $14l^2$ is equal to................
[27-Jan-2024 Shift 2]

Answer: 108

Solution:

$\frac{x-2}{1}=\frac{y}{-1}=\frac{z-7}{8}=\lambda$

$\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}=k$

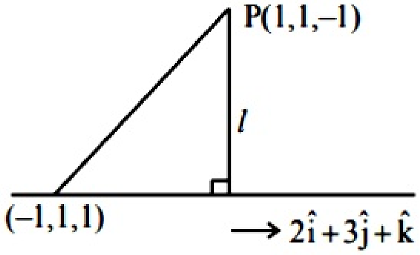
$\Rightarrow \lambda+2=4k-3$

$-\lambda=3k-2$

$\Rightarrow k=1, \lambda=-1$

$8\lambda+7=k-2$

$\therefore P=(1,1,-1)$



Projection of $2\hat{i}-2\hat{k}$ on $2\hat{i}+3\hat{j}+\hat{k}$ is

$=\frac{4-2}{\sqrt{4+9+1}}=\frac{2}{\sqrt{14}}$

$\therefore l^2=8-\frac{4}{14}=\frac{108}{14}$

$\Rightarrow 14l^2=108$

Question7

Let O be the origin and the position vector of A and B be $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$ respectively. If the internal bisector of ∠AOB meets the line AB at C, then the length of OC is
[29-Jan-2024 Shift 1]

Options:

A. $\frac{2}{3}\sqrt{31}$

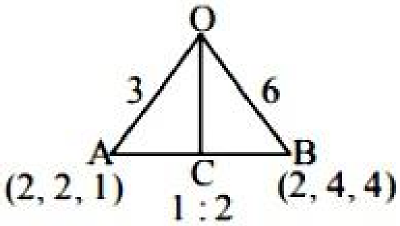
B. $\frac{2}{3}\sqrt{34}$

C. $\frac{3}{4}\sqrt{34}$

D. $\frac{3}{2}\sqrt{31}$

Answer: B

Solution:



length of $OC = \frac{\sqrt{136}}{3} = \frac{2\sqrt{34}}{3}$

Question8

Let PQR be a triangle with R(−1, 4, 2). Suppose M(2, 1, 2) is the mid point of PQ. The distance of the centroid of △PQR from the point of intersection of the line $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is [29-Jan-2024 Shift 1]

Options:

A. 69

B. 9

C. $\sqrt{69}$

D. $\sqrt{99}$

Answer: C

Solution:

Solution:

Centroid G divides MR in $1:2$

$G(1, 2, 2)$

Point of intersection A of given lines is $(2, -6, 0)$

$AG = \sqrt{69}$

Question9

A line with direction ratios 2, 1, 2 meets the lines x = y + 2 = z and x + 2 = 2y = 2z respectively at the point P and Q. if the length of the perpendicular from the point (1, 2, 12) to the line PQ is l, then l^2 is

[29-Jan-2024 Shift 1]

Answer: 65

Solution:

Solution:

Let $P(t, t-2, t)$ and $Q(2s-2, s, s)$

D.R's of PQ are 2, 1, 2

$$\frac{2s-2-t}{2}=\frac{s-t+2}{1}=\frac{s-t}{2}$$

$\Rightarrow t=6$ and $s=2$

$\Rightarrow P(6, 4, 6)$ and $Q(2, 2, 2)$

$$PQ: \frac{x-2}{2}=\frac{y-2}{1}=\frac{z-2}{2}=\lambda$$

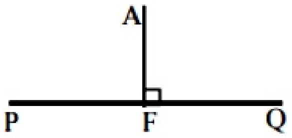
Let $F(2\lambda+2, \lambda+2, 2\lambda+2)$

$A(1, 2, 12)$

$\overrightarrow{AF}\cdot\overrightarrow{PQ}=0$

$\therefore \lambda=2$

So $F(6, 4, 6)$ and $AF=\sqrt{65}$



Question10

Let P(3, 2, 3), Q(4, 6, 2) and R(7, 3, 2) be the vertices of $\triangle$PQR. Then, the angle $\angle$QPR is
[29-Jan-2024 Shift 2]

Options:

A. $\frac{\pi}{6}$

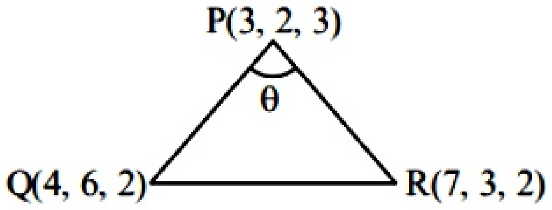
B. $\cos^{-1}\left(\frac{7}{18}\right)$

C. $\cos^{-1}\left(\frac{1}{18}\right)$

D. $\frac{\pi}{3}$

Answer: D

Solution:



Direction ratio of $PR = (4, 1, -1)$

Direction ratio of $PQ = (1, 4, -1)$

Now, $\cos\theta = \left|\frac{4+4+1}{\sqrt{18}\cdot\sqrt{18}}\right|$

$\theta = \frac{\pi}{3}$

Question11

Let O be the origin, and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\vec{OM} \cdot \vec{ON}$ is equal to [29-Jan-2024 Shift 2]

Answer: 9

Solution:

$L_1 : \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda$ drs$(4, 1, 3) = b_1$

$M(4\lambda+5, \lambda+4, 3\lambda+5)$

$L_2 : \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \mu$

$N(12\mu-8, 5\mu-2, 9\mu-11)$

$MN = (4\lambda-12\mu+13, \lambda-5\mu+6, 3\lambda-9\mu+16)$ ....(1)

Now

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k} \text{ ....(2)}$$

Equation (1) and (2)

$$\therefore \frac{4\lambda - 12\mu + 13}{-6} = \frac{\lambda - 5\mu + 6}{0} = \frac{3\lambda - 9\mu + 16}{8}$$

I and II

$\lambda - 5\mu + 6 = 0$ ....(3)

I and III

$\lambda - 3\mu + 4 = 0$ ....(4)

Solve (3) and (4) we get

$\lambda = -1, \mu = 1$

$\therefore M(1, 3, 2)$

$N(4, 3, -2)$

$\therefore \overrightarrow{OM} \cdot \overrightarrow{ON} = 4 + 9 - 4 = 9$

Question12

Let (α, β, γ) be the foot of perpendicular from the point (1, 2, 3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. then 19(α + β + γ) is equal to :
[30-Jan-2024 Shift 1]

Options:

A. 102

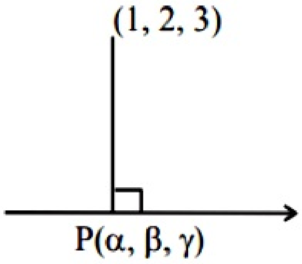
B. 101

C. 99

D. 100

Answer: B

Solution:



Let foot $P(5k-3, 2k+1, 3k-4)$

DR's $\longrightarrow$ AP: $5k-4, 2k-1, 3k-7$

DR's $\longrightarrow$ Line: 5, 2, 3

Condition of perpendicular lines $(25k-20)+(4k-2)+(9k-21)=0$

Then $k=\frac{43}{38}$

Then $19(\alpha+\beta+\gamma)=101$

Question13

If d_1 is the shortest distance between the lines $x+1=2y=-12z$, $x=y+2=6z-6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$, $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}d_1}{d_2}$ is :__

[30-Jan-2024 Shift 1]

Answer: 16

Solution:

$L_1 : \frac{x+1}{1}=\frac{y}{1/2}=\frac{z}{-1/12}, L_2 : \frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{\frac{1}{6}}$

$d_1 =$ shortest distance between L_1 & L_2

$$=\left|\frac{\left(\vec{a}_2-\vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)}{\left|\left(\vec{b}_1\times\vec{b}_2\right)\right|}\right|$$

$d_1=2$

$L_3 : \frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}, L_4 : \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$

$d_2 =$ shortest distance between L_3&L_4

$d_2=\frac{12}{\sqrt{3}}$ Hence

$$=\frac{32\sqrt{3}d_1}{d_2}=\frac{32\sqrt{3}\times 2}{\frac{12}{\sqrt{3}}}=16$$

Question14

Let $L_1 : \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k}), \lambda \in R$
$L_2 : \vec{r} = (\hat{j} - \hat{k}) + \mu(3\hat{i} + \hat{j} + p\hat{k}), \mu \in R$ and
$L_3 : \vec{r} = \delta(\ell\hat{i} + m\,\mathrm{hat}\,j + n\hat{k})\delta \in R$
Be three lines such that L_1 is perpendicular to L_2 and L_3 is perpendicular to both L_1 and L_2. Then the point which lies on L_3 is
[30-Jan-2024 Shift 2]

Options:

A. $(-1, 7, 4)$

B. $(-1, -7, 4)$

C. $(1, 7, -4)$

D. $(1, -7, 4)$

Answer: A

Solution:

$L_1 \perp L_2$

$L_3 \perp L_1, L_2$

$3 - 1 + 2P = 0$

$P = -1$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 4\hat{k}$$

$\therefore (-\delta, 7\delta, 4\delta)$ will lie on L_3

For $\delta = 1$ the point will be $(-1, 7, 4)$

Question15

Let a line passing through the point $(-1, 2, 3)$ intersect the lines $L_1 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2}$ at $M(\alpha, \beta, \gamma)$ and $L_2 : \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4}$ at $N(a, b, c)$. Then the value of $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}$ equals
[30-Jan-2024 Shift 2]

Answer: 196

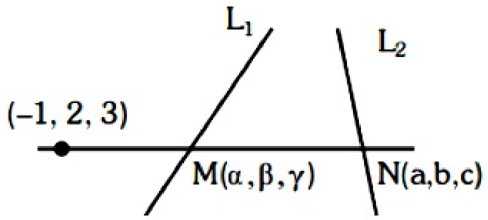
Solution:

$M(3\lambda+1, 2\lambda+2, -2\lambda-1)$

$\therefore \alpha+\beta+\gamma = 3\lambda+2$

$N(-3\mu-2, -2\mu+2, 4\mu+1)$

$\therefore a+b+c = -\mu+1$



$$\frac{3\lambda+2}{-3\mu-1} = \frac{2\lambda}{-2\mu} = \frac{-2\lambda-4}{4\mu-2}$$

$3\lambda\mu + 2\mu = 3\lambda\mu + \lambda$

$2\mu = \lambda$

$2\lambda\mu - \lambda = \lambda\mu + 2\mu$

$\lambda\mu = \lambda + 2\mu$

$\Rightarrow \lambda\mu = 2\lambda$

$\Rightarrow \mu = 2 \ (\lambda \neq 0)$

$\therefore \lambda = 4$

$\alpha+\beta+\gamma = 14$

$a+b+c = -1$

$$\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2} = 196$$

Question16

The distance of the point Q(0, 2, −2) form the line passing through the point P(5, −4, 3) and perpendicular to the lines $\vec{r} = \left(-3\hat{i} + 2\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$, $\lambda \in \mathbb{R}$ and $\vec{r} = \left(\hat{i} - 2\hat{j} + \hat{k}\right) + \mu\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$, $\mu \in \mathbb{R}$ is [31-Jan-2024 Shift 1]

Options:

A. $\sqrt{86}$

B. $\sqrt{20}$

C. $\sqrt{54}$

D. $\sqrt{74}$

Answer: D

Solution:

Solution:
A vector in the direction of the required line can be obtained by cross product of

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$$

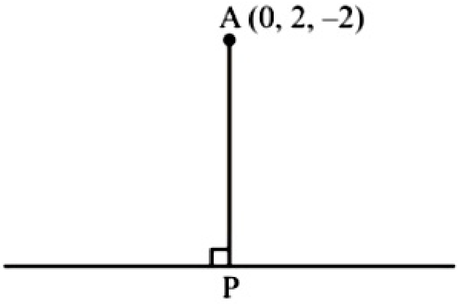
$$= -9\hat{i} - 9\hat{j} + 9\hat{k}$$

Required line,

$$\vec{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda'(-9\hat{i} - 9\hat{j} + 9\hat{k})$$

$$\vec{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

Now distance of $(0, 2, -2)$



$$\text{P.V. of } P \equiv (5+\lambda)\hat{i} + (\lambda - 4)\hat{j} + (3-\lambda)\hat{k}$$

$$\overrightarrow{AP} = (5+\lambda)\hat{i} + (\lambda-6)\hat{j} + (5-\lambda)\hat{k}$$

$$\overrightarrow{AP} \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

$$5 + \lambda + \lambda - 6 - 5 + \lambda = 0$$

$$\lambda = 2$$

$$|\overrightarrow{AP}| = \sqrt{49 + 16 + 9}$$

$$|\overrightarrow{AP}| = \sqrt{74}$$

Question17

Let Q and R be the feet of perpendiculars from the point P(a, a, a) on the lines x = y, z = 1 and x = −y, z = −1 respectively. If ∠QPR is a right angle, then $12a^2$ is equal to____
[31-Jan-2024 Shift 1]

Answer: 12

Solution:

$$\frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = r \longrightarrow Q(r, r, 1)$$

$$\frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0} = k \longrightarrow R(k, -k, -1)$$

$$\overline{PQ} = (a-r)\hat{i} + (a-r)\hat{j} + (a-1)\hat{k}$$

$$a = r + a - r = 0$$

$$2a = 2r \longrightarrow a = r$$

$$\overline{PR} = (a-k)i + (a+k)\hat{j} + (a+1)\hat{k}$$

$$a - k - a - k = 0 \Rightarrow k = 0$$

As, PQ $\perp PR$

$$(a-r)(a-k) + (a-r)(a+k) + (a-1)(a+1) = 0$$

$$a = 1 \text{ or } -1$$

$$12a^2 = 12$$

Question18

Let (α, β, γ) be mirror image of the point (2, 3, 5) in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.
Then 2α + 3β + 4γ is equal to
[31-Jan-2024 Shift 2]

Options:

A. 32

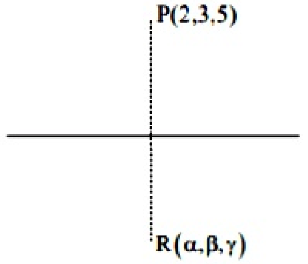
B. 33

C. 31

D. 34

Answer: B

Solution:



$\because \overrightarrow{PR} \perp (2, 3, 4)$

$\therefore \overrightarrow{PR} \cdot (2, 3, 4) = 0$

$(\alpha - 2, \beta - 3, \gamma - 5) \cdot (2, 3, 4) = 0$

$\Rightarrow 2\alpha + 3\beta + 4\gamma = 4 + 9 + 20 = 33$

Question19

The shortest distance between lines L_1 and L_2, where $L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$ and L_2 is the line passing through the points $A(-4, 4, 3) \cdot B(-1, 6, 3)$ and perpendicular to the line $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$, is [31-Jan-2024 Shift 2]

Options:

A. $\frac{121}{\sqrt{221}}$

B. $\frac{24}{\sqrt{117}}$

C. $\frac{141}{\sqrt{221}}$

D. $\frac{42}{\sqrt{117}}$

Answer: C

Solution:

$$L_2 = \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$$

$$\therefore S.D = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\overrightarrow{n_1} \times \overrightarrow{n_2}|}$$

$$= \frac{\begin{vmatrix} 5 & -5 & -7 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\overrightarrow{n_1} \times \overrightarrow{n_2}|}$$

$$= \frac{141}{|-4\hat{i} + 6\hat{j} + 13\hat{k}|}$$

$$= \frac{141}{\sqrt{16 + 36 + 169}}$$

$$= \frac{141}{\sqrt{221}}$$

Question20

A line passes through A(4, −6, −2) and B(16, −2, 4). The point P(a, b, c) where a, b, c are non-negative integers, on the line AB lies at a distance of 21 units, from the point A. The distance between the points P(a, b, c) and Q(4, −12, 3) is equal to
[31-Jan-2024 Shift 2]

Answer: 22

Solution:

$$\frac{x-4}{12} = \frac{x+6}{4} = \frac{z+2}{6}$$

$$\frac{x-4}{\frac{6}{7}} = \frac{y+6}{\frac{2}{7}} = \frac{z+2}{\frac{3}{7}} = 21$$

$$\left(21 \times \frac{6}{7} + 4,\ \frac{2}{7} \times 21 - 6,\ \frac{3}{7} \times 21 - 2\right)$$

$$= (22, 0, 7) = (a, b, c)$$

$$\therefore \sqrt{324 + 144 + 16} = 22$$

Question21

If the shortest distance between the lines $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ is 1 , then the sum of all possible values of λ is :____
[1-Feb-2024 Shift 1]

Options:

A. 0

B. $2\sqrt{3}$

C. $3\sqrt{3}$

D. $-2\sqrt{3}$

Answer: B

Solution:

Solution:

Passing points of lines $L_1 \& L_2$ are

$(\lambda, 2, 1) \& (\sqrt{3}, 1, 2)$

$$S.D = \frac{\begin{vmatrix} \sqrt{3}-\lambda & -1 & 1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}$$

$$1 = \left| \frac{\sqrt{3}-\lambda}{\sqrt{3}} \right|$$

$\lambda = 0, \lambda = 2\sqrt{3}$

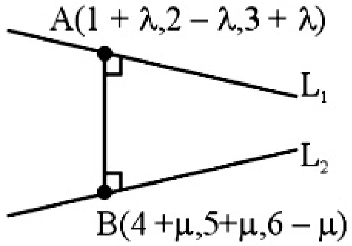
Question22

Let the line of the shortest distance between the lines
$L_1 : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and
$L_2 : \vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$
intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the midpoint of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to____
[1-Feb-2024 Shift 1]

Answer: 21

Solution:

Solution:



$\vec{b} = \hat{i} - \hat{j} + \hat{k}$(DR's of L_1)

$\vec{d} = \hat{i} + \hat{j} - \hat{k}$(DR's of L_2)

$$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$= 0\hat{i} + 2\hat{j} + 2\hat{k}$ (DR's of Line perpendicular to L_1 and L_2)

DR of AB line

$= (0, 2, 2) = (3 + \mu - \lambda, 3 + \mu + \lambda, 3 - \mu - \lambda)$

$$\frac{3 + \mu - \lambda}{0} = \frac{3 + \mu + \lambda}{2} = \frac{3 - \mu - \lambda}{2}$$

Solving above equation we get $\mu = -\frac{3}{2}$ and $\lambda = \frac{3}{2}$

point $A = \left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right)$

$B = \left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$

Point of $AB = \left(\frac{5}{2}, 2, 6\right) = (\alpha, \beta, \gamma)$

$2(\alpha + \beta + \gamma) = 5 + 4 + 12 = 21$

Question23

Let P and Q be the points on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ which are at a distance of 6 units from the point R(1, 2, 3). If the centroid of the triangle PQR is (α, β, γ), then $\alpha^2 + \beta^2 + \gamma^2$ is:
[1-Feb-2024 Shift 2]

Options:

A. 26

B. 36

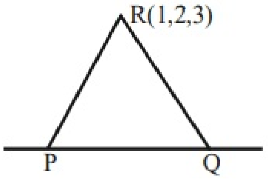
C. 18

D. 24

Answer: C

Solution:

Solution:



$P(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$
$PR = 6$
$(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$
$\lambda = 0, 1$
Hence $P(-3, 4, -1)$ & $Q(5, 6, 1)$
Centroid of Δ PQR $= (1, 4, 1) \equiv (\alpha, \beta, \gamma)$
$\alpha^2 + \beta^2 + \gamma^2 = 18$

Question24

If the mirror image of the point P(3, 4, 9) in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ), then $14(\alpha + \beta + \gamma)$ is :
[1-Feb-2024 Shift 2]

Options:

A. 102

B. 138

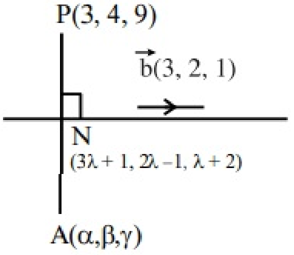
C. 108

D. 132

Answer: C

Solution:

Solution:



$\overrightarrow{PN} \cdot \vec{b} = 0?$
$3(3\lambda - 2) + 2(2\lambda - 5) + (\lambda - 7) = 0$
$14\lambda = 23 \Rightarrow \lambda = \frac{23}{14}$
$N\left(\frac{83}{14}, \frac{32}{14}, \frac{51}{14}\right)$
$\therefore \frac{\alpha + 3}{2} = \frac{83}{14} \Rightarrow \alpha = \frac{62}{7}$
$\frac{\beta + 4}{2} = \frac{32}{14} \Rightarrow \beta = \frac{4}{7}$

$\frac{\gamma+9}{2} = \frac{51}{14} \Rightarrow \gamma = \frac{-12}{7}$
Ans. $14(\alpha+\beta+r) = 108$

Question25

The distance of the point $(-1, 9, -16)$ from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is [24-Jan-2023 Shift 1]

Options:

A. $13\sqrt{2}$

B. 31

C. 26

D. $20\sqrt{2}$

Answer: C

Solution:

Solution:
Equation of line
$\frac{x+1}{3} = \frac{y-9}{-4} = \frac{z+16}{12}$
G.P on line $(3\lambda - 1, -4\lambda + 9, 12\lambda - 16)$
point of intersection of line & plane
$6\lambda - 2 - 12\lambda + 27 - 12\lambda + 16 = 5$
$\lambda = 2$
Point $(5, 1, 8)$
Distance $= \sqrt{36 + 64 + 576} = 26$

Question26

The distance of the point $(7, -3, -4)$ from the plane passing through the points $(2, -3, 1)$, $(-1, 1, -2)$ and $(3, -4, 2)$ is : [24-Jan-2023 Shift 1]

Options:

A. 4

B. 5

C. $5\sqrt{2}$

D. $4\sqrt{2}$

Answer: C

Solution:

Solution:
Equation of Plane is

$$= \begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix} = 0$$

$x - z - 1 = 0$
Distance of $P(7, -3, -4)$ from Plane is

$$d = \left| \frac{7+4-1}{\sqrt{2}} \right| = 5\sqrt{2}$$

Question27

The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is equal to
[24-Jan-2023 Shift 1]

Answer: 14

Solution:

Solution:

$$= \frac{\begin{vmatrix} 4 & 2 & -14 \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}}$$

$$= \frac{16+12+168}{\left|-4\hat{i} + 6\hat{j} - 12k\right|} = \frac{196}{14} = 14$$

Question28

If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to
[24-Jan-2023 Shift 2]

Options:

A. -1

B. 3

C. 1

D. 5

Answer: D

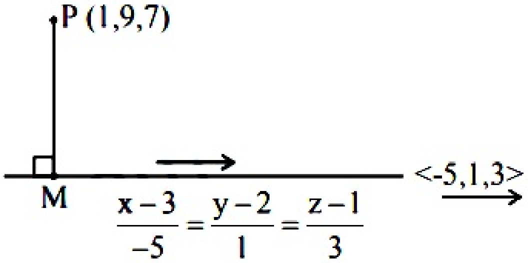
Solution:

Solution:

Direction ratio of line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$

$= -5\hat{i} + \hat{j} + 3\hat{k}$



$M(-5\lambda + 3, \lambda + 2, 3\lambda + 1)$

$\overrightarrow{PM} \perp (-5\hat{i} + \hat{j} + 3\hat{k})$

$-5(-5\lambda + 2) + (\lambda - 7) + 3(3\lambda - 6) = 0$

$\Rightarrow 25\lambda + \lambda + 9\lambda - 10 - 7 - 18 = 0$

$\Rightarrow \lambda = 1$

Point $M = (-2, 3, 4) = (\alpha, \beta, \gamma)$

$\alpha + \beta + \gamma = 5$

Question29

Let the plane containing the line of intersection of the planes
P1 : $x + (\lambda + 4)y + z = 1$ and
P2 : $2x + y + z = 2$ pass through the points (0, 1, 0) and (1, 0, 1).
Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P2 is
[24-Jan-2023 Shift 2]

**Let the plane containing the line of intersection of the planes
P1 : $x + (\lambda + 4)y + z = 1$ and
P2 : $2x + y + z = 2$ pass through the points (0, 1, 0) and (1, 0, 1).
Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P2 is
[24-Jan-2023 Shift 2]**

Options:

A. $5\sqrt{6}$

B. $4\sqrt{6}$

C. $2\sqrt{6}$

D. $3\sqrt{6}$

Answer: D

Solution:

Solution:
Equation of plane passing through point of intersection of P1 and P2
$P = P1 + kP2$
$(x + (\lambda + 4)y + z - 1) + k(2x + y + z - 2) = 0$
Passing through (0, 1, 0) and (1, 0, 1)
$(\lambda + 4 - 1) + k(1 - 2) = 0$
$(\lambda + 3) - k = 0 \ldots (1)$
Also passing (1, 0, 1)
$(1 + 1 - 1) + k(2 + 1 - 2) = 0$
$1 + k = 0$
$k = -1$
put in (1)
$\lambda + 3 + 1 = 0$
$\lambda = -4$
Then point $(2\lambda, \lambda, -\lambda)$
$d = \left| \frac{-16 - 4, -4, 4)}{\sqrt{6}} \right|$
$d = \frac{18}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = 3\sqrt{6}$

Question30

If the shortest between the lines $\frac{x + \sqrt{6}}{2} = \frac{y - \sqrt{6}}{3} = \frac{z - \sqrt{6}}{4}$ and $\frac{x - \lambda}{3} = \frac{y - 2\sqrt{6}}{4} = \frac{z + 2\sqrt{6}}{5}$ is 6 , then the square of sum of all possible values of λ is
[24-Jan-2023 Shift 2]

Answer: 384

Solution:

Shortest distance between the lines
$\frac{x + \sqrt{6}}{2} = \frac{y - \sqrt{6}}{3} = \frac{z - \sqrt{6}}{4}\ \frac{x - \lambda}{3} = \frac{y - 2\sqrt{6}}{4} = \frac{2 + 2\sqrt{6}}{5}$ is 6
Vector along line of shortest distance

$= \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$, $\Rightarrow -\hat{i} + 2\hat{j} - k$ (its magnitude is $\sqrt{6}$)

Now $\frac{1}{\sqrt{6}} \begin{vmatrix} \sqrt{6} + \lambda & \sqrt{6} & -3\sqrt{6} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \pm 6$

$\Rightarrow \lambda = -2\sqrt{6}, 10\sqrt{6}$
So, square of sum of these values is 384 .

Question31

Consider the lines L_1 and L_2 given by

$$L_1 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2}$$

$$L_2 : \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

A line L_3 having direction ratios 1, −1, −2, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is
[25-Jan-2023 Shift 1]

Options:

A. $2\sqrt{6}$

B. $3\sqrt{2}$

C. $4\sqrt{3}$

D. 4

Answer: A

Solution:

Solution:

Let P = $(2\lambda + 1, \lambda + 3, 2\lambda + 2)$
Let Q = $(\mu + 2, 2\mu + 2, 3\mu + 3)$

$$\Rightarrow \frac{2\lambda - \mu - 1}{1} = \frac{\lambda - 2\mu + 1}{-1} = \frac{2\lambda - 3\mu - 1}{-2}$$

$\Rightarrow \lambda = \mu = 3 \Rightarrow P(7, 6, 8)$ and $Q(5, 8, 12)$

$PQ = 2\sqrt{6}$

Question32

The distance of the point P(4, 6, −2) from the line passing through the point (−3, 2, 3) and parallel to a line with direction ratios 3, 3, −1 is equal to :
[25-Jan-2023 Shift 1]

Options:

A. 3

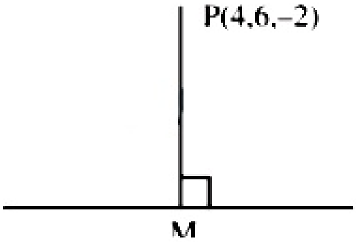
B. $\sqrt{6}$

C. $2\sqrt{3}$

D. $\sqrt{14}$

Answer: D

Solution:



Equation of line is $\frac{x+3}{3} = \frac{y-2}{3} = \frac{z-3}{-1} = \lambda$
$M(3\lambda - 3, 3\lambda + 2, 3 - \lambda)$
D.R of $PM(3\lambda - 7, 3\lambda - 4, 5 - \lambda)$
Since PM is perpendicular to line
$\Rightarrow 3(3\lambda - 7) + 3(3\lambda - 4) - 1(5 - \lambda) = 0$
$\Rightarrow \lambda = 2$
$\Rightarrow M(3, 8, 1) \Rightarrow PM = \sqrt{14}$

Question33

Let the equation of the plane passing through the line $x - 2y - z - 5 = 0 = x + y + 3z - 5$ and parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ be $ax + by + cz = 65$. Then the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is _______.
[25-Jan-2023 Shift 1]

Answer: 9

Solution:

Solution:
Equation of plane is
$(x - 2y - z - 5) + b(x + y + 3z - 5) = 0$

$$\begin{vmatrix} 1+b & -2+b & -1+3b \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow b = 12$
$\therefore$ plane is $13x + 10y + 35z = 65$
Distance from given point to plane $= 9$

Question34

The foot of perpendicular of the point (2, 0, 5) on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ). Then. Which of the following is NOT correct?
[25-Jan-2023 Shift 2]

Options:

A. $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$

B. $\frac{\alpha}{\beta} = -8$

C. $\frac{\beta}{\gamma} = -5$

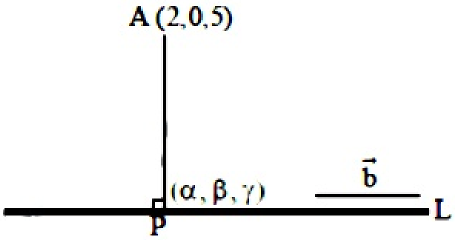
D. $\frac{\gamma}{\alpha} = \frac{5}{8}$

Answer: C

Solution:

Solution:

L : $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1} = \lambda$ (let)



Let foot of perpendicular is

$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1)$

$\overrightarrow{PA} = (3 - 2\lambda)\hat{i} - (5\lambda + 1)\hat{j} + (6 + \lambda)\hat{k}$

Direction ratio of line $\Rightarrow \vec{b} = 2\hat{i} + 5\hat{j} - \hat{k}$

$\text{Now,} \quad \Rightarrow \overrightarrow{PA} \cdot \vec{b} = 0$

$\Rightarrow 2(3 - 2\lambda) - 5(5\lambda + 1) - (6 + \lambda) = 0$

$\Rightarrow \lambda = \frac{-1}{6}$

$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1) \equiv P(\alpha, \beta, \gamma)$

$\Rightarrow \alpha = 2\left(-\frac{1}{6}\right) - 1 = -\frac{4}{3} \Rightarrow \alpha = -\frac{4}{3}$

$\Rightarrow \beta = 5\left(-\frac{1}{6}\right) + 1 = \frac{1}{6} \Rightarrow \beta = \frac{1}{6}$

$\Rightarrow \gamma = -\lambda - 1 = \frac{1}{6} - 1 \Rightarrow \gamma = -\frac{5}{6}$

∴ Check options

Question35

The shortest distance between the lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ is [25-Jan-2023 Shift 2]

Options:

A. 2

B. 3

C. $\frac{5}{2}$

D. $\frac{3}{2}$

Answer: A

Solution:

Solution:

$\frac{x+1}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{\frac{-1}{12}}$ and $\frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{\frac{1}{6}}$

$\Rightarrow$ Shortest distance $=\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$

S.D. $=\left(-\hat{i}+2\hat{j}-\hat{k}\right)\cdot\frac{(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$

$\left\{\vec{p}\times\vec{q}\equiv\left|\hat{i},\hat{j},\hat{k};1,\frac{1}{2},\frac{-1}{12};1,1,\frac{1}{6}\right|=\frac{1}{6}\hat{i}-\frac{1}{4}\hat{j}+\frac{1}{2}\hat{k}\text{ or }2\hat{i}-3\hat{j}+6\hat{k}\right\}$

S.D. $=\frac{\left(-\hat{i}+2\hat{j}-\hat{k}\right)\cdot\left(2\hat{i}-3\hat{j}+6\hat{k}\right)}{\sqrt{2^2+3^2+6^2}}=\left|\frac{-14}{7}\right|=2$

Question36

If the shortest distance between the line joining the points (1, 2, 3) and (2, 3, 4), and the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to _______. [25-Jan-2023 Shift 2]

Answer: 18

Solution:

Solution:

$\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$ $\vec{r}=\vec{a}+\lambda\vec{p}$

$\vec{r}=\left(+\hat{i}-\hat{j}+2\hat{k}\right)+\mu\left(2\hat{i}-\hat{j}\right)$ $\vec{r}=\vec{b}+\mu\vec{q}$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&1\\2&-1&0\end{vmatrix}=\hat{i}+2\hat{j}-3\hat{k}$$

$$d=\left|\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$

$$d=\left|\frac{\left(-3\hat{j}-\hat{k}\right)\cdot\left(\hat{i}+2\hat{j}-3\hat{k}\right)}{\sqrt{14}}\right|$$

$$=\left|\frac{-6+3}{\sqrt{14}}\right|=\frac{3}{\sqrt{14}}$$

$\alpha=\frac{3}{\sqrt{14}}$

Now, $28\alpha^2=2/2\times\frac{9}{14}=18$

Question37

Let the co-ordinates of one vertex of △ABC be A(0, 2, α) and the other two vertices lie on the line $\frac{x+\alpha}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. For $\alpha\in\mathbb{Z}$, if the area of

$\triangle ABC$ is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to _______.
[29-Jan-2023 Shift 1]

Answer: 9

Solution:

Solution:
A. $(O_1 2, \alpha)$

$\sqrt{(2\alpha+5)^2+(2\alpha+20)^2+(2\alpha-5)^2} = \sqrt{21}\sqrt{38}$
$\Rightarrow 12\alpha^2 + 80\alpha + 450 = 798$
$\Rightarrow 12\alpha^2 + 80\alpha - 348 = 0$
$\Rightarrow \alpha = 3 \Rightarrow \alpha^2 = 9$

Question38

Let the equation of the plane P containing the line $x + 10 = \frac{8-y}{2} = z$ be $ax + by + 3z = 2(a + b)$ and the distance of the plane P from the point (1, 27, 7) be c. Then $a^2 + b^2 + c^2$ is equal to ________.
[29-Jan-2023 Shift 1]

Answer: 355

Solution:

The line $\frac{x+10}{1} = \frac{y-8}{-2} = \frac{z}{1}$ have a point $(-10, 8, 0)$ with d. r. $(1, -2, 1)$
$\because$ the plane $ax + by + 3z = 2(a + b)$
$\Rightarrow b = 2a$
& dot product of d.r.'s is zero
$\therefore a - 2b + 3 = 0$
$\therefore a = 1 \& b = 2$
Distance from (1, 27, 7) is
$c = \frac{1+54+21-6}{\sqrt{14}} = \frac{70}{\sqrt{14}} = 5\sqrt{14}$
$\therefore a^2 + b^2 + c^2 = 1 + 4 + 350$
$= 355$

Question39

The plane $2x - y + z = 4$ intersects the line segment joining the points

$A(a, -2, 4)$ and $B(2, b, -3)$ at the point C in the ratio 2 : 1 and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point $(a-b, b, 2b-a)$ then CP^2 is equal to :
[29-Jan-2023 Shift 2]

Options:

A. $\frac{17}{3}$

B. $\frac{16}{3}$

C. $\frac{73}{3}$

D. $\frac{97}{3}$

Answer: A

Solution:

Solution:
$A(a, -2, 4), B(2, b, -3)$
$AC : CB = 2 : 1$
$\Rightarrow C \equiv \left(\frac{a+4}{3}, \frac{2b-2}{3}, \frac{-2}{3}\right)$
C lies on $2x - y + 2 = 4$
$\Rightarrow \frac{2a+8}{3} - \frac{2b-2}{3} - \frac{2}{3} = 4$
$\Rightarrow a - b = 2 \ldots (1)$
Also $OC = \sqrt{5}$
$\Rightarrow \left(\frac{a+4}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \frac{4}{9} = 5 \ldots (2)$
Solving, (1) and (2)
$(b+6)^2 + (2b-2)^2 = 41$
$\Rightarrow 5b^2 + 4b - 1 = 0$
$\Rightarrow b = -1$ or $\frac{1}{5}$
$\Rightarrow a = 1$ or $\frac{11}{5}$
But $ab < 0 \Rightarrow (a, b) = (1, -1)$
$C \equiv \left(\frac{5}{3}, \frac{-4}{3}, \frac{-2}{3}\right), P \equiv (2, -1, -3)$
$CP^2 = \frac{1}{9} + \frac{1}{9} + \frac{49}{9} = \frac{51}{9} = \frac{17}{3}$

Question40

Shortest distance between the lines $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$ and $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$ is
[29-Jan-2023 Shift 2]

Options:

A. $2\sqrt{3}$

B. $4\sqrt{3}$

C. $3\sqrt{3}$

D. $5\sqrt{3}$

Answer: B

Solution:

Solution:

$\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$ $\vec{a} = i - 8\hat{j} + 4\hat{k}$

$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$ $\vec{b} = \hat{i} + 2\hat{j} + 6\hat{k}$

$\vec{p} = 2\hat{i} - 7\hat{j} + 5\hat{k}, \vec{q} = 2\hat{i} + \hat{j} - 3\hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$= \hat{i}(16) - \hat{j}(-16) + \hat{k}(16)$

$= 16(\hat{i} + \hat{j} + \hat{k})$

$= \left|\frac{-12}{\sqrt{3}}\right| = 4\sqrt{3}|$

Question41

If the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{1}$ and $\frac{x-a}{2} = \frac{y+2}{3} = \frac{z-3}{1}$ intersects at the point P, then the distance of the point P from the plane z = a is :
[29-Jan-2023 Shift 2]

Options:

A. 16

B. 28

C. 10

D. 22

Answer: B

Solution:

Solution:
Point on $L_1 \equiv (\lambda + 1, 2\lambda + 2, \lambda - 3)$
Point on $L_2 \equiv (2\mu + a, 3\mu - 2, \mu + 3)$
$\lambda - 3 = \mu + 3 \Rightarrow \lambda = \mu + 6 \ldots (1)$
$2\lambda + 2 = 3\mu - 2 \Rightarrow 2\lambda = 3\mu - 4 \ldots (2)$
Solving, (1) and (2)
$\Rightarrow \lambda = 22 \& \mu = 16$
$\Rightarrow P \equiv (23, 46, 19)$
$\Rightarrow a = -9$
Distance of P from $z = -9$ is 28

Question42

Let a unit vector $\widehat{OP}$ make angle α, β, γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$ $\widehat{OP}$ is perpendicular to the plane through points (1, 2, 3), (2, 3, 4) and (1, 5, 7), then which one of the following is true ?
[30-Jan-2023 Shift 1]

Options:

A. $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

B. $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

C. $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

D. $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

Answer: A

Solution:

Solution:
Equation of plane :-

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{vmatrix} = 0$$

$\Rightarrow [x-1] - 4[y-2] + 3[z-3] = 0$
$\Rightarrow x - 4y + 3z = 2$
D.R's of normal of plane $<1, -4, 3>$
D.C's of $\left\langle \pm\frac{1}{\sqrt{26}}, \mp\frac{4}{\sqrt{26}}, \pm\frac{3}{\sqrt{26}} \right\rangle$

$\cos\beta = \frac{4}{\sqrt{26}}$

$\cos\alpha = \frac{-1}{\sqrt{26}} \quad \frac{\pi}{2} < \alpha < \pi$

$\cos\gamma = \frac{-3}{\sqrt{26}} \quad \frac{\pi}{2} < \gamma < \pi$

Question43

The line l_1 passes through the point (2, 6, 2) and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is :
[30-Jan-2023 Shift 1]

Options:

A. 7

B. $\frac{19}{3}$

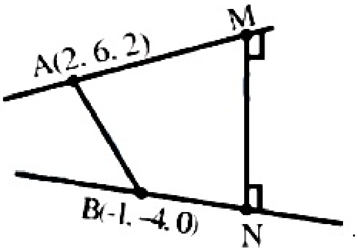
C. $\frac{19}{3}$

D. 9

Answer: D

Solution:

Solution:
Line ℓ, is given by
$L_1 : \frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$
Given,
$L_2 : \frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$



$$\text{Shortest distance} = \left| \overrightarrow{AB} \cdot \frac{\overrightarrow{MN}}{MN} \right|$$

$\overline{AB} = 3\hat{i} + 10\hat{j} + 2\hat{k}$

$$\overline{MN} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix} = -4\hat{i} - 8\hat{j} - 8\hat{k}$$

$MN = \sqrt{16 + 64 + 64} = 12$

$$\therefore \text{ Shortest distance} = \left| \frac{-12 - 80 - 16}{12} \right| = 9$$

$\therefore$ Option (4) is correct.

Question44

If the equation of the plane passing through the point (1, 1, 2) and perpendicular to the line $x - 3y + 2z - 1 = 0$ $4x - y + z$ is $Ax + By + Cz = 1$, then $140(C - B + A)$ is equal to _______. [30-Jan-2023 Shift 1]

Answer: 15

Solution:

Solution:
$x - 3y + 2z - 1 = 0$
$4x - y + z = 0$

$$\bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 4 & -1 & 1 \end{vmatrix}$$

$= -\hat{i} + 7\hat{j} + 11\hat{k}$

Dre of normal to the plane is $-1, 7, 11$

Equation of plane :

$-1(x-1) + 7(y-1) + 11(z-2) = 0$

$-x + 7y + 11z = 28$

$\frac{-1}{28}x + \frac{7y}{28} + \frac{11z}{28} = 1$

$Ax + By + Cz = 1$

$140(C - B + A) = 140\left(\frac{11}{28} - \frac{7}{28} - \frac{1}{28}\right)$

$= 140 \times \frac{3}{28} = 15$

Question45

If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1 : \bar{r}(3\hat{i} - 5\hat{j} + \hat{k}) = 7$ and $P_2 : \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is ______.

[30-Jan-2023 Shift 1]

Answer: 315

Solution:

$P_1 = \vec{r} \cdot (3\hat{i} - 5\hat{j} + \hat{k}) = 7$

$P_2 = \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$

$\theta = \sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$

$\Rightarrow \sin\theta = \frac{2\sqrt{6}}{5}$

$\therefore \cos\theta = \frac{1}{5}$

$\cos\theta = \vec{r} \cdot \frac{\vec{r}}{|\bar{r}_1|\left|\vec{r}_2\right|}$

$= \frac{(3i - 5j + K)(\lambda i + j - 3K)}{\sqrt{35} \cdot \sqrt{\lambda^2 + 10}}$

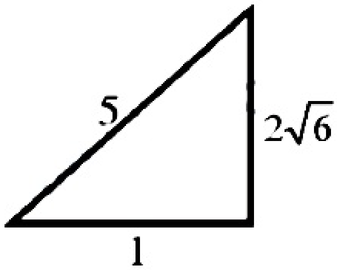
$\frac{1}{5} = \left|\frac{3\lambda - 8}{\sqrt{35} \cdot \sqrt{\lambda^2 + 10}}\right|$

Square $\Rightarrow \frac{1}{25} = \frac{9\lambda^2 + 64 - 48\lambda}{35(\lambda^2 + 10)}$

$\Rightarrow 19\lambda^2 - 120\lambda + 125 = 0$

$\Rightarrow 19\lambda^2 - 95\lambda - 25\lambda + 125 = 0$

$\Rightarrow x = 5, \frac{25}{19}$



Perpendicular distance of point
$(38\lambda_1, 10\lambda_2, 2) \equiv (50, 50, 2)$ from plane P_1

$$= \frac{|3 \times 50 - 5 \times 50 + 2 - 7|}{\sqrt{35}} = \frac{105}{\sqrt{35}}$$

$$\text{Square} = \frac{105 \times 105}{35} = 315$$

Question46

A vector $\vec{v}$ in the first octant is inclined to the x axis at 60°, to the y-axis at 45° and to the z-axis at an acute angle. If a plane passing through the points $(\sqrt{2}, -1, 1)$ and (a, b, c), is normal to $\vec{v}$, then
[30-Jan-2023 Shift 2]

Options:

A. $\sqrt{2}a + b + c = 1$

B. $a + b + \sqrt{2}c = 1$

C. $a + \sqrt{2}b + c = 1$

D. $\sqrt{2}a - b + c = 1$

Answer: C

Solution:

Solution:

$\hat{v} = \cos 60^\circ \hat{i} + \cos 45^\circ \hat{j} + \cos \gamma \hat{k}$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1 \ (\gamma \rightarrow \text{ Acute })$

$\Rightarrow \cos\gamma = \frac{1}{2}$

$\Rightarrow \gamma = 60^\circ$

Equation of plane is

$\frac{1}{2}(x - \sqrt{2}) + \frac{1}{\sqrt{2}}(y + 1) + \frac{1}{2}(z - 1) = 0$

$\Rightarrow x + \sqrt{2}y + z = 1$

(a, b, c) lies on it.

