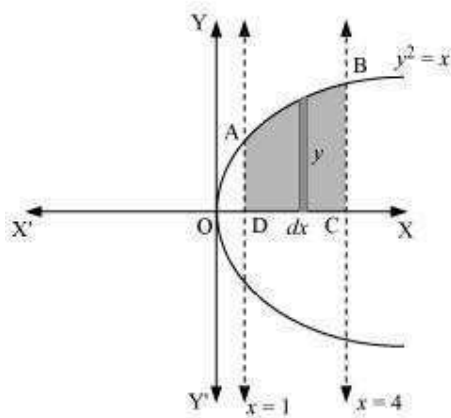


## Exercise 8.1

Question 1:

Find the area of the region bounded by the curve  $y^2 = x$  and the lines  $x = 1$ ,  $x = 4$  and the x-axis.

Answer



The area of the region bounded by the curve,  $y^2 = x$ , the lines,  $x = 1$  and  $x = 4$ , and the x-axis is the area ABCD.

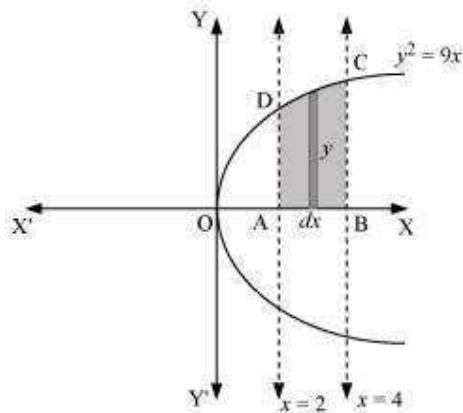
$$\begin{aligned}\text{Area of ABCD} &= \int_1^4 y \, dx \\ &= \int_1^4 \sqrt{x} \, dx \\ &= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \\ &= \frac{2}{3} \left[ (4)^{\frac{3}{2}} - (1)^{\frac{3}{2}} \right] \\ &= \frac{2}{3} [8 - 1] \\ &= \frac{14}{3} \text{ units}\end{aligned}$$

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Question 2:

Find the area of the region bounded by  $y^2 = 9x$ ,  $x = 2$ ,  $x = 4$  and the x-axis in the first quadrant.

Answer



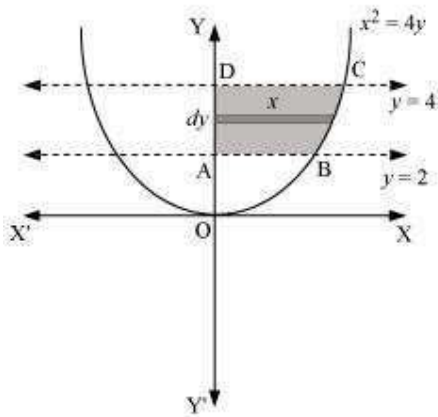
The area of the region bounded by the curve,  $y^2 = 9x$ ,  $x = 2$ , and  $x = 4$ , and the x-axis is the area ABCD.

$$\begin{aligned}\text{Area of ABCD} &= \int_2^4 y \, dx \\ &= \int_2^4 3\sqrt{x} \, dx \\ &= 3 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_2^4 \\ &= 2 \left[ x^{\frac{3}{2}} \right]_2^4 \\ &= 2 \left[ (4)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right] \\ &= 2 \left[ 8 - 2\sqrt{2} \right] \\ &= (16 - 4\sqrt{2}) \text{ units}\end{aligned}$$

Question 3:

Find the area of the region bounded by  $x^2 = 4y$ ,  $y = 2$ ,  $y = 4$  and the y-axis in the first quadrant.

Answer



The area of the region bounded by the curve,  $x^2 = 4y$ ,  $y = 2$ , and  $y = 4$ , and the y-axis is the area ABCD.