$\Rightarrow a + \sqrt{2}b + c = 1$

Question47

If a plane passes through the points $(-1, k, 0)$, $(2, k, -1)$, $(1, 1, 2)$ and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is

[30-Jan-2023 Shift 2]

Options:

A. $\frac{17}{5}$

B. $\frac{5}{17}$

C. $\frac{6}{13}$

D. $\frac{13}{6}$

Answer: D

Solution:

Solution:

$$\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$$

$$\frac{x-1}{1} = \frac{y+\frac{1}{2}}{1} = \frac{z+1}{-1}$$

Points : A $(-1, k, 0)$, B$(2, k, -1)$, C$(1, 1, 2)$

$\overrightarrow{CA} = -2\hat{i} + (k-1)\hat{j} - 2\hat{k}$

$\overrightarrow{CB} = \hat{i} + (k-1)\hat{j} - 3\hat{k}$

$$\overrightarrow{CA} \times \overrightarrow{CB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & k-1 & -2 \\ 1 & k-1 & -3 \end{vmatrix}$$

$= \hat{i}(-3k+3+2k-2) - \hat{j}(6+2) + \hat{k}(-2k+2-k+1)$

$= (1-k)\hat{i} - 8\hat{j} + (3-3k)\hat{k}$

The line $\frac{x-1}{1} = \frac{y+\frac{1}{2}}{1} = \frac{z+1}{-1}$ is perpendicular to normal vector.

$\therefore 1\cdot(1-k) + 1(-8) + (-1)(3-3k) = 0$

$\Rightarrow 1-k-8-3+3k = 0$

$\Rightarrow 2k = 10 \Rightarrow k = 5$

$\therefore \frac{k^2+1}{(k-1)(k-2)} = \frac{26}{4\cdot 3} = \frac{13}{6}$

Question48

Let a line L pass through the point P(2, 3, 1) and be parallel to the line $x+3y-2z-2=0=x-y+2z$. If the distance of L from the point (5, 3, 8) is α, then $3\alpha^2$ is equal to _______.
[30-Jan-2023 Shift 2]

Answer: 158

Solution:

Solution:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = 4\hat{i} - 4\hat{j} - 4\hat{k}$$

$\therefore$ Equation of line is $\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1}$

Let Q be (5, 3, 8) and foot of $\perp$ from Q on this line be R.

Now, $R \equiv (k+2, -k+3, -k+1)$

DR of QR are $(k-3, -k, -k-7)$

$\therefore (1)(k-3) + (-1)(-k) + (-1)(-k-7) = 0$

$\Rightarrow k = -\frac{4}{3}$

$\therefore \alpha^2 = \left(\frac{13}{3}\right)^2 + \left(\frac{4}{3}\right)^2 + \left(\frac{17}{3}\right)^2 = \frac{474}{9}$

$\therefore 3\alpha^2 = 158$

Question49

Let the shortest distance between the lines L : $\frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}$, $\lambda \geq 0$ and $L_1 : x + 1 = y - 1 = 4 - z$ be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is NOT possible?
[31-Jan-2023 Shift 1]

Options:

A. $\alpha + 2\gamma = 24$

B. $2\alpha + \gamma = 7$

C. $2\alpha - \gamma = 9$

D. $\alpha - 2\gamma = 19$

Answer: A

Solution:

Solution:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -\hat{i} - \hat{j} - 2\hat{k}$$

$a_2 - a_1 = 6\hat{i} + (\lambda - 1)\hat{j} + (-\lambda - 4)\hat{k}$

$2\sqrt{6} = \left|\frac{-6 - \lambda + 1 + 2\lambda + 8}{\sqrt{1+1+4}}\right|$

$|\lambda + 3| = 12 \Rightarrow \lambda = 9, -15$

$\alpha = -2k + 5, \gamma = k - \lambda$ where $k \in R$

$\Rightarrow \alpha + 2\gamma = 5 - 2\lambda = -13, 35$

Question50

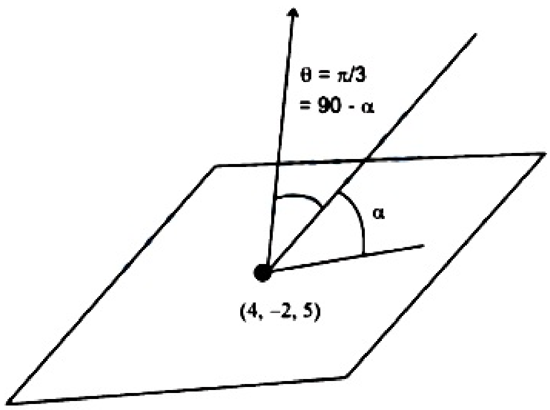
Let θ be the angle between the planes $P_1 = \vec{r} \cdot (\hat{i} + \hat{i} + 2\hat{k}) = 9$ and

$P_2 = \vec{r} \cdot \left(2\hat{i} - \hat{i} + \hat{k}\right) = 15.$
Let L be the line that meets P_2 at the point (4, −2, 5) and makes an angle θ with the normal of P_2. If α is the angle between L and P_2 then $(\tan^2\theta)(\cot^2\alpha)$ is equal to ______.
[31-Jan-2023 Shift 1]

Answer: 9

Solution:

Solution:



$\cos\theta = \frac{\left(\hat{i} + \hat{j} + 2\hat{k}\right) \cdot \left(2\hat{i} - \hat{j} + \hat{k}\right)}{6} = \frac{2 - 1 + 2}{6} = \frac{1}{2}$

$\theta = \pi / 3 \quad \alpha = \pi / 6$

$(\tan^2\theta)(\cot^2\alpha)$

(3) (3)=9

Question51

Let the line L : $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane 2x + y + 3z = 16 at the point P. Let the point Q be the foot of perpendicular from the point R(1, −1, −3) on the line L. If α is the area of triangle PQR. then α^2 is equal to _______.
[31-Jan-2023 Shift 1]

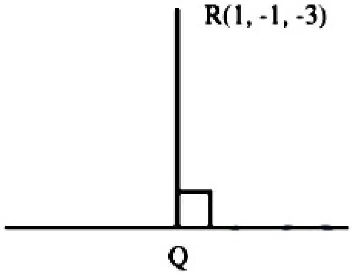
Answer: 180

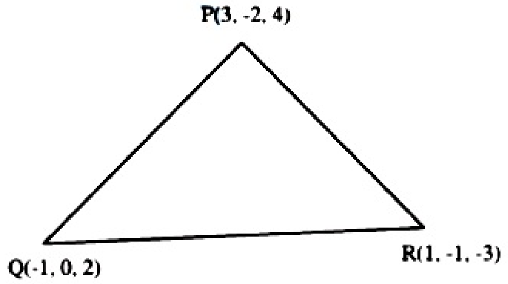
Solution:

Any point on L((2λ + 1), (−λ − 1), (λ + 3))

$2(2\lambda + 1) + (-\lambda - 1) + 3(\lambda + 3) = 16$

$6\lambda + 10 = 16 \Rightarrow \lambda = 1$
$\therefore P = (3, -2, 4)$
DR of QR $= \langle 2\lambda, -\lambda, \lambda + 6 \rangle$
DR of L $= \langle 2, -1, 1 \rangle$
$4\lambda + \lambda + \lambda + 6 = 0 \quad 6\lambda + 6 = 0 \Rightarrow \lambda = -1$
$Q = (-1, 0, 2)$





$\overrightarrow{QR} = 2\hat{i} - \hat{j} - 5\hat{k} \quad \overrightarrow{QP} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$$\overrightarrow{QR} \times \overrightarrow{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$$\alpha = \frac{1}{2} \times \sqrt{144 + 576} \Rightarrow \alpha^2 = \frac{720}{4} = 180$$

Question52

If a point P(α, β, γ) satisfying $(\alpha \beta \gamma)\begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} = (0 \ 0 \ 0)$ lies on the plane $2x + 4y + 3z = 5$, then $6\alpha + 9\beta + 7\gamma$ is equal to:
[31-Jan-2023 Shift 2]

Options:

A. -1

B. $\frac{11}{5}$

C. $\frac{5}{4}$

D. 11

Answer: D

Solution:

Solution:
$2\alpha + 4\beta + 3\gamma = 5 \ . \ . \ . \ (1)$

$2\alpha + 9\beta + 8\gamma = 0 \ldots (2)$
$10\alpha + 3\beta + 4\gamma = 0 \ldots (3)$
$8\alpha + 8\beta + 8\gamma = 0 \ldots (4)$
Subtract (4) from (2)
$-6\alpha + \beta = 0$
$\beta = 6\alpha \ldots (5)$
From equation (4)
$8\alpha + 48\alpha + 8\gamma = 0$
$\gamma = -7\alpha \ldots (6)$
From equation (1)
$2\alpha + 24\alpha - 21\alpha = 5$
$5\alpha = 5$
$\alpha = 1$
$\beta = +6, \quad \gamma = -7$
$\therefore 6\alpha + 9\beta + 7\gamma$
$= 6 + 54 - 49$
$= 11$

Question53

Let the plane P : $8x + \alpha_1 y + \alpha_2 z + 12 = 0$ be parallel to the line L : $\frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1 , then the distance between P and L is :
[31-Jan-2023 Shift 2]

Options:

A. $\sqrt{14}$

B. $\frac{6}{\sqrt{14}}$

C. $\sqrt{\frac{2}{7}}$

D. $\sqrt{\frac{7}{2}}$

Answer: A

Solution:

Solution:
P: $8x + \alpha_1 y + \alpha_2 z + 12 = 0$
L: $\frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$
$\because$P is parallel to L
$\Rightarrow 8(2) + \alpha_1(3) + 5(\alpha_2) = 0$
$\Rightarrow 3\alpha_1 + 5(\alpha_2) = -16$
Also y-intercept of plane P is 1
$\Rightarrow \alpha_1 = -12$
And $\alpha_2 = 4$
$\Rightarrow$ Equation of plane P is $2x - 3y + z + 3 = 0$
$\Rightarrow$ Distance of line L from Plane P is
$= \left| \frac{0 - 3(6) + 1 + 3}{\sqrt{4 + 9 + 1}} \right|$
$= \sqrt{14}$

Question54

Let P be the plane, passing through the point (1, −1, −5) and perpendicular to the line joining the points (4, 1, −3) and (2, 4, 3). Then the distance of P from the point (3, −2, 2) is
[31-Jan-2023 Shift 2]

Options:

A. 6

B. 4

C. 5

D. 7

Answer: C

Solution:

Equation of Plane :
$2(x-1)-3(y+1)-6(z+5)=0$
Or $2x-3y-6z=35$
$\Rightarrow$ Re quired distance $=$
$\frac{|2(3)-3(-2)-6(2)-35|}{\sqrt{4+9+36}}$
$=5$

Question55

The shortest distance between the lines $\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$ **and** $\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$ **is**
[1-Feb-2023 Shift 1]

Options:

A. $7\sqrt{3}$

B. $5\sqrt{3}$

C. $6\sqrt{3}$

D. $4\sqrt{3}$

Answer: C

Solution:

Solution:
Shortest distance between two lines
$\frac{x-x_1}{a_1}=\frac{y-y_1}{a_2}=\frac{z-z_1}{a_3\&}$

$\frac{x - x_2}{b_1} = \frac{y - y_2}{b_2} = \frac{z - z_2}{b_3}$ is given as

$$\frac{\left|\begin{matrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{matrix}\right|}{\sqrt{(a_1b_3 - a_3b_2)^2 + (a_1b_3 - a_3b_1)^2 + (a_1b_2 - a_2b_1)^2}}$$

$$\frac{\left|\begin{matrix} 5-(3) & 2-(-5) & 4-1 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{matrix}\right|}{\sqrt{(-10+12)^2 + (-5+3)^2 + (4-2)^2}}$$

$$\frac{\left|\begin{matrix} 8 & 7 & 3 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{matrix}\right|}{\sqrt{(2)^2 + (2)^2 + (2)^2}}$$

$$= \frac{|8(-10+12) - 7(-5+3) + 3(4-2)|}{\sqrt{4+4+4}}$$

$$= \frac{16+14+6}{\sqrt{12}} = \frac{36}{\sqrt{12}} = \frac{36}{2\sqrt{3}}$$

$$= \frac{18}{\sqrt{3}} = 6\sqrt{3}$$

Question56

Let the image of the point P(2, −1, 3) in the plane x + 2y − z = 0 be Q. Then the distance of the plane 3x + 2y + z + 29 = 0 from the point Q is [1-Feb-2023 Shift 1]

Options:

A. $\frac{22\sqrt{2}}{7}$

B. $\frac{24\sqrt{2}}{7}$

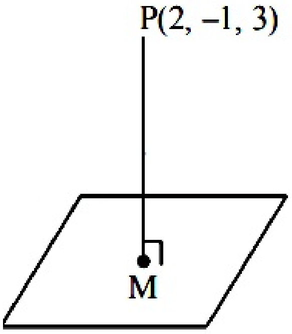
C. $2\sqrt{14}$

D. $3\sqrt{14}$

Answer: D

Solution:

Solution:



eq. of line PM $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-3}{-1}=\lambda$
any point on line $=(\lambda+2, 2\lambda-1, -\lambda+3)$
for point ' m ' $(\lambda+2)+2(2\lambda-1)-(3-\lambda)=0$
$\lambda=\frac{1}{2}$
Point m $\left(\frac{1}{2}+2, 2\times\frac{1}{2}-1, \frac{-1}{2}+3\right)$
$=\left(\frac{5}{2}, 0, \frac{5}{2}\right)$
For Image $Q(\alpha, \beta, \gamma)$
$\frac{\alpha+2}{2}=\frac{5}{2}, \frac{\beta-1}{2}=0,$
$\frac{\gamma+3}{2}=\frac{5}{2}$
$Q:(3, 1, 2)$
$d=\left|\frac{3(3)+2(1)+2+29}{\sqrt{3^2+2^2+1^2}}\right|$
$d=\frac{42}{\sqrt{14}}=3\sqrt{14}$

Question57

Let the plane P pass through the intersection of the planes $2x+3y-z=2$ and $x+2y+3z=6$, and be perpendicular to the plane $2x+y-z+1=0$. If d is the distance of P from the point $(-7, 1, 1)$, then d^2 is equal to :
[1-Feb-2023 Shift 2]

Options:

A. $\frac{250}{83}$

B. $\frac{15}{53}$

C. $\frac{25}{83}$

D. $\frac{250}{82}$

Answer: A

Solution:

Solution:
$P\equiv P_1+\lambda P_2=0$
$(2+\lambda)x+(3+2\lambda)y+(3\lambda-1)z-2-6\lambda=0$
Plane P is perpendicular to $P_3 \therefore \vec{n}\cdot\vec{n}_3=0$
$2(\lambda+2)+(2\lambda+3)-(3\lambda-1)=0$
$\lambda=-8$
$P\equiv-6x-13y-25z+46=0$
$6x+13y+25z-46=0$
Dist from $(-7, 1, 1)$
$d=\left|\frac{-42+13+25-46}{\sqrt{36+169+625}}\right|=\frac{50}{\sqrt{830}}$
$d^2=\frac{50\times50}{830}=\frac{250}{83}$

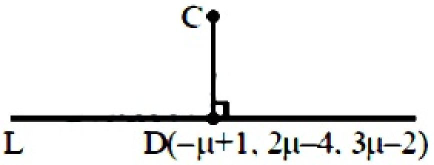
Question58

The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the points $A(-3, -6, 1)$ and $B(2, 4, -3)$ divides the line segment AB internally in the ratio $k : 1$. If a, b, c (|a|, | b | , | c| are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then $|a + b + c|$ is equal to _______.
[1-Feb-2023 Shift 2]

Answer: 10

Solution:

Solution:
Plane: $8x + y + 2z = 0$
Given line AB : $\frac{x-2}{5} = \frac{y-4}{10} = \frac{z+3}{-4} = \lambda$
Any point on line $(5\lambda + 2, 10\lambda + 4, -4\lambda - 3)$
Point of intersection of line and plane
$8(5\lambda + 2) + 10\lambda + 4 - 8\lambda - 6 = 0$
$\lambda = -\frac{1}{3}$
$C\left(\frac{1}{3}, \frac{2}{3}, -\frac{5}{3}\right)$
$L : \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = \mu$



$\overrightarrow{CD} = \left(-\mu + \frac{2}{3}\right)\hat{i} + \left(2\mu - \frac{14}{3}\right)\hat{j} + \left(3\mu - \frac{1}{3}\right)\hat{k}$
$\left(-\mu + \frac{2}{3}\right)(-1) + \left(2\mu - \frac{14}{3}\right)2 + \left(3\mu - \frac{1}{3}\right)3 = 0$
$\mu = \frac{11}{14}$
$\overrightarrow{CD} = \frac{-5}{42}, \frac{-130}{42}, \frac{85}{42}$
Direction ratios $\rightarrow (-1, -26, 17)$
$|a + b + c| = 10$

Question59

Let $\alpha x + \beta y + yz = 1$ be the equation of a plane passing through the point $(3, -2, 5)$ and perpendicular to the line joining the points $(1, 2, 3)$ and $(-2, 3, 5)$. Then the value of $\alpha\beta y$ is equal to ______.
[1-Feb-2023 Shift 2]

Answer: 6

Solution:

Solution:
Given Equation is not equation of plane as yz is present. If we consider y is γ then answer would be 6 .

Normal vector of plane $= 3\hat{i} - \hat{j} - 2\hat{k}$
Plane : $3x - y - 2z + \lambda = 0$
Point $(3, -2, 5)$ satisfies the plane
$\lambda = -1$
$3x - y - 2z = 1$
$\alpha\beta y = 6$

Question60

One vertex of a rectangular parallelepiped is at the origin O and the lengths of its edges along x, y and z axes are 3,4 and 5 units respectively. Let P be the vertex (3, 4, 5). Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is : [6-Apr-2023 shift 1]

Options:

A. $\frac{12}{5\sqrt{5}}$

B. $12\sqrt{5}$

C. $\frac{12}{5}$

D. $\frac{12}{\sqrt{5}}$

Answer: C

Solution:

Equation of OP is $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$
$a_1 = (0, 0, 0)$ $a_2 = (3, 0, 5)$
$b_1 = (3, 4, 5)$ $b_2 = (0, 0, 1)$
Equation of edge parallel to z axis

$$\frac{x-3}{0} = \frac{y-0}{0} = \frac{z-5}{1}$$

$$S.D = \frac{\left(\vec{a}_2 \cdot \vec{a}_1\right) \cdot \left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}$$

$$\frac{\begin{vmatrix} 3 & 0 & 5 \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix}} = \frac{3(4)}{|4\hat{i} - 3\hat{j}|} = \frac{12}{5}$$

Question61

If the equation of the plane passing through the line of intersection of the planes $2x - y + z = 3$, $4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ is $ax + by + cz + 6 = 0$, then $a + b + c$ is equal to :
[6-Apr-2023 shift 1]

Options:

A. 15

B. 14

C. 13

D. 12

Answer: B

Solution:

Solution:
Using family of planer
$P : P_1 + \lambda P_2 = 0 \Rightarrow P(2 + 4\lambda)x - (1 + 3\lambda)y + (1 + 5\lambda)z = (3 - 9\lambda)$
P is | to $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$
Then for $\lambda : \vec{n}_p \cdot \vec{v}_L = 0$
$-2(2 + 4\lambda) - 4(1 + 3\lambda) + 5(1 + 5\lambda) = 0$
$-3 + 5\lambda = 0 \Rightarrow \lambda = \frac{3}{5}$
Hence : $P : 22x - 14y + 20z = -12$
$P : 11x - 7y + 10z + 6 = 0$
$\Rightarrow a = 11$
$b = -7$
$c = 10$
$\Rightarrow a + b + c = 14$

Question62

Let the image of the point P(1, 2, 3) in the plane $2x - y + z = 9$ be Q. If the coordinates of the point R are (6, 10, 7). then the square of the area of the triangle PQR is _______.
[6-Apr-2023 shift 1]

Answer: 594

Solution:

Solution:
Let $Q(\alpha, \beta, \gamma)$ be the image of P, about the plane
$2x - y + z = 9$
$\frac{\alpha - 1}{2} = \frac{\beta - 2}{-1} = \frac{\gamma - 3}{1} = 2$
$\Rightarrow \alpha = 5, \beta = 0, \gamma = 5$
Then area of triangle PQR is $= \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right|$
$= \left|-12\hat{i} - 3\hat{j} + 21\hat{k}\right| = \sqrt{144 + 9 + 441} = \sqrt{594}$
Square of area $= 594$

Question63

Let the line L pass through the point (0, 1, 2), intersect the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and be parallel to the plane $2x + y - 3z = 4$. Then the distance of the point $P(1, -9, 2)$ from the line L is : [6-Apr-2023 shift 2]

Options:

A. 9

B. $\sqrt{54}$

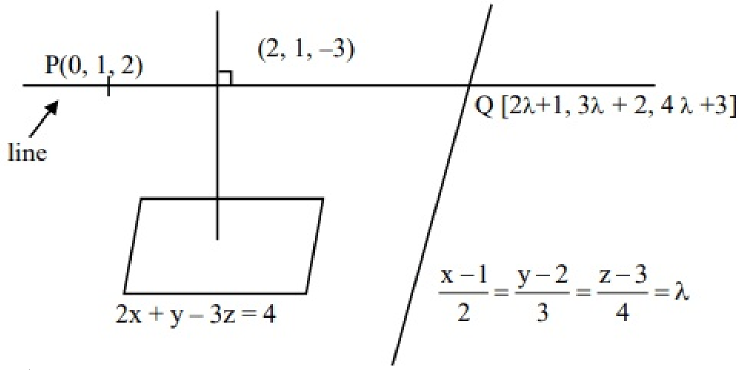
C. $\sqrt{69}$

D. $\sqrt{74}$

Answer: D

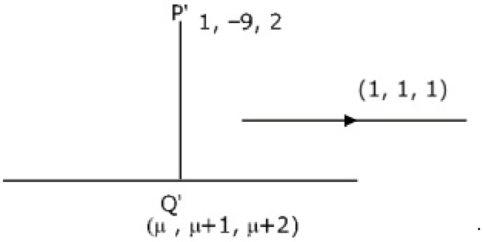
Solution:

Solution:



$\overrightarrow{PQ} = (2\lambda + 1, 3\lambda + 1, 4\lambda + 1)$
$\overrightarrow{PQ} \cdot \vec{n} = 0 \Rightarrow (2\lambda + 1) \cdot (2) + (3\lambda + 1)(1) + (4\lambda + 1)(-3) = 0$
$\Rightarrow -5\lambda = 0$
$\Rightarrow \lambda = 0$
$Q = (1, 2, 3)$
$\frac{x - 0}{1} = \frac{y - 1}{1} = \frac{z - 2}{1} = \mu$

distance of line from $(1, -9, 2)$
$(P'Q) \cdot (1, 1, 1) = 0$
$\Rightarrow [\mu - 1, \mu + 10, \mu] \cdot [1, 1, 1] = 0$
$\Rightarrow \mu - 1 + \mu + 10 + \mu = 0$
$\mu = -3$
$Q' = (-3, -2, 1)$
$P'Q' = \sqrt{16 + 49 + 9} = \sqrt{74}$



Question64

A plane P contains the line of intersection of the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$. If P passes through the point $(0, 2, -2)$, then the square of distance of the point $(12, 12, 18)$ from the plane P is : [6-Apr-2023 shift 2]

Options:

A. 620

B. 1240

C. 310

D. 155

Answer: A

Solution:

Solution:
eq ^{n} of plane $P_1 + \lambda P_2 = 0$
$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$
pass th. $(0, 2, -2)$
$(-6) + \lambda(6 - 8 + 5) = 0$
$(-6) + \lambda[3] = 0 \Rightarrow \lambda = 2$
eq^{n} of plane
$5x + 7y + 9z + 4 = 0$
distance from $(12, 12, 18)$
$d = \left| \dfrac{60 + 84 + 162 + 4}{\sqrt{25 + 49 + 81}} \right|$
$d = \dfrac{310}{\sqrt{155}}$
$d^2 = \dfrac{310 \times 310}{155}$
$d^2 = 620$
Ans. Option 1

Question65

If the lines $\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{\alpha}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$ intersect, then the magnitude of the minimum value of $8\alpha\beta$ is _______.
[6-Apr-2023 shift 2]

Answer: 18

Solution:

Solution:

If the lines $\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{\alpha}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$ intersect Point of first line $(1, 2, 3)$ and point on second line $(4, 1, 0)$.

Vector joining both points is $-3\hat{i} + \hat{j} + 3k$

Now vector along second line is $2\hat{i} + 3\hat{j} + \alpha k$

Also vector along second line is $5\hat{i} + 2\hat{j} + \beta k$

Now these three vectors must be coplanar

$$\Rightarrow \begin{vmatrix} 2 & 3 & \alpha \\ 5 & 2 & \beta \\ -3 & 1 & 3 \end{vmatrix}$$

$\Rightarrow 2(6-\beta) - 3(15+3\beta) + \alpha(11) = 0$

$\Rightarrow \alpha - \beta = 3$

Now $\alpha = 3 + \beta$

Given expression $8(3+\beta)\cdot\beta = 8(\beta^2 + 3\beta)$

$= 8\left(\beta^2 + 3\beta + \frac{9}{4} - \frac{9}{4}\right) = 8\left(\beta + \frac{3}{2}\right)^2 - 18$

So magnitude of minimum value $= 18$

Question66

The shortest distance between the lines $\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is
[8-Apr-2023 shift 1]

Options:

A. $2\sqrt{6}$

B. $3\sqrt{6}$

C. $6\sqrt{3}$

D. $6\sqrt{2}$

Answer: B

Solution:

$$S_d = \left| \frac{(\vec{a} - \vec{b}) \times (\vec{n}_1 \times \vec{n}_2)}{|\vec{n}_1 \times \vec{n}_2|} \right|$$

$\bar{a} = (4, -2, -3)$
$\bar{b} = (1, 3, 4)$
$\bar{n}_1 = (4, 5, 3)$
$\bar{n}_2 = (3, 4, 2)$

$$\bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} i & j & k \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = \hat{i}(-2) - \hat{j}(-1) + \hat{k}(1) = (-2, 1, 1)$$

$$S_d = \frac{(3, -5, -7) \cdot (-2, 1, 1)}{\sqrt{6}} = \left| \frac{-6 - 5 - 7}{\sqrt{6}} \right| = 3\sqrt{6}$$

Question67

If the eqation of the plane containing the line
$x + 2y + 3z - 4 = 02x + y - z + 5$ and perpendicular to the plane
$\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$ is ax+by +cz = 4, then
$(a - b + c)$ is equal to
[8-Apr-2023 shift 1]

Options:

A. 22

B. 24

C. 20

D. 18

Answer: A

Solution:

D.R's of line $\vec{n}_1 = -5\hat{i} + 7\hat{j} - 3\hat{k}$
D.R's of normal of second plane
$\vec{n}_2 = 5\hat{i} - 2\hat{j} - 3\hat{k}$
$\vec{n}_1 \times \vec{n}_2 = -27\hat{i} - 30\hat{j} - 25\hat{k}$
A point on the required plane is $\left(0, -\frac{11}{5}, \frac{14}{5}\right)$
The equation of required plane is
$27x + 30y + 25z = 4$
$\therefore a - b + c = 22$

Question68

Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2}, 1, \lambda\right)$ and
$(-2, 0, 1)$ are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If

$\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ is ________.
[8-Apr-2023 shift 1]

Answer: 9

Solution:

Solution:

$2x + 3y - 6z + 7 = 0\left(\frac{5}{2}, 1, \lambda\right), (-2, 0, 1)$

$d_1 = \left|\frac{5+3-6\lambda+7}{7}\right| = d_2 = \left|\frac{-4-6+7}{7}\right|$

$\Rightarrow |15 - 6\lambda| = |3|$

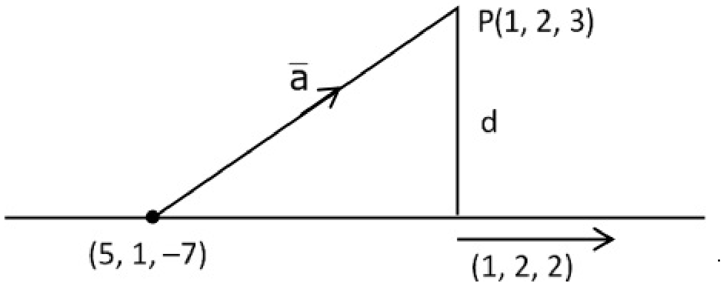
$15 - 6\lambda = 3$ or $15 - 6\lambda = -3$

$6\lambda = 12$ $6\lambda = 18$

$\lambda = 2$ $\lambda = 3$

$\lambda_1 = 3, \quad \lambda_2 = 2$

$\therefore P(1, 2, 3)$ $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$



$d = \left|\frac{(4, -1, -10) \times (1, 2, 2)}{3}\right| = \left|\frac{18\hat{i} - 18\hat{j} + 9\hat{k}}{3}\right| = 9$

Question69

For a, b ∈ Z and $|a - b| \le 10$, let the angle between the plane P : $ax + y - z = b$ and the line 1 : $x - 1 = a - y = z + 1$ be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point $(6, -6, 4)$ from the plane P is $3\sqrt{6}$, then $a^4 + b^2$ is equal to
[8-Apr-2023 shift 2]

Options:

A. 85

B. 48

C. 25

D. 32

Answer: D

Solution:

$\theta = \cos^{-1}\frac{1}{3} \therefore \sin\theta = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$

$\sin\theta = \frac{a\cdot 1 + 1(-1) + (-1)\cdot 1}{\sqrt{a^2+1+1}\cdot\sqrt{3}} = \frac{2\sqrt{2}}{3}$

$\Rightarrow \{3(a-2)\}^2 = 24(a^2+2)$

$\Rightarrow 3(a^2-4a+4) = 8a^2+16$

$\Rightarrow 5a^2+12a+4 = 0$

$\Rightarrow 5a^2+10a+2a+4 = 0$

$\therefore a = -2, \frac{-2}{5} \because a \in z$

$\therefore a = -2$

Distance of $(6, -6, 4)$ from

$-2x+y-z-b = 0$ is $3\sqrt{6}$

$\therefore \left|\frac{-12-6-4-b}{\sqrt{4+1+1}}\right| = 3\sqrt{6}$

$\Rightarrow |b+22| = 18 \therefore b = -40, -4$

$\because |a-b| \le 10$

$\therefore b = -4$

$\therefore a^4+b^2$

$= 32$ Ans.

Question70

Let P_1 be the plane $3x-y-7z = 11$ and P_2 be the plane passing through the points $(2, -1, 0)$, $(2, 0, -1)$, and $(5, 1, 1)$. If the foot of the perpendicular drawn from the point $(7, 4, -1)$ on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha+\beta+\gamma$ is equal to _______.
[8-Apr-2023 shift 2]

Answer: 11

Solution:

Solution:

P_2 is given by

$$\begin{vmatrix} x-5 & y-1 & z-1 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

$x-y-z = 3$

DR of line intersection of $P_1 \& P_2$

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 3 & -1 & -7 \end{vmatrix}$$

$+6\hat{i} + 4\hat{j} + 2\hat{k}$

Let

$z = 0, \quad x-y = 3$

$3x-y = 112x = 8$

$x = 4$
$y = 1$
So Line is

$$\frac{x-4}{6} = \frac{y-1}{4} = \frac{z-0}{2} = r$$

$(\alpha, \beta, \gamma) = (6r + 4, 4r + 1, 2r)$
$6(\alpha - 7) + 4(\beta - 4) + 2(\gamma + 1) = 0$
$6\alpha - 42 + 4\beta - 16 + 2\gamma + 2 = 0$
$36r + 24 + 16r + 4 + 4r - 56 = 0$
$56r = 28$
$r = \frac{1}{2}$ $\alpha + \beta + \gamma = 12r + 5$
$= 6 + 5 = 11$

Question71

Let P be the plane passing through the line $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z+5}{7}$ and the point $(2, 4, -3)$. If the image of the point $(-1, 3, 4)$ in the plane P is (α, β, γ) then $\alpha + \beta + \gamma$ is equal to
[8-Apr-2023 shift 2]

Options:

A. 12

B. 9

C. 10

D. 11

Answer: C

Solution:

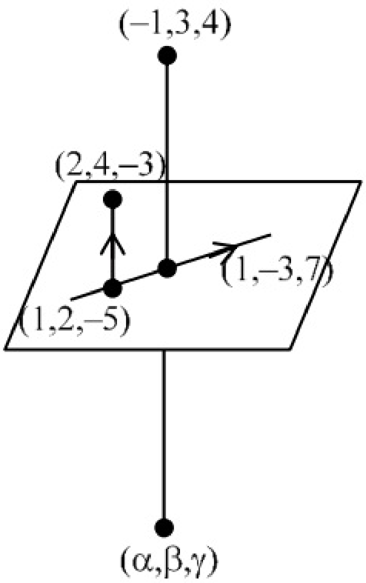
Solution:
Equation of plane is given by

$$\begin{vmatrix} x-1 & y-2 & z+5 \\ 1 & 2 & 2 \\ 1 & -3 & 7 \end{vmatrix} = 0$$

$4x - y - z = 7$

$$\frac{\alpha+1}{4} = \frac{\beta-3}{-1} = \frac{\gamma-4}{-1} = \frac{-2(-4-3-4-7)}{16+1+1} = 2$$

$\alpha = 7, \beta = 1, \gamma = 2$
$\alpha + \beta + \gamma = 10(\text{ Option } 3)$



Question72

Let O be the origin and the position vector of the point P be $-\hat{i}-2\hat{j}+3\hat{k}$. If the position vectors of the A, B and C are $-2\hat{i}+\hat{j}-3\hat{k}$, $2\hat{i}+4\hat{j}-2\hat{k}$ and $-4\hat{i}+2\hat{j}-\hat{k}$ respectively, then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is :
[10-Apr-2023 shift 1]

Options:

A. $\frac{10}{3}$

B. $\frac{8}{3}$

C. $\frac{7}{3}$

D. 3

Answer: D

Solution:

Solution:
Position vector of the point $P(-1,-2,3)$, $A(-2,1,-3)B(2,4,-2)$, and $C(-4,2,-1)$

Then $\overrightarrow{OP}\cdot\dfrac{\overrightarrow{AB}\times\overrightarrow{AC}}{|\overrightarrow{AB}\times\overrightarrow{AC})|}$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\4&3&1\\-2&1&2\end{vmatrix}$$

$=\hat{i}(5)-\hat{j}(8+2)+\hat{k}(4+6)$
$=5\hat{i}-10\hat{j}+10\hat{k}$
Now

$$\vec{OP}\cdot\frac{\vec{AB}\times\vec{AC}}{|\vec{AB}\times\vec{AC}|}=\left(-\hat{i}-2\hat{j}+3\hat{k}\right)\cdot\frac{\left(\hat{i}-10\hat{j}+10\hat{k}\right)}{\sqrt{(5)^2+(-10)^2+(10)^2}}$$
$$=\frac{-5+20+30}{\sqrt{25+100+100}}$$
$$=\frac{45}{\sqrt{225}}=\frac{45}{15}=3$$

Question73

Let two vertices of a triangle ABC be (2, 4, 6) and (0, −2, −5), and its centroid be (2, 1, −1). If the image of the third vertex in the plane $x+2y+4z=11$ is (α, β, γ), then $\alpha\beta+\beta\gamma+\gamma\alpha$ is equal to :
[10-Apr-2023 shift 1]

Options:

A. 76

B. 74

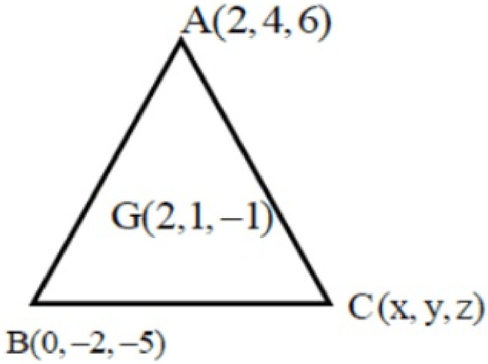
C. 70

D. 72

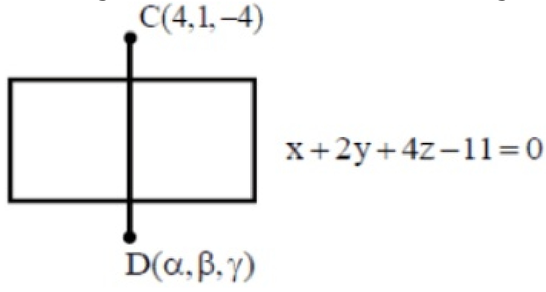
Answer: B

Solution:

Solution:



Given Two vertices of Triangle A(2, 4, 6) and B(0, −2, −5) and if centroid G(2, 1, −1)
Let Third vertices be (x, y, z)
Now $\frac{2+0+x}{3}=2,\ \frac{4-2+y}{3}=1,\ \frac{6-5+z}{3}=-1$
$x=4, y=1,\ \ z=-1$
Third vertices C(4, 1, −4)
Now, Image of vertices C(4, 1, −4) in the given plane is D(α, β, γ)



Now
$$\frac{\alpha-4}{1}=\frac{\beta-1}{2}=\frac{\gamma+4}{4}=-2\frac{(4+2-16-11)}{1+4+16}$$

$\frac{\alpha-4}{1}=\frac{\beta-1}{2}=\frac{\gamma+4}{4}=\frac{42}{21}\Rightarrow 2$

$\alpha=6, \beta=5, \gamma=4$

Then $\alpha\beta+\beta\gamma+\gamma\alpha$

$=(6\times5)+(5\times4)+(4\times6)$

$=30+20+24$

$=74$

Question74

The shortest distance between the lines $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ is :
[10-Apr-2023 shift 1]

Options:

A. 8

B. 7

C. 6

D. 9

Answer: D

Solution:

Solution:

$\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$

$=\frac{|-54|}{\left|-4\hat{i}+2\hat{j}+4k\right|}$

$=\frac{54}{\sqrt{16+4+16}}$

$=\frac{54}{6}$

$=9$

Question75

Let P be the point of intersection of the line $\frac{x+3}{3}=\frac{y+2}{1}=\frac{1-z}{2}$ and the plane $x+y+z=2$. If the distance of the point P from the plane $3x-4y+12z=32$ is q, then q and 2q are the roots of the equation :
[10-Apr-2023 shift 1]

Options:

A. $x^2+18x-72=0$

B. $x^2+18x+72=0$

C. $x^2-18x-72=0$

D. $x^2 - 18x + 72 = 0$

Answer: D

Solution:

Solution:

$\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2} = \lambda$

$x = 3\lambda - 3, y = \lambda - 2, z = 1 - 2\lambda$

$P(3\lambda - 3, \lambda - 2, 1 - 2\lambda)$ will satisfy the equation of plane $x + y + z = 2$.

$3\lambda - 3 + \lambda - 2 + 1 - 2\lambda = 2$

$2\lambda - 4 = 2$

$\lambda = 3$

$P(6, 1, -5)$

Perpendicular distance of P from plane $3x - 4y + 12z - 32 = 0$ is

$q = \left| \frac{3(6) - 4(1) + 12(-5) - 32}{\sqrt{9 + 16 + 144}} \right|$

$q = 6$

$2q = 12$

Sum of roots $= 6 + 12 = 18$

Product of roots $= 6(12) = 72$

$\therefore$ Quadratic equation having q and 2q as roots is $x^2 - 18x + 72$.

Question76

Let time image of the point P(1, 2, 6) in the plane passing through the points A(1, 2, 0), B(1, 4, 1) and C(0, 5, 1) be $Q(\alpha, \beta, \gamma)$. Then $(\alpha^2 + \beta^2 + \gamma^2)$ is equal to [10-Apr-2023 shift 2]

Options:

A. 70

B. 76

C. 62

D. 65

Answer: D

Solution:

Equation of plane $A(x - 1) + B(y - 2) + C(z - 0) = 0$

Put $(1, 4, 1) \Rightarrow 2B + C = 0$

Put $(0, 5, 1) \Rightarrow -A + 3B + C = 0$

Sub : $B - A = 0 \Rightarrow A = B, C = -2B$

$1(x - 1) + 1(y - 2) - 2(z - 0) = 0$

$x + y - 2z - 3 = 0$

Image is (α, β, γ)pt $\equiv (1, 2, 6)$

$\frac{\alpha - 1}{1} = \frac{\beta - 2}{1} = \frac{\gamma - 6}{-2} = \frac{-2(1 + 2 - 12 - 3)}{6}$

$\frac{\alpha - 1}{1} = \frac{\beta - 2}{1} = \frac{\gamma - 6}{-2} = 4$

$\alpha = 5, \beta = 6, \gamma = -2 \Rightarrow \alpha^2 + \beta^2 + \gamma^2$

$= 25 + 36 + 4 = 65$

Question77

Let the line $\frac{x}{1} = \frac{6-y}{2} = \frac{z+8}{5}$ intersect the lines $\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1}$ and $\frac{x+3}{6} = \frac{3-y}{3} = \frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x - 2y + z = 14$ is
[10-Apr-2023 shift 2]

Options:

A. 3

B. $\frac{10}{3}$

C. 4

D. $\frac{11}{3}$

Answer: C

Solution:

Solution:

$\frac{x}{1} = \frac{y-6}{-2} = \frac{z+8}{5} = \lambda \ldots (1)$

$\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1} = \mu \ldots (2)$

$\frac{x+3}{6} = \frac{y-3}{-3} = \frac{z-6}{1} = \gamma \ldots (3)$

Intersection of (1)&(2) "A"

$(\lambda, -2\lambda+6, 5\lambda-8)\&(4\mu+5, 3\mu+7, \mu-2)$

$\lambda = 1, \mu = -1$

$A(1, 4, -3)$

Intersection (1) & (3) B " "

$(\lambda, -2\lambda+6, 5\lambda-8)\&(6\gamma-3, -3\gamma+3, \gamma+6)$

$\lambda = 3$

$\gamma = 1$

$B(3, 0, 7)$

Mod point of A & B $\Rightarrow (2, 2, 2)$

Perpendicular distance from the plane

$2x - 2y + z = 14$

$\frac{2(2) - 2(2) + 2 - 14}{\sqrt{4+4+1}}| = 4$

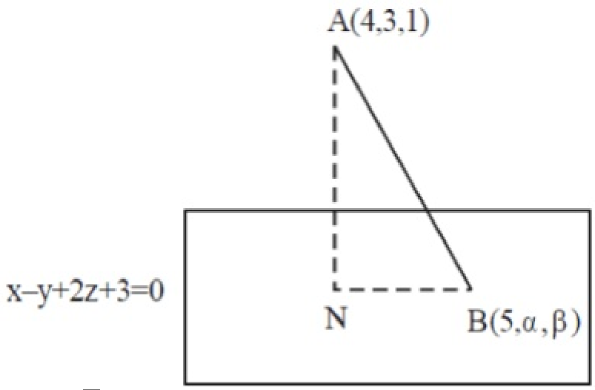
Question78

Let the foot of perpendicular from the point A(4, 3, 1) on the plane P : $x - y + 2z + 3 = 0$ be N. If B(5, α, β), α, β ∈ Z is a point on plane P such that the area of the triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to ______.
[10-Apr-2023 shift 2]

Answer: 7

Solution:

Solution:



$AN = \sqrt{6}$

$5 - \alpha + 2\beta + 3 = 0$

$\Rightarrow \alpha = 8 + 2\beta$

N is given by

$\frac{x-4}{1} = \frac{y-3}{-1} = \frac{z-1}{2} = \frac{-(4-3+2+3)}{1+1+4}$

$x = 3, y = 4, z = -1$

N

$(3, 4, -1)$

$BN = \sqrt{4 + (\alpha - 4)^2 + (\beta + 1)^2}$

$= \sqrt{4 + (2\beta + 4)^2 + (\beta + 1)^2}$

Area of $\triangle ABN = \frac{1}{2} AN \times BN = 3\sqrt{2}$

$\frac{1}{2} \times \sqrt{6} \times BN = 3\sqrt{2}$

$BN = 2\sqrt{3}$

$4 + (2\beta + 4)^2 + (\beta + 1)^2 = 12$

$(2\beta + 4)^2 + (\beta + 1)^2 - 8 = 0$

$5\beta^2 + 18\beta + 9 = 0$

$(5\beta + 3)(\beta + 3) = 0$

$\beta = -3$

$\alpha = 2$

$\alpha^2 + \beta^2 + \alpha\beta = 9 + 4 - 6 = 7$

Question79

Let (α, β, γ) be the image of the point $P(2, 3, 5)$ in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to : [11-Apr-2023 shift 1]

Options:

A. 5

B. 9

C. 10

D. 12

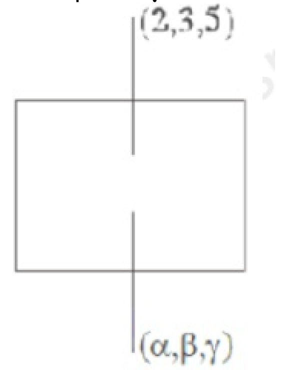
Answer: C

Solution:

$\frac{\alpha-2}{2}=\frac{\beta-3}{1}=\frac{\gamma-5}{-3}=-2\left(\frac{2\times2+3-3\times5-6}{2^2+1^2+1-3^2}\right)=2$

$\frac{\alpha-2}{2}=2\ \beta-3=2\ \gamma-5=-6$

$\alpha=6\ \beta=5\ \gamma=-1$



$\alpha+\beta+\gamma=10$

Question80

If the equation of the plane that contains the point $(-2, 3, 5)$ and is perpendicular to each of the planes $2x+4y+5z=8$ and $3x-2y+3z=5$ is $\alpha x+\beta y+\gamma z+97=0$ then $\alpha+\beta+\gamma=$:
[11-Apr-2023 shift 1]

Options:

A. 15

B. 18

C. 17

D. 16

Answer: A

Solution:

Solution:
The equation of plane through $(-2, 3, 5)$ is
$a(x+2)+b(y-3)+c(z-5)=0$
it is perpendicular to $2x+4y+5z=8\&3x-2y+3z=5$
$\therefore 2a+4b+5c=0$
$3a-2b+3c=0$

$$\therefore \frac{a}{\begin{vmatrix}4 & 5\\-2 & 3\end{vmatrix}}=\frac{-b}{\begin{vmatrix}2 & 5\\3 & 3\end{vmatrix}}=\frac{c}{\begin{vmatrix}2 & 4\\3 & -2\end{vmatrix}}$$

$\Rightarrow \frac{a}{22}=\frac{b}{9}=\frac{c}{-16}$
$\therefore$ Equation of plane is
$22(x+2)+9(y-3)-16(z-5)=0$
$\Rightarrow 22x+9y-16z+97=0$
Comparing with $\alpha x+\beta y+\gamma x+97=0$
We get $\alpha+\beta+\gamma=22+9-16=15$

Question81

Let a line l pass through the origin and be perpendicular to the lines $l_1 : \vec{r} = \hat{i} - 11\hat{j} - 7\hat{k} + \lambda\hat{i} + 2\hat{j} + 3\hat{k}, \lambda \in \mathbb{R}$ and
$l_2 : \vec{r} = -\hat{i} + \hat{k} + \mu 2\hat{i} + 2\hat{j} + \hat{k}, \mu \in \mathbb{R}$.
If P is the point of intersection of l and l_1, and $Q(\alpha, \beta, \gamma)$ is the foot of perpendicular from P on l_2, then $9(\alpha + \beta + \gamma)$ is equal to ______.
[11-Apr-2023 shift 1]

Answer: 5

Solution:

Solution:

Let $\ell = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \gamma(a\hat{i} + b\hat{j} + c\hat{k})$
$= \gamma(a\hat{i} + b\hat{j} + c\hat{k})$

$$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(2-6) - \hat{j}(1-6) + \hat{k}(2-4)$
$= -4\hat{i} - 5\hat{j} - 2\hat{k}$
$\ell = \gamma(-4\hat{i} + 5\hat{j} - 2\hat{k})$
P is intersection of ℓ and ℓ_1
$-4\gamma = 1 + \lambda, 5\gamma = -11 + 2\lambda, -2\gamma = -7 + 3\lambda$
By solving these equation $\gamma = -1$, $P(4, -5, 2)$
Let $Q(-1 + 2\mu, 2\mu, 1 + \mu)$
$\vec{PQ} \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 0$
$-2 + 4\mu + 4\mu + 1 + \mu = 0$
$9\mu = 1$
$\mu = \frac{1}{9}$
$Q\left(\frac{-7}{9}, \frac{2}{9}, \frac{10}{9}\right)$
$9(\alpha + \beta + \gamma) = 9\left(\frac{-7}{9} + \frac{2}{9} + \frac{10}{9}\right)$
$= 5$

Question82

Let P be the plane passing through the points (5, 3, 0), (13, 3, −2) and (1, 6, 2). For $\alpha \in N$, if the distances of the points $A(3, 4, \alpha)$ and $B(2, \alpha, a)$ from the plane P are 2 and 3 respectively, then the positive value of a is
[11-Apr-2023 shift 2]

Options:

A. 5

B. 6

C. 4

D. 3

Answer: C

Solution:

Solution:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & -2 \\ 4 & -3 & -2 \end{vmatrix} = \hat{i}(-6) + 8\hat{j} - 24\hat{k}$$

Normal of the plane $= 3\hat{i} - 4\hat{j} + 12\hat{k}$

Plane : $3x - 4y + 12z = 3$

Distance from $A(3, 4, \alpha)$

$$\left| \frac{9 - 16 + 12\alpha - 3}{13} \right| = 2$$

$\alpha = 3$

$\alpha = -8$ (rejected)

Distance from $B(2, 3, a)$

$$\left| \frac{6 - 12 + 12a - 3}{13} \right| = 3$$

$a = 4$

Question83

Let the line passing through the point P(2, −1, 2) and Q(5, 3, 4) meet the plane $x - y + z = 4$ at the point T. Then the distance of the point R from the plane $x + 2y + 3z + 2 = 0$ measured parallel to the line $\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$ is equal to [11-Apr-2023 shift 2]

Options:

A. 3

B. $\sqrt{61}$

C. $\sqrt{31}$

D. $\sqrt{189}$

Answer: A

Solution:

Solution:

Line: $\frac{x-5}{3} = \frac{y-3}{4} = \frac{z-4}{2} = \lambda$

$R(3\lambda + 5, 4\lambda + 3, 2\lambda + 4)$

$\therefore 3\lambda + 5 - 4\lambda - 3 + 2\lambda + 4 = 4$

$\lambda + 6 = 4 \quad \therefore \lambda = -2$

$\therefore R \equiv (-1, -5, 0)$

Line: $\frac{x+1}{2} = \frac{y+5}{2} = \frac{z-0}{1} = \mu$
Point $T = (2\mu - 1, 2\mu - 5, \mu)$
It lies on plane
$2\mu - 1 + 2(2\mu - 5) + 3\mu + 2 = 0$
$\mu = 1$
$\therefore T = (1, -3, 1)$
$\therefore RT = 3$

Question84

Let the line $\ell : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}, \lambda \in R$ meet the plane $P : x + 2y + 3z = 4$ at the point (α, β, γ). If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to ______.
[11-Apr-2023 shift 2]

Answer: 11

Solution:

Solution:
$\ell : x = \frac{y-1}{2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$
Dr's of line $\ell(1, 2, \lambda)$
Dr's of normal vector of plane $P : x + 2y + 3z = 4$ are $(1, 2, 3)$
Now, angle between line ℓ and plane P is given by

$$\sin\theta = \left|\frac{1+4+3\lambda}{\sqrt{5+\lambda^2}\cdot\sqrt{14}}\right| = \frac{3}{\sqrt{14}}\left(\text{ given } \cos\theta = \sqrt{\frac{5}{14}}\right)$$

$\Rightarrow \lambda = \frac{2}{3}$

Let variable point on line ℓ is $\left(t, 2t+1, \frac{2}{3}t + 3\right)$
line of plane P.
$\Rightarrow t = -1$
$\Rightarrow \left(-1, -1, \frac{7}{3}\right) \equiv (\alpha, \beta, \gamma)$
$\Rightarrow \alpha + 2\beta + 6\gamma = 11$

Question85

Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point $P(a, b, c)$ on l_1 is nearest to the point $Q(-4, -3, 2)$, then $|a| + |b| + |c|$ is equal to
[12-Apr-2023 shift 1]

Options:

A. 10

B. 8

C. 12

D. 14

Answer: A

Solution:

Solution:

$(3x + 2y + z - 2) + \mu(x - 3y + 2z - 13) = 0$

$3(3 + \mu) + 1 \cdot (2 - 3\mu) - 2(1 + 2\mu) = 0$

$9 - 4\mu = 0$

$\mu = \frac{9}{4}$

$4(-15 - 8 + \alpha - 2) + 9(-5 + 12 + 2\alpha - 13) = 0$

$-100 + 4\alpha - 54 + 18\alpha = 0$

$\Rightarrow \alpha = 7$

Let $P \equiv (3\lambda - 5, \lambda - 4, -2\lambda + 7)$

Direction ratio of PQ $(3\lambda - 1, \lambda - 1, -2\lambda + 5)$

But $PQ \perp \ell_1$

$\Rightarrow 3(3\lambda - 1) + 1 \cdot (\lambda - 1) - 2(-2\lambda + 5) = 0$

$\Rightarrow \lambda = 1$

$P(-2, -3, 5) \Rightarrow |a| + |b| + |c| = 10$

Question86

Let the plane P: $4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If α is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then 35α is equal to [12-Apr-2023 shift 1]

Options:

A. 90

B. 105

C. 85

D. 126

Answer: D

Solution:

Solution:

Let equation in new position is

$(4x - y + z - 10) + \lambda(x + y - z - 4) = 0$

$4(4 + \lambda) - 1 \cdot (-1 + \lambda) + 1 \cdot (1 - \lambda) = 0$

$\Rightarrow \lambda = -9$

So equation in new position is

$-5x - 10y + 10z + 26 = 0$

$\Rightarrow \alpha = \frac{54}{15}$

Question87

Let the equation of plane passing through the line of intersection of the planes $x + 2y + az = 2$ and $x - y + z = 3$ be $5x - 11y + bz = 6a - 1$. For $c \in Z$, if the distance of this plane from the point $(a, -c, c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to
[13-Apr-2023 shift 1]

Options:

A. -4

B. 2

C. -2

D. 4

Answer: A

Solution:

Solution:

$(x + 2y + az - 2) + \lambda(x - y + z - 3) = 0$

$\frac{1+\lambda}{5} = \frac{2-\lambda}{-11} = \frac{a+\lambda}{b} = \frac{2+3\lambda}{6a-1}$

$\lambda = -\frac{7}{2}, a = 3, b = 1$

$\frac{2}{\sqrt{a}} = \left| \frac{5a + 11c + bc - 6a + 1}{\sqrt{25 + 121 + 1}} \right|$

$c = -1$

$\therefore \frac{a+b}{c} = \frac{3+1}{-1} = -4$

Question88

The distance of the point $(-1, 2, 3)$ from the plane $\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) = 10$ parallel to the line of the shortest distance between the lines $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})$ is
[13-Apr-2023 shift 1]

Options:

A. $2\sqrt{5}$

B. $3\sqrt{5}$

C. $3\sqrt{6}$

D. $2\sqrt{6}$

Answer: D

Solution:

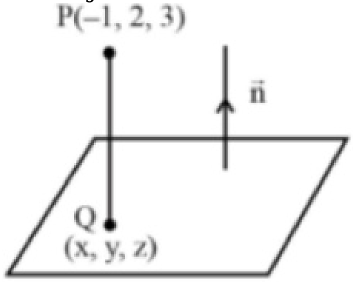
Solution:

Let $L_1 : \vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$

$L_2 : \vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$\vec{n} = \hat{i} - \hat{j} - 2\hat{k}$



Equation of line along shortest distance of L_1 and L_2

$\frac{x+1}{1} = \frac{y-2}{-1} = \frac{z-3}{-2} = r$

$\Rightarrow (x, y, z) \equiv (r-1, 2-r, 3-2r)$

$\Rightarrow (r-1) - 2(2-r) + 3(3-2r) = 10$

$\Rightarrow r = -2$

$\Rightarrow Q(x, y, z) \equiv (-3, 4, 7)$

$\Rightarrow PQ = \sqrt{4+4+16} = 2\sqrt{6}$

Question89

Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point $Q(6, -2, \alpha)$, $\alpha > 0$, from P is 13 , then α is equal to ________.
[13-Apr-2023 shift 1]

Answer: 15

Solution:

Image of point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$

$$\frac{x-\frac{5}{3}}{1} = \frac{y-\frac{5}{3}}{-2} = \frac{z-\frac{8}{3}}{1} = \frac{-2\left(1\times\frac{5}{3} + (-2)\times\frac{8}{3} + 1\times\frac{8}{3} - 2\right)}{1^2 + 2^2 + 1^2}$$

$= \frac{1}{3}$

$\therefore x = 2, y = 1, z = 3$

$13^2 = (6-2)^2 + (-2-1)^2 + (\alpha - 3)^2$

$\Rightarrow (\alpha - 3)^2 = 144 \Rightarrow \alpha = 15 (\because \alpha > 0)$

Question90

The plane, passing through the points $(0, -1, 2)$ and $(-1, 2, 1)$ and parallel to the line passing through $(5, 1, -7)$ and $(1, -1, -1)$, also passes through the point
[13-Apr-2023 shift 2]

Options:

A. $(0, 5, -2)$

B. $(-2, 5, 0)$

C. $(2, 0, 1)$

D. $(1, -2, 1)$

Answer: B

Solution:

Plane passing through $(0, -1, 0)$ and $(-1, 2, 1)$
Then vector in plane $\langle -1, 3, -1\rangle$ vector parallel to plane is $\langle 4, 2, -6\rangle$

Normal vector to plane $(\vec{n}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 4 & 2 & -6 \end{vmatrix}$

$= \hat{i}(16) - \hat{j}(10) + \hat{k}(-14)$
$\vec{n} = \langle 8, 5, 7\rangle$
Equation of plane
$8(x - 0) + 5(y + 1) + 7(z - 2) = 0$
$\Rightarrow 8x + 5y + 7z = 9$
From given options point $(-2, 5, 0)$ lies on plane.

Question91

The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$, is
[13-Apr-2023 shift 2]

Options:

A. $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

B. $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$

C. $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

D. $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

Answer: A

Solution:

Condition of co-planarity

$$\begin{vmatrix} x_2 - x_1 & a_1 & a_2 \\ y_2 - y_1 & b_1 & b_2 \\ z_2 - z_1 & c_1 & c_2 \end{vmatrix} = 0$$

Where a1, b1, c1 are direction cosine of 1^{st} line and a2, b2, c2 are direction cosine of 2^{nd} line. Now. Solving options
Point $(-3, 1, 5)$& point $(-1, 2, 5)$

$$(1)\ \begin{vmatrix} -3 & 1 & 5 \\ 1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$$

$= -3(5) - (10) + 5(-1 + 4)$
$= -15 - 10 + 15 = -10$

(2) point $(-1, 2, 5)$

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$$

$= 3(5) - (10) + 5(1 + 4)$
$- 25 + 5 = 0$

(3) point $(-1, 2, 5)$

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 4 \\ -2 & -1 & 0 \end{vmatrix}$$

$- 3(4) - (8) + 5(1 + 4)$
$- 12 - 8 + 25 = 5$

(4) point $(-1, 2, 5)$

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ 4 & 1 & 0 \end{vmatrix}$$

$- 3(-5) - (-20) + 5(-1 - 8)$
$15 + 20 - 45 = -10$

Question92

Let N be the foot of perpendicular from the point P(1, −2, 3) on the line passing through the points (4, 5, 8) and (1, −7.5). Then the distance of N from the plane 2x − 2y + z + 5 = 0 is [13-Apr-2023 shift 2]

Options:

A. 6

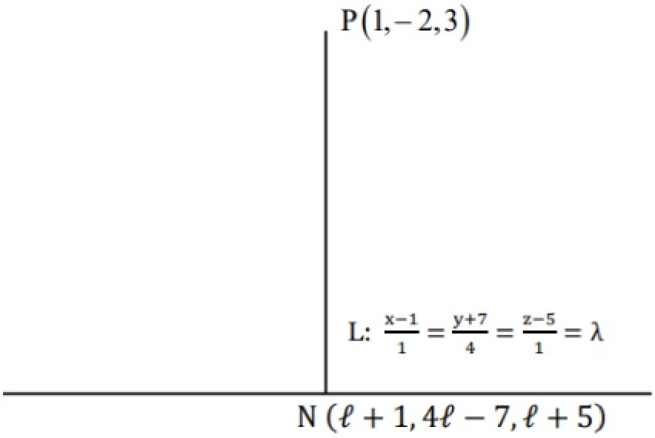
B. 7

C. 9

D. 8

Answer: B

Solution:



$\overrightarrow{PN} = \langle\lambda, 4\lambda-5, \lambda+2\rangle$
$\overrightarrow{PN}. < 1, 4, 1 > = 0$
$\Rightarrow \lambda + 16\lambda - 20 + \lambda + 2 = 0$
$\Rightarrow \lambda = 1$
$N(2, -3, 6)$
Distance of N from $2x - 2y + z + 5 = 0$ is

$$d = \left| \frac{2(2) - 2(-3) + 6 + 5}{\sqrt{2^2 + (-2)^2 + (1)^2}} \right|$$

$$= \left| \frac{21}{3} \right| = 7$$

Question93

Let the foot of perpendicular of the point $P(3, -2, -9)$ on the plane passing through the points $(-1, -2, -3)$, $(9, 3, 4)$, $(9, -2, 1)$ be $Q(\alpha, \beta, \gamma)$. Then the distance of Q from the origin is [15-Apr-2023 shift 1]

Options:

A. $\sqrt{29}$

B. $\sqrt{38}$

C. $\sqrt{42}$

D. $\sqrt{35}$

Answer: C

Solution:

$P(3, -2, -9)$
Equation of plane through A,B,C.

$$\begin{vmatrix} x+1 & y+2 & z+3 \\ 10 & 5 & 7 \\ 10 & 0 & 4 \end{vmatrix} = 0$$

$2x + 3y - 5z - 7 = 0$
Foot of I^r of $P(3, -2, -9)$ is

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -\frac{(6-6+45-7)}{4+9+25}$$

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -1$$

$Q(1,-5,-4) \equiv (\alpha, \beta, \gamma)$
$OQ = \sqrt{\alpha^2+\beta^2+\gamma^2} = \sqrt{42}$

Question94

Let S be the set of all values of λ, for which the shortest distance between the lines $\frac{x-\lambda}{0} = \frac{y-3}{4} = \frac{z+6}{1}$ and $\frac{x+\lambda}{3} = \frac{y}{-4} = \frac{z-6}{0}$ is 13 . Then $8\left|\sum_{\lambda \in S} \lambda\right|$ is equal to
[15-Apr-2023 shift 1]

Options:

A. 302

B. 306

C. 304

D. 308

Answer: B

Solution:

Solution:

$$\text{Short test distance} = \frac{\left|\begin{vmatrix} 0 & 4 & 1 \\ 3 & -4 & 0 \\ 2\lambda & 3 & -12 \end{vmatrix}\right|}{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 1 \\ 3 & -4 & 0 \end{vmatrix}\right|}$$

$13 = \frac{|153+8\lambda|}{|4\hat{i}+3\hat{j}-12\hat{k}|}$
$= \frac{|153+8\lambda|}{13}$
$|153+8\lambda| = 169$
$153+8\lambda = 169, -169$
$\lambda = \frac{16}{8}, \frac{-322}{8}$
$8\left|\sum_{\lambda \in S} \lambda\right| = 306$

Question95

If the line x = y = z intersects the line $x\sin A + y\sin B + z\sin C - 18 = 0 = x\sin 2A + y\sin 2B + z\sin 2C - 9$, where A, B, C are the angles of a triangle ABC, then $80\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$ is equal to ________
[15-Apr-2023 shift 1]

Answer: 5

Solution:

$\sin A + \sin B + \sin C = \frac{18}{x}$

$\sin 2A + \sin 2B + \sin 2C = \frac{9}{x}$

$\therefore \sin A + \sin B + \sin C = 2(\sin 2A + \sin 2B + \sin 2C)$

$4 \cos A / 2 \cos B / 2 \cos C / 2 = 2(4 \sin A \sin B \sin C)$

$16 \sin A / 2 \sin B / 2 \sin C / 2 = 1$

Hence Ans. $= 5$.

Question96

Let the plane P contain the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$ Then the distance of the point A(8, −1, −19) from the plane P measured parallel to the line $\frac{x}{-3} = \frac{y-5}{4} = \frac{2-z}{-12}$ is equal to _______

[15-Apr-2023 shift 1]

Answer: 26

Solution:

Plane $\equiv P_1 = \lambda P_2 = 0$

$(2x + y - z - 3) + \lambda(5x - 3y) + 4z + 9) = 0$

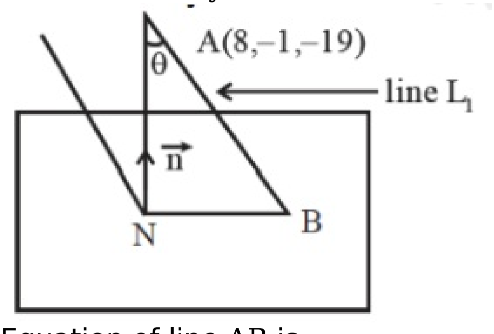
$(5\lambda + 2)x + (1 - 3\lambda)y + (4\lambda - 1)z + 9\lambda - 3 = 0$

$\vec{n} \cdot \vec{b} = 0$ where $\vec{b}(2, 4, 5)$

$2(5\lambda + 2) + 4(1 - 3\lambda) + 5(4\lambda - 1) = 0$

$\lambda = -\frac{1}{6}$

Plane $7x + 9y - 10z - 27 = 0$



Equation of line AB is

$\frac{x-8}{-3} = \frac{y+1}{4} = \frac{z+19}{12} = \lambda$

Let $B = (8 - 3\lambda, -1 + 4\lambda, -19 + 12\lambda)$ lies on plane P

$\therefore 7(8 - 3\lambda) + 9(4\lambda - 1) - 10(12\lambda - 19) = 27$

$\lambda = 2$

$\therefore$ Point $B = (2, 7, 5)$

AB = $\sqrt{6^2 + 8^2 + 24^2} = 26$

Question97

If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{\lambda}$ and $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible value of λ is :
[24-Jun-2022-Shift-2]

Options:

A. 16

B. 6

C. 12

D. 15

Answer: A

Solution:

Let $\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a_2} = 2\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{p} = 2\hat{i} + 3\hat{j} + \lambda\hat{k}$, $\vec{q} = \hat{i} + 4\hat{j} + 5\hat{k}$

$\therefore \vec{p} \times \vec{q} = (15 - 4\lambda)\hat{i} - (10 - \lambda)\hat{j} + 5\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$

∴ Shortest distance

$= \left| \frac{(15 - 4\lambda) - 2(10 - \lambda) + 10}{\sqrt{(15 - 4\lambda)^2 + (10 - \lambda)^2 + 25}} \right| = \frac{1}{\sqrt{3}}$

$\Rightarrow 3(5 - 2\lambda)^2 = (15 - 4\lambda)^2 + (10 - \lambda)^2 + 25$

$\Rightarrow 5\lambda^2 - 80\lambda + 275 = 0$

∴ Sum of values of $\lambda = \frac{80}{5} = 16$

Question98

Let the points on the plane P be equidistant from the points (−4, 2, 1) and (2, −2, 3). Then the acute angle between the plane P and the plane 2x + y + 3z = 1 is
[24-Jun-2022-Shift-2]

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{5\pi}{12}$

Answer: C

Solution:

Let $P(x, y, z)$ be any point on plane P_1

Then $(x-4)^2-(y-2)^2+(z-1)^2=(x-2)^2+(y+2)^2+(z-3)^2$

$\Rightarrow 12x-8y+4z+4=0$

$\Rightarrow 3x-2y+z+1=0$

And $P_2 : 2x+y-3z=0$

$\therefore$ angle between P_1 and P_2

$\cos\theta \left| \frac{6-2+3}{14} \right| \Rightarrow \theta = \frac{\pi}{3}$

Question99

Let Q be the mirror image of the point P(1, 0, 1) with respect to the plane S : x + y + z = 5. If a line L passing through (1, −1, −1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to :
[25-Jun-2022-Shift-1]

Options:

A. 2

B. 5

C. 7

D. 11

Answer: B

Solution:

Solution:

As L is parallel to PQ d.r.s of S is $(1, 1, 1)$

$\therefore L \equiv \frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{1}$

Point of intersection of L and S be λ

$\Rightarrow (\lambda+1)+(\lambda-1)+(\lambda-1) = S$

$\Rightarrow \lambda = 2$

$\therefore R \equiv (3, 1, 1)$

Let $Q(\alpha, \beta, \gamma)$

$\Rightarrow \frac{\alpha-1}{1} = \frac{\beta}{1} = \frac{\gamma-1}{1} = \frac{-2(-3)}{3}$

$\Rightarrow \alpha = 3, \beta = 2, \gamma = 3$

$\Rightarrow Q \equiv (3, 2, 3)$

$(QR)^2 = 0^2 + (1)^2 + (2)^2 = 5$

Question100

Let the lines

$L_1 : \vec{r} = \lambda\left(\hat{i} + 2\hat{j} + 3\,\text{widehat}\,k\right), \lambda \in R$

$L_2 : \vec{r} = \left(\hat{i} + 3\hat{j} + \text{widehat}\,k\right) + \mu\left(\hat{i} + \hat{j} + 5\,\text{widehat}\,k\right); \mu \in R$

intersect at the point S. If a plane $ax + by - z + d = 0$ passes through S and is parallel to both the lines L_1 and L_2, then the value of $a + b + d$ is equal to

[25-Jun-2022-Shift-1]

Answer: 5

Solution:

As plane is parallel to both the lines we have d.r's of normal to the plane as $<7, -2, -1>$

$$\left(\text{from} \begin{vmatrix} \hat{i} & \hat{j} & \text{widehat}\,k \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix} = 7\hat{i} - \hat{j}(2) + \text{widehat}\,k(-1)\right)$$

Also point of intersection of lines is $2\hat{i} + 4\hat{j} + 6\hat{k}$

$\therefore$ Equation of plane is

$7(x-2) - 2(y-4) - 1(z-6) = 0$

$\Rightarrow 7x - 2y - z = 0$

$a + b + d = 7 - 2 + 0 = 5$

Question101

Let p be the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 5$ and $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 3$, and the point $(2, 1, -2)$. Let the position vectors of the points X and Y be $\hat{i} - 2\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 2\hat{k}$ respectively. Then the points
[25-Jun-2022-Shift-2]

Options:

A. X and $X + Y$ are on the same side of P

B. Y and $Y - X$ are on the opposite sides of P

C. X and Y are on the opposite sides of P

D. $X + Y$ and $X - Y$ are on the same side of P

Answer: C

Solution:

Solution:
Let the equation of required plane
$\pi : (x + 3y - z - 5) + \lambda(2x - y + z - 3) = 0$
$\because (2, 1, -2)$ lies on it so, $2 + \lambda(-2) = 0$
$\Rightarrow \lambda = 1$
Hence, $\pi : 3x + 2y - 8 = 0$
$\because \pi_x = -9$, $\pi_y = 5$, $\pi_{x+y} = 4$
$\pi_{x-y} = -22$ and $\pi_{y-x} = 6$
Clearly X and Y are on opposite sides of plane π

Question102

Let I_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and I_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line I_1 and I_2, then d^{-2} is equal to____
[25-Jun-2022-Shift-2]

Answer: 51

Solution:

Solution:

$$\frac{x - \frac{1}{8}}{\frac{1}{8}} = \frac{y}{-\frac{1}{4\sqrt{2}}} = \frac{z}{0} \quad L_1$$

or $\frac{x-\frac{1}{8}}{1}=\frac{y}{-\sqrt{2}}=\frac{z}{0}$...... (i)

Equation of L_2

$\frac{x+\frac{1}{8}}{-6\sqrt{3}}=\frac{y}{0}=\frac{z}{8}$...... (ii)

$$d = \left| \frac{(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})}{|\vec{b}\times\vec{d}|} \right|$$

$$= \frac{\left(\frac{1}{4}\hat{i}\right)\cdot\left(4\sqrt{2}\hat{i}+4\hat{j}+3\sqrt{6}\hat{k}\right)}{\sqrt{(4\sqrt{2})^2+4^2+(3\sqrt{6})^2}}$$

$$= \frac{\sqrt{2}}{\sqrt{32+16+54}} = \frac{1}{\sqrt{51}}$$

$d^{-2}=51$

Question103

If the two lines l_1 : $\frac{x-2}{3}=\frac{y+1}{-2}$, z = 2 and l_2 : $\frac{x-1}{1}=\frac{2y+3}{\alpha}=\frac{z+5}{2}$ are perpendicular, then an angle between the lines I_2 and l_3 : $\frac{1-x}{3}=\frac{2y-1}{-4}=\frac{z}{4}$ is :
[26-Jun-2022-Shift-1]

Options:

A. $\cos^{-1}\left(\frac{29}{4}\right)$

B. $\sec^{-1}\left(\frac{29}{4}\right)$

C. $\cos^{-1}\left(\frac{2}{29}\right)$

D. $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Answer: B

Solution:

$\because L_1$ and L_2 are perpendicular, so

$3\times1+(-2)\left(\frac{\alpha}{2}\right)+0\times2=0$

$\Rightarrow \alpha=3$

Now angle between I_2 and I_3,

$$\cos\theta = \frac{1(-3)+\frac{\alpha}{2}(-2)+2(4)}{\sqrt{1-\frac{\alpha^2}{4}+4}\sqrt{9+4+16}}$$

$\Rightarrow \cos\theta=\frac{2}{\frac{29}{2}} \Rightarrow \theta=\cos^{-1}\left(\frac{4}{29}\right)=\sec^{-1}\left(\frac{29}{4}\right)$

Question104

Let the plane 2x + 3y + z + 20 = 0 be rotated through a right angle about its line of intersection with the plane x − 3y + 5z = 8. If the mirror image of the point $\left(2, -\frac{1}{2}, 2\right)$ in the rotated plane is B(a, b, c), then : [26-Jun-2022-Shift-1]

Options:

A. $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$

B. $\frac{a}{4} = \frac{b}{5} = \frac{c}{-2}$

C. $\frac{a}{8} = \frac{b}{-5} = \frac{c}{4}$

D. $\frac{a}{4} = \frac{b}{5} = \frac{c}{2}$

Answer: A

Solution:

Consider the equation of plane,

$P : (2x + 3y + z + 20) + \lambda(x - 3y - 5z - 8) = 0$

$P : (2 + \lambda)x + 3(3 - 3\lambda)y - 1(1 + 5\lambda)z + (20 - 8\lambda) = 0$

$\because$ Plane P is perpendicular to $2x - 3y + z + 20 = 0$

So, $4 - 2\lambda + 9 - 9\lambda + 1 + 5\lambda = 0$

$\Rightarrow \lambda = 7$

$P : 9x - 18y + 36z - 36 = 0$

or $P : x - 2y + 4z = 4$

If image of $\left(2, -\frac{1}{2}, 2\right)$ in plane P is (a, b, c) then

$$\frac{a-2}{1} = \frac{b+\frac{1}{2}}{-2} = \frac{c-2}{4}$$

and $\left(\frac{a+2}{2}\right) - 2\left(\frac{b-\frac{1}{2}}{2}\right) - 4\left(\frac{c-2}{2}\right) = 4$

Clearly $a = \frac{4}{3}, b = \frac{5}{6}$ and $c = -\frac{2}{3}$

So, $a : b : c = 8 : 5 : -4$

Question105

If the plane 2x + y − 5z = 0 is rotated about its line of intersection with the plane 3x − y + 4z − 7 = 0 by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point :
[26-Jun-2022-Shift-2]

Options:

A. (2, −2, 0)

B. (−2, 2, 0)

C. (1, 0, 2)

D. (−1, 0, −2)

Answer: C

Solution:

$P_1 : 2x + y - 52 = 0, P_2 : 3x - y + 4z - 7 = 0$

Family of planes P_1 and P_2

$P : P_1 + \lambda P_2$

$\therefore P : (2 + 3\lambda)x + (1 - \lambda)y + (-5 + 4\lambda)z - 7\lambda = 0$

$\because P \perp P_1$

$\therefore 4 + 6\lambda + 1 - \lambda + 25 - 20\lambda = 0$

$\lambda = 2$

$\therefore P : 8x - y + 32 - 14 = 0$

It passes through the point (1, 0, 2)

Question106

If the lines $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{j} - \hat{k})$ and $\vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{i} - 3\hat{k})$ are co-planar, then the distance of the plane containing these two lines from the point (α, 0, 0) is :
[26-Jun-2022-Shift-2]

Options:

A. $\frac{2}{9}$

B. $\frac{2}{11}$

C. $\frac{4}{11}$

D. 2

Answer: B

Solution:

Solution:

∵ Both lines are coplanar, so

$$\begin{vmatrix} \alpha-1 & 0 & -1 \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix} =$$

$$\Rightarrow \alpha = \frac{5}{3}$$

Equation of plane containing both lines

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix} = 0$$

$$\Rightarrow 9x + 2y + 6z = 13$$

So, distance of $\left(\frac{5}{3}, 0, 0\right)$ from this plane

$$= \frac{2}{\sqrt{81+4+36}} = \frac{2}{11}$$

Question107

If two straight lines whose direction cosines are given by the relations $l + m - n = 0$, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is :
[27-Jun-2022-Shift-1]

Options:

A. 6

B. 4

C. 3

D. 2

Answer: A

Solution:

$l + m - n = 0 \Rightarrow n = l + m$
$3l^2 + m^2 + cnl = 0$
$3l^2 + m^2 + cl(l + m) = 0$
$= (3 + c)l^2 + cl\,m + m^2 = 0$
$= (3 + c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0$
$\therefore$ Lines are parallel
$D = 0$
$c^2 - 4(3 + c) = 0$
$c^2 - 4c - 12 = 0$
$(c - 4)(c + 3) = 0$
$c = 4(\text{ as } c > 0)$

Question108

Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be $(a - 6, \beta, \gamma)$. If $a + b + c = 5$, then $7\beta - 9\gamma$ is equal to
[27-Jun-2022-Shift-1]

Answer: 137

Solution:

$\frac{x-a}{3} = \frac{y-b}{-4} = \frac{z-c}{12} = \frac{-2(3a-4b+12c+19)}{3^2+(-4)^2+12^2}$

$\frac{x-a}{3} = \frac{y-b}{-4} = \frac{z-c}{12} = \frac{-6a+8b-24c-38}{169}$

$(x, y, z) \equiv (a-6, \beta, \gamma)$

$\frac{(a-6)-a}{3} = \frac{\beta-b}{-4} = \frac{\gamma-c}{12} = \frac{-6a+8b-24c-38}{169}$

$\frac{\beta-b}{-4} = -2 \Rightarrow \beta = 8+b$

$\frac{\gamma-c}{12} = -2 \Rightarrow \gamma = -24+c$

$\frac{-6a+8b-24c-38}{169} = -2$

$\Rightarrow 3a-4b+12c = 150$

$a+b+c = 5$

$3a+3b+3c = 15$

Applying (1) - (2)

$-7b+9c = 135$

$7b-9c = -135$

$7\beta-9\gamma = 7(8+b)-9(-24+c)$

$= 56+216+7b-9c$

$= 56+216-135 = 137$

Question109

Let the foot of the perpendicular from the point (1, 2, 4) on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P. Then the distance of P from the plane $3x+4y+12z+23=0$ is
[27-Jun-2022-Shift-2]

Options:

A. 5

B. $\frac{50}{13}$

C. 4

D. $\frac{63}{13}$

Answer: A

Solution:

$L: \frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = t$

Let $P = (4t-2, 2t+1, 3t-1)$

$\because$P is the foot of perpendicular of $(1, 2, 4)$

$\therefore 4(4t-3)+2(2t-1)+3(3t-5) = 0$

$\Rightarrow 29t = 29 \Rightarrow t = 1$

$\therefore P = (2, 3, 2)$

Now, distance of P from the plane

$3x+4y+12z+23 = 0$, is

$\left| \frac{6+12+24+23}{\sqrt{9+16+144}} \right| = \frac{65}{13} = 5$

Question110

The shortest distance between the lines $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$ and $\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$, is
[27-Jun-2022-Shift-2]

Options:

A. $\frac{18}{\sqrt{5}}$

B. $\frac{22}{3\sqrt{5}}$

C. $\frac{46}{3\sqrt{5}}$

D. $6\sqrt{3}$

Answer: A

Solution:

Solution:

$L_1 : \frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$

$L_2 : \frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$

Now, $\vec{p} \times \vec{q} = \mid$begin array ccc $\hat{i}\ \hat{j}\ \hat{k}$
2 3 −1
2 1 3 end array $\cdot \Big| = 10\hat{i} - 8\hat{j} - 4\hat{k}$

and $\vec{a}_2 - \vec{a}_1 = 6\hat{i} - 4\hat{j} - 4\hat{k}$

$\therefore S.D. = \left| \frac{60+32+16}{\sqrt{100+64+16}} \right| = \frac{108}{\sqrt{180}} = \frac{18}{\sqrt{5}}$

Question111

If two distinct point Q, R lie on the line of intersection of the planes $-x + 2y - z = 0$ and $3x - 5y + 2z = 0$ and $PQ = PR = \sqrt{18}$ where the point P is $(1, -2, 3)$, then the area of the triangle PQR is equal to
[28-Jun-2022-Shift-1]

Options:

A. $\frac{2}{3}\sqrt{38}$

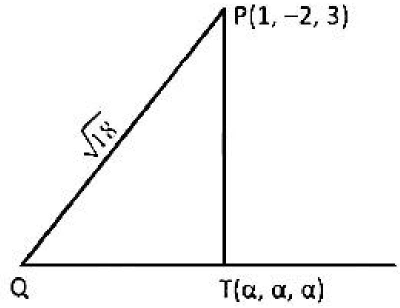
B. $\frac{4}{3}\sqrt{38}$

C. $\frac{8}{3}\sqrt{38}$

D. $\sqrt{\frac{152}{3}}$

Answer: B

Solution:



Line L is $x = y = z$

$\overrightarrow{PQ} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 0$

$\Rightarrow (\alpha - 3) + \alpha + 2 + \alpha - 1 = 0$

$\Rightarrow \alpha = \frac{2}{3}$ so, $T = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$PT = \sqrt{\frac{38}{3}}$

$\Rightarrow QT = \frac{4}{\sqrt{3}}$

So, Area $= \left(\frac{1}{2} \times \frac{4}{\sqrt{3}} \times \frac{\sqrt{38}}{\sqrt{3}}\right) \cdot 2$

$= \frac{4\sqrt{38}}{3}$ sq. units

Question112

The acute angle between the planes P_1 and P_2, when P_1 and P_2 are the planes passing through the intersection of the planes $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the points (2, 1, 3) and (0, 1, 2), respectively, is
[28-Jun-2022-Shift-1]

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{12}$

Answer: A

Solution:

Family of Plane's equation can be given by

$(5 + 8\lambda)x + (8 - 7\lambda)y + (13 + \lambda)z - (29 + 20\lambda) = 0$

P_1 passes through $(2, 1, 3)$

$\Rightarrow(10+16\lambda)+(8-7\lambda)+(39+3\lambda)-(29+20\lambda)=0$
$\Rightarrow -8\lambda+28=0 \Rightarrow \lambda=\frac{7}{2}$
d.r, s of normal to P_1
$\left\langle 33, \frac{-33}{2}, \frac{33}{2}\right\rangle$ or $\left\langle 1, -\frac{1}{2}, \frac{1}{2}\right\rangle$
P_2 passes through $(0, 1, 2)$
$\Rightarrow 8-7\lambda+26+2\lambda-(29+20\lambda)=0$
$\Rightarrow 5-25\lambda=0$
$\Rightarrow \lambda=\frac{1}{5}$
d.r, s of normal to P_2
$\left\langle \frac{33}{5}, \frac{33}{5}, \frac{66}{5}\right\rangle$ or $\langle 1, 1, 2\rangle$
Angle between normals

$$=\frac{\left(\hat{i}-\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}\right)\cdot\left(\hat{i}+\hat{j}+2\hat{k}\right)}{\frac{\sqrt{3}}{2}}$$

$$\cos\theta=\frac{1-\frac{1}{2}+1}{3}=\frac{1}{2}$$

$$\theta=\frac{\pi}{3}$$

Question113

Let the plane P : $\vec{r}\cdot\vec{a}=d$ contain the line of intersection of two planes $\vec{r}\cdot\left(\hat{i}+3\hat{j}-\hat{k}\right)=6$ and $\vec{r}\cdot\left(-6\hat{i}+5\hat{j}-\hat{k}\right)=7$ If the plane P passes through the point $\left(2, 3, \frac{1}{2}\right)$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to
[28-Jun-2022-Shift-1]

Options:

A. 90

B. 93

C. 95

D. 97

Answer: B

Solution:

$P_1: x+3y-z=6$
$P_2: -6x+5y-z=7$
Family of planes passing through line of intersection of P_1 and P_2 is given by
$x(1-6\lambda)+y(3+5\lambda)+z(-1-\lambda)-(6+7\lambda)=0$
It passes through $\left(2, 3, \frac{1}{2}\right)$
So, $2(1-6\lambda)+3(3+5\lambda)+\frac{1}{2}(-1-\lambda)-(6+7\lambda)=0$
$\Rightarrow 2-12\lambda+9+15\lambda-\frac{1}{2}-\frac{\lambda}{2}-6-7\lambda=0$
$\Rightarrow \frac{9}{2}-\frac{9\lambda}{2}=0 \Rightarrow \lambda=1$

Required plane is
$-5x + 8y - 2z - 13 = 0$

Or $\vec{r} \cdot \left(-5\hat{i} + 8\hat{j} - 2\hat{k}\right) = 13$

$\frac{|13\vec{a}|^2}{|d|^2} = \frac{13^2}{(13)^2} \cdot \left|\vec{a}\right|^2 = 93$

Question114

Let the plane $ax + by + cz = d$ pass through $(2, 3, -5)$ and is perpendicular to the planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$. If a, b, c, d are integers $d > 0$ and $\gcd(|a|, |b|, |c|, d) = 1$, then the value of $a + 7b + c + 20d$ is equal to :
[28-Jun-2022-Shift-2]

Options:

A. 18

B. 20

C. 24

D. 22

Answer: D

Solution:

Equation of pane through point $(2, 3, -5)$ and perpendicular to planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$ is

$$\begin{vmatrix} x-2 & y-3 & z+5 \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 0$$

∴ Equation of plane is $(x - 2)(-7 + 25) - (y - 3)$
$(-14 + 15) + (z + 5) \cdot 7 = 0$
$\therefore 18x - y + 7z + 2 = 0$
$\Rightarrow 18x - y + 7z = -2$
$\therefore -18x + y - 7z = 2$
On comparing with $ax + by + cz = d$ where $d > 0$ is $a = -18, b = 1, c = -7, d = 2$
$\therefore a + 7b + c + 20d = 22$

Question115

Let the image of the point P(1, 2, 3) in the line L : $\frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be Q. Let R(α, β, γ) be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of 22(α + β + γ) is equal to___
[28-Jun-2022-Shift-2]

Options:

Answer: 125

Solution:

The point dividing PQ in the ratio 1 : 3 will be mid-point of P& foot of perpendicular from P on the line.
∴ Let a point on line be λ
$\Rightarrow \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$
$\Rightarrow P'(3\lambda + 6, 2\lambda + 1, 3\lambda + 2)$
as P' is foot of perpendicular
$(3\lambda + 5)3 + (2\lambda - 1)2 + (3\lambda - 1)3 = 0$
$\Rightarrow 22\lambda + 15 - 2 - 3 = 0$
$\Rightarrow \lambda = \frac{-5}{11}$
$\therefore P'\left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11}\right)$
$$\text{Mid-point of } PP' \equiv \left(\frac{\frac{51}{11}+1}{2}, \frac{\frac{1}{11}+2}{2}, \frac{\frac{7}{11}+3}{2}\right)$$
$\equiv\left(\frac{62}{22}, \frac{23}{22}, \frac{40}{22}\right) \equiv (\alpha, \beta, \gamma)$
$\Rightarrow 22(\alpha, \beta, \gamma) = 62 + 23 + 40 = 125$

Question116

If the mirror image of the point (2, 4, 7) in the plane 3x − y + 4z = 2 is (a, b, c), then 2a + b + 2c is equal to: [29-Jun-2022-Shift-1]

Options:

A. 54

B. 50

C. −6

D. −42

Answer: C

Solution:

Solution:
We know mirror image of point (x_1, y_1, z_1) in the plane $ax + by + cz = d$
$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \frac{-2(ax_1 + by_1 + cz_1 - d)}{a^2 + b^2 + c^2}$$
Here given point (2, 4, 7) and plane $3x - y + 4z = 2$ then mirror image is
$$\frac{x-2}{3} = \frac{y-4}{-1} = \frac{z-7}{4} = \frac{-2(6-4+28-2)}{9+1+16}$$
$\Rightarrow \frac{x-2}{3} = \frac{y-4}{-1} = \frac{z-7}{4} = -\frac{28}{13}$
$\therefore x = -\frac{58}{13} = a$
$y = \frac{80}{13} = b$
$z = -\frac{21}{13} = c$
$\therefore 2a + b + 2c$

$= 2\left(-\frac{58}{13}\right) + \frac{80}{13} + 2\left(-\frac{21}{13}\right)$
$= \frac{-116+80-42}{13} = \frac{-78}{13} = -6$

Question117

Let d be the distance between the foot of perpendiculars of the points P(1, 2, −1) and Q(2, −1, 3) on the plane −x + y + z = 1. Then d^2 is equal to___
[29-Jun-2022-Shift-1]

Answer: 26

Solution:

Foot of perpendicular from P
$\frac{x-1}{-1} = \frac{y-2}{1} = \frac{z+1}{1} = \frac{-(-1+2-1-1)}{3}$
$\Rightarrow p' \equiv \left(\frac{2}{3}, \frac{7}{3}, \frac{-2}{3}\right)$
and foot of perpendicular from Q
$\frac{x-2}{-1} = \frac{y+1}{1} = \frac{z-3}{1} = \frac{-(-2-1+3-1)}{3}$
$\Rightarrow Q' \equiv \left(\frac{5}{3}, \frac{-2}{3}, \frac{10}{3}\right)$
$P'Q' = \sqrt{(1)^2+(3)^2+(4)^2} = d = \sqrt{26}$
$\Rightarrow d^2 = 26$

Question118

Let $P_1 : \vec{r} \cdot \left(2\hat{i} + \hat{j} - 3\hat{k}\right) = 4$ be a plane. Let P_2 be another plane which passes through the points (2, −3, 2), (2, −2, −3) and (1, −4, 2). If the direction ratios of the line of intersection of P_1 and P_2 be 16, α, β, then the value of α + β is equal to____
[29-Jun-2022-Shift-1]

Answer: 28

Solution:

Direction ratio of normal to $P_1 \equiv < 2, 1, -3 >$

and that of $P_2 \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -5 \\ -1 & -2 & 5 \end{vmatrix} = -5\hat{i} - \hat{j}(-5) + \hat{k}(1)$

i.e. $\langle -5, 5, 1 \rangle$

d.r's of line of intersection are along vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ -5 & 5 & 1 \end{vmatrix} = \hat{i}(16) - \hat{j}(-13) + \hat{k}(15)$$

i.e. $< 16, 13, 15 >$

$\therefore \alpha + \beta = 13 + 15 = 28$

Question119

Let $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z+3}{-1}$ lie on the plane $px - qy + z = 5$, for some $p, q \in R$. The shortest distance of the plane from the origin is :
[29-Jun-2022-Shift-2]

Options:

A. $\sqrt{\frac{3}{109}}$

B. $\sqrt{\frac{5}{142}}$

C. $\frac{5}{\sqrt{71}}$

D. $\frac{1}{\sqrt{142}}$

Answer: B

Solution:

Solution:

$(2, -1, -3)$ satisfy the given plane.