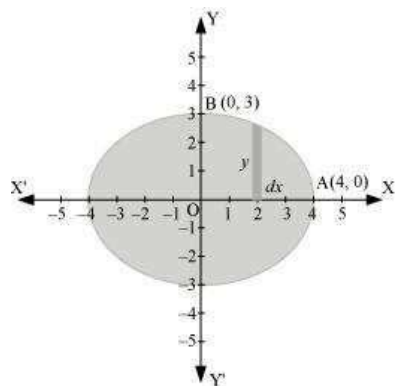
$$\begin{aligned}\text{Area of ABCD} &= \int_2^4 x \, dy \\&= \int_2^4 2\sqrt{y} \, dy \\&= 2 \int_2^4 \sqrt{y} \, dy \\&= 2 \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_2^4 \\&= \frac{4}{3} \left[ (4)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right] \\&= \frac{4}{3} [8 - 2\sqrt{2}] \\&= \left( \frac{32 - 8\sqrt{2}}{3} \right) \text{ units}\end{aligned}$$

Question 4:

Find the area of the region bounded by the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Answer

The given equation of the ellipse,  $\frac{x^2}{16} + \frac{y^2}{9} = 1$



, can be represented as  
It can be observed that the ellipse is symmetrical about x-axis and y-axis.

∴ Area bounded by ellipse = 4 × Area of OAB

$$\begin{aligned}
 \text{Area of OAB} &= \int_0^4 y \, dx \\
 &= \int_0^4 3 \sqrt{1 - \frac{x^2}{16}} \, dx \\
 &= \frac{3}{4} \int_0^4 \sqrt{16 - x^2} \, dx \\
 &= \frac{3}{4} \left[ \frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_0^4 \\
 &= \frac{3}{4} \left[ 2\sqrt{16 - 16} + 8\sin^{-1}(1) - 0 - 8\sin^{-1}(0) \right] \\
 &= \frac{3}{4} \left[ \frac{8\pi}{2} \right] \\
 &= \frac{3}{4} [4\pi] \\
 &= 3\pi
 \end{aligned}$$

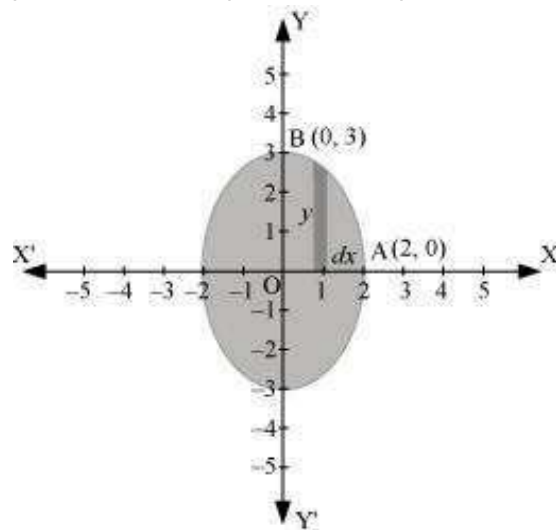
Therefore, area bounded by the ellipse = 4 × 3π = 12π units

Question 5:

Find the area of the region bounded by the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Answer

The given equation of the ellipse can be represented as



$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\Rightarrow y = 3\sqrt{1 - \frac{x^2}{4}} \quad \dots(1)$$

It can be observed that the ellipse is symmetrical about x-axis and y-axis.

$\therefore$  Area bounded by ellipse = 4  $\times$  Area OAB

$$\begin{aligned}\therefore \text{Area of OAB} &= \int_0^2 y \, dx \\&= \int_0^2 3\sqrt{1 - \frac{x^2}{4}} \, dx \quad [\text{Using (1)}] \\&= \frac{3}{2} \int_0^2 \sqrt{4 - x^2} \, dx \\&= \frac{3}{2} \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\&= \frac{3}{2} \left[ \frac{2\pi}{2} \right] \\&= \frac{3\pi}{2}\end{aligned}$$

Therefore, area bounded by the ellipse  $= 4 \times \frac{3\pi}{2} = 6\pi$  units