So $2p + q = 8$

Also given line is perpendicular to normal plane so $3p + 2q - 1 = 0$

$\Rightarrow p = 15, q = -22$

Eq. of plane $15x - 22y + z - 5 = 0$

its distance from origin $= \frac{6}{\sqrt{710}} = \sqrt{\frac{5}{142}}$

Question120

Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane $x + 2y + 2z = 16$. Let T be a plane passing through the point Q and contains the line $\vec{r} = -\hat{k} + \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right)$, $\lambda \in R$. Then, which of the following points lies on T ?
[29-Jun-2022-Shift-2]

Options:

A. (2, 1, 0)

B. (1, 2, 1)

C. (1, 2, 2)

D. (1, 3, 2)

Answer: B

Solution:

Solution:
Image of P(1, 2, 1) in $x + 2y + 2z - 16 = 0$

is given by Q(4, 8, 7)

Eq. of plane T = $\begin{vmatrix} x & y & z+1 \\ 4 & 8 & 6 \\ 1 & 1 & 2 \end{vmatrix} = 0$

$\Rightarrow 2x - z = 1$ so B(1, 2, 1) lies on it.

Question121

Let a line having direction ratios 1, −4, 2 intersect the lines $\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ and $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ at the points A and B. Then $(AB)^2$ is equal to__ [24-Jun-2022-Shift-1]

Answer: 84

Solution:

Let $A(3\lambda + 7, -\lambda + 1, \lambda - 2)$ and $B(2\mu, 3\mu + 7, \mu)$

So, DR's of $AB \propto 3\lambda - 2\mu + 7, -(\lambda + 3\mu + 6), \lambda - \mu - 2$

Clearly $\frac{3\lambda - 2\mu + 7}{1} = \frac{\lambda + 3\mu + 6}{4} = \frac{\lambda - \mu - 2}{2}$

$\Rightarrow 5\lambda - 3\mu = -16$

And $\lambda - 5\mu = 10$

From (i) and (ii) we get $\lambda = -5$, $\mu = -3$

So, A is $(-8, 6, -7)$ and B is $(-6, -2, -3)$

$AB = \sqrt{4 + 64 + 16} \Rightarrow (AB)^2 = 84$

Question122

Let α be the angle between the lines whose direction cosines satisfy the equations $I + m - n = 0$ and $I^2 + m^2 - n^2 = 0$. Then, the value of $\sin^4\alpha + \cos^4\alpha$ is
[25 Feb 2021 Shift 1]

Options:

A. $\frac{3}{4}$

B. $\frac{3}{8}$

C. $\frac{5}{8}$

D. $\frac{1}{2}$

Answer: C

Solution:

Given, $l + m - n = 0 \ldots$ (i)
and $I^2 + m^2 - n^2 = 0 \ \ldots$ (ii)
On squaring Eq. (i), we get
$(l + m)^2 = n^2$
$\Rightarrow l^2 + m^2 + 2lm = n^2 \ldots$ (iii)
From Eqs. (ii) and (iii),

$$\begin{array}{r} I^2 + m^2 - n^2 = 0 \\ I^2 + m^2 + 2lm = n^2 \\ - \ - \quad - \quad - \\ \hline -n^2 - 2lm = -n^2 \end{array}$$

$\Rightarrow 2lm = 0 \Rightarrow Im = 0$
$\Rightarrow I = 0$ or $m = 0$
Case I When $I = 0$
$\Rightarrow 0 + m - n = 0$
$\Rightarrow m = n$
and $I^2 + m^2 + n^2 = 1$
$\Rightarrow m^2 + m^2 = 1$ [$\because n = m$ and $l = 0$]
$\Rightarrow m^2 = \frac{1}{2}$
$m = \pm\frac{1}{\sqrt{2}} = n$
$\therefore (I, m, n) = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$
Case II When $m = 0$
then, $l + m - n = 0$
$\Rightarrow I = n$ and $l^2 + m^2 + n^2 = 1$
[$\because n = I$ and $m = 0$]
$\Rightarrow I^2 + 0 + I^2 = 1$
$I = \pm\frac{1}{\sqrt{2}}$ [$\because n = l$ and $m = 0$]
$\Rightarrow \therefore (I, m, n) = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ or $\left(\frac{-1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$
$\Rightarrow a = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $b = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$
Then $\cos\alpha = \frac{a \cdot b}{|a|\,|b|} = \frac{1}{2}$
$\therefore \sin\alpha = \pm\frac{\sqrt{3}}{2}$

Now, $\cos^4\alpha + \sin^4\alpha = \frac{1}{16} + \frac{9}{16} = \frac{10}{16} = \frac{5}{8}$

Question123

A line ' l ' passing through origin is perpendicular to the lines

$l_1 : r = (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}$

$l_2 : r(3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}$

If the coordinates of the point in the first octant on ' I_2 ' at a distance of $\sqrt{17}$ from the point of intersection of ' l ' and ' l_1 ' are (a, b, c), then 18(a + b + c) is equal to ....
[25 Feb 2021 Shift 2]

Answer: 44

Solution:

Let $L_1 \Rightarrow \frac{x-3}{1} = \frac{y-3}{2} = \frac{z-4}{2} = u$ (say)
$\Rightarrow$ Direction ratios of $L_1 = 1, 2, 2$
$L_2 \Rightarrow \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} = v$ (say)
Direction ratios of $L_2 = 2, 2, 1$
Line L passing through origin is perpendicular to L_1 and L_2.
Hence, direction ratios of L is parallel to $(L_1 \times L_2)$.
$\Rightarrow (-2, 3, -2)$
Equation of $L \Rightarrow \frac{x}{2} = \frac{y}{-3} = \frac{z}{2} = \lambda$ (say)
Solve L and L_1, we get
$(2\lambda, -3\lambda, 2\lambda) = (\mu + 3, 2\mu - 1, 2\mu + 4)$
Gives, $\lambda = 1, \mu = -1$
So, intersection point $P(2, -3, 2)$.
Let $Q(2v + 3, 2v + 3, v + 2)$ be required point on L_2.
Now, $PQ = \sqrt{17}$ (given)
Now, $PQ = \sqrt{17}$ (given)
$PQ = \sqrt{(2v+1)^2 + (2v+6)^2 + (v)^2}$
$= \sqrt{17}$
$\Rightarrow (2v+1)^2 + (2v+6)^2 + v^2 = 17$ (squaring on both sides)
$\Rightarrow 9v^2 + 28v + 20 = 0$
On solving, we get $v = -2$ (rejected), $\frac{-10}{9}$ (accepted)
$\therefore Q$ is $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$
$\therefore 18(a + b + c) = 18\left(\frac{7}{9} + \frac{7}{9} + \frac{8}{9}\right) = 44$

Question124

The equation of the line through the point (0, 1, 2) and perpendicular to

the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is
[25 Feb 2021 Shift 1]

Options:

A. $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

B. $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

C. $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

D. $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

Answer: D

Solution:

Solution:
Given, line $\Rightarrow \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = \lambda$ (let)
Any point on this line is $B(2\lambda + 1, 3\lambda - 1, -2\lambda + 1)$ and direction ratios of this line $= (2, 3, -2) = d_1$
Let given point be $A(0, 1, 2)$.
Then direction ratio of
$AB = (2\lambda + 1, 3\lambda - 2, -2\lambda - 1) = d_2$
$\because$ Both lines are perpendicular to each other.
$\therefore\ d_1 \cdot d_2 = 0$
$2(2\lambda + 1) + 3(3\lambda - 2) - 2(-2\lambda - 1) = 0$
$\Rightarrow\ 4\lambda + 2 + 9\lambda - 6 + 4\lambda + 2 = 0$
$\Rightarrow\ 17\lambda = 2$
$\lambda\ \ \lambda = 2 / 17$
$\therefore$ Direction ratio of required line $d_2 = (21, -28, -21)$
$= (3, -4, -3) = (-3, 4, 3)$
This line passes through $A(0, 1, 2)$.
$\therefore$ Required equation of line $\Rightarrow \frac{x-0}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

Question125

Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is ....
[24 Feb 2021 Shift 2]

Answer: 1

Solution:

Given, equation of line $\Rightarrow x - \lambda = 2y - 1 =$

$\Rightarrow \quad \frac{x-\lambda}{1} = \frac{y-1/2}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$

or $\quad \frac{x-\lambda}{2} = \frac{y-1/2}{1} = \frac{z}{-1}$

Point on this line through which it passes is $\left(\lambda, \frac{1}{2}, 0\right)$.

Equation of another line $\Rightarrow x = y + 2\lambda = z - \lambda$

$\Rightarrow \frac{x}{1} = \frac{y-(-2\lambda)}{1} = \frac{z-\lambda}{1}$ ... (ii)

A point through which this line passes is $(0, -2\lambda, \lambda)$.

Now, distance between two skew lines

$= \frac{|(a_2 - a_1)\cdot(b_1 \times b_2)|}{|b_1 \times b_2|}$

According to the question, $\frac{\left|-5\lambda - \frac{3}{2}\right|}{\sqrt{14}} = \frac{\sqrt{7}}{2\sqrt{2}}$

$\Rightarrow \; |10\lambda + 3| \; = 7$

$10\lambda + 3 \; = \pm 7$

$\Rightarrow \; 10\lambda \; = 4, -10$

$\Rightarrow \; \lambda \; = \frac{2}{5}$ and $\lambda = -1$

$\therefore \; \lambda = -1$

$(\; \lambda = \frac{2}{5}$ is not possible as λ is an integer)

$\therefore$

Hence, $|\lambda| = |-1| = 1$

Question126

Let a, b ∈ R. If the mirror image of the point P(a, 6, 9) with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is (20, b, −a − 9), then |a + b| is equal to [24 Feb 2021 Shift 2]

Options:

A. 88

B. 86

C. 84

D. 90

Answer: A

Solution:

Solution:

Given, P(a, 6, 9)

Equation of line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$

Image of point P with respect to line is point $Q(20, b, -a-9)$

Mid-point of P and Q $= \left(\frac{a+20}{2}, \frac{6+b}{2}, \frac{-a}{2}\right)$

This point lies on line

$\therefore \quad \frac{\frac{a+20}{2} - 3}{7} = \frac{\frac{6+b}{2} - 2}{5} = \frac{\frac{-a}{2} - 1}{-9}$

$\Rightarrow \frac{a+14}{14} = \frac{b+2}{10} = \frac{a+2}{18}$

$\Rightarrow \quad \frac{a+14}{14} = \frac{a+2}{18}$ and $\frac{b+2}{10} = \frac{a+2}{18}$
Solving, we get $a = -56, b = -32$
$\therefore \quad | a + b | = | -56 - 32 | = 88$

Question127

Consider the three planes $P_1 : 3x + 15y + 21z = 9$, $P_2 : x - 3y - z = 5$ and $P_3 : 2x + 10y + 14z = 5$ Then, which one of the following is true? [26 Feb 2021 Shift 1]

Options:

A. P_1 and P_2 are parallel

B. P_1 and P_3 are parallel

C. P_2 and P_3 are parallel

D. P_1, P_2 and P_3 all are parallel

Answer: B

Solution:

Solution:
Given, $P_1 \Rightarrow 3x + 15y + 21z = 9$
$P_2 \Rightarrow x - 3y - z = 5$
$P_3 \Rightarrow 2x + 10y + 14z = 5$
Consider plane P_1, it can be written as
$3x + 15y + 21z = 9$ or $x + 5y + 7z = 3$
Again, consider plane P_3, it can be written as,
$2x + 10y + 14z = 5$ or $x + 5y + 7z = 5 / 2$
Hence, P_1 and P_3 are parallel.

Question128

If the mirror image of the point (1, 3, 5) with respect to the plane $4x - 5y + 2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals [26 Feb 2021 Shift 2]

Options:

A. 47

B. 43

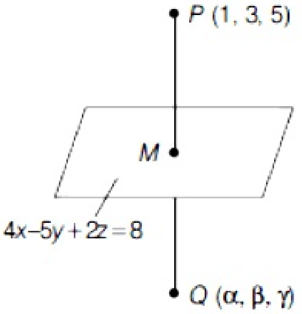
C. 39

D. 41

Answer: A

Solution:

Solution:
Given, point $P(1, 3, 5)$ has the mirror image $Q(\alpha, \beta, \gamma)$ with respect to plane $4x - 5y + 2z = 8$



Then, M be the mid-point of line joining point P and Q. and M lies on given plane.
Coordinates of $M(a, b, c)$ are as follows

$a = \frac{\alpha+1}{2}, b = \frac{\beta+3}{2}, c = \frac{\gamma+5}{2}$

∵ M lies on plane, then

$4a - 5b + 2c = 8$

$4\left(\frac{\alpha+1}{2}\right) - 5\left(\frac{\beta+3}{2}\right) + 2\left(\frac{\gamma+5}{2}\right) = 8...$ (i)

Also, PQ is perpendicular to plane

$\Rightarrow \frac{\alpha-1}{4} = \frac{\beta-3}{-5} = \frac{\gamma-5}{2} = \lambda$ (say)

$\Rightarrow \alpha = 4\lambda + 1, \beta = 3 - 5\lambda, \gamma = 2\lambda + 5 ...$ (ii)

Use Eq. (ii) in Eq. (i), we get

$2(4\lambda + 2) - 5\left(\frac{6-5\lambda}{2}\right) + 2\lambda + 10 = 8$

$\Rightarrow 8\lambda + 4 - 15 + \frac{25\lambda}{2} + 2\lambda + 10 = 8 \Rightarrow \lambda = \frac{2}{5}$

$\therefore \alpha = 4\lambda + 1 = 4\left(\frac{2}{5}\right) + 1 = \frac{13}{5}$

$\beta = 3 - 5\left(\frac{2}{5}\right) = \frac{5}{5} = 1$

and $\gamma = 5 + 2\left(\frac{2}{5}\right) = \frac{29}{5}$

$\therefore 5(\alpha + \beta + \gamma) = 5\left(\frac{13}{5} + 1 + \frac{29}{5}\right) = 47$

Question129

Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from $(3, 2, 1)$ on L, then the value of $21(\alpha + \beta + \gamma)$ equals [26 Feb 2021 Shift 2]

Options:

A. 142

B. 68

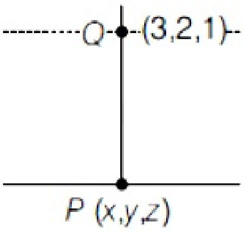
C. 136

D. 102

Answer: D

Solution:

Given, $x + 2y + z = 6$ . . . (i)
and $y + 2z = 4$ . . . (ii)
Put $y = 4 - 2z$ from Eq. (ii) Eq. in (i), we get
$x + 8 - 4z + z = 6$
$\Rightarrow x = -2 + 3z$
$\Rightarrow \frac{x+2}{3} = z$ . . . (iii)
$y = 4 - 2z$
$\Rightarrow \frac{y-4}{-2} = z$ . . . (iv)
From Eqs. (iii) and (iv), line of intersection of two planes is
$\frac{x+2}{3} = \frac{y-4}{-2} = \frac{z}{1} = \lambda$
Then, Direction ratios of PQ is
$x - 3\lambda - 4 - 2\lambda, z = \lambda$
$(3\lambda - 5, -2\lambda + 2, \lambda - 1)$
$(3\lambda - 5, -2\lambda + 2, \lambda - 1)$



Since, PQ is perpendicular to the line, then
$3(3\lambda - 5) - 2(-2\lambda + 2) + 1(\lambda - 1) = 0$
$\therefore \lambda = \frac{10}{7}$
$\therefore P\left(\frac{16}{7}, \frac{8}{7}, \frac{10}{7}\right)$
Then, $21(\alpha + \beta + \gamma) = 21\left(\frac{16}{7} + \frac{8}{7} + \frac{10}{7}\right)$
$= 21\left(\frac{34}{7}\right) = 3 \times 34 = 102$

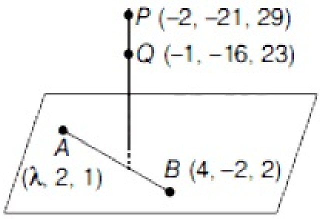
Question130

Let (λ, 2, 1) be a point on the plane which passes through the point (4, −2, 2). If the plane is perpendicular to the line joining the points (−2, −21, 29) and (−1, −16, 23), then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is ....
[26 Feb 2021 Shift 1]

Answer: 8

Solution:

Solution:
Given $(\lambda, 2, 1)$ be point on the plane which passes through $(4, -2, 2)$ and plane is perpendicular to line joining P and Q.



Given, AB is perpendicular to PQ i.e., $AB \cdot PQ = 0$

Now, AB $= (4-\lambda)\hat{i} + (-2-2)\hat{j} + \left(2 - \hat{k}\right.$

$= (4-\lambda)\hat{i} - 4\hat{j} + \hat{k}$

PQ $= (-1+2)\hat{i} + (-16+21)\hat{j} + (23-29)\hat{k}$

$= \hat{i} + 5\hat{j} - 6\hat{k}$

$= (4-\lambda)\hat{i} - 4\hat{j} + \hat{k}$

PQ $= (-1+2)\hat{i} + (-16+21)\hat{j} + (23-29)\hat{k}$

$= \hat{i} + 5\hat{j} - 6\hat{k}$

Hence, $AB \cdot PQ = 0$

$\Rightarrow (4-\lambda)(1) + (-4)(5) + (1)(-6) = 0$

$\Rightarrow 4 - \lambda - 20 - 6 = 0$

$\Rightarrow \lambda = -22$

Then, $\left(\frac{\lambda}{11}\right)^2 - \left(\frac{4\lambda}{11}\right) - 4 = \left(\frac{-22}{11}\right)^2 - \left(\frac{4 \times (-22)}{11}\right) - 4$

$= 4 - (-8) - 4 = 8$

Question131

A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If 0 is the origin and P is (2, −1, 1), then the projection of OP on this plane is of length
[2021 25 Feb Shift 2]

Options:

A. $\sqrt{\frac{2}{3}}$

B. $\sqrt{\frac{2}{11}}$

C. $\sqrt{\frac{2}{7}}$

D. $\sqrt{\frac{2}{5}}$

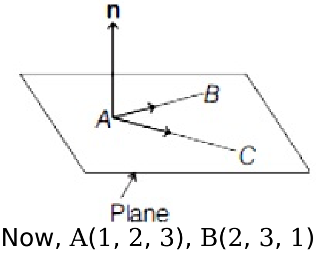
Answer: B

Solution:

Solution:

Refer diagram, the normal vector be n and it is perpendicular to both AB and AC

$AB \times AC = n$



Now, A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2)

Then, $AB = (2-1)\hat{i} + (3-2)\hat{j} + (1-3)\hat{k}$

$= \hat{i} + \hat{j} - 2\hat{k}$

$-1)\hat{i} + (4-2)\hat{j} + (2-3)\hat{k}$

AC $= (2-1)\hat{i} + (4$

$= \hat{i} + 2\hat{j} - \hat{k}$

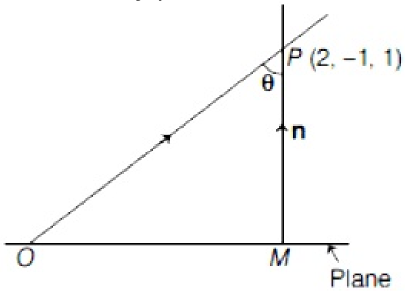
Now, $AB \times AC = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix}$

$= \hat{i}(-1+4) - \hat{j}(-1+2) + \hat{k}(2-1)$

$= 3\hat{i} - \hat{j} + \hat{k}$

$n = 3\hat{i} - \hat{j} + \hat{k}$

Let P be any point on normal vector and O be origin. Then refer the diagram, projection of OP on plane have length OM .



$OP = 2\hat{i} - \hat{j} + \hat{k}$ and $n = 3\hat{i} - \hat{j} + \hat{k}$

$OP.\ n = |OP|\,|n|\cos\theta$

$(6+1+1) = \sqrt{4+1+1}(\sqrt{9+1+1})\cos\theta$

$8 = \sqrt{6}\sqrt{11}\cos\theta \Rightarrow \cos\theta = \frac{8}{\sqrt{66}}$

Again, $\sin\theta = \frac{|OM|}{|OP|}$, gives $\left|OM\right| = \sin\theta \cdot \left|OP\right|$

$\Rightarrow |OM| = \sqrt{1-\cos^2\theta}\,|OP|$

$= \sqrt{1 - \frac{64}{66}}\sqrt{4+1+1}$ (use $\cos\theta = 8\sqrt{66}$)

$= \sqrt{\frac{2}{66}} \cdot \sqrt{6}$

$\therefore |OM| = \sqrt{\frac{2}{11}}$

Question132

The vector equation of the plane passing through the intersection of the planes $r \cdot (\hat{i} + j + \hat{k}) = 1$ and $r \cdot (\hat{i} - 2\hat{j}) = -2$ and the point (1, 0, 2) is [24 Feb 2021 Shift 2]

Options:

A. $r \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = \frac{7}{3}$

B. $r \cdot (3\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

C. $r \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

D. $r \cdot (\hat{i} - 7\hat{j} + 3\hat{k}) = \frac{7}{3}$

Answer: C

Solution:

Solution:
Given, point (1, 0, 2)
Equation of plane =
$r \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $r \cdot (\hat{i} - 2\hat{j}) = -2$
Equation of plane passing through the intersection of given planes is
$[r \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] + \lambda[r \cdot (\hat{i} - 2\hat{j}) + 2] = 0$
∵ This plane passes through point (1, 0, 2) i.e.,
vector $(\hat{i} + 2\hat{k})$
$\therefore [(\hat{i} + 2\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] + \lambda[(\hat{i} + 2\hat{k}) \cdot (\hat{i} - 2\hat{j}) + 2] = 0$
$\Rightarrow (3 - 1) + \lambda(1 + 2) = 0$
$\Rightarrow 2 + \lambda \times 3 = 0$
$\Rightarrow \lambda = -2/3$
Hence, equation of required plane is
$[r \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] + \left(\frac{-2}{3}\right)[r \cdot (\hat{i} - 2\hat{j}) + 2] = 0$
or $3[r \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] - 2[r \cdot (\hat{i} - 2\hat{j}) + 2] = 0$
or $r \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

Question133

The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z = 17$ is :
24 Feb 2021 Shift 1

Options:

A. $2\sqrt{19}$

B. $19\sqrt{2}$

C. 38

D. $\sqrt{38}$

Answer: D

Solution:

Solution:
Let $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = t$
$\Rightarrow x = 3 + t, y = 2t + 4, z = 2t + 5$
$3 + t + 2t + 4 + 2t + 5 = 17$
$\Rightarrow 5t = 5 \Rightarrow t = 1$
⇒ Point of intersection is (4, 6, 7)
Distance between (1, 1, 9) and (4, 6, 7) is
$\sqrt{(4-1)^2 + (6-1)^2 + (7-9)^2}$
$= \sqrt{9 + 25 + 4} = \sqrt{38}$.

Question134

The equation of the plane passing through the point (1, 2, −3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is

24 Feb 2021 Shift 1

Options:

A. $3x - 10y - 2z + 11 - 0$

B. $6x - 5y - 2z - 2 = 0$

C. $11x + y + 17z + 38 = 0$

D. $6x - 5y + 2z + 10 = 0$

Answer: C

Solution:

Solution:
Normal vector:

$$\begin{vmatrix} \hat{i} & j & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$$

So, direction ratios of normal to the required plane are $<11, 1, 17>$
Plane passes through $(1, 2, -3)$
So, equation of plane :
$11(x - 1) + 1(y - 2) + 17(z + 3) = 0$
$\Rightarrow 11x + y + 17z + 38 = 0$

Question135

Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are $(4, -1, 2)$ and $(-2, 1, -2)$, respectively. Let lines PR and QS intersect at T . If the vector TA is perpendicular to both PR and QS and the length of vector TA is $\sqrt{5}$ units, then the modulus of a position vector of A is [16 Mar 2021 Shift 1]

Options:

A. $\sqrt{482}$

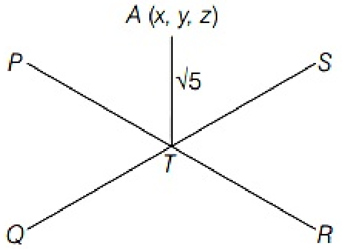
B. $\sqrt{171}$

C. $\sqrt{5}$

D. $\sqrt{227}$

Answer: B

Solution:



$P = \left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and $Q = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right)$
$V_{PR} = (4, -1, 2)$ and $V_{QS}(-2, 1, -2)$
Equation of line PR = $\left(3\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(4\hat{i} - \hat{j} + 2\hat{k}\right)$
Equation of line QS = $\left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \mu\left(-2\hat{i} + \hat{j} - 2\hat{k}\right)$
Let T be the point of intersection.
$T = (3 + 4\lambda, -1 - \lambda, 2 + 2\lambda)$
$T = (1 - 2\mu, 2 + \mu, -4 - 2\mu)$
$3 + 4\lambda = 1 - 2\mu$
$\Rightarrow 2\lambda + \mu = -1$.......(i)
$-1 - \lambda = 2 + \mu$
$\Rightarrow \lambda + \mu = -3$.....(ii)
From Eqs. (i) and (ii),
$\lambda = 2$ and $\mu = -5$
$T = [3 + 4(2)], -1 - (2), 2 + 2(2) = (11, -3, 6)$
Now, DC of TA will be $V_{PR} \times V_{QS}$
$\left| \hat{i}, \hat{j}, \hat{k}; -2, 1, -2; 4, -1, 2 \right| = 0\hat{i} - 4\hat{j} - 2\hat{k}$
$L_{TA} \Rightarrow \left(11\hat{i} - 3\hat{j} + 6\hat{k}\right) + x\left(-4\hat{j} - 2\hat{k}\right)$
Let $A = (11, -3 - 4x, 6 - 2x)$
$TA = \sqrt{5}$
$\Rightarrow \sqrt{(11 - 11)^2 + (-3 - 4x + 3)^2 + (6 - 2x - 6)^2} = \sqrt{5}$
$\Rightarrow (4x)^2 + (2x)^2 = 5 \Rightarrow 20x^2 = 5$
$\Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm \frac{1}{2}$
$A = [11, -3 - 4(1/2), 6 - 2(1/2)]$
$A = (11, -5, 5)$
Or
$A = [11, -3 + 4(1/2), 6 + 2(1/2)]$
$A = (11, -1, 7)$
$\therefore |A| = \sqrt{11^2 + 5^2 + 5^2}$ or
$\Rightarrow |A| = \sqrt{11^2 + 1^2 + 7^2}$
$\Rightarrow |A| = \sqrt{171}$ or $\sqrt{171}$
$\therefore |A| = \sqrt{171}$

Question136

If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{1} = \frac{y-2}{3} = \frac{z-b}{4}$, I $\neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to [16 Mar 2021 Shift 2]

Options:

A. 1/2

B. $1/\sqrt{6}$

C. $\sqrt{2/3}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

Solution:

Solution:

$L_1 \Rightarrow \frac{x-a}{I} = \frac{y-2}{3} = \frac{z-b}{4}$

Foot of perpendicular from $A(4, 3, 8)$ to L_1 is $B(3, 5, 7)$.

$AB = OB - OA$

$= (3\hat{i} + 5\hat{j} + 7\hat{k}) - (4\hat{i} + 3\hat{j} + 8\hat{k})$

$= -\hat{i} + 2\hat{j} - \hat{k}$

Now, AB is perpendicular to direction cosine of L_1,

So, $(-\hat{i} + 2\hat{j} - \hat{k}) \cdot (\hat{i} + 3\hat{j} + 4\hat{k}) = 0$

$\Rightarrow -I + 6 - 4 = 0 \Rightarrow I = 2$

As, $(3, 5, 7)$ lies on L_1,

$\frac{3-a}{2} = \frac{5-2}{3} = \frac{7-b}{4}$

$3 - a = 2$

So, $a = 1$, $7 - b = 4$

So, $b = 3$

$L_1 \Rightarrow \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

and $L_2 = \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

∴ Shortest distance will be along common normal.

So, common normal $= \left| \hat{i}, \hat{j}, \hat{k}; 2, 3, 4; 3, 4, 5 \right|$

$\Rightarrow n = -\hat{i} + 2\hat{j} - \hat{k} \Rightarrow \hat{n} = \frac{1}{\sqrt{6}}(-\hat{i} + 2\hat{j} - \hat{k})$

Shortest distance will be the projection of $(2-1)\hat{i} + (4-2)\hat{j} + (5-3)\hat{k}$ or $\hat{i} + 2\hat{j} + 2\hat{k}$ along $\hat{n}$

$\Rightarrow (\hat{i} + 2\hat{j} + 2\hat{k}) \frac{(-\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{6}} = \frac{-1 + 4 - 2}{\sqrt{6}} = \frac{1}{\sqrt{6}}$

Question137

Let P be an arbitrary point having sum of the squares of the distance from the planes $x + y + z = 0$, $I x - nz = 0$ and $x - 2y + z = 0$, equal to 9 . If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of $I - n$ is equal to
[17 Mar 2021 Shift 2]

Answer: 0

Solution:

Let $P = (\alpha, \beta, \gamma)$

Distance of point P from the plane $x + y + z = 0$ is

$= \frac{\alpha + \beta + \gamma}{\sqrt{3}}$

Distance of point P from the plane $l x - nz = 0$ is

$= \dfrac{l\alpha - n\gamma}{\sqrt{1^2 + n^2}}$

and distance of point P from the plane $x - 2y + z = 0$ is

$= \dfrac{\alpha - 2\beta + \gamma}{\sqrt{6}}$

According to the question,

$$\left(\frac{\alpha + \beta + \gamma}{\sqrt{3}}\right)^2 + \left(\frac{lx - nz}{\sqrt{I^2 + n^2}}\right)^2 + \left(\frac{\alpha - 2\beta + \gamma}{\sqrt{6}}\right)^2 = 9$$

∴ Locus is

$$\frac{(x + y + z)^2}{3} + \frac{(lx - nz)^2}{I^2 + n^2} + \frac{(x - 2y + z)^2}{6} = 9$$

$$\Rightarrow x^2\left(\frac{1}{2} + \frac{l^2}{l^2 + n^2}\right) + y^2 + z^2\left(\frac{1}{2} + \frac{n^2}{l^2 + n^2}\right) + xz\left(1 - \frac{2ln}{l^2 + n^2}\right) = 0$$

Comparing it with the given equation of locus, we get

$2ln = I^2 + n^2$

$\Rightarrow (I - n)^2 = 0$

$\Rightarrow I - n = 0$

Question138

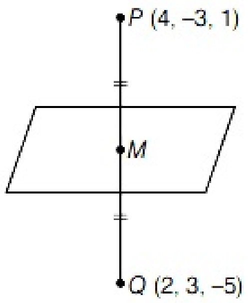
Let the plane, $ax + by + cz + d = 0$ bisect the line joining the points $(4, -3, 1)$ and $(2, 3, -5)$ at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is ....
[18 Mar 2021 Shift 1]

Answer: 28

Solution:

Let $P \equiv (4, -3, 1)$

and $Q \equiv (2, 3, -5)$



$\therefore M = \dfrac{P + Q}{2}$

$\Rightarrow M \equiv \left(\dfrac{4 + 2}{2}, \dfrac{-3 + 3}{2}, \dfrac{1 - 5}{2}\right)$

$\Rightarrow M \equiv (3, 0, -2)$

Also, direction ratios of $PQ = \{4 - 2, -3 - 3, 1 + 5\}$

$= \{2, -6, 6\}$

⇒ Direction ratios of $PQ = \{1, -3, 3\}$ = direction ratios of normal to the plane.

∴ Equation of the plane is

$1(x - 3) - 3(y - 0) + 3(z + 2) = 0$

$\Rightarrow x - 3y + 3z + 3 = 0$

Comparing this to $ax + by + cz + d = 0$, we get

$a = 1, b - 3, c = 3, d = 3$

∴ Minimum value of $(a^2 + b^2 + c^2 + d^2) = 28$

Question139

The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distance from the point $(1, 2, 3)$ is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is ....
[18 Mar 2021 Shift 1]

Answer: 4

Solution:

Equation of any plane parallel to the plane $x - 2y + 2z - 3 = 0$ is
$x - 2y + 2z + \lambda = 0$....(i)
Given, distance from $(1, 2, 3)$ is 1.
$\therefore \left| \frac{1 - 2 \times 2 + 2 \times 3 + \lambda}{\sqrt{(1)^2 + (-2)^2 + (2)^2}} \right| = 1$
$\Rightarrow |\lambda + 3| = 3$
$\Rightarrow \lambda + 3 = \pm 3$
$\Rightarrow \lambda = 0, -6$
Consider, $\lambda = -6$
$\therefore$ Equation of required plane is
$x - 2y + 2z - 6 = 0$
On comparing this equation to
$ax + by + cz + d = 0$, we get
$a = 1, b = -2, c = 2$ and $d = -6$
$\because (b - d) = k(c - a)$
$\therefore 4 = k \times (1)$
$\Rightarrow k = 4$

Question140

The equation of the plane which contains the Y-axis and passes through the point $(1, 2, 3)$ is
[17 Mar 2021 Shift 1]

Options:

A. $x + 3z = 10$

B. $x + 3z = 0$

C. $3x + z = 6$

D. $3x - z = 0$

Answer: D

Solution:

Equation of plane passing through a point (x_1, y_1, z_1) is
$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Here, $(x_1, y_1, z_1) = (1, 2, 3)$
So, $a(x-1) + b(y-2) + c(z-3) = 0$
Now, Y-axis lies on it.
Direction ratio of Y-axis is $(0, 1, 0)$.
Normal vector to the plane $= a\hat{i} + b\hat{j} + c\hat{k}$
So, the normal vector of the plane will be perpendicular to direction ratio of Y-axis.
$a \cdot 0 + b \cdot 1 + c \cdot 0 = 0 \Rightarrow b = 0$
Equation of plane becomes
$a(x-1) + c(z-3) = 0$
Now, $x = 0$, $z = 0$ also satisfies the equation.
$a(0-1) + c(0-3) = 0$
$\Rightarrow\ t\ \ -a - 3c = 0 \Rightarrow a = -3c$
So, $-3c(x-1) + c(z-3) = 0$
$-3x + 3 + z - 3 = 0$
$[\text{as, } C \neq 0]$
$\Rightarrow\ 3x - z = 0$

Question141

If for $a > 0$, the feet of perpendiculars from the points $A(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $\mathrm{I}x + my + nz = 0$ are points $C(0, -a, -1)$ and D respectively, then, the length of line segment CD is equal to [16 Mar 2021 Shift 1]

Options:

A. $\sqrt{31}$

B. $\sqrt{41}$

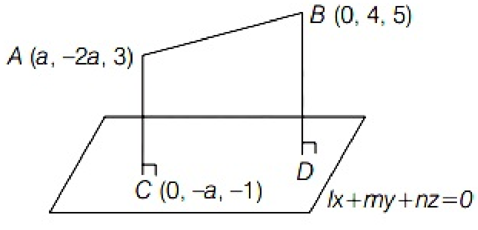
C. $\sqrt{55}$

D. $\sqrt{66}$

Answer: D

Solution:

Solution:



Given, $A \Rightarrow (a, -2a, 3)$
$B \Rightarrow (0, 4, 5)$
$C \Rightarrow (0, -a, -1)$
Equation of plane $P \Rightarrow \mathrm{I}x + my + nz = 0$
As, C is foot of perpendicular from A to plane P. So, CA $|N$, where N is the normal vector to the plane.
$CA = (a-0)\hat{i} + (-2a+a)\hat{j} + (3+1)\hat{k}$
$= a\hat{i} - a\hat{j} + 4\hat{k}$
Now, CA $|N$
So, $\frac{a}{l} = \frac{-a}{m} = \frac{4}{n} = \lambda$
where λ is any real number.
$P \Rightarrow \left(\frac{a}{\lambda}\right)x - \left(\frac{a}{\lambda}\right)y + \left(\frac{4}{\lambda}\right)z = 0$
$P \Rightarrow ax - ay + 4z = 0$

C lies on plane.
So, $a \cdot 0 - a(-a) + 4(-1) = 0$
$a^2 - 4 = 0 \Rightarrow a = \pm 2$
As per the question, $a > 0$, so $a = 2$
So, equation of plane $P \Rightarrow 2x - 2y + 4z = 0$
$P \Rightarrow x - y + 2z = 0$
Coordinates of D
$$\frac{x-0}{1} = \frac{y-4}{-1} = \frac{z-5}{2} = \frac{-(0-4+10)}{[1^2+(-1)^2+2^2]}$$
If (x, y, z) be the foot of perpendicular drawn from (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$.
Then,
$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \frac{-(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$
Here, $(x, y, z) = (0, 4, 5)$
$$\Rightarrow x - 0 = -(y-4) = \frac{z-5}{2} = \frac{-6}{6}$$
$\therefore\ x = -1, y = 5, z = 3$
$C = (0, -2, -1) \Rightarrow D = (-1, 5, 3)$
$\therefore\ CD = \sqrt{(0+1)^2 + (-2-5)^2 + (-1-3)^2}$
$= \sqrt{1+49+16}$
$CD = \sqrt{66}$

Question142

If (x, y, z) be an arbitrary point lying on a plane P, which passes through the points (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of expression
$$3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$$
is equal to
[16 Mar 2021 Shift 2]

Options:

A. 0

B. 3

C. 39

D. -45

Answer: B

Solution:

Solution:
Equation of plane passing through A(42, 0, 0), B(0, 42, 0) and C(0, 0, 42) will be
$$\frac{x}{42} + \frac{y}{42} + \frac{z}{42} = 1$$
$\Rightarrow\ x + y + z = 42$
$(x-11) + (y-19) + (z-12) = 0$
Now,
$$3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} + \frac{y-19}{(x-11)^2(z-12)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

$$\Rightarrow \frac{3(x-11)^2(y-19)^2(z-12)^2+(x-11)^3+(y-19)^3+(z-12)^3}{(x-11)^2(y-19)^2(z-12)^2}-\frac{42}{14(x-11)(y-19)(z-12)}$$

$$\Rightarrow \frac{(x-11)^3+(y-19)^3+(z-12)^3-3(x-11)(y-19)(z-12)+3(x-11)^2(y-19)^2(z-12)^2}{(x-11)^2(y-19)^2(z-12)^2}$$

⇒ If $A + B + C = 0$

Then, $A^3 + B^3 + C^3 = 3ABC$

$\Rightarrow (x-11)^3 + (y-19)^3 + (z-12)^3 = 3(x-11)(y-19)(z-12)$

$$\Rightarrow \frac{3(x-11)(y-19)(z-12)-3(x-11)(y-19)(z-12)+3(x-11)^2(y-19)^2(z-12)^2}{(x-11)^2(y-19)^2(z-12)^2}$$

$$\Rightarrow \frac{3(x-11)^2(y-19)^2(z-12)^2}{(x-11)^2(y-19)^2(z-12)}=3$$

Question143

If the equation of the plane passing through the line of intersection of the planes $2x-7y+4z-3=0,\ 3x-5y+4z+11=0$ and the point $(-2, 1, 3)$ is $ax+by+cz-7=0$, then the value of $2a+b+c-7$ is ....
[17 Mar 2021 Shift 1]

Answer: 4

Solution:

Solution:

Equation of the plane passing through the line of intersections of planes $2x-7y+4z-3=0$ and $3x-5y+4z+11=0$ is

$(2x-7y+4z-3)+\lambda(3x-5y+4z+11)=0$

Since this plane passes thought the point $(-2, 1, 3)$

$\therefore (-4-7+12-3)+\lambda(-6-5+12+11)=0$

$-2+12\lambda=0 \Rightarrow \lambda=1/6$

∴ Equation of plane is

$(2x-7y+4z-3)+\frac{1}{6}(3x-5y+4z+11)=0$

$15x-47y+28z-7=0$

$\therefore a=15, b=-47, c=28$

$2a+b+c-7=30-47+28-7=4$

Question144

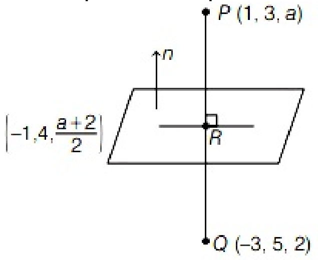
Let the mirror image of the point $(1, 3, a)$ with respect to the plane $\mathbf{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)-b=0$ be $(-3, 5, 2)$. Then the value of $|a+b|$ is equal to
[18 Mar 2021 Shift 2]

Answer: 1

Solution:

Solution:

Given equation of plane in vector form is $r \cdot \left(2\hat{i} - \hat{j} + \hat{k}\right) - b = 0$



Its Cartesian form will be

$2x - y + z = b$ ...(i)

$\because$R is the mid-point of PQ.

$\therefore\ R \equiv \dfrac{P+Q}{2} \Rightarrow R \equiv \left(-1, 4, \dfrac{a+2}{2}\right)$

$\because$R lies on the plane (i).

$\therefore\ -2 - 4 + \dfrac{a+2}{2} = b \Rightarrow a + 2 = 2b + 12$

$\Rightarrow\ a = 2b + 10$...(ii)

$\because$ Direction ratio's of QP is $(1 - (-3), 3 - 5, a - 2)$

i.e. $(4, -2, a - 2)$

and direction ratios of normal to the given plane are $(2, -1, 1)$

$\because$n and QP are parallel.

$\dfrac{2}{4} = \dfrac{-1}{-2} = \dfrac{1}{a-2}$

$\therefore\ a - 2 = 2 \Rightarrow a = 4$

From Eq. (ii), $b = -3$

$\therefore\ |a + b| = |4 - 3| = |1| = 1$

Question145

Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to ....

[18 Mar 2021 Shift 2]

Answer: 38

Solution:

Solution:

Equation of required plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$

Since, $(1, -1, \infty)$ lies on it,

So, replace x by 1, y by (-1) and z and α

$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$

$\Rightarrow\ 5\alpha + 38 = 0 \Rightarrow 5\alpha = -38$

$\therefore |5\alpha| = |-38| = 38$

Question146

Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(−3, −6, 1) and B(2, 4, −3) in ratio k : 1, then the value of k is equal to
[16 Mar 2021 Shift 1]

Options:

A. 1.5

B. 3

C. 2

D. 4

Answer: C

Solution:

Solution:
$P \Rightarrow lx + my + nz = 0$
P contains L_1
$L_1 \Rightarrow \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$
So, $(1, -4, -2)$ lies on plane.
$1 - 4m - 2n = 0 \ldots (i)$
And $(-1, 2, 3)$ will be perpendicular to (I, m, n).
$-1 + 2m + 3n = 0 \ldots . (ii)$
Adding Eqs. (i) and (ii),
$-2m + n = 0$
$n = 2m$
$l - 4m - 4m = 0$
$l = 8m$
So, $l = 8m$ and $n = 2m$
Plane $\Rightarrow 8x + y + 2z = 0$
Now, $A(-3, -6, 1)$ and $B(2, 4, -3)$
Plane P divides AB in the ratio of $k : 1$.
Let plane P intersect the line AB at point O.
So, $O = \left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$
And O lies on plane P,
So, $8(2k-3) + (4k-6) + 2(-3k+1) = 0$
$\Rightarrow 14k - 28 = 0$
$\therefore k = 2$

Question147

If the distance of the point (1, −2, 3) from the plane $x + 2y - 3z + 10 = 0$ measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of $|m|$ is equal to.........
[16 Mar 2021 Shift 2]

Answer: 2

Solution:

Solution:
Given, point $A \Rightarrow (1, -2, 3)$
Plane $\Rightarrow x + 2y - 3z + 10 = 0$
Distance of point from plane along the vector $\left(3\hat{i} - m\hat{j} + \hat{k}\right)$ is $\sqrt{7/2}$.
Line passing through $(1, -2, 3)$ in the direction of
$\left(3\hat{i} - m\hat{j} + \hat{k}\right)$ is $\frac{x-1}{3} = \frac{y+2}{-m} = \frac{z-3}{1} = \lambda$
Any general point B will be $(3\lambda + 1, -m\lambda - 2, \lambda + 3)$
Now, this point B lies on plane
So, $x + 2y - 3z + 10 = 0$
$(3\lambda + 1) + 2(-m\lambda - 2) - 3(\lambda + 3) + 10 = 0$
$= (3 - 2m - 3)\lambda = 2$
$\Rightarrow \lambda = -1/m$
Now, $A = (1, -2, 3)$
$B = (3\lambda + 1, -m\lambda - 2, \lambda + 3)$
$|AB|^2 = (3\lambda + 1 - 1)^2 + (-m\lambda - 2 + 2)^2 + (\lambda + 3 - 3)^2$
$\Rightarrow 7/2 = 9\lambda^2 + m^2\lambda^2 + \lambda^2$
$\Rightarrow 7/2 = 10\lambda^2 + 1 \quad [\because m\lambda = -1]$
$\Rightarrow 10\lambda^2 = 5/2 \Rightarrow \lambda^2 = 1/4$
$\Rightarrow = \pm 1/2 \Rightarrow m = -1/\lambda$
$\therefore m = \pm 2$
and $|m| = 2$

Question148

If the lines $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are co-planar, then the value of k is _______.
[25 Jul 2021 Shift 2]

Answer: 1

Solution:

Solution:
$$\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$$
$(k + 1)[2 - 6] - 4[1 - 9] + 6[2 - 6] = 0$
$k = 1$

Question149

If the shortest distance between the straight lines $3(x-1)=6(y-2)=2(z-1)$ and $4(x-2)=2(y-\lambda)=(z-3)$, $\lambda \in R$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to :
[22 Jul 2021 Shift 2]

Options:

A. 3

B. 2

C. 5

D. −1

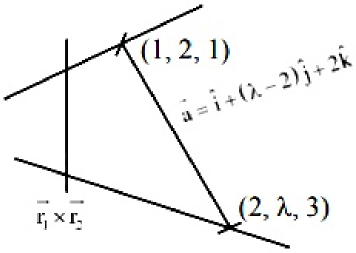
Answer: A

Solution:

Solution:

$L_1 : \frac{(x-1)}{2} = \frac{(y-2)}{1} = \frac{(z-1)}{3}$ $\vec{r}_1 = 2\hat{i} + \hat{j} + 3\hat{k}$

$L_2 : \frac{(x-2)}{1} = \frac{y-\lambda}{2} = \frac{z-3}{4}$ $\vec{r}_2 = \hat{i} + 2\hat{j} + 4\hat{k}$



Shortest distance = Projection of $\vec{a}$ on $\vec{r}_1 \times \vec{r}_2$

$= \frac{|\vec{a} \cdot (\vec{r}_1 \times \vec{r}_2)|}{|\vec{r}_1 \times \vec{r}_2|}$

$|\vec{a} \cdot (\vec{r}_1 \times \vec{r}_2)| = \begin{vmatrix} 1 & \lambda - 2 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = |14 - 5\lambda|$

$|\vec{r}_1 \times \vec{r}_2| = \sqrt{38}$

$\therefore \frac{1}{\sqrt{38}} = \frac{|14 - 5\lambda|}{\sqrt{38}}$

$\Rightarrow |14 - 5\lambda| = 1$

$\Rightarrow 14 - 5\lambda = 1$ or $14 - 5\lambda = -1$

$\Rightarrow \lambda = \frac{13}{5}$ or 3

$\therefore$ Integral value of $\lambda = 3$

Question150

If the shortest distance between the lines $\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$, $\lambda \in R$, $\alpha > 0$ and overrightarrow mathrm $r_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$, $\mu \in R$ is 9, then α is equal to ________.
[20 Jul 2021 Shift 1]

Answer: 6

Solution:

Solution:

If $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{c} + \lambda \vec{d}$
then shortest distance between two lines is

$$L = \frac{(\vec{a} - \vec{c}) \cdot (\vec{b} \times \vec{d})}{|b \times d|}$$

$$\therefore \vec{a} - \vec{c} = \left((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$\frac{\vec{b} \times \vec{d}}{|b \times d|} = \frac{\left(2\hat{i} + 2\hat{j} + \hat{k}\right)}{3}$$

$$\therefore \left((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}\right) \cdot \frac{\left(2\hat{i} + 2\hat{j} + \hat{k}\right)}{3} = 9$$

or $\alpha = 6$

Question151

The lines $x = ay - 1 = z - 2$ and $x = 3y - 2 = bz - 2$, $(ab \neq 0)$ are coplanar, if :
[20 Jul 2021 Shift 2]

Options:

A. $b = 1, a \in R - \{0\}$

B. $a = 1, b \in R - \{0\}$

C. $a = 2, b = 2$

D. $a = 2, b = 3$

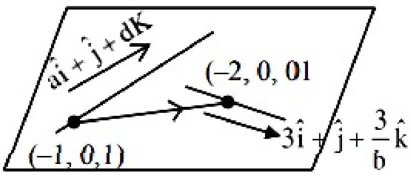
Answer: A

Solution:

Solution:

$$\frac{x + 1}{a} = y = \frac{z - 1}{a}$$

$$\frac{x + 2}{3} = y = \frac{z}{3 / b}$$



lines are Co-planar

$$\begin{vmatrix} a & 1 & a \\ 3 & 1 & \frac{3}{b} \\ -1 & 0 & -1 \end{vmatrix} = 0 \Rightarrow -\left(\frac{3}{b} - a\right) - 1(a - 3) = 0$$

$a - \frac{3}{b} - a + 3 = 0$
$\sim b = 1, a \in R - \{0\}$

Question152

Let the plane passing through the point $(-1, 0, -2)$ and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$ be $ax + by + cz + 8 = 0$. Then the value of $a + b + c$ is equal to: [27 Jul 2021 Shift 1]

Options:

A. 3

B. 8

C. 5

D. 4

Answer: D

Solution:

Solution:
Normal of req. plane $\left(2\hat{i} + \hat{j} - \hat{k}\right) \times \left(\hat{i} - \hat{j} - \hat{k}\right)$
$= -2\hat{i} + \hat{j} - 3\hat{k}$
Equation of plane
$-2(x + 1) + 1(y - 0) - 3(z + 2) = 0$
$-2x + y - 3z - 8 = 0$
$2x - y + 3z + 8 = 0$
$a + b + c = 4$

Question153

Let a, b and c be distinct positive numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are co-planar, then c is equal to: [25 Jul 2021 Shift 2]

Options:

A. $\frac{2}{\frac{1}{a} + \frac{1}{b}}$

B. $\frac{a + b}{2}$

C. $\frac{1}{a} + \frac{1}{b}$

D. $\sqrt{ab}$

Answer: D

Solution:

Solution:
Because vectors are coplanar

Hence $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

$\Rightarrow c^2 = ab \Rightarrow c = \sqrt{ab}$

Question154

Let a plane P pass through the point (3, 7, −7) and contain the line, $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 is equal to ________.
[27 Jul 2021 Shift 1]

Answer: 3

Solution:

Solution:
$\overrightarrow{BA} = (\hat{i} + 4\hat{j} - 5\hat{k})$
$\overrightarrow{BA} = (\hat{i} + 4\hat{j} - 5\hat{k})$

$\overrightarrow{BA} \times \vec{l} = \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 1 \\ 1 & 4 & -5 \end{vmatrix}$

$a\hat{i} + b\hat{j} + c\hat{k} = -14\hat{i} - \hat{j}(14) + \hat{k}(-14)$
$a = 1, b = 1, c = 1$
Plane is $(x-2) + (y-3) + (z+2) = 0$
$x + y + z - 3 = 0$
$d = \sqrt{3} \Rightarrow d^2 = 3$

Question155

For real numbers α and β ≠ 0, if the point of intersection of the straight lines $\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and $\frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3}$ lies on the plane $x + 2y - z = 8$, then $\alpha - \beta$ is equal to :
[27 Jul 2021 Shift 2]

Options:

A. 5

B. 9

C. 3

D. 7

Answer: D

Solution:

First line is $(\phi + \alpha, 2\phi + 1, 3\phi + 1)$
and second line is $(q\beta + 4, 3q + 6, 3q + 7)$.
For intersection
$\phi + \alpha = q\beta + 4$ .....(i)
$2\phi + 1 = 3q + 6$ ......(ii)
$3\phi + 1 = 3q + 7$........(iii)
for (ii) & (iii) $\phi = 1, q = -1$
So, from (i) $\alpha + \beta = 3$
Now, point of intersection is $(\alpha + 1, 3, 4)$
It lies on the plane.
Hence, $\alpha = 5 \& \beta = -2$

Question156

The distance of the point $P(3, 4, 4)$ from the point of intersection of the line joining the points. $Q(3, -4, -5)$ and $R(2, -3, 1)$ and the plane $2x + y + z = 7$, is equal to ______.
[27 Jul 2021 Shift 2]

Answer: 7

Solution:

Solution:
$\overrightarrow{QR} : -\frac{x-3}{1} = \frac{y+4}{-1} = \frac{z+5}{-6} = r$
$\Rightarrow (x, y, z) \equiv (r + 3, -r - 4, -6r - 5)$
Now, satisfying it in the given plane.
We get $r = -2$.
so, required point of intersection is $T(1, -2, 7)$.
Hence, $PT = 7$.

Question157

Let the foot of perpendicular from a point $P(1, 2, -1)$ to the straight line $L : \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ be N.
Let a line be drawn from P parallel to the plane $x + y + 2z = 0$ which meets L at point Q. If α is the acute angle between the lines PN and PQ, then $\cos\alpha$ is equal to ________.
[25 Jul 2021 Shift 1]

Options:

A. $\frac{1}{\sqrt{5}}$

B. $\frac{\sqrt{3}}{2}$

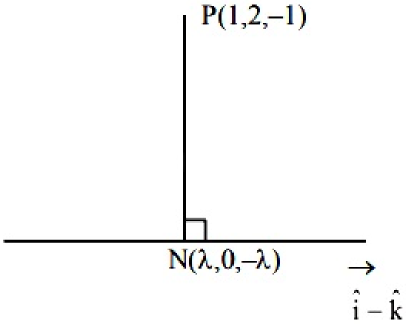
C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{2\sqrt{3}}$

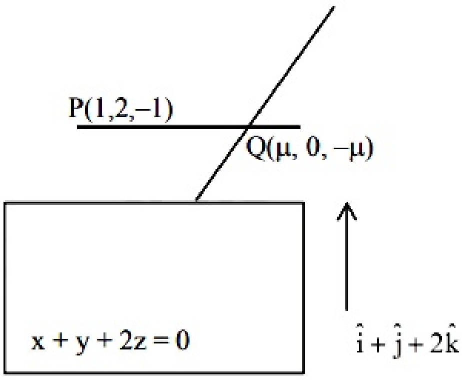
Answer: C

Solution:

Solution:



$\overrightarrow{PN} \cdot \left(\hat{i} - \hat{k}\right) = 0$
$\Rightarrow N\ (1, 0, -1)$
Now,



$\overrightarrow{PQ} \cdot \left(\hat{i} + \hat{j} + 2\hat{k}\right) = 0$
$\Rightarrow \mu = -1$
$\Rightarrow Q(-1, 0, 1)$
$\overrightarrow{PN} = 2\hat{j}$ and $\overrightarrow{PQ} = 2\hat{i} + 2\hat{j} - 2\hat{k}$
$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

Question158

Let L be the line of intersection of planes $\vec{r} \cdot \left(\hat{i} - \hat{j} + 2\hat{k}\right) = 2$ and $\vec{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) = 2$. If P(α, β, γ) is the foot of perpendicular on L from the point (1, 2, 0), then thevalue of 35(α + β + γ) is equal to :
[22 Jul 2021 Shift 2]

Options:

A. 101

B. 119

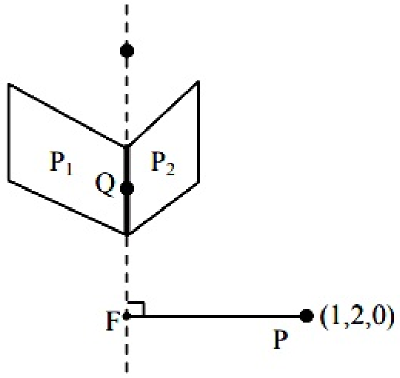
C. 143

D. 134

Answer: B

Solution:

Solution:
$P_1 : x - y + 2z = 2$
$P_2 = 2x + y - 3 = 2$



Let line of Intersection of planes P_1 and P_2 cuts xy plane in point Q.
⇒ z-coordinate of point Q is zero

$$\Rightarrow \left.\begin{array}{c} x - y = 2 \\ \text{and } 2x + y = 2 \end{array}\right\} \Rightarrow x = \frac{4}{3}, y = \frac{-2}{3}$$

$$\Rightarrow Q\left(\frac{4}{3}, \frac{-2}{3}, 0\right)$$

Vector parallel to the line of intersection

$$\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix} = -\hat{i} + 5\hat{j} + 3\hat{k}$$

Equation of Line of intersection

$$\frac{x - \frac{4}{3}}{-1} = \frac{y + \frac{2}{3}}{5} = \frac{z - 0}{3} = \lambda(\text{ say })$$

Let coordinates of foot of perpendicular be

$$F\left(-\lambda + \frac{4}{3}, 5\lambda - \frac{2}{3}, 3\lambda\right)$$

$$\overrightarrow{PF} = \left(-\lambda + \frac{1}{3}\right)\hat{i} + \left(5\lambda - \frac{8}{3}\right)\hat{j} + (3\lambda)\hat{k}$$

$$\overrightarrow{PF} \cdot \vec{a} = 0$$

$$\Rightarrow \lambda - \frac{1}{3} + 25\lambda \frac{-40}{3} + 9\lambda = 0$$

$$\Rightarrow 35\lambda = \frac{41}{3} \Rightarrow \lambda = \frac{41}{105}$$

Now, $\alpha = -\lambda + \frac{4}{3}, \beta = 5\lambda - \frac{2}{3}, \gamma = 3\lambda$

$$\Rightarrow \alpha + \beta + \gamma = 7\lambda + \frac{2}{3}$$

$$= 7\left(\frac{41}{105}\right) + \frac{2}{3}$$

$$= \frac{51}{15}$$

$$\Rightarrow 35(\alpha + \beta + \gamma) = \frac{51}{15} \times 35 = 119$$

Question159

Let P be a plane passing through the points (1, 0, 1), (1, −2, 1) and (0, 1, −2). Let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to theplane P, perpendicular to $\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\vec{a} \cdot \left(\hat{i} + \hat{j} + 2\hat{k}\right) = 2$, then $(\alpha - \beta + \gamma)^2$ equals _______.
[20 Jul 2021 Shift 1]

Answer: 81

Solution:

Equation of plane :

$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 1-1 & 2 & 1-1 \\ 1-0 & 0-1 & 1+2 \end{vmatrix} = 0$$

$\Rightarrow 3x - z - 2 = 0$
$\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} \parallel$ to $3x - z - 2 = 0$
$\Rightarrow 3\alpha - 8 = 0$ ....(1)
$\vec{a} \perp \hat{i} + 2\hat{j} + 3\hat{k}$
$\Rightarrow \alpha + 2\beta + 38 = 0$.......(2)
$\vec{a} \cdot \left(\hat{i} + \hat{j} + 2\hat{k}\right) = 0$
$\Rightarrow \alpha + \beta + 28 = 2$ ....(3)
on solving 1, 2 & 3
$\alpha = 1, \beta = -5, 8 = 3$
So $(\alpha - \beta + 8) = 81$

Question160

Consider the line L given by the equation $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$. Let Q be the mirror image of the point (2, 3, −1) with respect to L. Let a plane Pbe such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P ?
[20 Jul 2021 Shift 2]

Options:

A. $(-1, 1, 2)$

B. $(1, 1, 1)$

C. $(1, 1, 2)$

D. $(1, 2, 2)$

Answer: D

Solution:

Solution:
Plane $\mathrm{mathrm\,p}$ is $\perp^{r}$ to line
$\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$
& passes through pt. $(2, 3)$ equation of plane p
$2(x-2)+1(y-3)+1(z+1)=0$
$2x+y+z-6=0$
pt $(1, 2, 2)$ satisfies above equation

Question161

The angle between the straight lines, whose direction cosines are given by the equations 2I + 2m − n = 0 and mn + nI + Im = 0, is [27 Aug 2021 Shift 2]

Options:

A. $\frac{\pi}{2}$

B. $\pi-\cos^{-1}\left(\frac{4}{9}\right)$

C. $\cos^{-1}\left(\frac{8}{9}\right)$

D. $\frac{\pi}{3}$

Answer: A

Solution:

Given,
2l + 2m − n = 0 ....(i)
mn + nl + lm = 0 ...(ii)
From Eq. (i), we get
n = 2l + 2m ...(iii)
Substituting, n = 2l + 2m in Eq. (ii), we have
m(2l + 2m) + l(2l + 2m) + lm = 0
$\Rightarrow 2\,\mathrm{Im}+2m^{2}+2I^{2}+2\,\mathrm{Im}+\mathrm{Im}=0$
$\Rightarrow 2I^{2}+4\,\mathrm{Im}+\mathrm{Im}+2m^{2}=0$
⇒ 2l(l + 2m) + m(l + 2m) = 0
⇒ (2l + m) (l + 2m) = 0
When
2l = − m
From Eq (iii),
n = m
$\Rightarrow\frac{2I}{-1}=\frac{m}{1}=\frac{n}{1}$
$\Rightarrow\frac{I}{-\frac{1}{2}}=\frac{m}{1}=\frac{n}{1}$
or $\frac{I}{1}=\frac{m}{-2}=\frac{n}{-2}$
⇒ (l, m, n) = (1, − 2, − 2)
When
l = − 2m
From Eq. (iii),

n = − 2m
⇒ l = − 2m = n
$\Rightarrow \frac{l}{-2} = \frac{m}{1} = \frac{n}{-2}$
⇒ (l, m, n) = (− 2,1, − 2)
∴ Angles between straight lines
$$\cos\theta = \frac{(\hat{i} - 2\hat{j} - 2\hat{k})(-2\hat{i} + \hat{j} - 2\hat{k})}{|\hat{i} - 2\hat{j} - 2\hat{k}|-2\hat{i} + \hat{j} - 2\hat{k}|}$$
$$\cos\theta = \frac{-2-2+4}{9} = 0$$
$$\Rightarrow \theta = \frac{\pi}{2}$$

Question162

The square of the distance of the point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ and the plane $2x - y + z = 6$ from the point $(-1, -1, 2)$ is
[31 Aug 2021 Shift 1]

Answer: 61

Solution:

Solution:
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6} = \lambda$
$$\begin{cases} x = 2\lambda + 1 \\ y = 3\lambda + 2 \\ z = 6\lambda - 1 \end{cases}$$
Equation of plane is $2x - y + z = 6$
$\Rightarrow 2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1) = 6$
$7\lambda = 7$
$\lambda = 1$
P(3, 5, 5)
$(\text{Distance})^2 = (3 + 1) + (5 + 1)^2 + (5 - 2)^2$
$= 16 + 36 + 9 = 61$

Question163

The distance of the point $(-1, 2, -2)$ from the line of intersection of the planes $2x + 3y + 2z = 0$ and $x - 2y + z = 0$ is
[31 Aug 2021 Shift 2]

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{5}{2}$

C. $\frac{\sqrt{42}}{2}$

D. $\frac{\sqrt{34}}{2}$

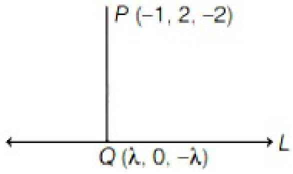
Answer: D

Solution:

Solution:
Let L be the line of intersection of $2x + 3y + 2z = 0$ and $x - 2y + z = 0$
If $z = 0$, then $x = y = 0$
The line L is parallel to $r_1 \times r_2$, where $r_1 = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $r_2 = \hat{i} - 2\hat{j} + \hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ 1 & -2 & 1 \end{vmatrix} = 7\hat{i} - 7\hat{k}$$

DR's of is $(1, 0, -1)$
and equation of L is $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1} = \lambda$



Let PQ be the distance from the point $P(-1, 2, -2)$ to the line L.
DRs of PQ $= \lambda + 1, -2, 2 - \lambda$
$\therefore$ **PQ** $\perp$ **r**
$\Rightarrow (\lambda + 1)(1) + (-2)(0) + (2 - \lambda)(-1) = 0$
$\Rightarrow \lambda + 1 - 2 + \lambda = 0$
$\Rightarrow \lambda = \frac{1}{2}$
Coordinate of Q is $\left(\frac{1}{2}, 0, \frac{-1}{2}\right)$
$\Rightarrow PQ = \sqrt{\left(-1 - \frac{1}{2}\right)^2 + (2 - 0)^2 + \left(-2 + \frac{1}{2}\right)^2}$
$= \sqrt{\frac{9}{4} + 4 + \frac{9}{4}} = \frac{\sqrt{34}}{2}$

Question164

If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$ and containing the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha + \beta + \gamma$ is equal to [17 Mar 2021 Shift 2]

Options:

A. 20

B. 19

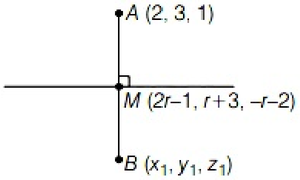
C. 18

D. 21

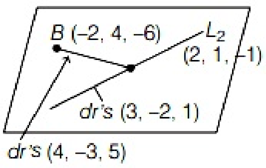
Answer: B

Solution:

Solution:
Let A = (2, 3, 1)
$L_1 \Rightarrow \frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$
$L_2 \Rightarrow \frac{x-2}{3} = \frac{y-1}{-2} = \frac{z+1}{1}$
Any point M taken on L_1 is $(2r-1, r+3, -r-2)$



$\therefore$ Direction ratios of AM are $(2r-3, r, -r-3)$
$\because AM \perp L_1$
$\therefore 2(2r-3) + 1 \times r + (-1)(-r-3) = 0$
$\Rightarrow 4r - 6 + r + r + 3 = 0$
$\Rightarrow 6r = 3 \Rightarrow r = \frac{1}{2}$
$\therefore M = \left(0, \frac{7}{2}, \frac{-5}{2}\right)$
$\therefore M = \left(0, \frac{7}{2}, \frac{-5}{2}\right)$
$\therefore B \equiv \left((2 \times 0) - 2, \left(2 \times \frac{7}{2}\right) - 3, \left(2 \times \left(\frac{-5}{2}\right)\right) - 1\right)$
$\Rightarrow B = (-2, 4, -6)$
Now, equation of plane containing $B(-2, 4, -6)$ and the line L_2 is



$$\begin{vmatrix} x-2 & y-1 & z+1 \\ 3 & -2 & 1 \\ 4 & -3 & 5 \end{vmatrix} = 0$$
$\Rightarrow (x-2)(-10+3) - (y-1)(15-4) + (z+1)(-9+8) = 0$
$\Rightarrow -7(x-2) - 11(y-1) - 1(z+1) = 0$
$\Rightarrow -7x - 11y - z = -14 - 11 + 1$
$\Rightarrow 7x + 11y + z = 24$ comparing this to
$\alpha x + \beta y + \gamma z = 24$
We get, $\alpha = 7, \beta = 11, \gamma = 1$
$\therefore \alpha + \beta + \gamma = 7 + 11 + 1 = 19$

Question165

Suppose the line $\frac{x-2}{\alpha} + \frac{y-2}{-5} = \frac{z+2}{2}$ lies on the plane $x + 3y - 2z + \beta = 0$. Then, $(\alpha + \beta)$ is equal to
[31 Aug 2021 Shift 2]

Answer: 7

Solution:

Given equation of line
$\frac{x-2}{\alpha}=\frac{y-2}{-5}=\frac{z+2}{2}$ ...(i)
and plane $x+3y-2z+\beta=0$ ...(ii)
Line (i) pases through $(2, 2, -2)$
which lies on plane (ii).
$\therefore 2+6+4+\beta=0$
$\Rightarrow \beta=-12$
Also, given line is perpendicular to normal of the plane
$\alpha(1)-5(3)+2(-2)=0$
$\Rightarrow \alpha=19$
$\therefore \alpha+\beta=7$

Question166

Let the equation of the plane, that passes through the point $(1, 4, -3)$ and contains the line of intersection of the planes $3x-2y+4z-7=0$ and $x+5y-2z+9=0$ be $\alpha x+\beta y+\gamma z+3=0$, then $\alpha+\beta+\gamma$ isequal to [31 Aug 2021 Shift 1]

Options:

A. -23

B. -15

C. 23

D. 15

Answer: A

Solution:

Solution:
Equation of plane is
$(3x-2y+4z-7)+\lambda(x+5y-2z+9)=0$
$(\lambda+3)x+(5\lambda-2)y+(4-2\lambda)z+9\lambda-7=0$
Passing through $(1, 4, -3)$
$(\lambda+3)+4(5\lambda-2)-3(4-2\lambda)+9\lambda-7=0$
$\Rightarrow 36\lambda-24=0$
$\lambda=\frac{2}{3}$
$\Rightarrow$ Equation of plane
$\left(\frac{2}{3}+3\right)x+\left(\frac{10}{3}-2\right)y+\left(4-\frac{4}{3}\right)z+6-7=0$
$\Rightarrow 11x+4y+8z-3=0$
$\alpha=-11, \beta=-4, \gamma=-8$
$\alpha+\beta+\gamma=-23$

Question167

The distance of the point $(1, -2, 3)$ from the plane $x-y+z=5$ measured parallel to a line, whose direction ratios are $2, 3, -6$ is [27 Aug 2021 Shift 1]

Options:

A. 3

B. 5

C. 2

D. 1

Answer: D

Solution:

Solution:
Let A be any point on the plane $x - y + z = 5$ and $B(1, -2, 3)$.
Then equation of the line AB whose direction ratios are $2, 3, -6$
$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$ (Let)
$\Rightarrow x = 1 + 2\lambda, y = -2 + 3\lambda, z = 3 - 6\lambda$
$A(1 + 2\lambda, -2 + 3\lambda, 3 - 6\lambda)$
A lies on plane.
Then, $1 + 2\lambda - (-2 + 3\lambda) + 3 - 6\lambda = 5$
$\Rightarrow 1 + 2\lambda + 2 - 3\lambda + 3 - 6\lambda = 5$
$\Rightarrow \lambda = \frac{1}{7}$
$\therefore A\left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$
$\text{Distance AB} = \sqrt{\left(1 - \frac{9}{7}\right)^2 + \left(-2 + \frac{11}{7}\right)^2 + \left(3 - \frac{15}{7}\right)^2}$
$= \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$

Question168

Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes $x - y - z - 1 = 0$ and $2x + y - 3z + 4 = 0$, is [27 Aug 2021 Shift 1]

Options:

A. $3x - y - 5z + 2 = 0$

B. $3x - 4z + 3 = 0$

C. $-x + 2y + 2z - 3 = 0$

D. $4x - y - 5z + 2 = 0$

Answer: D

Solution:

Given planes,
$x - y - z - 1 = 0$ ...(i)
$2x + y - 3z + 4 = 0$ ...(ii)
Equation of plane passing through line of intersection of planes (i) and (ii) is given by

$(x - y - z - 1) + \lambda(2x + y - 3z + 4) = 0$
$\Rightarrow (2\lambda + 1)x + (\lambda - 1)y + (-3\lambda - 1)z + (4\lambda - 1) = 0 \ldots$ (iii)
Distance of plane (iii) from origin $= \frac{2}{21}$(given)
$\Rightarrow \frac{|4\lambda - 1|}{\sqrt{(2\lambda + 1)^2 + (\lambda - 1)^2 + (-3\lambda + 1)^2}} = \sqrt{\frac{2}{21}}$
Squaring both sides
$\frac{(4\lambda - 1)^2}{(2\lambda + 1)^2 + (\lambda - 1)^2 + (3\lambda + 1)^2} = \frac{2}{21}$
$\Rightarrow 21(16\lambda^2 - 8\lambda + 1) = 2(14\lambda^2 + 8\lambda + 3)$
$\Rightarrow 308\lambda^2 - 184\lambda + 15 = 0$
$308\lambda^2 - 154\lambda - 30\lambda + 15 = 0$
$(2\lambda - 1)(154\lambda - 15) = 0$
$\Rightarrow \lambda = \frac{1}{2}$ or $\lambda = \frac{15}{154}$
Putting $\lambda = \frac{1}{2}$ in Eq. (iii), we have
$4x - y - 5z + 2 = 0$

Question169

The equation of the plane passing through the line of intersection of the planes $r.\left(\hat{i} + \hat{j} + \hat{k}\right) = 1$ and $r.\left(2\hat{i} + 3\hat{j} - \hat{k}\right) + 4 = 0$ and parallel to the X-axis is [27 Aug 2021 Shift 2]

Options:

A. $r.\left(\hat{j} - 3\hat{k}\right) + 6 = 0$

B. $r.\left(\hat{i} + 3\hat{k}\right) + 6 = 0$

C. $r.\left(\hat{i} - 3\hat{k}\right) + 6 = 0$

D. $r.\left(\hat{j} - 3\hat{k}\right) - 6 = 0$

Answer: A

Solution:

Solution:
Given, equation of planes
$r.\left(\hat{i} + \hat{j} + \hat{k}\right) = 1$ ...(i)
$r.\left(2\hat{i} + 3\hat{j} - \hat{k}\right) + 4 = 0$ ...(ii)
Equation of plane passing through the intersection of the planes Eqs. (i) and (ii) is given by
$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$
or $(1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + (-1 + 4\lambda) = 0$ ...(iii)
Plane (iii) in parallel to X-axis
$1 + 2\lambda = 0$ [Coefficient of x = 0]
$\Rightarrow \lambda = \frac{-1}{2}$
∴ From Eq. (iii) becomes
$y - 3z + 6 = 0$
or $r.\left(\hat{j} - 3\hat{k}\right) + 6 = 0$

Question170

Let S be the mirror image of the point Q (1, 3, 4) with respect to the plane$2x - y + z + 3 = 0$ and let R(3, 5, γ) be a point of this plane. Then the square of the length of the line segment SR is
[27 Aug 2021 Shift 2]

Answer: 72

Solution:

Let point S(a, b, c)
Then,
$\frac{a-1}{2} = \frac{b-3}{-1} = \frac{c-4}{1} = \frac{-2(2-3+4+3)}{4+1+1} = -2$
$\Rightarrow a = -3, b = 5, c = 2$
$\therefore$ S $(-3, 5, 2)$
and point R(3, 5, γ) lies on the plane $2x - y + z + 3 = 0$
$\Rightarrow 6 - 5 + \gamma + 3 = 0$
$\Rightarrow \gamma = -4$
$\therefore$ R $(3, 5, -4)$
Now, $SR^2 = 6^2 + 0^2 + (6)^2$
$= 36 + 0 + 36 = 72$

Question171

A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$ and is perpendicular to the plane $-2x + y + z + 8 = 0$. Then which of the following points lies on P?
[26 Aug 2021 Shift 1]

Options:

A. $(-1, 1, 2)$

B. $(0, 1, 1)$

C. $(1, 0, 1)$

D. $(2, -1, 1)$

Answer: B

Solution:

Equation of plane containing the given planes is
$(x + 2y + 3z + 1) + \lambda(x - y - z - 6) = 0$
$(1 + \lambda)x + (2 - \lambda)y + (3 - \lambda)z + (1 - 6\lambda) = 0$
This plane is perpendicular to the plane $-2x + y + z + 8 = 0$
So, $-2(1 + \lambda) + (2 - \lambda) + (3 - \lambda) = 0$
$-2 - 2\lambda + 2 - \lambda + 3 - \lambda = 0$

$\lambda = \frac{3}{4}$
So, equation of required plane is $7x + 5y + 9z = 14$
Now, $(0, 1, 1)$ satisfies the above plane.

Question172

Let P be the plane passing through the point (1, 2, 3) and the line of intersection of the planes $r \cdot (\hat{i} + \hat{j} + 4\hat{k})$ and $r \cdot (-\hat{i} + \hat{j} + \hat{k}) = 6$ Then which of the following points does not lie on P ? [26 Aug 2021 Shift 2]

Options:

A. $(3, 3, 2)$

B. $(6, -6, 2)$

C. $(4, 2, 2)$

D. $(-8, 8, 6)$

Answer: C

Solution:

P is a plane passing through the intersection of P_1 and P_2.
Equation of $P : P_1 + \lambda P_2 = 0$
$(x + y + 4z - 16) + \lambda(-x + y + z - 6) = 0$ ...(i)
Since plane P passes through $(1, 2, 3)$, then
$(1 + 2 + 12 - 16) + \lambda(-1 + 2 + 3 - 6) = 0$
$\Rightarrow -1 + \lambda(-2) = 0$
$\Rightarrow \lambda = \frac{-1}{2}$
On putting $\lambda = \frac{-1}{2}$ in Eq. (i), we get
$P : 3x + y + 7z - 26 = 0$
Clearly $(4, 2, 2)$ not lie on the plane.

Question173

Let the line L be the projection of the line $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$ in the plane $x - 2y - z = 3$.If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to [26 Aug 2021 Shift 1]

Answer: 26

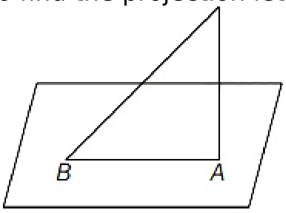
Solution:

Solution:

Given line, $\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$

Given plane, $x-2y-z=3$

To find the projection let's find the foot of perpendicular from $(1, 3,, 4)$ to plane $x-2y-z=3$



$\frac{x-1}{1}=\frac{y-3}{-2}=\frac{z-4}{-1}=\lambda_1$

$(\lambda_1+1)-2(-2\lambda_1+3)-(-\lambda_1+4)=3$

$\Rightarrow 6\lambda_1=12$

$\Rightarrow \lambda_1=2$

So, foot of perpendicular from (1, 3, 4) to plane $x-2y-z=3$ is A $(3,-1,2)$.

Let us also find the intersection point of the plane and line

$\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}=\lambda_2$

$(2\lambda_2+1)-2(\lambda_2+3)-(2\lambda_2+4)=3-2\lambda_2 \ =12$

$\Rightarrow \lambda_2=-6$

The intersection point of the plane and line is $B(-11,-3,-8)$

Line passing through A and B is

$\frac{x-3}{-14}=\frac{y+1}{-2}=\frac{z-2}{-10}=\mu$

$\frac{x-3}{7}=\frac{y+1}{1}=\frac{z-2}{5}=\mu$

Now, let's find the distance from O(0, 0, 6) to this line L.

Let's say $C(7\mu+3,\mu-1,5\mu+2)$ is any point on L.

Then,

$\{(7\mu+3)-0\}.7+\{(\mu-1)-0\}.1+\{(5\mu+2)-6\}.5=0$

$\Rightarrow 49\mu+21+\mu-1+25\mu-20=0$

$\Rightarrow \mu=0$

$C(3,-1,2)$

Distance $=\sqrt{(3-0)^2+(-1-0)^2+(2-6)^2}=\sqrt{26}$

$d^2=26$

Question174

Let Q be the foot of the perpendicular from the point $P(7,-2,13)$ on the plane containing the lines $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x+1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$. Then $(PQ)^2$ is equal to [26 Aug 2021 Shift 2]

Answer: 96

Solution:

Plane containing the lines would be

$$\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix}=0$$

$\Rightarrow (x+1)(49-40)-(y-1)(42-24)+(z-3)(30-21)=0$
$\Rightarrow 9(x+1)-18(y-1)+9(z-3)=0$
$\Rightarrow x-2y+z=0$
Now, PQ will be equal to the perpendicular distance of the point $P(7,-2,13)$ from the plane $x-2y+z=0$

$$\therefore PQ = \left|\frac{7-2(-2)+13}{\sqrt{1^2+(-2)^2+(1)^2}}\right|$$

$$= \left|\frac{7+4+13}{\sqrt{1+4+1}}\right| = \left|\frac{24}{\sqrt{6}}\right| = 4\sqrt{6}$$

$PQ^2 = (4\sqrt{6})^2 = 16 \times 6 = 96$

Question175

The distance of line $3y-2z-1=0 \; = 3x-z+4$ from the point $(2,-1,6)$ is
[1 Sep 2021 Shift 2]

Options:

A. $\sqrt{26}$

B. $2\sqrt{5}$

C. C. $2\sqrt{6}$

D. $4\sqrt{2}$

Answer: C

Solution:

Solution:
Equation of line
$3y-2z-1=0=3x-z+4$

$$\Rightarrow \frac{3y-1}{2} = \frac{z-0}{1} = \frac{3x+4}{1}$$

$$\Rightarrow \frac{x+\frac{4}{3}}{1/3} = \frac{y-\frac{1}{3}}{2/3} = \frac{z-0}{1}$$

$$PR = |PQ|\cos\theta = |PQ|\frac{PQ.P}{|PQ||P|} = \frac{PQ.PQ}{|PR|}$$

$$PR = \left|\frac{\frac{1}{3}\left(2+\frac{4}{3}\right)+\frac{2}{3}\left(-1-\frac{1}{3}\right)+1(6-0)}{\sqrt{\frac{1}{9}+\frac{4}{9}+1}}\right| = 4\sqrt{\frac{14}{9}}$$

$OR^2 = PQ^2 - PR^2$

$$= \frac{100}{9}+\frac{16}{9}+36-\frac{224}{9}$$

$$= \frac{100}{9}+\frac{16}{9}+36-\frac{224}{9}$$

$QR = \sqrt{24} = 2\sqrt{6}$

Question176

Let the acute angle bisector of the two planes $x-2y-2z+1=0$ and $2x-3y-6z+1=0$ be the plane P. Then, which of the following points lies on P ?

[1 Sep 2021 Shift 2]

Options:

A. $\left(3, 1, -\frac{1}{2}\right)$

B. $\left(-2, 0, -\frac{1}{2}\right)$

C. $(0, 2, -4)$

D. $(4, 0, -2)$

Answer: B

Solution:

Equation of angle bisectors

$\frac{x-2y-2z+1}{\sqrt{1+4+4}} = \pm\frac{2x-3y-6z+1}{\sqrt{4+9+36}}$

$\Rightarrow x-5y+4z+4 \ = 0$ and $13x-23y-32z+10 \ = 0$

Then, $\cos\theta = \frac{1+10-8}{\sqrt{1+4+4}\sqrt{1+25+16}} \ = \frac{1}{\sqrt{42}}$

$\Rightarrow \tan\theta = \sqrt{41} > 1$

$\Rightarrow \theta > 45°$

Then, acute angle bisector in plane

$P : 13x - 23y - 32z + 10 = 0$

$\therefore$ Point $\left(-2, 0, \frac{-1}{2}\right)$ lies on the plane P.