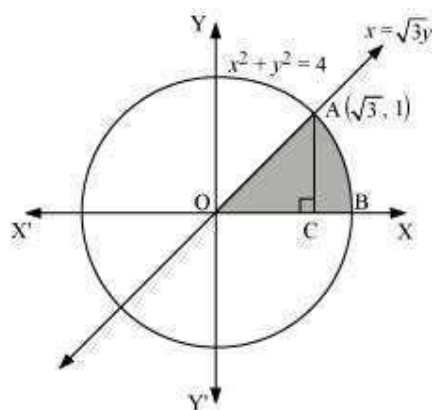
Question 6:

Find the area of the region in the first quadrant enclosed by x-axis, line  $x = \sqrt{3}y$  and the circle  $x^2 + y^2 = 4$

Answer

The area of the region bounded by the circle,  $x^2 + y^2 = 4$ ,  $x = \sqrt{3}y$ , and the x-axis is the area OAB.





The point of intersection of the line and the circle in the first quadrant is  $(\sqrt{3}, 1)$ .

$$\text{Area OAB} = \text{Area } \triangle OCA + \text{Area ACB}$$

$$\text{Area of OAC} = \frac{1}{2} \times \text{OC} \times \text{AC} = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2} \quad \dots(1)$$

$$\begin{aligned} \text{Area of ABC} &= \int_{\sqrt{3}}^2 y \, dx \\ &= \int_{\sqrt{3}}^2 \sqrt{4-x^2} \, dx \\ &= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_{\sqrt{3}}^2 \\ &= \left[ 2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} \sqrt{4-3} - 2 \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \right] \\ &= \left[ \pi - \frac{\sqrt{3}\pi}{2} - 2 \left( \frac{\pi}{3} \right) \right] \\ &= \left[ \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} \right] \\ &= \left[ \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right] \quad \dots(2) \end{aligned}$$

Therefore, area enclosed by x-axis, the line  $x = \sqrt{3}y$ , and the circle  $x^2 + y^2 = 4$  in the first

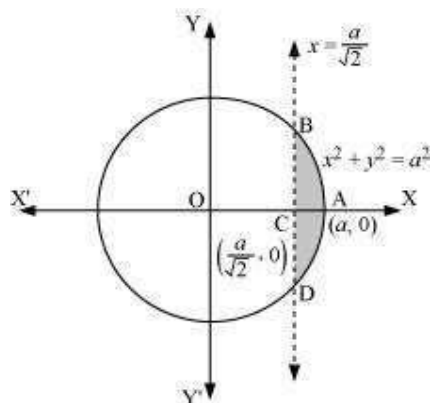
$$\text{quadrant} = \frac{\sqrt{3}\pi}{2} + \frac{3\sqrt{3}}{2} - \frac{2\pi}{3} = \frac{\pi}{3} \text{ units}$$

Question 7:

Find the area of the smaller part of the circle  $x^2 + y^2 = a^2$  cut off by the line  $x = \frac{a}{\sqrt{2}}$

Answer

The area of the smaller part of the circle,  $x^2 + y^2 = a^2$ , cut off by the line,  $x = \frac{a}{\sqrt{2}}$ , is the area ABCDA.



It can be observed that the area ABCD is symmetrical about x-axis.