Question177

The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is:

[Jan. 08, 2020 (I)]

Options:

A. $2\sqrt{30}$

B. $\frac{7}{2}\sqrt{30}$

C. $3\sqrt{30}$

D. 3

Answer: C

Solution:

$\overrightarrow{AB} = 6\hat{i} + 15\hat{j} + 3\hat{k}$

$\vec{p} = \hat{i} + 4\hat{j} + 22\hat{k}$

$\vec{q} = \hat{i} + \hat{j} + 7\hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 22 \\ 1 & 1 & 7 \end{vmatrix} = 6\hat{i} + 15\hat{j} - 3\hat{k}$$

Shortest distance between the lines is

$$= \frac{\left|\vec{AB} \cdot (\vec{p} \times \vec{q})\right|}{\left|\vec{p} \times \vec{q}\right|} = \frac{|36 + 225 + 9|}{\sqrt{36 + 225 + 9}} = 3\sqrt{30}$$

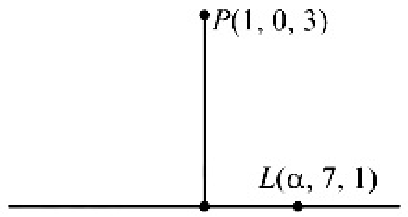
Question178

If the foot of the perpendicular drawn from the point (1,0,3) on a line passing through (α, 7, 1) is, then α is equal to _______. [NAJan.07, 2020 (II)]

Answer: 4

Solution:

Since, PQ is perpendicular to L



$$\therefore \left(1 - \frac{5}{3}\right)\left(\alpha - \frac{5}{3}\right) + \left(\frac{-7}{3}\right)\left(7 - \frac{7}{3}\right) + \left(3 - \frac{17}{3}\right)\left(1 - \frac{17}{3}\right) = 0$$

$$\Rightarrow \frac{-2\alpha}{3} + \frac{10}{9} - \frac{98}{9} + \frac{112}{9} = 0$$

$$\Rightarrow \frac{2\alpha}{3} = \frac{24}{9} \Rightarrow \alpha = 4$$

Question179

If for some α and β in R, the intersection of the following three planes $x + 4y - 2z = 1$ $x + 7y - 5z = \beta$ $x + 5y + \alpha z = 5$ is a line in R^3, then α + β is equal to: [Jan. 9, 2020 (I)]

Options:

A. 0

B. 10

C. 2

D. –10

Answer: B

Solution:

Solution:

$$\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$$

$\Rightarrow (7\alpha + 25) - (4\alpha + 10) + (-20 + 14) = 0$
$\Rightarrow 3\alpha + 9 = 0 \Rightarrow \alpha = -3$

$$\text{Also, } D_z = 0 \Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0$$

$\Rightarrow 1(35 - 5\beta) - (15) + 1(4\beta - 7) = 0 \Rightarrow \beta = 13$
Hence, $\alpha + \beta = -3 + 13 = 10$

Question180

If the distance between the plane, $23x - 10y - 2z + 48 = 0$and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$($\lambda \in R$)is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ________.
[NA Jan. 9, 2020 (II)]

Answer: 3

Solution:

Solution:
Since, the line $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ contains the point (-1,3,-1) and line $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ contains the point(-3,-2,1)
Then, the distance between the plane$23x - 10y - 2z + 48 = 0$ and the plane containing the lines= perpendicular distance of plane
$23x - 10y - 2z + 48 = 0$ either from (-1,3,-1) or (-3,-2,1)

$$= \left| \frac{23(-1) - 10(3) - 2(-1)}{\sqrt{(23)^2 + (10)^2 + (-2)^2}} \right| = \frac{3}{\sqrt{633}}$$

It is given that distance between the planes

$$= \frac{k}{\sqrt{633}} \Rightarrow \frac{k}{\sqrt{633}} = \frac{3}{\sqrt{633}} \Rightarrow k = 3$$

Question181

The mirror image of the point (1,2,3) in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane?
[Jan. 8, 2020 (II)]

Options:

A. (1, 1, 1)

B. (1, –1, 1)

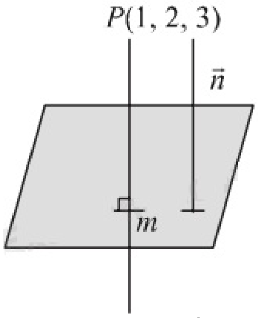
C. (–1, –1, 1)

D. (–1, –1, –1)

Answer: B

Solution:

Solution:



$\vec{n} = \frac{-7}{3} - 1, \frac{-4}{3} - 2, \frac{-1}{3} - 3$

$\vec{n} = \frac{10}{3}, \frac{10}{3}, \frac{10}{3}$

D. r of normal to the plane (1,1,1)

Midpoint of P and Q is $\left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

∴ Equation of required plane Q

$\vec{r} \cdot \hat{n} = \hat{a} \cdot \hat{n}$

$\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = \frac{-2}{3} + \frac{1}{3} + \frac{4}{3}$

∴ Equation of plane is $x + y + z = 1$

Question182

Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is: [Jan. 7, 2020 (I)]

Options:

A. (6, 5, 2)

B. (6, 5, –2)

C. (4, 3, 2)

D. (3, 4, –2)

Answer: B

Solution:

Equation of plane is $x + y - 2z = 3$

$\Rightarrow \frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \frac{-2(2+1-12-3)}{6}$

$\Rightarrow (x, y, z) = (6, 5, -2)$

Question183

A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1,1,2) . Then the equation of the line through this centroid and perpendicular to the plane P is: [Sep. 06, 2020 (II)]

Options:

A. $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

B. $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

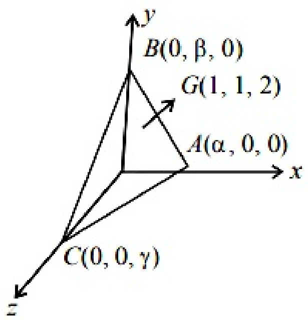
C. $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

D. $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

Answer: C

Solution:

Solution:



$\therefore \alpha = 3, \beta = 3$ and $\gamma = 6$ as G is centroid.

$\therefore$ The equation of plane is

$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$

$\Rightarrow \frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1 \Rightarrow 2x + 2y + z = 6$

$\therefore$ The required line is, $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

Question184

If (a, b, c) is the image of the point (1,2,-3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then $a + b + c$ is equals to: [Sep. 05, 2020 (I)]

Options:

A. 2

B. – 1

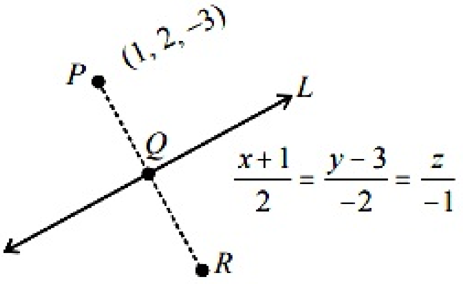
C. 3

D. 1

Answer: A

Solution:

$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$
Any point on line $= Q(2\lambda - 1, -2\lambda + 3, -\lambda)$



$\therefore$ D.r. of PQ $= [2\lambda - 2, -2\lambda + 1, -\lambda + 3]$
D.r. of given line $= [2, -2, -1]$
$\because$PQ is perpendicular to line L
$\therefore 2(2\lambda - 2) - 2(-2\lambda + 1) - 1(-\lambda + 3) = 0$
$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$
$\Rightarrow 9\lambda - 9 = 0 \Rightarrow \lambda = 1$
$\because$Q is mid point of PR $= Q = (1, 1, -1)$
$\therefore$ Coordinate of image $R = (1, 0, 1) = (a, b, c)$
$\therefore a + b + c = 2$

Question185

The lines $\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$ [Sep. 03, 2020 (I)]

Options:

A. do not intersect for any values of l and m

B. intersect for all values of l and m

C. intersect when $l = 2$ and $m = \frac{1}{2}$

D. intersect when $l = 1$ and $m = 2$

Answer: A

Solution:

Solution:
$L_1 \equiv \vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$
$L_2 \equiv \vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$
Equating coeff. of $\hat{i}$, $\hat{j}$ and $\hat{k}$ of L_1 and L_2

$2l + 1 = m + 2$ ......(i)
$-1 = -1 + m \Rightarrow m = 0$ ....(ii)
$l = -m$ ....(iii)
$\Rightarrow m = l = 0$ which is not satisfy eqn. (i) hence lines do not intersect for any value of l and m.

Question186

The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x + y + z + 1 = 0, 2x - y + z + 3 = 0$ is :
[Sep. 06, 2020 (I)]

Options:

A. 1

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2}$

Answer: B

Solution:

Solution:
For line of intersection of planes $x + y + z + 1 = 0$ and $2x - y + z + 3 = 0$:

$$\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 2\hat{i} + \hat{j} - 3\hat{k}$$

Put $y = 0$, we get $x = -2$ and $z = 1$
$L_2 : \bar{r} = \left(-2\hat{i} + \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - 3\hat{k}\right)$ and $L_1 : \bar{r} = \left(\hat{i} - \hat{j}\right) + \mu\left(-\hat{j} + \hat{k}\right)($ Given)
Now, $\bar{b}_1 \times \bar{b}_2 = -2\left[\hat{i} + \hat{j} + \hat{k}\right]$ and $\bar{a}_2 - oc\, a_1 = -3\hat{i} + \hat{j} + \hat{k}$
$\therefore$ Shortest distance $= \frac{1}{\sqrt{3}}$

Question187

If for some $\alpha \in R$, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2passes through the point :
[Sep. 05, 2020 (II)]

Options:

A. (10,2,2)

B. (2,-10,-2)

C. (10,-2,-2)

D. (-2,10,2)

Answer: B

Solution:

Solution:

$$\therefore \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(-1-5+\alpha) - 3(2-\alpha) + 2(10-2\alpha+\alpha) = 0$
$\therefore \alpha = -4$
$\therefore$ Equation of $L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$
$\therefore$ Point (2,-10,-2) lies on line L_2

Question188

If the equation of a plane P, passing through the intersection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for some $a, b \in R$, then the distance of the point (3,2,-1) from the plane P is ________.
[Sep. 04, 2020 (I)]

Answer: 3

Solution:

Solution:
Equation of plane P is
$(x + 4y - z + 7) + \lambda(3x + y + 5z - 8) = 0$
$\Rightarrow x(1 + 3\lambda) + y(4 + \lambda) + z(-1 + 5\lambda) + (7 - 8\lambda) = 0$
$\Rightarrow \frac{1+3\lambda}{a} = \frac{4+\lambda}{b} = \frac{5\lambda-1}{6} = \frac{7-8\lambda}{-15}$
From last two ratios, $\lambda = -1$
$\Rightarrow \frac{-2}{a} = \frac{3}{b} = -1$
$\therefore a = 2, b = -3$
$\therefore$ Equation of plane is, $2x - 3y + 6z - 15 = 0$
Distance $= \frac{|6-6-6-15|}{7} = \frac{21}{7} = 3.$

Question189

The distance of the point (1,-2,3) from the plane$x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is:
[NA Sep. 04, 2020 (II)]

Options:

A. $\frac{7}{5}$

B. 1

C. $\frac{1}{7}$

D. 7

Answer: B

Solution:

Solution:

Equation of line through point $P(1, -2, 3)$ and parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$

So, any point on line $= Q(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

Since, this point lies on plane $x - y + 2 = 5$

$\therefore 2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5 \Rightarrow \lambda = \frac{1}{7}$

$\therefore$ Point of intersection line and plane, $Q = \left(\frac{9}{7}, \frac{11}{7}, \frac{15}{7}\right)$

$\therefore$ Required distance PQ

$= \sqrt{\left(\frac{9}{7} - 1\right)^2 + \left(-\frac{11}{7} + 2\right)^2 + \left(\frac{15}{7} - 3\right)^2} = 1$

Question190

The foot of the perpendicular drawn from the point (4,2,3) to the line joining the points (1,-2,3) and (1,1,0) lies on the plane: [Sep. 03, 2020 (I)]

Options:

A. $2x + y - z = 1$

B. $x - y - 2z = 1$

C. $x - 2y + z = 1$

D. $x + 2y - z = 1$

Answer: A

Solution:

Equation of line through points (1,-2,3) and (1,1,0) is

$\frac{x-1}{0} = \frac{y-1}{-3} = \frac{z-0}{3-0}$($= \lambda$ say)
$\therefore M(1, -\lambda + 1, \lambda)$
Direction ratios of PM $= [-3, -\lambda - 1, \lambda - 3]$
$\because PM \perp AB$
$\therefore (-3) . 0 + (-1 - \lambda)(-1) + (\lambda - 3) . 1 = 0$
$\therefore \lambda = 1$
$\therefore$ Foot of perpendicular $= (1, 0, 1)$
This point satisfy the plane $2x + y - z = 1$

Question191

The plane which bisects the line joining the points (4, – 2, 3) and (2, 4, – 1) at right angles also passes through the point: [Sep. 03, 2020 (II)]

Options:

A. (4, 0, 1)

B. (0, –1, 1)

C. (4, 0, –1)

D. (0, 1, –1)

Answer: C

Solution:

Direction ratios of normal to the plane are $<1, -3, 2>$.
Plane passes through (3,1,1) .
Equation of plane is,
$1(x - 3) - 3(y - 1) + 2(z - 1) = 0$
$\Rightarrow x - 3y + 2z - 2 = 0$

Question192

Let a plane P contain two lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in R$ and $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in R$ If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point M (1, 0, 1) to P, then $3(\alpha + \beta + \gamma)$ equals _______. [NA Sep. 03, 2020 (II)]

Answer: 5

Solution:

Normal of plane = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$\vec{n} = -\hat{i} + \hat{j} + \hat{k}$
Direction ratios of normal to the plane = $< -1, 1, 1 >$
Equation of plane
$-1(x-1) + 1(y-0) + 1(z-0) = 0$
$\Rightarrow x - y - z - 1 = 0$
If (x, y, z) is foot of perpendicular of $M(1, 0, 1)$ on the plane then
$\Rightarrow \frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = \frac{-(1-0-1-1)}{3}$
$\therefore x = \frac{4}{3}, y = -\frac{1}{3}, z = \frac{2}{3}$
$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$
$\therefore 3(\alpha + \beta + \gamma) = 3 \times \frac{5}{3} = 5$

Question193

The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, 2x = 3y, z = 1 also through the point : [Sep. 02, 2020 (I)]

Options:

A. (0, 6, –2)

B. (–2, 0, 1)

C. (0, –6, 2)

D. (2, 0, –1)

Answer: B

Solution:

Let plane passes through (2,1,2) be
$a(x-2) + b(y-1) + (z-2) = 0$
It also passes through (1,2,1)
$\therefore -a + b - c = 0 \Rightarrow a - b + c = 0$
The given line is
$\frac{x}{3} = \frac{y}{2} = \frac{z-1}{0}$ is parallel to plane
$\therefore 3a + 2b + c(0) = 0$
$\Rightarrow \frac{a}{0-2} = \frac{b}{3-0} = \frac{c}{2+3}$
$\Rightarrow \frac{a}{2} = \frac{b}{-3} = \frac{c}{2+3}$
$\Rightarrow \frac{a}{2} = \frac{b}{-3} = \frac{c}{-5}$
$\therefore$ plane is $2x - 4 - 3y + 3 - 5z + 10 = 0$
$\Rightarrow 2x - 3y - 5z + 9 = 0$
The plane satisfies the point (-2,0,1) .

Question194

A plane passing through the point (3, 1, 1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point (α , -3, 5), then α is equal to :
[Sep. 02, 2020 (II)]

Options:

A. 5

B. –10

C. 10

D. –5

Answer: A

Solution:

∵ Plane contains two lines

$\therefore \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix}$

$= \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$
$= -4\hat{i} + 5\hat{j} + 7\hat{k}$
So, equation of plane is
$-4(x-3) + 5(y-1) + 7(z-1) = 0$
$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$
$\Rightarrow -4x + 5y + 7z = 0$
This also passes through $(\alpha, -3, 5)$
So, $-4\alpha - 15 + 35 = 0$
$\Rightarrow -4\alpha = -20 \Rightarrow \alpha = 5$

Question195

Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point- (-1,-1,1) . Then S is equal to:
[Jan. 12, 2019 (II)]

Options:

A. $\{\sqrt{3}\}$

B. $\{\sqrt{3}, -\sqrt{3}\}$

C. $\{1, -1\}$

D. $\{3, -3\}$

Answer: B

Solution:

Let $A(-\lambda^2, 1, 1), B(1, -\lambda^2, 1), C(1, 1, -\lambda^2), D(-1, -1, 1)$
lie on same plane, then

$$\begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & 1-\lambda^2 & 0 \\ 2 & 2 & -\lambda^2-1 \end{vmatrix} = 0$$

$\Rightarrow(\lambda^2+1)((1-\lambda^2)^2-4) = 0$
$\Rightarrow(3-\lambda^2)(\lambda^2+1) = 0 \Rightarrow \lambda^2 = 3$
$\lambda = \pm\sqrt{3}$
Hence, $S = \{-\sqrt{3}, \sqrt{3}\}$

Question196

The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points? [Jan. 11, 2019 (I)]

Options:

A. (2,2,0)

B. (-2,2,2)

C. (0,-2,2)

D. (2,0,-2)

Answer: D

Solution:

Let normal to the required plane is $\vec{n}$
$\Rightarrow \vec{n}$ is perpendicular to both vector $2\hat{i} - \hat{j} + 3\hat{k}$ and $2\hat{i} + 3\hat{j} - 3\hat{k}$

$$\Rightarrow \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 2 & 3 & -1 \end{vmatrix} = -8\hat{i} + 8\hat{j} + 8\hat{k}$$

$\Rightarrow$ Equation of the required plane is
$\Rightarrow(x-3)(-8) + (y+2) \times 8 + (z-1) \times 8 = 0$
$\Rightarrow(x-3)(-1) + (y+2) \times 1 + (z-1) \times 1 = 0$
$\Rightarrow x - 3 - y - 2 - z + 1 = 0$
$\because x - y - z = 4$ passes through (2,0,-2)
$\therefore$ plane contains (2,0,-2)

Question197

The direction ratios of normal to the plane through the points (0,-1,0) and (0,0,1) and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$ are: [Jan. 11, 2019 (I)]

Options:

A. 2,-1,1

B. $2, \sqrt{2}, -\sqrt{2}$

C. $\sqrt{2}, 1, -1$

D. $2\sqrt{3}, 1, -1$

Answer: B & D

Solution:

(b,c) Let the d.r's of the normal be $\langle a, b, c\rangle$
Equation of the plane is
$a(x-0)+b(y+1)+c(z-0)=0$
∵ It passes through (0,0,1)
$\therefore b+c=0$
Also $\frac{0 . a+b-c}{\sqrt{a^2+b^2+c^2} . \sqrt{2}} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$
$\Rightarrow b-c = \sqrt{a^2+b^2+c^2}$
And $b+c=0$
$\Rightarrow b = \pm\frac{1}{\sqrt{2}}a$
∴ The d.r's are $\sqrt{2}, 1, -1$ or $2, \sqrt{2}, -\sqrt{2}$

Question198

If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points (3,4,2) and (7,0,6) and is perpendicular to the plane $2x-5y=15$, then $2\alpha-3\beta$ is equal to :
[Jan. 11, 2019(II)]

Options:

A. 12

B. 7

C. 5

D. 17

Answer: B

Solution:

Let the normal to the required plane is $\vec{n}$, then

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -4 & 4 \\ 2 & -5 & 0 \end{vmatrix} = 20\hat{i}+8\hat{j}-12\hat{k}$$

∴ Equation of the plane
$(x-3)\times 20+(y-4)\times 8+(z-2)\times(-12)=0$
$5x-15+2y-8-3z+6=0$
$5x+2y-3z-17=0$ ....(1)

Since, equation of plane (1) passes through (2, α, β), then $10 + 2\alpha - 3\beta - 17 = 0 \Rightarrow 2\alpha - 3\beta = 7$

Question199

The plane which bisects the line segment joining the points (– 3, – 3, 4) and (3, 7, 6) at right angles, passes through which one of the following points?
[Jan. 10, 2019 (II)]

Options:

A. (–2, 3, 5)

B. (4, – 1, 7)

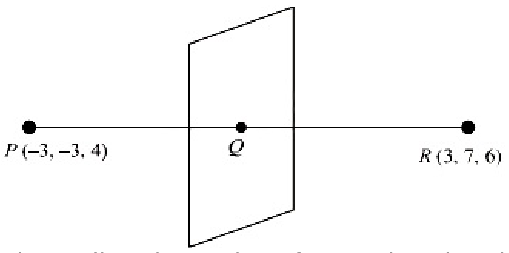
C. (2, 1, 3)

D. (4, 1, – 2)

Answer: D

Solution:

Solution:



Since, direction ratios of normal to the plane is
$\vec{n} = 6\hat{i} + 10\hat{j} + 2\hat{k}$
Then, equation of the plane is
$(x - 0)6 + (y - 2)10 + (z - 5)2 = 0$
$3x + 5y - 10 + z - 5 = 0$
$3x + 5y + z = 15$
Since, plane (1) satisfies the point (4,1,-2)
Hence, required point is (4,1,-2)

Question200

On which of the following lines lies the point of in-ter-section of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $x + y + z = 2$?
[Jan. 10, 2019 (II)]

Options:

A. $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

B. $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

C. $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$

D. $\frac{x-2}{2}=\frac{y-3}{2}=\frac{z+3}{3}$

Answer: C

Solution:

Solution:
Let any point on the line $\frac{x-4}{2}=\frac{y-5}{2}=\frac{z-3}{1}$ be$A(2\lambda+4, 2\lambda+5, \lambda+3)$ which lies on the plane $x+y+z=2$
$\Rightarrow 2\lambda+4+2\lambda+5+\lambda+3=2$
$\Rightarrow 5\lambda=-10 \Rightarrow \lambda=-2$
Then, the point of intersection is (0,1,1)
which lies on the line $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$

Question201

The system of linear equations
$x+y+z=2$
$2x+3y+2z=5$
$2x+3y+(a^2-1)z=a+1$
[Jan 09 2019I]

Options:

A. is inconsistent when $a=4$

B. has a unique solution for $|a|=\sqrt{3}$

C. has infinitely many solutions for $a=4$

D. is inconsistent when $|a|=\sqrt{3}$

Answer: D

Solution:

Solution:
Since the system of linear equations are
$x+y+z=2$ .....(1)
$2x+3y+2z=5$ .....(2)
$2x+3y+(a^2-1)z=a+1$ .....(1)

Now, $\Delta=\begin{vmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ 2 & 3 & a^2-1\end{vmatrix}$

(Applying $R_3 \to R_3-R_2$)

$\Rightarrow \Delta=\begin{vmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ 0 & 0 & a^2-3\end{vmatrix}$

$=a^2-3$
When, $\Delta=0 \Rightarrow a^2-3=0 \Rightarrow |a|=\sqrt{3}$
If $a^2=3$, then plane represented by eqn (2) and eqn (3) are parallel.

Hence, the given system of equation is inconsistent.

Question202

The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:
[Jan 09 2019I]

Options:

A. $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

B. $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

C. $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

D. $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

Answer: C

Solution:

Solution:
Let any point on the intersecting line

$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1} = \lambda$ (say)
is $(-3\lambda - 1, 2\lambda + 3, -\lambda + 2)$
Since, the above point lies on a line which passes through the point (-4,3,1)
Then, direction ratio of the required line
$= < -3\lambda - 1 + 4, 2\lambda + 3 - 3, -\lambda + 2 - 1 >$
or $< -3\lambda + 3, 2\lambda, -\lambda + 1 >$
Since, line is parallel to the plane
$x + 2y - z - 5 = 0$
Then, perpendicular vector to the line is $\hat{i} + 2\hat{j} - \hat{k}$
Now $(-3\lambda + 3)(1) + (2\lambda)(2) + (-\lambda + 1)(-1) = 0$
$\Rightarrow \lambda = -1$
Now direction ratio of the required line $= < 6, -2, 2 >$ or $< 3, -1, 1 >$
Hence required equation of the line is
$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

Question203

The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y -axis also passes through the point:
[Jan 09 2019I]

Options:

A. (-3,0,-1)

B. (-3,1,1)

C. (3,3,-1)

D. (3,2,1)

Answer: D

Solution:

Solution:
Since, equation of plane through intersection of planes
$x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is
$(2x + 3y - z + 4) + \lambda (x + y + z - 1) = 0$
$(2 + \lambda)x + (3 + \lambda)y + (-1 + \lambda)z + (4 - \lambda) = 0$ ......(1)
But, the above plane is parallel to y -axis then
$(2 + \lambda) \times 0 + (3 + \lambda) \times 1 + (-1 + \lambda) \times 0 = 0$
$\Rightarrow \lambda = -3$
Hence, the equation of required plane is
$-x - 4z + 7 = 0$
$\Rightarrow x + 4z - 7 = 0$
Therefore, (3,2,1) the passes through the point.

Question204

The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is: [Jan. 09, 2019 (II)]

Options:

A. $x - 2y + z = 0$

B. $3x + 2y - 3z = 0$

C. $x + 2y - 2z = 0$

D. $5x + 2y - 4z = 0$

Answer: A

Solution:

Solution:
Let the direction ratios of the plane containing lines
$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is $<a, b, c>$
$\therefore 3a + 4b + 2c = 0$
$4a + 2b + 3c = 0$
$\therefore \frac{a}{12 - 4} = \frac{b}{8 - 9} = \frac{c}{6 - 16}$
$\frac{a}{8} = \frac{b}{-1} = \frac{c}{-10}$
$\therefore$ Direction ratio of plane $= < -8, 1, 10 >$
Let the direction ratio of required plane is $<l, m, n>$
Then $-8l + m + 10n = 0$ ....(1)
and $2l + 3m + 4n = 0$ .... (2)
From (1) and (2),
$\frac{l}{-26} = \frac{m}{52} = \frac{n}{-26}$

$\therefore$ D.R.s are $<1, -2, 1>$
$\therefore$ Equation of plane: $x - 2y + z = 0$

Question205

A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(−1, 1, 2)and O(0, 0, 0). The angle between the faces OPQ and PQR is: [Jan.12, 2019 (I)]

Options:

A. $\cos^{-1}\left(\frac{17}{31}\right)$

B. $\cos^{-1}\left(\frac{19}{35}\right)$

C. $\cos^{-1}\left(\frac{9}{35}\right)$

D. $\cos^{-1}\left(\frac{7}{31}\right)$

Answer: B

Solution:

Solution:
Let $\vec{v}_1$ and $\vec{v}_2$ be the vectors perpendicular to the plane OPQ and PQR respectively.

$$\vec{v}_1 = \overline{PQ} \times \overline{OQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5\hat{i} - \hat{j} - 3\hat{k}$$

$$\vec{v}_2 = \overline{PQ} \times \overline{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = \hat{i} - 5\hat{j} - 3\hat{k}$$

$$\because \cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1||\vec{v}_2|} = \frac{5 + 5 + 9}{25 + 1 + 9} = \frac{19}{35}$$

$$\therefore \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

Question206

Two lines $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1}$ and $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$intersect at the point R. The reflection of R in the xy - plane has coordinates : [Jan. 11, 2019 (II)]

Options:

A. (2, –4, –7)

B. (2, 4, 7)

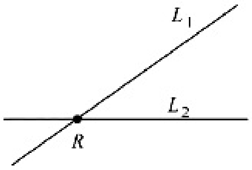
C. (2, –4, 7)

D. (–2, 4, 7)

Answer: A

Solution:

Solution:
Let the coordinate of P with respect to line



$\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1} = \lambda$
$\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4} = \mu$
$L_1 = (\lambda + 3, 3\lambda - 1, -\lambda + 6)$
and coordinate of P w.r.t.
line $L_2 = (7\mu - 5, -6\mu + 2, 4\mu + 3)$
$\therefore \lambda - 7\mu = -8, 3\lambda + 6\mu = 3, \lambda + 4\mu = 3$
From above equation : $\lambda = -1, \mu = 1$
$\therefore$ Coordinate of point of intersection $R = (2, -4, 7)$
Image of R w.r.t. xy plane $= (2, -4, -7)$

Question207

If the lines x = ay + b, z = cy + d and x = a' z + b', y = c' z + d' are perpendicular, then :
[Jan. 09, 2019 (II)]

Options:

A. ab' + bc' + 1 = 0

B. cc' + a + a' = 0

C. bb' + cc' + 1 = 0

D. aa' + c + c' = 0

Answer: D

Solution:

Solution:
First line is: $x = ay + b, z = cy + d$
$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$
and another line is: $x = a'z + b', y = c'z + d'$
$\Rightarrow \frac{x-b'}{a'} = \frac{y-d'}{c'} = \frac{z}{1}$
$\because$ Both lines are perpendicular to each other
$\therefore aa' + c' + c = 0$

Question208

The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is :
[Jan. 12, 2019 (I)]

Options:

A. $11\sqrt{6}$

B. $11/\sqrt{6}$

C. 11

D. $6\sqrt{11}$

Answer: B

Solution:

Solution:
∵ plane containing both lines.

D.R. of plane = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 7\hat{i} - 14\hat{j} + 7\hat{k}$

Now, equation of plane is,
$7(x-1)-14(y-4)+7(z+4)=0$
$\Rightarrow x-1-2y+8+z+4=0$
$\Rightarrow x-2y+z+11=0$
Hence, distance from (0,0,0) to the plane,
$=\frac{11}{\sqrt{1+4+1}}=\frac{11}{\sqrt{6}}$

Question209

If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, $x-2y-kx=3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is
[Jan. 12, 2019 (II)]

Options:

A. $\sqrt{\frac{5}{3}}$

B. $\sqrt{\frac{3}{5}}$

C. $-\frac{3}{5}$

D. $-\frac{5}{3}$

Answer: A

Solution:

Solution:
Let angle between line and plane is θ, then

$\sin\theta = \left| \frac{\vec{b}\,.\,\vec{n}}{|\vec{b}|\,.\,|\vec{n}|} \right|$

$= \left| \frac{(2\hat{i} + \hat{j} - 2\hat{k})\,.\,(\hat{i} - 2\hat{j} - K\hat{k})}{\sqrt{9}\,.\,\sqrt{1 + 4 + K^2}} \right|$

$= \left| \frac{2 - 2\_2K}{3\sqrt{5 + K^2}} \right| = \frac{2|K|}{3\sqrt{4} + K^2}$

Since, $\cos\theta = \frac{2\sqrt{2}}{3} \Rightarrow \sin\theta = \frac{1}{3}$

Then, $\frac{2|K|}{3\sqrt{5} + K^2} = \frac{1}{3} \Rightarrow 4K^2 = 5 + K^2$

$3K^2 = 5 \Rightarrow K = \pm\sqrt{\frac{5}{3}}$

Question210

If the length of the perpendicular from the point $(\beta, 0, \beta)(\beta \neq 0)$ to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to:

[April 10, 2019 (I)]

Options:

A. 1

B. 2

C. –1

D. –2

Answer: C

Solution:

Solution:
Given, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1} = p$(let) and point $P(\beta, 0, \beta)$
Any point on line $A = (p, 1, -p - 1)$
Now, DR of AP a" $<p - \beta, 1 - 0, -p - 1 - \beta>$
Which is perpendicular to line.
$\therefore (p - \beta)1 + 0.1 - 1(-p - 1 - \beta) = 0$

$\Rightarrow p - \beta + p + 1 + \beta = 0 \Rightarrow p = \frac{-1}{2}$

$\therefore$ Point $A\left(\frac{-1}{2}, 1 - \frac{1}{2}\right)$

Given that distance AP = $\sqrt{\frac{3}{2}} \Rightarrow AP^2 = \frac{3}{2}$

$\Rightarrow \left(\beta + \frac{1}{2}\right)^2 + 1 + \left(\beta + \frac{1}{2}\right)^2 = \frac{3}{2}$ or $2\left(\beta + \frac{1}{2}\right)^2 = \frac{1}{2}$

$\Rightarrow \left(\beta + \frac{1}{2}\right)^2 = \frac{1}{4} \Rightarrow \beta = 0, -1, (\beta \neq 0)$
$\therefore \beta = -1$

Question211

The vertices B and C of a "ABC lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, −1, 2), is:
[April 09, 2019 (II)]

Options:

A. $5\sqrt{17}$

B. $2\sqrt{34}$

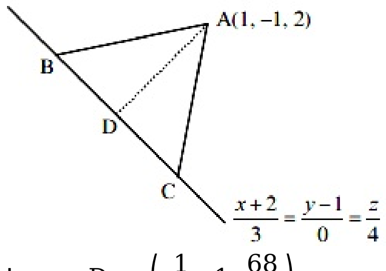
C. 6

D. $\sqrt{34}$

Answer: D

Solution:

Solution:
Let a point D on BC = $(3\lambda - 2, 1, 4\lambda)$
$\overline{AD} = (3\lambda - 3)\hat{i} + 2\hat{j} + (4\lambda - 2)\hat{k}$
$\because \overline{AD} \perp \overline{BC}$,
$\therefore \overline{AD} \cdot \overline{BC} = 0$
$\Rightarrow (3\lambda - 3) + 3 + 2(0) + (4\lambda - 2)4 = 0$
$\Rightarrow \lambda = \frac{17}{25}$



Hence, D = $\left(\frac{1}{25}, 1, \frac{68}{25}\right)$
$|\overline{AD}| = \sqrt{\left(\frac{1}{25} - 1\right)^2 + (2)^2 + \left(\frac{68}{25} - 2\right)^2}$
$= \sqrt{\frac{(24)^2 + 4(25)^2 + (18)^2}{25}} = \sqrt{\frac{3400}{25}} = \frac{2\sqrt{34}}{5}$
Area of triangle $= \frac{1}{2} \times \left|\overline{BC}\right| \times \left|\overline{AD}\right|$
$= \frac{1}{2} \times 5 \times \frac{2\sqrt{34}}{5} = \sqrt{34}$

Question212

If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4)

and Q(8, 0, 10), then distance of R from the origin is :
[April 08, 2019 (II)]

Options:

A. $2\sqrt{14}$

B. $2\sqrt{21}$

C. 6

D. $\sqrt{53}$

Answer: A

Solution:

Solution:
Here, P, Q, R are collinear
$\therefore \overline{PR} = \lambda \overline{PQ}$
$2\hat{i} + (y+3)\hat{j} + (z-4)\hat{k} = \lambda\left[6\hat{i} + 3\hat{j} + 6\hat{k}\right]$
$\Rightarrow 6\lambda = 2, y + 3 = 3\lambda, z - 4 = 6\lambda$
$\Rightarrow \lambda = \frac{1}{3}, y = -2, z = 6$
$\therefore$ Point R$(4, -2, 6)$
Now, OR = $\sqrt{(4)^2 + (-2)^2 + (6)^2} = \sqrt{56} = 2\sqrt{14}$

Question213

A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are : [April 10, 2019 (II)]

Options:

A. (1, 0, 2)

B. (2, 0, 1)

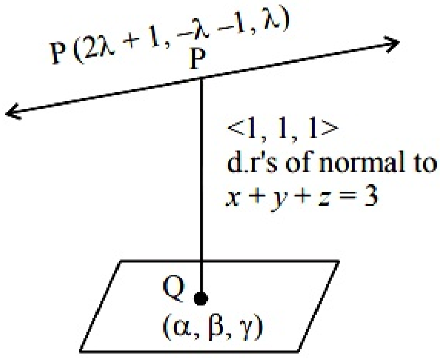
C. (–1, 0, 4)

D. (4, 0, –1)

Answer: B

Solution:

Solution:



Let co-ordinates of Q be (α, β, γ), then
$\alpha + \beta + \gamma = 3$ .....(i)
$\alpha - \beta + \gamma = 3$ .....(ii)
$\Rightarrow \alpha + \gamma = 3$ and $\beta = 0$
Equating direction ratio's of PQ, we get
$\frac{\alpha - 2\lambda - 1}{1} = \frac{\lambda + 1}{1} = \frac{\gamma - \lambda}{1}$
$\Rightarrow \alpha = 3\lambda + 2, \gamma = 2\lambda + 1$
Substituting the values of α and γ in equation (i), we get
$\Rightarrow 5\lambda + 3 = 3 \Rightarrow \lambda = 0$
Hence, point is $Q(2, 0, 1)$

Question214

The length of the perpendicular from the point (2,-1,4) on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is :
[April 08, 2019 (I)]

Options:

A. greater than 3 but less than 4

B. less than 2

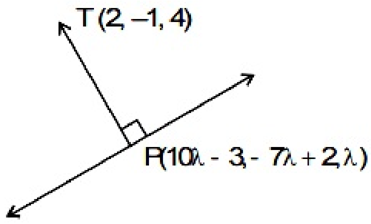
C. greater than 2 but less than 3

D. greater than 4

Answer: A

Solution:

Solution:
Let P be the foot of perpendicular from point $T(2, -1, 4)$ on the given line. So P can be assumed as $P(10\lambda - 3, -7\lambda + 2, \lambda)$



DR's of TP propto to $10\lambda - 5, -7\lambda + 3, \lambda - 4$
$\because TP$ and given line are perpendicular, so $10(10\lambda - 5) - 7(-7\lambda + 3) + 1(\lambda - 4) = 0$
$\Rightarrow \lambda = \frac{1}{2}$
$\Rightarrow TP = \sqrt{(10\lambda - 5)^2 + (-7\lambda + 3)^2 + (\lambda - 4)^2}$
$= \sqrt{0 + \frac{1}{4} + \frac{49}{4}} = \sqrt{12.5} = 3.54$
Hence, the length of perpendicular is greater than 3 but less than 4

Question215

If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the plane$2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to: [April 12, 2019 (I)]

Options:

A. 14

B. $\sqrt{14}$

C. $2\sqrt{7}$

D. $2\sqrt{14}$

Answer: D

Solution:

Solution:
Let points $P(3\lambda + 2, 2\lambda - 1, -\lambda + 1)$ and $Q(3\mu + 2, 2\mu - 1, -\mu + 1)$
$\because$P lies on $2x + 3y - z + 13 = 0$
$\therefore 6\lambda + 4 + 6\lambda - 3 + \lambda - 1 + 13 = 0$
$\Rightarrow 13\lambda = -13 \Rightarrow \lambda = -1$
Hence, $P(-1, -3, 2)$
Similarly, Q lies on $3x + y + 4z = 16$
$\therefore 9\mu + 6 + 2\mu - 1 - 4\mu + 4 = 16$
$\Rightarrow 7\mu = 7 \Rightarrow \mu = 1$
Hence, Q is (5,1,0)
Now, $PQ = \sqrt{36 + 16 + 4} = \sqrt{56} = 2\sqrt{14}$

Question216

A plane which bisects the angle between the two given planes 2x – y + 2z – 4 = 0 and x + 2y + 2z – 2 = 0, passes through the point : [April 12, 2019 (II)]

Options:

A. (1, –4, 1)

B. (1, 4, –1)

C. (2, 4, 1)

D. (2, –4, 1)

Answer: D

Solution:

Solution:
The equations of angle bisectors are,

$x + 2y + 2z - 23 = \pm \frac{2x - y + 2z - 4}{3}$
$\Rightarrow x - 3y - 2 = 0$
or $3x + y + 4z - 6 = 0$
(2,-4,1) lies on the second plane.

Question217

The length of the perpendicular drawn from the point (2, 1,4) to the plane containing the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ is : [April 12, 2019 (II)]

Options:

A. 3

B. $\frac{1}{3}$

C. $\sqrt{3}$

D. $\frac{1}{\sqrt{3}}$

Answer: C

Solution:

Solution:

The equation of plane containing two given lines is, $\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = 0$

On expanding, we get $x - y - z = 0$
Now, the length of perpendicular from (2,1,4) to this plane
$= \left| \frac{2 - 1 - 4}{\sqrt{1^2 + 1^2 + 1^2}} \right| = \sqrt{3}$

Question218

If Q(0, −1, −3) is the image of the point P in the plane $3x - y + 4z = 2$ and R is the point (3, −1, −2), then the area (in sq. units) of Δ PQR is : [April 10, 2019 (I)]

Options:

A. $2\sqrt{13}$

B. $\frac{\sqrt{91}}{4}$

C. $\frac{\sqrt{91}}{2}$

D. $\frac{\sqrt{65}}{2}$

Answer: C

Solution:

Solution:
Image of Q$(0,-1,-3)$ in plane is,
$\frac{(x-0)}{3} = \frac{(y+1)}{-1} = \frac{z+3}{+4} = \frac{-2(1-12-2)}{9+1+16} = 1$
$\Rightarrow x = 3, y = -2, z = 1$
$\Rightarrow P(3,-2,1), Q(0,-1,-3), R(3,-1,-2)$
$\therefore$ Area of ΔPQR is

$$\frac{1}{2}\left|\overrightarrow{QP} \times \overrightarrow{QR}\right| = \frac{1}{2}\left\| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 4 \\ 3 & 0 & 1 \end{vmatrix} \right\|$$

$$= \frac{1}{2}\left| \left\{ \hat{i}(-1) - \hat{j}(3-12) + \hat{k}(3) \right\} \right|$$

$$= \frac{1}{2}\sqrt{(1+81+9)} = \frac{\sqrt{91}}{2}$$

Question219

If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{2}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to:
[April 10,2019 (II)]

Options:

A. 9

B. 15

C. 5

D. 13

Answer: D

Solution:

Solution:
Let, $P_1 : 2x - y + 2z + 3 = 0$

$P_2 : 2x - y + 2z + \frac{\lambda}{2} = 0$

$P_3 : 2x - y + 2z + \mu = 0$

Given, distance between P_1 and P_2 is $\frac{1}{3}$

$$\frac{1}{3} = \frac{\left|3 - \frac{\lambda}{2}\right|}{\sqrt{9}} \Rightarrow \left|3 - \frac{\lambda}{2}\right| = 1 \Rightarrow \lambda_{max} = 8$$

And distance between P_1 and P_3 is $\frac{2}{3}$

$$\frac{2}{3} = \frac{|\mu - 3|}{\sqrt{9}} \Rightarrow \mu_{max} = 5$$

$\Rightarrow (\lambda + \mu)_{max} = 13$

Question220

If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is:
[April 09 2019 I]

Options:

A. $\sqrt{5} / 2$

B. $2\sqrt{5}$

C. $9 / 2$

D. $7 / 2$

Answer: C

Solution:

Solution:
Let point on line be $P(2k + 1, 3k - 1, 4k + 2)$
Since, point P lies on the plane $x + 2y + 3z = 15$
$\therefore 2k + 1 + 6k - 2 + 12k + 6 = 15$
$\Rightarrow k = \frac{1}{2}$
$\therefore P \equiv \left(2, \frac{1}{2}, 4\right)$
Then the distance of the point P from the origin is $OP = \sqrt{4 + \frac{1}{4} + 16} = \frac{9}{2}$

Question221

A plane passing through the points (0,-1,0) and (0,0,1)and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$, also passes through the point:
[April 09 2019 I]

Options:

A. $(-\sqrt{2}, 1, -4)$

B. $(\sqrt{2}, -1, 4)$

C. $(-\sqrt{2}, -1, -4)$

D. $(\sqrt{2}, 1, 4)$

Answer: D

Solution:

Solution:

Let the required plane passing through the points(0,-1,0) and (0,0,1) be $\frac{x}{\lambda} + \frac{y}{-1} + \frac{z}{1} = 1$ and the given plane is $y - z + 5 = 0$

$\therefore \cos\frac{\pi}{4} = \frac{-1-1}{\sqrt{\left(\frac{1}{\lambda^2}+1+1\right)}\sqrt{2}}$

$\Rightarrow \lambda^2 = \frac{1}{2} \Rightarrow \frac{1}{\lambda} = \pm\sqrt{2}$

Then, the equation of plane is $\pm\sqrt{2}x - y + z = 1$

Then the point $(\sqrt{2}, 1, 4)$ satisfies the equation of plane

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Question222

Let P be the plane, which contains the line of intersection of the planes, $x + y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$ and it is perpendicular to the xy - plane. Then the distance of the point (0,0,256) from P is equal to: [April 09, 2019 (II)]

Options:

A. $17 / \sqrt{5}$

B. $63\sqrt{5}$

C. $205\sqrt{5}$

D. $11 / \sqrt{5}$

Answer: D

Solution:

Solution:

Let the plane be

$P \equiv (2x + 3y + z + 5) + \lambda(x + y + z - 6) = 0$

$\because$ above plane is perpendicular to xy plane.

$\therefore \left((2+\lambda)\hat{i} + (3+\lambda)\hat{j} + (1+\lambda)\hat{k}\right) . \hat{k} = 0 \Rightarrow \lambda = -1$

Hence, the equation of the plane is,

$P \equiv x + 2y + 11 = 0$

Distance of the plane P from (0,0,256)

$\left|\frac{0+0+11}{\sqrt{5}}\right| = \frac{11}{\sqrt{5}}$

Question223

The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point (1,1,0) is: [April 08 2019 I]

Options:

A. $x - 3y - 2z = -2$

B. $2x - z = 2$

C. $x - y - z = 0$

D. $x + 3y + z = 4$

Answer: C

Solution:

Solution:
Let the equation of required plane be;
$(2x - y - 4) + \lambda(y + 2z - 4) = 0$
∵ This plane passes through the point (1,1,0) then $(2 - 1 - 4) + \lambda(1 + 0 - 4) = 0$
$\Rightarrow \lambda = -1$
Then, equation of required plane is,
$(2x - y - 4) - (y + 2z - 4) = 0$
$\Rightarrow 2x - 2y - 2z = 0 \Rightarrow x - y - z = 0$

Question224

The vector equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$which is perpendicular to the plane $x - y + z = 0$ is:
[April 08,2019 (II)]

Options:

A. $\vec{r} \times \left(\hat{i} - \hat{k}\right) + 2 = 0$

B. $\vec{r} . \left(\hat{i} - \hat{k}\right) - 2 = 0$

C. $\vec{r} \times \left(\hat{i} + \hat{k}\right) + 2 = 0$

D. $\vec{r} . \left(\hat{i} - \hat{k}\right) + 2 = 0$

Answer: D

Solution:

Solution:
Equation of the plane passing through the line of intersection of $x + y + z = 1$ and $2x + 3y + 4z = 5$ is
$(2x + 3y + 4z - 5) + \lambda(x + y + z - 1) = 0$
$\Rightarrow (2 + \lambda)x + (3 + \lambda)y + (4 + \lambda)z + (-5 - \lambda) = 0$ ....(i)
∵ plane(i) is perpendicular to the plane $x - y + z = 0$
$\therefore (2 + \lambda)(1) + (3 + \lambda)(-1) + (4 + \lambda)(1) = 0$
$2 + \lambda - 3 - \lambda + 4 + \lambda = 0 \Rightarrow \lambda = -3$
Hence, equation of required plane is
$-x + z - 2 = 0$ or $x - z + 2 = 0$
$\Rightarrow \bar{r} . \left(\hat{i} - \hat{k}\right) + 2 = 0$

Question225

The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x + y + z = 7$ is:

[2018]

Options:

A. $\frac{2}{3}$

B. $\frac{1}{3}$

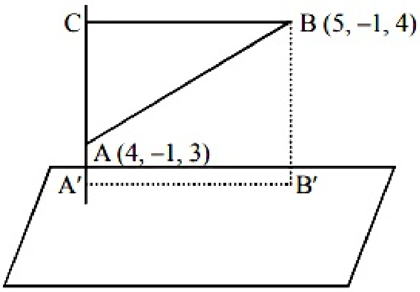
C. $\sqrt{\frac{2}{3}}$

D. $\frac{2}{\sqrt{3}}$

Answer: C

Solution:

Solution:



$AC = \vec{AB} . \hat{AC} = (\hat{i} + \hat{k}) . \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

Now, $A'B' = BC = \sqrt{AB^2 - AC^2} = \sqrt{2 - \frac{4}{3}} = \sqrt{\frac{2}{3}}$

$\therefore$ Length of projection $= \sqrt{\frac{2}{3}}$

Question226

An angle between the lines whose direction cosines are given by the equations, $l + 3m + 5n = 0$ and $5lm - 2mn + 6nl = 0$, is [Online April 15, 2018]

Options:

A. $\cos^{-1}\left(\frac{1}{8}\right)$

B. $\cos^{-1}\left(\frac{1}{6}\right)$

C. $\cos^{-1}\left(\frac{1}{3}\right)$

D. $\cos^{-1}\left(\frac{1}{4}\right)$

Answer: B

Solution:

Given
$l + 3m + 5n = 0$ ....(1)
and $5lm - 2mn + 6nl = 0$ ....(2)
From eq. (1) we have
$l = -3m - 5n$
Put the value of l in eq. (2), we get;
Put the value of l in eq. (2), we get;
$5(-3m - 5n)m - 2mn + 6n(-3m - 5n) = 0$
$\Rightarrow 15m^2 + 45mn + 30n^2 = 0$
$\Rightarrow m^2 + 3mn + 2n^2 = 0$
$\Rightarrow m^2 + 2mn + mn + 2n^2 = 0$
$\Rightarrow (m + n)(m + 2n) = 0$
$\therefore m = -n$ or $m = -2n$
For $m = -n, l = -2n$
And for $m = -2n, l = n$
$\therefore (l, m, n) = (-2n, -n, n)$ Or $(l, m, n) = (n, -2n, n)$
$\Rightarrow (l, m, n) = (-2, -1, 1)$ Or $(l, m, n) = (1, -2, 1)$
Therefore, angle between the lines is given as:
$\cos(\theta) = \frac{(-2)(1) + (-1).(-2) + (1)(1)}{\sqrt{6}.\sqrt{6}}$
$\Rightarrow \cos(\theta) = \frac{1}{6} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6}\right)$

Question227

If the angle between the lines, $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{5-x}{-2} = \frac{7y-14}{P} = \frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then P is equal to
[Online April 16, 2018]

Options:

A. $-\frac{7}{4}$

B. $\frac{2}{7}$

C. $-\frac{4}{7}$

D. $\frac{7}{2}$

Answer: D

Solution:

Solution:
Let θ be the angle between the two lines
Here direction cosines of $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ are 2,2,1
Also second line can be written as:

$$\frac{x-5}{2} = \frac{y-2}{\frac{P}{7}} = \frac{z-3}{4}$$

$\therefore$ its direction cosines are $2, \frac{P}{7}, 4$

Also, $\cos\theta = \frac{2}{3}$ (Given)

$$\because \cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

$$\Rightarrow \frac{2}{3} = \left| \frac{(2 \times 2) + \left(2 \times \frac{P}{7}\right) + (1 \times 4)}{\sqrt{2^2 + 2^2 + 1^2}\sqrt{2^2 + \frac{P^2}{49} + 4^2}} \right|$$

$$= \frac{4 + \frac{2P}{7} + 4}{3 \times \sqrt{2^2 + \frac{P^2}{49} + 4^2}}$$

$$\Rightarrow \left(4 + \frac{P}{7}\right)^2 = 20 + \frac{P^2}{49} \Rightarrow 16 + \frac{8P}{7} + \frac{P^2}{49} = 20 + \frac{P^2}{49}$$

$$\Rightarrow \frac{8P}{7} = 4 \Rightarrow P = \frac{7}{2}$$

Question228

If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is: [2018]

Options:

A. $\frac{1}{3\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{4\sqrt{2}}$

Answer: A

Solution:

Solution:
Equation of plane passing through the line of intersection of first two planes is:

$(2x - 2y + 3z - 2) + \lambda(x - y + z + 1) = 0$
or $x(\lambda + 2) - y(2 + \lambda) + z(\lambda + 3) + (\lambda - 2) = 0$ ......(i)
is having infinite number of solution with
$x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$, then

$$\begin{vmatrix} (\lambda + 2) & -(\lambda + 2) & (\lambda + 3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

Now put $\lambda = 5$ in (i), we get
$7x - 7y + 8z + 3 = 0$

Now perpendicular distance from (0,0,0) to the place containing L_1 and L_2 = $\frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$

Question229

The sum of the intercepts on the coordinate axes of the plane passing through the point (-2,-2,2) and containing the line joining the points (1,-1,2) and (1,1,1) is [Online April 16, 2018]

Options:

A. 12

B. -8

C. -4

D. 4

Answer: C

Solution:

Solution:
Equation of plane passing through three given points is:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+2 & y+2 & z-2 \\ 1+2 & -1+2 & 2-2 \\ 1+2 & 1+2 & 1-2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+2 & y+2 & z-2 \\ 3 & 1 & 0 \\ 3 & 3 & -1 \end{vmatrix} = 0$$

$\Rightarrow -x+3y+6z-8=0$

$\Rightarrow \frac{x}{8}-\frac{3y}{8}-\frac{6z}{8}+\frac{8}{8}=0$

$\Rightarrow \frac{x}{8}-\frac{y}{\frac{8}{3}}-\frac{z}{\frac{8}{6}}=-1$

$\Rightarrow \frac{x}{-8}+\frac{y}{\frac{8}{3}}+\frac{z}{\frac{8}{6}}=1$

$\therefore$ Sum of intercepts $=-8+\frac{8}{3}+\frac{8}{6}=-4$

Question230

A variable plane passes through a fixed point (3,2,1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz− plane through A, a second plane is drawn parallel zx− plane through B and a third plane is drawn parallel to xy− plane through C. Then the locus of

the point of intersection of these three planes, is
[Online April 15, 2018]

Options:

A. $x + y + z = 6$

B. $\frac{x}{3} + \frac{y}{2} + \frac{z}{1} = 1$

C. $\frac{3}{x} + \frac{2}{y} + \frac{1}{z} = 1$

D. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{11}{6}$

Answer: C

Solution:

Solution:
If a, b, c are the intercepts of the variable plane on the x, y, z axes respectively, then the equation of the plane is
$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
And the point of intersection of the planes parallel to the xy, yz and zx planes is (a, b, c).
As the point (3,2,1) lies on the variable plane, so
$\frac{3}{a} + \frac{2}{b} + \frac{1}{c} = 1$
Therefore, the required locus is $\frac{3}{x} + \frac{2}{y} + \frac{1}{z} = 1$

Question231

An angle between the plane, $x + y + z = 5$ and the line of intersection of the planes, $3x + 4y + z - 1 = 0$ and $5x + 8y + 2z + 14 = 0$, is [Online April 15, 2018]

Options:

A. $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$

B. $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$

C. $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$

D. $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

Answer: D

Solution:

Solution:
Normal to $3x + 4y + z = 1$ is $3\hat{i} + 4\hat{j} + \hat{k}$.

Normal to $5x + 8y + 2z = -14$ is $5\hat{i} + 8\hat{j} + 2\hat{k}$
The line of intersection of the planes is perpendicular to both normals, so, direction ratios of the intersection line are directly proportional to the cross product of the normal vectors.
Therefore the direction ratios of the line is $-\hat{j} + 4\hat{k}$
Hence the angle between the plane $x + y + z + 5 = 0$ and the intersection line is $\sin^{-1}\left(\frac{-1+4}{\sqrt{17}\sqrt{3}}\right) = \sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

Question232

A plane bisects the line segment joining the points (1, 2, 3) and (– 3, 4, 5) at right angles. Then this plane also passes through the point. [Online April 15, 2018]

Options:

A. (– 3, 2, 1)

B. (3, 2, 1)

C. (1, 2, – 3)

D. (– 1, 2, 3)

Answer: A

Solution:

Solution:
Since the plane bisects the line joining the points (1,2,3) and (-3,4,5) then the plane passes through the midpoint of the line which is:
$\left(\frac{1-3}{2}, \frac{2+4}{2}, \frac{5+3}{2}\right) \equiv \left(\frac{-2}{2}, \frac{6}{2}, \frac{8}{2}\right) \equiv (-1, 3, 4)$
As plane cuts the line segment at right angle, so the direction cosines of the normal of the plane are
$(-3-1, 4-2, 5-3) = (-4, 2, 2)$
So the equation of the plane is : $-4x + 2y + 2z = \lambda$
As plane passes through (-1,3,4) so
$-4(-1) + 2(3) + 2(4) = \lambda \Rightarrow \lambda = 18$
Therefore, equation of plane is : $-4x + 2y + 2z = 18$
Now, only (-3,2,1) satisfies the given plane as
$-4(-3) + 2(2) + 2(1) = 18$

Question233

If the image of the point $P(1, -2, 3)$ in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to line, $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to : [2017]

Options:

A. $6\sqrt{5}$

B. $3\sqrt{5}$

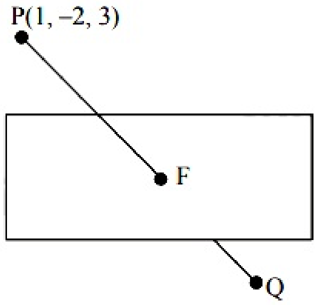
C. $2\sqrt{42}$

D. $\sqrt{42}$

Answer: C

Solution:

Solution:
Equation of line PQ is
$\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}$
Let F be $(\lambda+1, 4\lambda-2, 5\lambda+3)$



Since F lies on the plane
$\therefore 2(\lambda+1)+3(4\lambda-2)-4(5\lambda+3)+22=0$
$2\lambda+2+12\lambda-6-20\lambda-12+22=0$
$\Rightarrow -6\lambda+6=0 \Rightarrow \lambda=1$
$\therefore$F is (2,2,8)
$PQ=2PF=2\sqrt{1^2+4^2+5^2}=2\sqrt{42}$

Question234

The distance of the point (1,3,-7) from the plane passing through the point $(1,-1,-1)$, having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is : [2017]

Options:

A. $\frac{10}{\sqrt{74}}$

B. $\frac{20}{\sqrt{74}}$

C. $\frac{10}{\sqrt{83}}$

D. $\frac{5}{\sqrt{83}}$

Answer: C

Solution:

Solution:
Let the plane be
$a(x-1)+b(y+1)+c(z+1)=0$
Normal vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i} + 7\hat{j} + 3\hat{k}$$

So plane is $5(x-1) + 7(y+1) + 3(z+1) = 0$
$\Rightarrow 5x + 7y + 3z + 5 = 0$
Distance of point (1,3,-7) from the plane is
$\frac{5+21-21+5}{\sqrt{25+49+9}} = \frac{10}{\sqrt{83}}$

Question235

If x = a, y = b, z = c is a solution of the system of linear equations $x + 8y + 7z = 0$ $9x + 2y + 3z = 0$ $x + y + z = 0$ such that the point (a, b, c) lies on the plane $x + 2y + z = 6$, then $2a + b + c$ equals: [Online April 9, 2017]

Options:

A. -1

B. 0

C. 1

D. 2

Answer: C

Solution:

Solution:
$x + 8y + 7z = 0$
$9x + 2y + 3z = 0$
$x + y + z = 0$
$x = \lambda \quad y = 6\lambda \quad z = -7\lambda$

$x = \lambda \quad y = 6\lambda \quad z = -7\lambda$

Now, $\lambda + 12\lambda - 7\lambda = 6$
$6\lambda = 6$
$\lambda = 1$
$\therefore 2\lambda + 6\lambda - 7\lambda$
$= \lambda$
$= 1$

Question236

If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is : [Online April 9, 2017]

Options:

A. $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$

B. $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=3$

C. $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{9}$

D. $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$

Answer: A

Solution:

Solution:
Suppose centroid be (h, k, l)
$\therefore x-\operatorname{int} p=3h, y-\operatorname{int} p=3k, z-\operatorname{int} p=3l$
Equation $\frac{x}{3h}+\frac{y}{3k}+\frac{z}{3l}=1$
$\therefore$ Distance from (0,0,0)

$$\left|\frac{-1}{\sqrt{\frac{1}{9h^2}+\frac{1}{9k^2}+\frac{1}{9l^2}}}\right|=3$$

$\Rightarrow \frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$

Question237

If the line, $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+\lambda}{-2}$ lies in the plane, $2x-4y+3z=2$, then the shortest distance between this line and the line, $\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}$ is : [Online April 9, 2017]

Options:

A. 2

B. 1

C. 0

D. 3

Answer: C

Solution:

Solution:
Point $(3,-2,-\lambda)$ on p line $2x-4y+3z-2=0 \;\; =6+8-3\lambda-2 \;\; =0=3\lambda=12$
$\lambda=4$
Now,
$\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+4}{-2}=k_1$ ......(i)
$\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}=k_2$ ....(ii)

Point on equation (i) $P(k_1 + 3, -k_1 - 2, -2k_1 - 4)$
Point on equation (ii) $Q(12k_2 + 1, 9k_2, 4k_2)$
$k_1 + 3 = 12k_2 + 1 \mid -k_1 - 2 = 9k_2 \mid -2k_1 - 4 = 4k_2$
$k_2 = 0$
$k_1 = -2$
$p(1, 0, 0)$ lie on equation of a line 1
gives shortest distance $= 0$

Question238

The coordinates of the foot of the perpendicular from the point (1,-2,1) on the plane containing the lines, $\frac{x+1}{6} = \frac{y-1}{7} = \frac{z-3}{8}$ and $\frac{x-1}{3} = \frac{y-2}{5} = \frac{z-3}{7}$, is : [Online April 8, 2017]

Options:

A. (2,-4,2)

B. (-1,2,-1)

C. (0,0,0)

D. (1,1,1)

Answer: C

Solution:

Solution:

$$\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix} = (9, -18, 9) = (1, -2, 1)$$

∴ Equation of plane is
$1(x + 1) - 2(y - 1) + (z - 3) = 0$
$\Rightarrow x - 2y + z = 0$
foot to z
$$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-1}{1} = -\frac{[1+4+1]}{6}$$
$x = 0, y = 0, z = 0$

Question239

The line of intersection of the planes $\vec{r} \cdot \left(3\hat{i} - \hat{j} + \hat{k}\right) = 1$ and $\vec{r} \cdot \left(\hat{i} + 4\hat{j} - 2\hat{k}\right) = 2$, is : [Online April 8, 2017]

Options:

A. $\frac{x - \frac{4}{7}}{-2} = \frac{y}{7} = \frac{z - \frac{5}{7}}{13}$

B. $\frac{x-\frac{4}{7}}{2}=\frac{y}{-7}=\frac{z+\frac{5}{7}}{13}$

C. $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$

D. $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{7}=\frac{z}{-13}$

Answer: C

Solution:

Solution:

$\vec{n}=\vec{n}_1\times\vec{n}_2\Rightarrow\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 1\\ 1 & 4 & -2\end{vmatrix}=\hat{i}(-2)-\hat{j}(-7)+\hat{k}(13)\Rightarrow\vec{n}=-2\hat{i}+7\hat{j}+13\hat{k}$

Now,
$3x-y+z=1$
$x+4y-2z=2$
but $z=0$& solving the given
$x=6/13$ & $y=5/13$
∴ required equation of a line is
$\frac{x-6/13}{2}=\frac{y-5/13}{-7}=\frac{z}{-13}$

Question240

ABC is triangle in a plane with vertices A(2, 3, 5), B(−1, 3,2) and C(λ, 5, μ). If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3+\mu^3+5)$ is:
[Online April 10, 2016]

Options:

A. 1130

B. 1348

C. 1077

D. 676

Answer: B

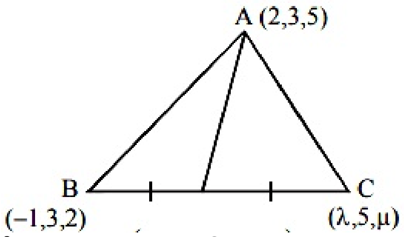
Solution:

Solution:

DR's of AD are $\frac{\lambda-1}{2}-2, 4-3, \mu+22-5$

i.e. $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$

∵ This median is making equal angles with coordinate axes, therefore,



$\frac{\lambda - 5}{2} = 1 = \frac{\mu - 8}{2}$
$\Rightarrow \lambda = 7 \ \& \ \mu = 10$
$\therefore \lambda^3 + \mu^3 = 5 = 1348$

Question241

The number of distinct real values of lambda for which the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{\lambda^2}$ and $\frac{x-3}{1} = \frac{y-2}{\lambda^2} = \frac{z-1}{2}$ are coplanar is :
[Online April 10, 2016]

Options:

A. 2

B. 4

C. 3

D. 1

Answer: C

Solution:

Solution:

Lines are coplanar $\begin{vmatrix} 3-1 & 2-2 & 1-(-3) \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 2 & 0 & 4 \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0$
$\Rightarrow 2(4 - \lambda^4) + 4(\lambda^2 - 2) = 0$
$\Rightarrow 4 - \lambda^4 + 2\lambda^2 - 4 = 0 \Rightarrow \lambda^2(\lambda^2 - 2) = 0$
$\Rightarrow \lambda = 0, \sqrt{2}, -\sqrt{2}$

Question242

The shortest distance between the lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ lies in the interval:
[Online April 9, 2016]

Options:

A. (3, 4]

B. (2, 3]

C. [1, 2)

D. [0, 1)

Answer: B

Solution:

Solution:
Shortest distance between two lines
$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ is given by,

$$\left| \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}} \right|$$

∴ The shortest distance between given lines are

$$\left| \frac{\begin{vmatrix} -2 & 4 & 5 \\ 2 & 2 & 1 \\ -1 & 8 & 4 \end{vmatrix}}{\sqrt{(8 - 8)^2 + (-1 - 8)^2 + (16 + 2)^2}} \right|$$

$= \left| \frac{0 - 36 + 90}{\sqrt{405}} \right| = \frac{54}{20.1} = 2.68$

Question243

If the line, $\frac{x - 3}{2} = \frac{y + 2}{-1} = \frac{z + 4}{3}$ lies in the plane, $1x + my - z = 9$, then $l^2 + m^2$ is equal to:
[2016]

Options:

A. 5

B. 2

C. 26

D. 18

Answer: B

Solution:

Solution:
Line lies in the plane ⇒$(3, -2, -4)$ lie in the plane
⇒$3l - 2m + 4 = 9$ or $3l - 2m = 5$ ....(1)
Also, $l, m, -1$ are dr's of line perpendicular to plane and 2 . -1,3 are dr's of line lying in the plane
⇒$2l - m - 3 = 0$ or $2l - m = 3$ ....(2)
Solving (1) and (2) we get $l = 1$ and $m = -1$

$\Rightarrow l^2 + m^2 = 2$

Question244

The distance of the point (1,-5,9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is: [2016]

Options:

A. $\frac{10}{\sqrt{3}}$

B. $\frac{20}{3}$

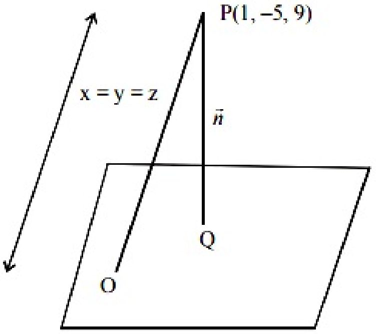
C. $3\sqrt{10}$

D. $10\sqrt{3}$

Answer: D

Solution:

Solution:



E q^n of PO : $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$
$\Rightarrow x = \lambda + 1;\ y = \lambda - 5;\ z = \lambda + 9$
Putting these in eq ^{n} of plane :
$\lambda + 1 - \lambda + 5 + \lambda + 9 = 5$
$\Rightarrow \lambda = -10$
$\Rightarrow$O is (-9,-15,-1)
$\Rightarrow$ distance OP = $10\sqrt{3}$

Question245

The distance of the point (1,-2,4) from the plane passing through the point (1,2,2) and perpendicular to the planes $x - y + 2z = 3$ and $2x - 2y + z + 12 = 0$, is [Online April 9, 2016]

Options:

A. 2

B. $\sqrt{2}$

C. $2\sqrt{2}$

D. $\frac{1}{\sqrt{2}}$

Answer: C

Solution:

Solution:
Let equation of plane be
$a(x-1)+b(y-2)+c(z-2)=0$ .... (1)
is perpendicular to given planes then
$a-b+2c=0$
$2a-2b+c=0$
Solving above equation $c=0$ and $a=b$
equation of plane (1) can be
$x+y-3=0$
distance from (1,-2,4) will be
$D=\frac{|1-2-3|}{\sqrt{1+1}}=\frac{4}{\sqrt{2}}=2\sqrt{2}$

Question246

The equation of the plane containing the line $2x-5y+z=3$; $x+y+4z=5$, and parallel to the plane, $x+3y+6z=1$, is: [2015]

Options:

A. $x+3y+6z=7$

B. $2x+6y+12z=-13$

C. $2x+6y+12z=13$

D. $x+3y+6z=-7$

Answer: A

Solution:

Solution:
Equation of the plane containing the lines
$2x-5y+z=3$ and $x+y+4z=5$ is
$2x-5y+z-3+\lambda\,(x+y+4z-5)=0$
$\Rightarrow(2+\lambda)x+(-5+\lambda)y+(1+4\lambda)\,z+(-3-5\lambda)=0$
Since the plane (i) parallel to the given plane
$x+3y+6z=1$
$\therefore\frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6}$
$\Rightarrow\lambda=-\frac{11}{2}$
Hence equation of the required plane is
$\left(2-\frac{11}{2}\right)x+\left(-5-\frac{11}{2}\right)y+\left(1-\frac{44}{2}\right)z+\left(-3+\frac{55}{2}\right)=0$
$\Rightarrow(4-11)x+(-10-11)y+(2-44)z+(-6+55)=0$
$\Rightarrow-7x-21y-42z+49=0$

$\Rightarrow x + 3y + 6z - 7 = 0$
$\Rightarrow x + 3y + 6z = 7$

Question247

The distance of the point (1,0,2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is [2015]

Options:

A. $3\sqrt{21}$

B. 13

C. $2\sqrt{14}$

D. 8

Answer: B

Solution:

Solution:
General point on given line $\equiv P(3r + 2, 4r - 1, 12r + 2)$
Point P must satisfy equation of plane
$(3r + 2) - (4r - 1) + (12r + 2) = 16$
$11r + 5 = 16$
$r = 1$
$P(3 \times 1 + 2, 4 \times 1 - 1, 12 \times 1 + 2) = P(5, 3, 14)$
distance between P and (1,0,2)
$D = \sqrt{(5-1)^2 + 3^2 + (14-2)^2} = 13$

Question248

The shortest distance between the z -axis and the line $x + y + 2z - 3 = 0 = 2x + 3y + 4z - 4$, is [Online April 11, 2015]

Options:

A. 1

B. 2

C. 4

D. 3

Answer: B

Solution:

The equation of any plane passing through given line is
$(x + y + 2z - 3) + \lambda(2x + 3y + 4z - 4) = 0$
$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (2 + 4\lambda)z - (3 + 4\lambda) = 0$
If this plane is parallel to z -axis then normal to the plane will be perpendicular to z-axis.
$\therefore (1 + 2\lambda)(0) + (1 + 3\lambda)(0) + (2 + 4\lambda)(1) = 0$
$\lambda = -\frac{1}{2}$
Thus, Required plane is
$(x + y + 2z - 3) - \frac{1}{2}(2x + 3y + 4z - 4) = 0 \Rightarrow y + 2 = 0$
$\therefore S . D = \frac{2}{\sqrt{(1)^2}} = 2$

Question249

A plane containing the point (3,2,0) and the line $\frac{x-1}{1} = \frac{y-2}{5} = \frac{z-3}{4}$ also contains the point:
[Online April 11, 2015]

Options:

A. (0,3,1)

B. (0,7,-10)

C. (0,-3,1)

D. 0,7,10

Answer: C

Solution:

Solution:
Equation of the plane containing the given line
$\frac{x-1}{1} = \frac{y-2}{5} = \frac{z-3}{4}$ is
$A(x - 1) + B(y - 2) + C(z - 3) = 0$ ......(i)
where $A + 5B + 4C = 0$ ....(ii)
Since the point (3,2,0) contains in the plane (i), therefore $2A + 0 . B - 3C = 0$ ....(iii)
From equations (ii) and (iii),
$\frac{A}{-15-0} = \frac{B}{6+3} = \frac{C}{0-10} = k$(let)
$\Rightarrow A = -15k, B = 9k$ and $C = -10k$
Putting the value of A, B and C in equation (i), we get
$-15(x - 1) + 9(y - 2) - 10(z - 3) = 0$ ....(iv)
Now the coordinates of the point (0,-3,1)
satisfy the equation of the plane (iv) as
$-15(0 - 1) + 9(-3 - 2) - 10(1 - 3)$
$= 15 - 45 + 20 = 0$
Hence the point (0, -3, 1) contains in the plane.

Question250

If the points (1, 1, λ) and (-3,0,1) are equidistant from the plane, $3x + 4y - 12z + 13 = 0$, then λ satisfies the equation:
[Online April 10, 2015]

Options:

A. $3x^2 + 10x - 13 = 0$

B. $3x^2 - 10x + 21 = 0$

C. $3x^2 - 10x + 7 = 0$

D. $3x^2 + 10x - 7 = 0$

Answer: C

Solution:

Solution:
$|3 + 4 - 12\lambda + 13| = |-9 + 0 - 12 + 13|$
$\Rightarrow |-12\lambda + 20| = |8| \Rightarrow 3\lambda - 5| = 2$
$\Rightarrow 9\lambda^2 + 25 - 30\lambda = 4 \Rightarrow 9\lambda^2 - 30\lambda + 21 = 0$
$\Rightarrow 3\lambda^2 - 10\lambda + 7 = 0$

Question251

If the shortest distance between the lines $\frac{x-1}{\alpha} = \frac{y+1}{-1} = \frac{z}{1}$, $(\alpha \neq -1)$ and $x + y + z + 1 = 0 = 2x - y + z + 3$ is $\frac{1}{\sqrt{3}}$, then a value α is:
[Online April 10, 2015]

Options:

A. $-\frac{16}{19}$

B. $-\frac{19}{16}$

C. $\frac{32}{19}$

D. $\frac{19}{32}$

Answer: C

Solution:

Solution:
Plane passing through $x + y + z + 1 = 0$ and $2x - y + z + 3 = 0$ is $x + y + z + 1 + \lambda(2x - y + z + 3) = 0$
$\Rightarrow (2\lambda + 1)x + (1 - \lambda)y + (1 + \lambda)z + 3\lambda + 1 = 0$
Parallel to the given line if
$\alpha(2\lambda + 1) - 1(1 - \lambda) + 1(1 + \lambda) = 0$
$\Rightarrow \alpha = \frac{-2\lambda}{2\lambda + 1}$....... (i)
Also, $\left|\frac{2\lambda + 1 - (1 - \lambda) + 0 + 3\lambda + 1}{\sqrt{(2\lambda + 1)^2 + (1 - \lambda)^2 + (1 + \lambda)^2}}\right| = \frac{1}{\sqrt{3}}$
$\Rightarrow \lambda = 0, \frac{-32}{102}$; $\alpha = 0$ or $\alpha = \frac{32}{19}$

Question252

The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ and $l^2 + m^2 + n^2$ is [2014]

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

Solution:
Given, $l + m + n = 0$ and $l^2 = m^2 + n^2$
Now, $(-m - n)^2 = m^2 + n^2$
$\Rightarrow mn = 0 \Rightarrow m = 0$ or $n = 0$
If $m = 0$ then $l = -n$
We know $l^2 + m^2 + n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$
i.e. $(l_1, m_1, n_1) = \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$
If $n = 0$ then $l = -m$
$l^2 + m^2 + n^2 = 1 \Rightarrow 2m^2 = 1$
$\Rightarrow m = \pm\frac{1}{\sqrt{2}}$
Let $m = \frac{1}{\sqrt{2}}$
$\Rightarrow l = -\frac{1}{\sqrt{2}}$ and $n = 0$
$(l_2, m_2, n_2) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$
$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

Question253

Let $A(2, 3, 5)$, $B(-1, 3, 2)$ and $C(\lambda, 5, \mu)$ be the vertices of a ΔABC. If the median through A is equally inclined to the coordinate axes, then: [Online April 11, 2014]

Options:

A. $5\lambda - 8\mu = 0$

B. $8\lambda - 5\mu = 0$

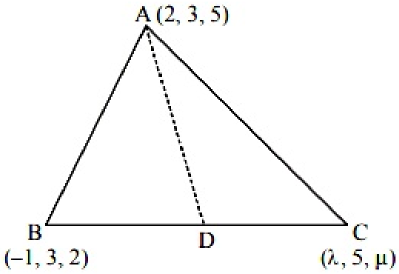
C. $10\lambda - 7\mu = 0$

D. $7\lambda - 10\mu = 0$

Answer: C

Solution:

Solution:
If D be the mid-point of BC, then
$D = \left(\frac{\lambda - 1}{2}, 4, \frac{\mu + 2}{2}\right)$



Direction ratios of AD are $\frac{\lambda - 5}{2}, 1, \frac{\mu - 8}{2}$
Since median AD is equally inclined with coordinate axes, therefore direction ratios of AD will be equal, i.e,

$$\frac{\left(\frac{\lambda-5}{2}\right)^2}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2} = \frac{1}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2}$$

$$= \frac{\left(\frac{\mu-8}{2}\right)^2}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2}$$

$\Rightarrow \left(\frac{\lambda-5}{2}\right)^2 = 1 = \left(\frac{\mu-8}{2}\right)^2$
$\Rightarrow \lambda = 7, 3$ and $\mu = 10, 6$
If $\lambda = 7$ and $\mu = 10$
Then $\frac{\lambda}{\mu} = \frac{7}{10} \Rightarrow 10\lambda - 7\mu = 0$

Question254

A line in the 3 -dimensional space makes an angle θ $\left(0 < \theta \leq \frac{\pi}{2}\right)$ with both the x and y axes. Then the set of all values of theta is the interval: [Online April 9, 2014]

Options:

A. $\left(0, \frac{\pi}{4}\right]$

B. $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

D. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right]$

Answer: C

Solution:

Solution:
It makes θ with x and y -axes.
$l = \cos\theta$, $m = \cos\theta$, $n = \cos(\pi - 2\theta)$
we have $l^2 + m^2 + n^2 = 1$
$\Rightarrow \cos^2\theta + \cos^2\theta + \cos^2(\pi - 2\theta) = 1$
$\Rightarrow 2\cos^2\theta + (-\cos 2\theta)^2 = 1$
$\Rightarrow 2\cos^2\theta - 1 + \cos^2 2\theta = 0$
$\Rightarrow \cos 2\theta - [1 + \cos 2\theta] = 0$
$\Rightarrow \cos 2\theta = 0$ or $\cos 2\theta = -1$
$\Rightarrow 2\theta = \pi / 2$ or $2\theta = \pi$
$\Rightarrow \theta = \pi / 4$ or $\theta = \frac{\pi}{2}$
$\Rightarrow \theta = \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

Question255

Equation of the line of the shortest distance between the lines $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ and $\frac{x-1}{0} = \frac{y+1}{-2} = \frac{z}{1}$ is:
[Online April 19, 2014]

Options:

A. $\frac{x}{1} = \frac{y}{-1} = \frac{z}{-2}$

B. $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{-2}$

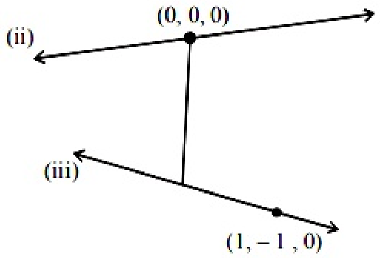
C. $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{1}$

D. $\frac{x}{-2} = \frac{y}{1} = \frac{z}{2}$

Answer: B

Solution:

Solution:
Let equation of the required line be
$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ ......(i)
Given two lines
$\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ ....(ii)
and $\frac{x-1}{0} = \frac{y+1}{0} = \frac{z}{1}$ ......(iii)
Since the line (i) is perpendicular to both the lines (ii) and (iii), therefore
$a - b + c = 0$ ......(iv)
$-2b + c = 0$ .....(v)
From (iv) and (v) $c = 2b$ and $a + b = 0$, which are not satisfy by options (c) and (d). Hence options (c) and (d) are rejected.
Thus point (x_1, y_1, z_1) on the required line will be either (0,0,0) or (1,-1,0).
Now foot of the perpendicular from point (0,0,0) to the line(iii) $= (1, -2r - 1, r)$



The direction ratios of the line joining the points (0,0,0) and $(1, -2r-1, r)$ are $1, -2r-1, r$
Since sum of the x and y -coordinate of direction ratio of the required line is 0 .
$\therefore 1-2r-1=0, \Rightarrow r=0$
Hence direction ratio are 1,-1,0
But the z-direction ratio of the required line is twice the y -direction ratio of the required line
i.e. $0=2(-1)$, which is not true.
Hence the shortest line does not pass through the point (0,0,0) . Therefore option (a) is also rejected.

Question256

The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane$2x-y+z+3=0$ is the line:
[2014]

Options:

A. $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

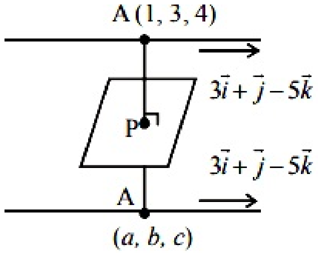
B. $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

C. $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

D. $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

Answer: C

Solution:

Solution:
$\frac{a-1}{2}=\frac{b-3}{-1}=\frac{c-4}{1}=\lambda$(let)
$a=2\lambda+1$
$b=3-\lambda$
$c=4+\lambda$


$P=\left(\frac{a+1}{2}, \frac{b+3}{2}, \frac{c+4}{2}\right)=\left(\lambda+1, \frac{6-\lambda}{2}, \frac{\lambda+8}{2}\right)$
$\therefore 2(\lambda+1)-\frac{6-\lambda}{2}+\frac{\lambda+8}{2}+3=0$
$3\lambda+6=0 \Rightarrow \lambda=-2$
$a=-3, b=5, c=2$
Required line is $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

Question257

If the angle between the line $2(x+1)=y=z+4$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is $\frac{\pi}{6}$, then the value of λ is:
[Online April 19, 2014]

Options:

A. $\frac{135}{7}$

B. $\frac{45}{11}$

C. $\frac{45}{7}$

D. $\frac{135}{11}$

Answer: C

Solution:

Solution:
Given equation of line can be written as
$\frac{x+1}{1}=\frac{y}{2}=\frac{z+4}{2}$
Eqn of plane is $2x-y+\sqrt{\lambda}z+4=0$
Since, angle between the line and the plane is $\frac{\pi}{6}$
therefore
$\sin\frac{\pi}{6}=\frac{2(1)+2(-1)+2(\sqrt{\lambda})}{\sqrt{1+4+4}\sqrt{4+1+\lambda}}$
$\frac{1}{2}=\frac{2-2+2\sqrt{\lambda}}{\sqrt{9}\sqrt{5+\lambda}}$
$\Rightarrow\frac{\sqrt{\lambda}}{\sqrt{5+\lambda}}=\frac{3}{4}\Rightarrow\frac{\lambda}{5+\lambda}=\frac{9}{16}$
$\Rightarrow 7\lambda=45\Rightarrow\lambda=\frac{45}{7}$

Question258

If the distance between planes, $4x-2y-4z+1=0$ and $4x-2y-4z+d=0$ is 7, then d is:
[Online April 12, 2014]

Options:

A. 41 or -42

B. 42 or -43

C. -41 or 43

D. -42 or 44

Answer: C

Solution:

Solution:
Given planes are
$4x - 2y - 4z + 1 = 0$
and $4x - 2y - 4z + d = 0$
They are parallel.
Distance between them is $\pm 7 = \frac{d - 1}{\sqrt{16 + 4 + 16}}$
$\Rightarrow \frac{d - 1}{6} = \pm 7 \Rightarrow d = 42 + 1$
or $-42 + 1$ i.e. $d = -41$ or 43 .

Question259

A symmetrical form of the line of intersection of the planes $x = ay + b$ and $z = cy + d$ is
[Online April 12, 2014]

Options:

A. $\frac{x - b}{a} = \frac{y - 1}{1} = \frac{z - d}{c}$

B. $\frac{x - b - a}{a} = \frac{y - 1}{1} = \frac{z - d - c}{c}$

C. $\frac{x - a}{b} = \frac{y - 0}{1} = \frac{z - c}{d}$

D. $\frac{x - b - a}{b} = \frac{y - 1}{0} = \frac{z - d - c}{d}$

Answer: B

Solution:

Solution:
Given two planes:
$x - ay - b = 0$ and $cy - z + d = 0$
Let, l, m, n be the direction ratio of the required line. Since the required line is perpendicular to normal of both the plane, therefore $l - am = 0$ and $cm - n = 0$
$\Rightarrow l - am + 0 . n = 0$ and $0 . l + cm - n = 0$
$\therefore \frac{l}{a - 0} = \frac{m}{0 + 1} = \frac{n}{c - 0}$
Hence, $d . R$ of the required line are $a, 1, c$.
Hence, options (c) and (d) are rejected.
Now, the point $(a + b, 1, c + d)$ satisfy the equation of the two given planes.
$\therefore$ Option(b) is correct.

Question260

The plane containing the line $\frac{x - 1}{1} = \frac{y - 2}{2} = \frac{z - 3}{3}$ and parallel to the line $\frac{x}{1} = \frac{y}{1} = \frac{z}{4}$ passes through the point:

[Online April 11, 2014]

Options:

A. (1,-2,5)

B. (1,0,5)

C. (0,3,-5)

D. (-1,-3,0)

Answer: B

Solution:

Solution:
Equation of the plane containing the line
$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ is
$a(x-1)+b(y-2)+c(z-3)=0$ ......(i)
where $a.1+b.2+c.3=0$
i.e., $a+2b+3c=0$ ....(ii)
Since the plane (i) parallel to the line
$\frac{x}{1}=\frac{y}{1}=\frac{z}{4}$
$\therefore a.1+b.1+c.4=0$
i.e., $a+b+4c=0$ ...... (iii)
From (ii) and (iii) ,
$\frac{a}{8-3}=\frac{b}{3-4}=\frac{c}{1-2}=k$ (let)
$\therefore a=5k, b=-k, c=-k$
On putting the value of a, b and c in equation (i),
$5(x-1)-(y-2)-(z-3)=0$
$\Rightarrow 5x-y-z=0$ ....(iv)
when $x=1$, $y=0$ and $z=5$; then
L.H.S. of equation (iv) $=5x-y-2$
$=5\times1-0-5=0$
$=R.H.S.$ of equation
Hence coordinates of the point (1, 0, 5) satisfy the equation plane represented by equations (iv),
Therefore the plane passes through the point (1,0,5)

Question261

Equation of the plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and has the largest distance from the origin is:
[Online April 9, 2014]

Options:

A. $7x+2y+4z=54$

B. $3x+4y+5z=49$

C. $4x+3y+5z=50$

D. $5x+4y+3z=57$

Answer: C

Solution:

Solution:
Given equation of lines are
$\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ ....(1)
and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ ......(2)
Any point on line (1) is $P(3\lambda+1, \lambda+2, 2\lambda+3)$ and on line (2) is $Q(\mu+3, 2\mu+1, 3\mu+2)$
On solving $3\lambda+1=\mu+3$ and $\lambda+2=2\mu+1$
we get $\lambda=1, \mu=1$
∴ Point of intersection of two lines is $R(4, 3, 5)$
So, equation of plane $\perp$ to OR where O is (0,0,0) and passing through R is
$4x+3y+5z=50$

Question262

Let ABC be a triangle with vertices at points A(2, 3, 5), B(−1, 3, 2) and C(λ, 5, μ) in three dimensional space. If the median through A is equally inclined with the axes, then (λ, μ) is equal to : [Online April 25, 2013]

Options:

A. (10,7)

B. (7,5)

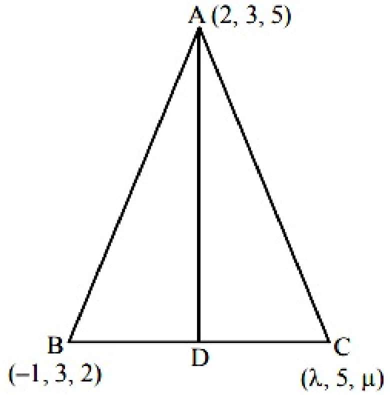
C. (7,10)

D. (5,7)

Answer: C

Solution:

Solution:
Since AD is the median



$\therefore D=\left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$
Now, d R 's of AD is
$a=\left(\frac{\lambda-1}{2}-2\right)=\frac{\lambda-5}{2}$
$b=4-3=1, c=\frac{\mu+2}{2}-5=\frac{\mu-8}{2}$
Also, a, b, c are dR's
$\therefore a=kl, b=km, c=kn$ where $l=m=n$

and $l^2 + m^2 + n^2 = 1$
$\Rightarrow l = m = n = \frac{1}{\sqrt{3}}$
Now, $a = 1$, $b = 1$ and $c = 1$
$\Rightarrow \lambda = 7$ and $\mu = 10$

Question263

If the protections of a line segment on the x, y and z-axes in 3-dimensional space are 2, 3 and 6 respectively, then the length of the line segment is :
[Online April 23, 2013]

Options:

A. 12

B. 7

C. 9

D. 6

Answer: B

Solution:

Solution:
Length of the line segment $= \sqrt{(2)^2 + (3)^2 + (6)^2} = 7$

Question264

The acute angle between two lines such that the direction cosines l, m, n, of each of them satisfy the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$ is
[Online April 22, 2013]

Options:

A. 15°

B. 30°

C. 60°

D. 45°

Answer: C

Solution:

Solution:
Let l_1, m_1, n_1 and l_2, m_2, n_2 be the d.c of line 1 and 2 respectively, then as given

$l_1 + m_1 + n_1 = 0$
and $l_2 + m_2 + n_2 = 0$
and $l_1^2 + m_1^2 - n_1^2 = 0$ and
$l_2^2 + m_2^2 - n_2^2 = 0$
$(\because l + m + n = 0$ and $l^2 + m^2 - n^2 = 0)$
Angle between lines, θ is
$\cos theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ .....(1)
As given $l^2 + m^2 = n^2$ and $l + m = -n$
$\Rightarrow (-n)^2 - 2lm = n^2 \Rightarrow 2lm = 0$ or $lm = 0$
So $l_1 m_1 = 0$, $l_2 m_2 = 0$
If $l_1 = 0$, $m_1 \neq 0$ then $l_1 m_2 = 0$
If $m_1 = 0$, $l_1 \neq 0$ then $l_2 m_1 = 0$
If $l_2 = 0$, $m_2 \neq 0$ then $l_2 m_1 = 0$
If $m_2 = 0$, $l_2 \neq 0$ then $l_1 m_2 = 0$
Also $l_1 l_2 = 0$ and $m_1 m_2 = 0$
$l^2 + m^2 - n^2 = l^2 + m^2 + n^2 - 2n^2 = 0$
$\Rightarrow 1 - 2n^2 = 0 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$
$\therefore n_1 = \pm\frac{1}{\sqrt{2}}, n_2 = \pm\frac{1}{\sqrt{2}}$
$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$(acute angle)

Question265

If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have
[2013]

Options:

A. any value

B. exactly one value

C. exactly two values

D. exactly three values

Answer: C

Solution:

Solution:
Given lines will be coplanar

If $\begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$
$\Rightarrow -1(1 + 2k) - (1 + k^2) + 1(2 - k) = 0$
$\Rightarrow k = 0, -3$

Question266

If two lines L_1 and L_2 in space, are defined by $L_1 = \{ x = \sqrt{\lambda}y + (\sqrt{\lambda} - 1)$,

$z = (\sqrt{\lambda} - 1)y + \sqrt{\lambda}$ } and L_2 = { $x = \sqrt{\mu}y + (1 - \sqrt{\mu})$, $z = (1 - \sqrt{\mu})y + \sqrt{\mu}$ } then L_1 is perpendicular to L_2, for all non-negative reals λ and μ, such that :
[Online April 23, 2013]

Options:

A. $\sqrt{\lambda} + \sqrt{\mu} = 1$

B. $\lambda \neq \mu$

C. $\lambda + \mu = 0$

D. $\lambda = \mu$

Answer: D

Solution:

Solution:
For L_1,
$x = \sqrt{\lambda}y + (\sqrt{\lambda} - 1) \Rightarrow y = \frac{x - (\sqrt{\lambda} - 1)}{\sqrt{\lambda}}$ ....(i)
$z = (\sqrt{\lambda} - 1)y + \sqrt{\lambda} \Rightarrow y = \frac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1}$ ....(ii)
From (i) and (ii)
$\frac{x - (\sqrt{\lambda} - 1)}{\sqrt{\lambda}} = \frac{y - 0}{1} = \frac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1}$ ....(A)
The equation (A) is the equation of line L_1.
Similarly equation of line L_2 is
$\frac{x - (1 - \sqrt{\mu})}{\sqrt{\mu}} = \frac{y - 0}{1} = \frac{z - \sqrt{\mu}}{1 - \sqrt{\mu}}$ ......(B)
Since $L_1 \perp L_2$, therefore
$\sqrt{\lambda}\sqrt{\mu} + 1 \times 1 + (\sqrt{\lambda} - 1)(1 - \sqrt{\mu}) = 0$
$\Rightarrow \sqrt{\lambda} + \sqrt{\mu} = 0 \Rightarrow \sqrt{\lambda} = -\sqrt{\mu}$
$\Rightarrow \lambda = \mu$

Question267

If the lines $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z+1}{3}$ and $\frac{x+2}{2} = \frac{y-k}{3} = \frac{z}{4}$ are coplanar, then the value of k is :
[Online April 9, 2013]

Options:

A. $\frac{11}{2}$

B. $-\frac{11}{2}$

C. $\frac{9}{2}$

D. $-\frac{9}{2}$

Answer: A

Solution:

Solution:
Two given planes are coplanar, if

$$\begin{vmatrix} -2-(-1) & k-1 & 0-(-1) \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & k-1 & 1 \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$\Rightarrow(-1)(4-9)-(k-1)(8-6)+6-2=0$

$\Rightarrow k = \frac{11}{2}$

Question268

Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is [2013]

Options:

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. $\frac{7}{2}$

D. $\frac{9}{2}$

Answer: C

Solution:

Solution:
$2x + y + 2z - 8 = 0$ ....(Plane 1)
$2x + y + 2z + \frac{5}{2} = 0$ ....(Plane 2)Distance between Plane 1 and 2

$$= \left| \frac{-8-\frac{5}{2}}{\sqrt{2^2+1^2+2^2}} \right| = \left| \frac{-21}{6} \right| = \frac{7}{2}$$

Question269

The equation of a plane through the line of intersection of the planes $x + 2y = 3$, $y - 2z + 1 = 0$, and perpendicular to the first plane is: [Online April 25, 2013]

Options:

A. $2x - y - 10z = 9$

B. $2x - y + 7z = 11$

C. $2x - y + 10z = 11$

D. $2x - y - 9z = 10$

Answer: C

Solution:

Solution:
Equation of a plane through the line of intersection of the planes
$x + 2y = 3$, $y - 2z + 1 = 0$ is
$(x + 2y - 3) + \lambda(y - 2z + 1) = 0$
$\Rightarrow x + (2 + \lambda)y - 2\lambda(z) - 3 + \lambda = 0$ ....(i)
Now, plane (i) is $\perp$ to $x + 2y = 3$
$\therefore$ Their dot product is zero
i.e. $1 + 2(2 + \lambda) = 0 \Rightarrow \lambda = -\frac{5}{2}$
Thus, required plane is
$x + \left(2 - \frac{5}{2}\right)y - 2 \times \frac{-5}{2}(z) - 3 - \frac{5}{2} = 0$
$\Rightarrow x - \frac{y}{2} + 5z - \frac{11}{2} = 0$
$\Rightarrow 2x - y + 10z - 11 = 0$

Question270

Let Q be the foot of perpendicular from the origin to the plane $4x - 3y + z + 13 = 0$ and R be a point (-1,-6) on the plane. Then length QR is:
[Online April 22, 2013]

Options:

A. $\sqrt{14}$

B. $\sqrt{\frac{19}{2}}$

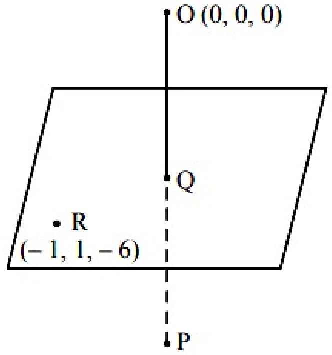
C. $3\sqrt{\frac{7}{2}}$

D. $\frac{3}{\sqrt{2}}$

Answer: C

Solution:

Solution:
Let P be the image of O in the given plane.



Equation of the plane, $4x - 3y + z + 13 = 0$
OP is normal to the plane, therefore direction ratio of OP are proportional to 4,-3,1
Since OP passes through (0,0,0) and has direction ratio proportional to $4, -3, 1$. Therefore equation of OP is
$\frac{x-0}{4} = \frac{y-0}{-3} = \frac{z-0}{1} = r$ (let)
$\therefore x = 4r, y = -3r, z = r$
Let the coordinate of P be $(4r, -3r, r)$
Since Q be the mid point of OP
$\therefore Q = \left(2r, -\frac{3}{2}r, \frac{r}{2}\right)$
Since Q lies in the given plane
$4x - 3y + z + 13 = 0$
$\therefore 8r + \frac{9}{2}r + \frac{r}{2} + 13 = 0$
$\Rightarrow r = \frac{-13}{8 + \frac{9}{2} + \frac{1}{2}} = \frac{-26}{26} = -1$
$\therefore Q = \left(-2, \frac{3}{2}, -\frac{1}{2}\right)$
$QR = \sqrt{(-1+2)^2 + \left(1 - \frac{3}{2}\right)^2 + \left(-6 + \frac{1}{2}\right)^2}$
$= \sqrt{1 + \frac{1}{4} + \frac{121}{4}} = 3\sqrt{\frac{7}{2}}$

Question271

A vector $\vec{n}$ is inclined to x -axis at 45°, to y -axis at 60° and at an acute angle to z -axis. If $\vec{n}$ is a normal to a plane passing through the point $(\sqrt{2}, -1, 1)$ then the equation of the plane is: [Online April 9, 2013]

Options:

A. $4\sqrt{2}x + 7y + z - 2$

B. $2x + y + 2z = 2\sqrt{2} + 1$

C. $3\sqrt{2}x - 4y - 3z = 7$

D. $\sqrt{2}x - y - z = 2$

Answer: B

Solution:

Solution:
Direction cosines of $\vec{n}$ are $\frac{1}{2}, \frac{1}{4}, \frac{1}{2}$.
Equation of the plane,

$\frac{1}{2}(x - \sqrt{2}) + \frac{1}{4}(y + 1) + \frac{1}{2}(z - 1) = 0$
$\Rightarrow 2(x - \sqrt{2}) + (y + 1) + 2(z - 1) = 0$
$\Rightarrow 2x + y + 2z = 2\sqrt{2} - 1 + 2$
$\Rightarrow 2x + y + 2z = 2\sqrt{2} + 1$

Question272

If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to: [2012]

Options:

A. -1

B. $\frac{2}{9}$

C. $\frac{9}{2}$

D. 0

Answer: C

Solution:

Solution:
Given lines are $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$
and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$
$\therefore \vec{a}_1 = \hat{i} - \hat{j} + \hat{k}, \vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$
$\vec{a}_2 = 3\hat{i} + k\hat{j}, \vec{b}_2 = \hat{i} + 2\hat{j} + \hat{k}$
Given lies are intersect if
$\frac{(\vec{a}_2 - \vec{a}_1) . (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1||\vec{b}_2|} = 0$
$\Rightarrow (\vec{a}_2 - \vec{a}_1) . (\vec{b}_1 \times \vec{b}_2) = 0$
$\Rightarrow \begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$
$\Rightarrow 2(3 - 8) - (k + 1)(2 - 4) - 1(4 - 3) = 0$
$\Rightarrow 2(-5) - (k + 1)(-2) - 1(1) = 0$
$\Rightarrow -10 + 2k + 2 - 1 = 0 \Rightarrow k = \frac{9}{2}$

Question273

The distance of the point $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line that passes through the point $2\hat{i} + 3\hat{j} - 4\hat{k}$ and is parallel to the vector $6\hat{i} + 3\hat{j} - 4\hat{k}$ is [Online May 26, 2012]

Options:

A. 9

B. 8

C. 7

D. 10

Answer: C

Solution:

Solution:
Point is (–1, 2, 6)
Line passes through the point (2, 3, –4) parallel to vector whose direction ratios is 6, 3, – 4.
Equation is $\frac{x-2}{6} = \frac{y-3}{3} = \frac{z+4}{-4} = \lambda$
Any point on this line is given by $x = 6\lambda + 2$, $y = 3\lambda + 3$, $z = -4\lambda - 4$
Now, d .Rs of line passing through (-1,2,6) and ⊥ to this line is $\{(x+1), (y-2), (z-6)\}$
So, $6(x+1) + 3(y-2) - 4(z-6) = 0$
$\Rightarrow 6x + 3y - 4z + 24 = 0$
Now, $6(6\lambda + 2) + 3(3\lambda + 3) + 4(4\lambda + 4) + 24 = 0$
$\Rightarrow 61\lambda + 61 = 0 \Rightarrow \lambda = -1$
So, $x = -4$, $y = 0$, $z = 0$
Now, distance between (-1,2,6) and (-4,0,0) is $\sqrt{9+4+36} = \sqrt{49} = 7$

Question274

Statement 1: The shortest distance between the lines $\frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$ and $\frac{x-1}{4} = \frac{y-1}{-2} = \frac{z-1}{4}$ is $\sqrt{2}$.
Statement 2 : The shortest distance between two parallel lines is the perpendicular distance from any point on one of the lines to the other line.
[Online May 19, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

C. Statement 1 is false, Statement 2 is true.

D. Statement 1 is true, Statement 2 is true, , Statement 2 is not a correct explanation for Statement 1

Answer: C

Solution:

Solution:
On solving we will get shortest distance $\neq \sqrt{2}$

Question275

The coordinates of the foot of perpendicular from the point (1, 0, 0) to the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ are
[Online May 12, 2012]

Options:

A. (2, – 3, 8)

B. (1, – 1, – 10)

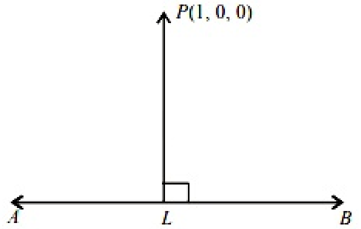
C. (5, – 8, – 4)

D. (3, – 4, – 2)

Answer: D

Solution:

Solution:
Let the equation of AB is
$\frac{x-1}{2} = \frac{y-(-1)}{-3} = \frac{z-(-10)}{8} = k$
Let L be the foot of the perpendicular drawn from P(1, 0, 0)



Now, direction ratio of PL = $(2k, -3k-1, 8k-10)$ and direction ratio of AB = $(2, -3, 8)$
Since, PL is perpendicular to AB
$\therefore 2(2k) - 3(-3k-1) + 8(8k-10) = 0$
Now, $k = \frac{2(1-1) + (-3)(0+1) + 8(0+10)}{(2)^2 + (-3)^2 + (8)^2}$
$= \frac{0-3+80}{4+9+64} = \frac{77}{77} = 1$
$\therefore$ Required co-ordinate $= L = (2+1, -3-1, 8-10)$
$= (3, -4, -2)$

Question276

A equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is :
[2012]

Options:

A. $x - 2y + 2z - 3 = 0$

B. $x - 2y + 2z + 1 = 0$

C. $x - 2y + 2z - 1 = 0$

D. $x - 2y + 2z + 5 = 0$

Answer: A

Solution:

Solution:
Given that, equation of a plane is
$x-2y+2z-5=0$
So, Equation of parallel plane is
$x-2y+2z+d=0$
Now, it is given that distance from origin to the parallel plane is 1 .
$\therefore\left|\frac{d}{\sqrt{1^2+2^2+2^2}}\right|=1 \Rightarrow d=\pm 3$
So equation of required plane $x-2y+2z\pm 3=0$

Question277

The equation of a plane containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and the point (0,7,-7) is
[Online May 26, 2012]

Options:

A. $x+y+z=0$

B. $x+2y+z=21$

C. $3x-2y+5z+35=0$

D. $3x+2y+5z+21=0$

Answer: A

Solution:

Solution:
The equation of the plane containing the line
$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ is a $(x+1)+b(y-3)+c(z+2)=0$
where $-3a+2b+c=0$ ....(A)
This passes through (0,7,-7)
$\therefore a(0+1)+b(7-3)+c(-7+2)=0$
$\Rightarrow a+4b-5c=0$.......(B)
On solving equation (A) and (B) we get
$a=1, b=1, c=1$
$\therefore$ Required plane is
$x+1+y-3+z+2=0$
$\Rightarrow x+y+z=0$

Question278

Consider the following planes
$P: x+y-2z+7=0$
$Q: x+y+2z+2=0$
$R: 3x+3y-6z-11=0$

[Online May 26, 2012]

Options:

A. P and R are perpendicular

B. Q and R are perpendicular

C. P and Q are parallel

D. P and R are parallel

Answer: D

Solution:

Solution:
Given planes are
$P : x + y - 2z + 7 = 0$
$Q : x + y + 2z + 2 = 0$
and $R : 3x + 3y - 6z - 11 = 0$
Consider Plane P and R.
Here $a_1 = 1, b_1 = 1, c_1 = -2$
and $a_2 = 3, b_2 = 3, c_2 = -6$
Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{3}$
therefore P and R are parallel.

Question279

If the three planes $x = 5$, $2x - 5ay + 3z - 2 = 0$ and $3bx + y - 3z = 0$ contain a common line, then (a, b) is equal to [Online May 19, 2012]

Options:

A. $\left(\frac{8}{15}, -\frac{1}{5}\right)$

B. $\left(\frac{1}{5}, -\frac{8}{15}\right)$

C. $\left(-\frac{8}{15}, \frac{1}{5}\right)$

D. $\left(-\frac{1}{5}, \frac{8}{15}\right)$

Answer: B

Solution:

Solution:
Let the direction ratios of the common line be l, m and n.
$\therefore l \times 1 + m \times 0 + n \times 0 = 0 \Rightarrow l = 0$
$2l - 5ma + 3n = 0 \Rightarrow 5ma - 3n = 0$
$3lb + m - 3n = 0 \Rightarrow m - 3n = 0$ ....(3)
Subtracting (3) from (1), we get $m(5a - 1) = 0$

Now, value of m can not be zero because if $m = 0$ then $n = 0$
$\Rightarrow l = m = n = 0$ which is not possible.

Hence, $5a - 1 = 0 \Rightarrow a = \frac{1}{5}$

Thus, option (b) is correct.

Question280

A line with positive direction cosines passes through the point $P(2, -1, 2)$ and makes equal angles with the coordinate axes. If the line meets the plane $2x + y + z = 9$ at point Q, then the length PQ equals [Online May 7, 2012]

Options:

A. $\sqrt{2}$

B. 2

C. $\sqrt{3}$

D. 1

Answer: C

Solution:

Solution:
Point P is (2,-1,2)
Let this line meet at $Q(h, k, w)$
Direction ratio of this line is
$(h - 2, k + 1, w - 2)$
Since, $d\ c_s$ are equal &$d\ r_s$ are also equal,
So, $h - 2 = k + 1 + w - 2$
$\Rightarrow k = h - 3$ and $w = h$
This line meets the plane
$2x + y + z = 9$ at Q, so,
$2h + k + w = 9$ or $2h + h - 3 + h = 9$
$\Rightarrow 4h - 3 = 9 \Rightarrow h = 3$
and $k = 0$ and $w = 3$
DistancePQ $= \sqrt{(3-2)^2 + (0-(-1))^2 + (3-2)^2}$
$= \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Question281

The values of a for which the two points $(1, a, 1)$ and $(-3, 0, a)$ lie on the opposite sides of the plane $3x + 4y - 12z + 13 = 0$, satisfy [Online May 7, 2012]

Options:

A. $0 < a < \frac{1}{3}$

B. $-1 < a < 0$

C. $a < -1$ or $a < \frac{1}{3}$

D. $a = 0$

Answer: D

Solution:

Solution:
Given equation of plane is
$3x + 4y - 12z + 13 = 0$
$(1, a, 1)$ and $(-3, 0, a)$ satisfy the equation of plane.
∴ We have
$3 + 4(a) - 12 + 13 = 0$ and$3(-3) - 12(a) + 13 = 0$
$\Rightarrow 4 + 4a = 0$ and $4 - 12a = 0$
$\Rightarrow a = -1$ and $a = \frac{1}{3}$
Since, $(1, a, 1)$ and $(-3, 0, a)$ lie on the opposite sides of the plane $\therefore a = 0$

Question282

The length of the perpendicular drawn from the point(3,-1,11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is: [2011RS]

Options:

A. $\sqrt{29}$

B. $\sqrt{33}$

C. $\sqrt{53}$

D. $\sqrt{66}$

Answer: C

Solution:

Solution:
Any point on line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \alpha$ is $(2\alpha, 3\alpha + 2, 4\alpha + 3)$
$\Rightarrow$ Direction ratio of the $\perp$ line is
$2\alpha - 3, 3\alpha + 3, 4\alpha - 8.$
and Direction ratio of the given line are 2,3,4
$\Rightarrow 2(2\alpha - 3) + 3(3\alpha + 3) + 4(4\alpha - 8) = 0$
$\Rightarrow 29\alpha - 29 = 0$
$\Rightarrow \alpha = 1$
$\Rightarrow$ Foot of $\perp$ is (2,5,7)
$\Rightarrow$ Length $\perp$ is $\sqrt{1^2 + 6^2 + 4^2} = \sqrt{53}$

Question283

Statement-1: The point A(1, 0, 7)) is the mirror image of the point

B(1, 6, 3) in the line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$
Statement-2: The line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A(1, 0, 7) and B(1, 6, 3).
[2011]

Options:

A. Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is false.

C. Statement-1 is false, Statement-2 is true.

D. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Answer: A

Solution:

Solution:
The direction ratio of the line segment AB is 0,6,-4 and the direction ratio of the given line is 1,2,3 .
Clearly $1 \text{ times } 0 + 2 \times 6 + 3 \times (-4) = 0$
So, the given line is perpendicular to line AB.
Also, the mid point of A and B is (1,3,5) which satisfy the given line.
So, the image of B in the given line is A statement- 1 and 2 both true but 2 is not correct explanation. of 1 .

Question284

The distance of the point (1,-5,9) from the plane x − y+ z = 5 measured along a straight x = y = z is
[2011RS]

Options:

A. $10\sqrt{3}$

B. $5\sqrt{3}$

C. $3\sqrt{10}$

D. $3\sqrt{5}$

Answer: A

Solution:

Solution:
Equation of line through $P(1, -5, 9)$ and parallel to the line $x = y = z$ is
$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda(\text{say})$
$Q = (x = 1 + \lambda, y = -5 + \lambda, z = 9 + \lambda)$
Since Q lies on plane $x - y + z = 5$
$\therefore 1 + \lambda + 5 - \lambda + 9 + \lambda = 5$
$\Rightarrow \lambda = -10$

$\therefore Q = (-9, -15, -1)$
$\therefore PQ = \sqrt{(1 + 9)^2 + (15 - 5)^2 + (9 + 1)^2}$
$= \sqrt{300} = 10\sqrt{3}$

Question285

If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals

[2011]

Options:

A. $\frac{3}{2}$

B. $\frac{2}{5}$

C. $\frac{5}{3}$

D. $\frac{2}{3}$

Answer: D

Solution:

Solution:
Let θ be the angle between the given line and plane, then
$\sin\theta = \frac{1 \times 1 + 2 \times 2 + \lambda \times 3}{\sqrt{1^2 + 2^2 + \lambda^2} \cdot \sqrt{1^2 + 2^2 + 3^2}} = \frac{5 + 3\lambda}{\sqrt{14} \cdot \sqrt{5 + \lambda^2}}$
$\Rightarrow \cos\theta = \sqrt{1 - \frac{(5 + 3\lambda)^2}{14(5 + \lambda^2)}}$
$\Rightarrow \sqrt{\frac{5}{14}} = \sqrt{1 - \frac{(5 + 3\lambda)^2}{14(5 + \lambda^2)}}$
Squaring both sides, we get
$\frac{5}{14} = \frac{5\lambda^2 - 30\lambda + 45}{14(5 + \lambda^2)}$
$\Rightarrow \lambda = \frac{2}{3}$

Question286

A line AB in three-dimensional space makes angles 45° and 120° with the positive x -axis and the positive y -axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals
[2010]

Options:

A. 45°

B. 60°

C. 75°

D. 30°

Answer: B

Solution:

Solution:

As per question, direction cosines of the line : $l = \cos 45° = \frac{1}{\sqrt{2}}$, $m = \cos 120° = \frac{-1}{2}$, $n = \cos\theta$

where θ is the angle, which line makes with positive z -axis.

We know that, $l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$

$\cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2} = \cos\frac{\pi}{2}$ (θ being acute)

$\Rightarrow \theta = \frac{\pi}{3}$

Question287

The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13,32) . The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is
[2010]

Options:

A. $\sqrt{17}$

B. $\frac{17}{\sqrt{15}}$

C. $\frac{23}{\sqrt{17}}$

D. $\frac{23}{\sqrt{15}}$

Answer: C

Solution:

Solution:

Slope of line $L = -\frac{b}{5}$

Slope of line $K = -\frac{3}{c}$

Line L is parallel to line k.

$\Rightarrow \frac{b}{5} = \frac{3}{c} \Rightarrow bc = 15$

(13,32) is a point on L.

$\therefore \frac{13}{5} + \frac{32}{b} = 1 \Rightarrow \frac{32}{b} = -\frac{8}{5}$

$\Rightarrow b = -20 \Rightarrow c = -\frac{3}{4}$
Equation of K :
$y - 4x = 3 \Rightarrow 4x - y + 3 = 0$
Distance between L and K $= \frac{|52 - 32 + 3|}{\sqrt{17}} = \frac{23}{\sqrt{17}}$

Question288

Statement -1 : The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane x – y + z = 5.
Statement -2: The plane x – y + z = 5 bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).
[2010]

Options:

A. Statement -1 is true, Statement -2 is true ; Statement - 2 is not a correct explanation for Statement -1.

B. Statement -1 is true, Statement -2 is false.

C. Statement -1 is false, Statement -2 is true .

D. Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

Answer: A

Solution:

Solution:
$A(3, 1, 6); B = (1, 3, 4)$
Putting coordinate of mid-point of $AB = (2, 2, 5)$ in plane $x - y + z = 5$ then $2 - 2 + 5 = 5$, satisfy
So, mid-point of $AB = (2, 2, 5)$ lies on the plane.
d.r's of $AB = (2, -2, 2)$
d.r's of normal to plane $= (1, -1, 1)$
Direction ratio of AB and normal to the plane are proportional therefore,
AB is perpendicular to the normal of plane
$\therefore$ A is image of B
Statement-1 is correct.
Statement-2 is also correct but it is not correct explanation.

Question289

The projections of a vector on the three coordinate axis are 6,-3,2 respectively. The direction cosines of the vector are
[2009]

Options:

A. $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$

B. $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

C. $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$

D. 6, −3, 2

Answer: B

Solution:

Solution:
Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be the initial and final points of the vector whose projections on the three coordinate axes are 6,-3,2 then
$x_2 - x_1, = 6; y_2 - y_1 = -3; z_2 - z_1 = 2$
So that direction ratios of $\overrightarrow{PQ}$ are 6,-3,2
∴ Direction cosines of $\overrightarrow{PQ}$ are $\frac{6}{\sqrt{6^2 + (-3)^2 + 2^2}}, \frac{-3}{\sqrt{6^2 + (-3)^2 + 2^2}}\ \frac{2}{\sqrt{6^2 + (-3)^2 + 2^2}} = \frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

Question290

Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane$x + 3y - \alpha z + \beta = 0$. Then (α, β) equals
[2009]

Options:

A. (-6,7)

B. (5,-15)

C. (-5,5)

D. (6,-17)

Answer: A

Solution:

Solution:
Given that, the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$
∴Pt$(2, 1, -2)$ lies on the plane
i.e. $2 + 3 + 2\alpha + \beta = 0$
or $2\alpha + \beta + 5 = 0$ ....(i)
Also normal to plane will be perpendicular to line,
$\therefore 3 \times 1 - 5 \times 3 + 2 \times (-\alpha) = 0$
$\Rightarrow \alpha = -6$
From equation (i) then, $\beta = 7$
$\therefore (\alpha, \beta) = (-6, 7)$

Question291

If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to

[2008]

Options:

A. -5

B. 5

C. 2

D. -2

Answer: A

Solution:

Solution:
hen the two lines intersect then shortest distance between them is zero i.e.

$$\frac{\left(\vec{a}_2 - \vec{a}_1\right) . \vec{b}_1 \times \vec{b}_2}{\left|\vec{b}_1 \times \vec{b}_2\right|} = 0$$

$\Rightarrow \left(\vec{a}_2 - \vec{a}_1\right) . \vec{b}_1 \times \vec{b}_2 = 0$

where $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b}_1 = k\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a}_2 = 2\hat{i} + 3\hat{j} + \hat{k}, \vec{b}_2 = 3\hat{i} + k\hat{j} + 2\hat{k}$

$$\Rightarrow \begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$\Rightarrow 1(4 - 3k) - 1(2k - 9) - 2(k^2 - 6) = 0$

$\Rightarrow -2k^2 - 5k + 25 = 0 \Rightarrow k = -5$ or $\frac{5}{2}$

$\because k$ is an integer, therefore $k = -5$

Question292

The line passing through the points (5, 1, a) and (3, b, 1)crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

[2008]

Options:

A. a = 2, b = 8

B. a = 4, b = 6

C. a = 6, b = 4

D. a = 8, b = 2

Answer: C

Solution:

Equation of line through (5, 1, a) and (3, b, 1) is $\frac{x-5}{-2}=\frac{y-1}{b-1}=\frac{z-a}{1-a}=\lambda$

$x=-2\lambda+5$
$y=(b-1)\lambda+1$
$z=(1-a)\lambda+a$
$\therefore$ Any point on this line is a
$[-2\lambda+5, (b-1)\lambda+1, (1-a)\lambda+a]$
Given that it crosses yz plane $\therefore -2\lambda+5=0$
$\lambda=\frac{5}{2}$

$\therefore \left(0, (b-1)\frac{5}{2}+1, (1-a)\frac{5}{2}+a\right) = \left(0, \frac{17}{2}, \frac{-13}{2}\right)$

$\Rightarrow (b-1)\frac{5}{2}+1=\frac{17}{2}$

and $(1-a)\frac{5}{2}+a=-\frac{13}{2}$

$\Rightarrow b=4$ and $a=6$

Question293

If a line makes an angle of π / 4 with the positive directions of each of x - axis and y -axis, then the angle that the line makes with the positive direction of the z -axis is
[2007]

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer: B

Solution:

Solution:

Let the line makes an angle θ with the positive direction of z -axis. Given that lines makes angle $\frac{\pi}{4}$ with x axis and y - axis.

$\therefore l = \cos\frac{\pi}{4}, m=\cos\frac{\pi}{4}, n=\cos\theta$

We know that , $l^2+m^2+n^2=1$

$\therefore \cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}+\cos^2\theta=1$

$\Rightarrow \frac{1}{2}+\frac{1}{2}+\cos^2\theta=1$

$\Rightarrow \cos^2\theta=0 \Rightarrow \theta=\frac{\pi}{2}$

Hence, angle with positive direction of the z -axis is $\frac{\pi}{2}$.

Question294

Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x -axis, then $\cos\alpha$ equals
[2007]

Options:

A. 1

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{2}$.

Answer: C

Solution:

Solution:
Let the direction cosines of line L be l, m, n. Since line L lies on both planes.
$\therefore 2l + 3m + n = 0$ ....(i)
and $l + 3m + 2n = 0$ ....(ii)
on solving equation (i) and (ii), we get
$\frac{l}{6-3} = \frac{m}{1-4} = \frac{n}{6-3} \Rightarrow \frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$
Now $\frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{3^2 + (-3)^2 + 3^2}}$
$\because l^2 + m^2 + n^2 = 1$
$\therefore \frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{1}{\sqrt{27}}$
$\Rightarrow l = \frac{3}{\sqrt{27}} = \frac{1}{\sqrt{3}}, m = -\frac{1}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}$
Line L, makes an angle α with + ve x -axis
$\therefore l = \cos\alpha \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

Question295

TOPIC 4 - Sphere and Miscellaneous Problems on Sphere
If (2,3,5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the cooordinates of the other end of the diameter are
[2007]

Options:

A. (4,3,5)

B. (4,3,-3)

C. (4,9,-3)

D. (4,-3,3)

Answer: C

Solution:

Solution:
We know that centre of sphere
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$
is $(-u, -v, -w)$
Given that, $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$
∴ Centre $\equiv (3, 6, 1)$
Coordinates of one end of diameter of the sphere are (2,3,5)
Let the coordinates of the other end of diameter are(α, β, γ)
$\therefore \frac{\alpha + 2}{2} = 3, \frac{\beta + 3}{2} = 6, \frac{\gamma + 5}{2} = 1$
$\Rightarrow \alpha = 4, \beta = 9$ and $\gamma = -3$
∴ Coordinate of other end of diameter are (4,9,-3)

Question296

The image of the point (-1,3,4) in the plane $x - 2y = 0$ is [2006]

Options:

A. $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$

B. (15,11,4)

C. $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$

D. None of these

Answer: D

Solution:

Solution:
Let (α, β, γ) be the image, then mid point of (α, β, γ) and (-1,3,4) must lie on $x - 2y = 0$
$\therefore \frac{\alpha - 1}{2} - 2\left(\frac{\beta + 3}{2}\right) = 0$
$\therefore \alpha - 1 - 2\beta - 6 = 0 \Rightarrow \alpha - 2\beta = 7$ ....(1)
Also line joining (α, β, γ) and (-1,3,4) should be parallel to the normal of the plane $x - 2y = 0$
$\therefore \frac{\alpha + 1}{1} = \frac{\beta - 3}{-2} = \frac{\gamma - 4}{0} = \lambda$
$\Rightarrow \alpha = \lambda - 1, \beta = -2\lambda + 3, \gamma = 4$ ....(2)
From (1) and (2)
$\alpha = \frac{9}{5}, \beta = -\frac{13}{5}, \gamma = 4$
None of the option matches.

Question297

If non zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point. That point is

[2005]

Options:

A. (-1,2)

B. (-1,-2)

C. (1,-2)

D. $\left(1, -\frac{1}{2}\right)$

Answer: C

Solution:

Solution:

a, b, c are in H.P. $\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$

$\therefore \frac{x}{a} + \frac{y}{a} + \frac{1}{c} = 0$ passes through (1,-2)

Question298

The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is [2005]

Options:

A. 0°

B. 90°

C. 45°

D. 30°

Answer: B

Solution:

Solution:

The given lines are $2x = 3y = -z$

or $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}\Big[$ Dividing by 6]

and $6x = -y = -4z$

or $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$ [Dividing by 12]

$\therefore$ Angle between two lines is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}}$$

$$\cos\theta = \frac{3.2 + 2\,.\,(-12) + (-6)\,.\,(-3)}{\sqrt{3^2 + 2^2 + (-6)^2}\sqrt{2^2 + (-12)^2 + (-3)^2}}$$

$= 6 - 24 + 18\sqrt{49}\sqrt{157} \ = 0 \Rightarrow \theta = 90°$

Question299

The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(i - j + 4k)$ and the plane $\vec{r}\,.\left(\hat{i} + 5\hat{j} + \hat{k}\right) = 5$ is
[2005]

Options:

A. $\frac{10}{9}$

B. $\frac{10}{3\sqrt{3}}$

C. $\frac{3}{10}$

D. $\frac{10}{3}$

Answer: B

Solution:

Solution:
The given line is $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(i - j + 4k)$
and the plane is $\vec{r}\,.\left(\hat{i} + 5\hat{j} + \hat{k}\right) = 5$
$\Rightarrow x + 5y + z = 5$
Required distance = $\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$
$= \left|\frac{2 - 10 + 3 - 5}{\sqrt{1 + 25 + 1}}\right| = \frac{10}{3\sqrt{3}}$

Question300

If the angle theta between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that$\sin\theta = \frac{1}{3}$ then the value of λ is
[2005]

Options:

A. $\frac{5}{3}$

B. $\frac{-3}{5}$

C. $\frac{3}{4}$

D. $\frac{-4}{3}$

Answer: A

Solution:

Solution:
Let θ is the angle between line and plane then

$\sin\theta = \dfrac{\vec{b}\,.\,\vec{n}}{|\vec{b}||\vec{n}|}$

$= \dfrac{(\hat{i} + 2\hat{j} + 2\hat{k})\,.\,(2\hat{i} - \hat{j} + \sqrt{\lambda}\hat{k})}{\sqrt{1+4+4}\sqrt{4+1+\lambda}} = \dfrac{2-2+2\sqrt{\lambda}}{3 \times \sqrt{5+\lambda}}$

$\Rightarrow \sin\theta = \dfrac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}} = \dfrac{1}{3} \Rightarrow 4\lambda = 5 + \lambda$

$\Rightarrow \lambda = \dfrac{5}{3}$

Question301

The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius [2005]

Options:

A. 3

B. 1

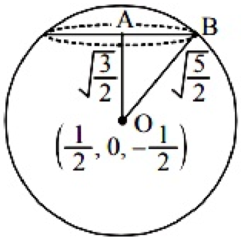
C. 2

D. $\sqrt{2}$

Answer: B

Solution:

Solution:



Centre of sphere $= \left(\frac{1}{2}, 0, -\frac{1}{2}\right)$ and radius of sphere

$= \sqrt{\frac{1}{4} + \frac{1}{4} + 2} = \sqrt{\frac{5}{2}}$

Perpendicular distance OA of centre from $x + 2y - z = 4$ is given by

$\dfrac{\left|\frac{1}{2} + \frac{1}{2} - 4\right|}{\sqrt{6}} = \sqrt{\frac{3}{2}}$

$\therefore$ radius of circle $AB = \sqrt{OB^2 - OA^2} = \sqrt{\frac{5}{2} - \frac{3}{2}} = 1$

Question302

If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the mid point of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals
[2005]

Options:

A. -1

B. 1

C. -2

D. 2

Answer: C

Solution:

Solution:
Plane $2ax - 3ay + 4az + 6 = 0$ passes through the mid point of the line joining the centres of spheres
$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and
$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$
respectively centre of spheres are $c_1(-3, 4, 1)$ and $c_2(5, -2, 1)$. Mid point of c_1c_2 is (1,1,1)
Satisfying this in the equation of plane, we get
$2a - 3a + 4a + 6 = 0$
$\Rightarrow a = -2.$

Question303

A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y -axis, is such that$\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals
[2004]

Options:

A. $\frac{2}{5}$

B. $\frac{1}{5}$

C. $\frac{3}{5}$

D. $\frac{2}{3}$

Answer: C

Solution:

Solution:
As per question the direction cosines of the line are $\cos\theta$, $\cos\beta$, $\cos\theta$
$\therefore \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$
$\because 2\cos^2\theta = 1 - \cos^2\theta$

$\Rightarrow 2\cos^2\theta = \sin^2\beta = 3\sin^2\theta$ (given)
$\Rightarrow 2\cos^2\theta = 3 - 3\cos^2\theta$
$\therefore \cos^2\theta = \frac{3}{5}$

Question304

If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = \frac{t}{2}$, $y = 1 + t$, $z = 2 - t$, with parameters s and t respectively, are co-planar, then λ equals.
[2004]

Options:

A. 0

B. -1

C. $-\frac{1}{2}$

D. -2

Answer: D

Solution:

Solution:
The given lines are
$x - 1 = \frac{y + 3}{-\lambda} = \frac{z - 1}{\lambda} = s$ ......(1)
and $2x = y - 1 = \frac{z - 2}{-1} = t$ .....(2)
The lines are coplanar, if

$$\begin{vmatrix} 0 - 1 & 1 - (-3) & 2 - 1 \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & 4 & 1 \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix} = 0$$

Apply $c_2 \to c_2 + c_3$; $\begin{vmatrix} -1 & 5 & 1 \\ 1 & 0 & \lambda \\ \frac{1}{2} & 0 & -1 \end{vmatrix} = 0$

$\Rightarrow -5\left(-1 - \frac{\lambda}{2}\right) = 0 \Rightarrow \lambda = -2$

Question305

A line with direction cosines proportional to 2,1,2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the

points of intersection are given by
[2004]

Options:

A. (2a, 3a, 3a), (2a, a, a)

B. (3a, 2a, 3a), (a, a, a)

C. (3a, 2a, 3a), (a, a, 2a)

D. (3a, 3a, 3a), (a, a, a)

Answer: B

Solution:

Solution:
Let a point on the line $x = y + a = z = \lambda$ is
$(\lambda, \lambda - a, \lambda)$ and a point on the line
$x + a = 2y = 2z = \mu$ is $\left(\mu - a, \frac{\mu}{2}, \frac{\mu}{2}\right)$, then Direction ratio of the line joining these points are
$\lambda - \mu + a, \lambda - a - \frac{\mu}{2}, \lambda - \frac{\mu}{2}$
If it respresents the required line whose d . r be $2, 1, 2$, then
$\frac{\lambda - \mu + a}{2} = \frac{\lambda - a - \frac{\mu}{2}}{1} = \frac{\lambda - \frac{\mu}{2}}{2}$
on solving we get $\lambda = 3a, \mu = 2a$
∴ The required points of intersection are
$(3a, 3a - a, 3a)$ and $\left(2a - a, \frac{2a}{2}, \frac{2a}{2}\right)$
or $(3a, 2a, 3a)$ and (a, a, a)

Question306

Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is
[2004]

Options:

A. $\frac{9}{2}$

B. $\frac{5}{2}$

C. $\frac{7}{2}$

D. $\frac{3}{2}$

Answer: C

Solution:

Solution:
The planes are $2x + y + 2z - 8 = 0$ ....(1)

and $4x + 2y + 4z + 5 = 0$
or $2x + y + 2z + \frac{5}{2} = 0$ ....(2)
Since, both planes are parallel
∴ Distance between (1) and (2)

$$= \left| \frac{\frac{5}{2} + 8}{\sqrt{2^2 + 1^2 + 2^2}} \right| = \left| \frac{21}{2\sqrt{9}} \right| = \frac{7}{2}$$

Question307

The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane [2004]

Options:

A. $2x - y - z = 1$

B. $x - 2y - z = 1$

C. $x - y - 2z = 1$

D. $x - y - z = 1$

Answer: A

Solution:

Solution:
Given that, the equations of spheres are
$S_1 : x^2 + y^2 + z^2 + 7x - 2y - z - 13 = 0$ and
$S_2 : x^2 + y^2 + z^2 - 3x + 3y + 4z - 8 = 0$
We know that eqn. of intersection plane be
$S_1 - S_2 = 0 \Rightarrow 10x - 5y - 5z - 5 = 0$
$\Rightarrow 2x - y - z = 1$

Question308

The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if [2003]

Options:

A. k = 3 or -2

B. k = 0 or -1

C. k = 1 or -1

D. k = 0 or -3 .

Answer: D

Solution:

Solution:
Two planes are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 + C_1$, $C_3 \to C_3 + C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 - k \\ k & k + 2 & 1 + k \end{vmatrix} = 0$$

$\Rightarrow 1[2 + 2k - (k + 2)(1 - k)] = 0$
$\Rightarrow 2 + 2k - (-k^2 - k + 2) = 0$
$k^2 + 3k = 0 \Rightarrow k(k + 3) = 0$
or $k = 0$ or -3

Question309

The two lines x = ay + b , z = cy + d and x = a'y + b', z = c'y + d' will be perpendicular, if and only if [2003]

Options:

A. aa' + cc' + 1 = 0

B. aa' + bb' + cc' + 1 = 0

C. aa' + bb' +cc' = 0

D. (a + a') (b + b') +(c + c') = 0.

Answer: A

Solution:

Solution:
$\frac{x - b}{a} = \frac{y}{1} = \frac{z - d}{c}$; $\frac{x - b'}{a'} = \frac{y}{1} = \frac{z - d'}{c'}$.
For perpendicularity of lines,
$aa' + 1 + cc' = 0$

Question310

Two system of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a′, b′, c′ from the origin then [2003]

Options:

A. $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

B. $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$

C. $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$

D. $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

Answer: A

Solution:

Solution:

Equation of planes in intercept form be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ & $\frac{x}{a'}+\frac{y}{b'}+\frac{z}{c'}=1$

($\perp r$ distance on plane from origin is same.)

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right|=\left|\frac{-1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}\right|$$

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$$

Question311

The radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+7=0$ is [2003]

Options:

A. 4

B. 1

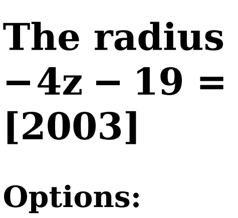
C. 2

D. 3

Answer: D

Solution:

Solution:



Centre of sphere $=(-1,1,2)$

Radius of sphere $\sqrt{1+1+4+19}=5$

Perpendicular distance from centre to the plane

$OC = d = \left|\dfrac{-1+2+4+7}{\sqrt{1+4+4}}\right| = \dfrac{12}{3} = 4$
In right, ΔAOC
$AC^2 = AO^2 - OC^2 = 5^2 - 4^2 = 9$
$\Rightarrow AC = 3$

Question312

The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is
[2003]

Options:

A. 39

B. 26

C. $11\frac{4}{13}$

D. 13 .

Answer: D

Solution:

Solution:
Centre of sphere be (-2,1,3) and radius 13
We know that,
Shortest distance = perpendicular distance between the plane and sphere = distance of plane from centre of sphere - radius
$= \left|\dfrac{-2 \times 12 + 4 \times 1 + 3 \times 3 - 327}{\sqrt{144+9+16}}\right| - 13$
$= 26 - 13 = 13$

Question313

The d.r. of normal to the plane through (1,0,0),(0,1,0) which makes an angle $\pi / 4$ with plane $x + y = 3$ are
[2002]

Options:

A. $1, \sqrt{2}, 1$

B. $1, 1, \sqrt{2}$

C. 1,1,2

D. $\sqrt{2}, 1, 1$

Answer: B

Solution:

Solution:
Equation of plane through (1,0,0) is
$a(x-1)+by+cz=0$ ....(i)
It is also passes through (0,1,0) .
$\therefore -a+b=0 \Rightarrow b=a$
$\cos 45° = \frac{a+a}{\sqrt{2(2a^2+c^2)}}$
$\Rightarrow 2a = \sqrt{2a^2+c^2} \Rightarrow 2a^2 = c^2 \Rightarrow c = \sqrt{2}a$
So d.r of normal are a, a $\sqrt{2}a$ i.e. $1, 1, \sqrt{2}$.

Question314

A plane which passes through the point (3,2,0) and the line $\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$ is
[2002]

Options:

A. $x - y + z = 1$

B. $x + y + z = 5$

C. $x + 2y - z = 1$

D. $2x - y + z = 5$

Answer: A

Solution:

Solution:
Since the point (3,2,0) lies on the given line
$\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$
∴ There can be infinite many planes passing through this line. We observed that only option (a) is satisfied by the coordinates of both the points (3,2,0) and (4,7,4)
$\therefore x-y+z=1$ is the required plane.