$$\therefore \text{Area ABCD} = 2 \times \text{Area ABC}$$

$$\begin{aligned}
 \text{Area of } ABC &= \int_{\frac{a}{\sqrt{2}}}^a y \, dx \\
 &= \int_{\frac{a}{\sqrt{2}}}^a \sqrt{a^2 - x^2} \, dx \\
 &= \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_{\frac{a}{\sqrt{2}}}^a \\
 &= \left[ \frac{a^2}{2} \left( \frac{\pi}{2} \right) - \frac{a}{2\sqrt{2}} \sqrt{a^2 - \frac{a^2}{2}} - \frac{a^2}{2} \sin^{-1} \left( \frac{1}{\sqrt{2}} \right) \right] \\
 &= \frac{a^2 \pi}{4} - \frac{a}{2\sqrt{2}} \cdot \frac{a}{\sqrt{2}} - \frac{a^2}{2} \left( \frac{\pi}{4} \right) \\
 &= \frac{a^2 \pi}{4} - \frac{a^2}{4} - \frac{a^2 \pi}{8} \\
 &= \frac{a^2}{4} \left[ \pi - 1 - \frac{\pi}{2} \right] \\
 &= \frac{a^2}{4} \left[ \frac{\pi}{2} - 1 \right] \\
 \Rightarrow \text{Area } ABCD &= 2 \left[ \frac{a^2}{4} \left( \frac{\pi}{2} - 1 \right) \right] = \frac{a^2}{2} \left( \frac{\pi}{2} - 1 \right)
 \end{aligned}$$

Therefore, the area of smaller part of the circle,  $x^2 + y^2 = a^2$ , cut off by the line,  $x = \frac{a}{\sqrt{2}}$ ,

is  $\frac{a^2}{2} \left( \frac{\pi}{2} - 1 \right)$  units.

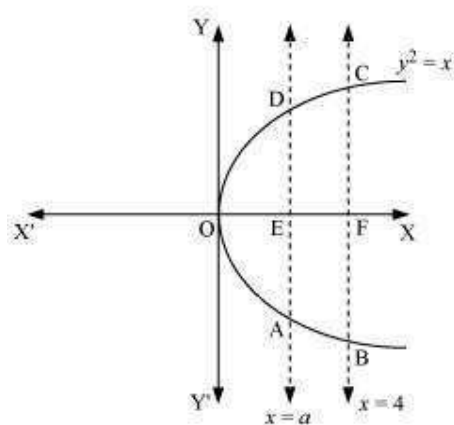
Question 8:

The area between  $x = y^2$  and  $x = 4$  is divided into two equal parts by the line  $x = a$ , find the value of  $a$ .

Answer

The line,  $x = a$ , divides the area bounded by the parabola and  $x = 4$  into two equal parts.

$\therefore$  Area OAD = Area ABCD



It can be observed that the given area is symmetrical about x-axis.

$\Rightarrow$  Area OED = Area EFCD

$$\begin{aligned} \text{Area } OED &= \int_0^a y \, dx \\ &= \int_0^a \sqrt{x} \, dx \\ &= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^a \\ &= \frac{2}{3} (a)^{\frac{3}{2}} \quad \dots(1) \end{aligned}$$

$$\begin{aligned} \text{Area of } EFCD &= \int_0^4 \sqrt{x} \, dx \\ &= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 \\ &= \frac{2}{3} \left[ 8 - a^{\frac{3}{2}} \right] \quad \dots(2) \end{aligned}$$

From (1) and (2), we obtain

$$\frac{2}{3} (a)^{\frac{3}{2}} = \frac{2}{3} \left[ 8 - (a)^{\frac{3}{2}} \right]$$

$$\Rightarrow 2 \cdot (a)^{\frac{3}{2}} = 8$$

$$\Rightarrow (a)^{\frac{3}{2}} = 4$$

$$\Rightarrow a = (4)^{\frac{2}{3}}$$

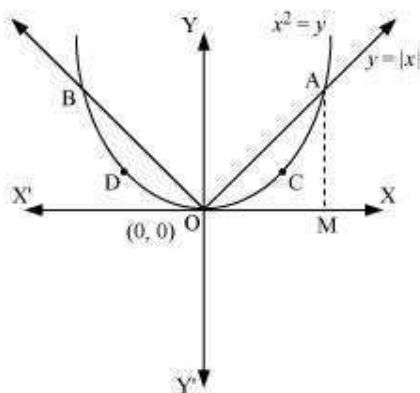
Therefore, the value of  $a$  is  $(4)^{\frac{2}{3}}$ .

Question 9:

Find the area of the region bounded by the parabola  $y = x^2$  and  $y = |x|$

Answer

The area bounded by the parabola,  $x^2 = y$ , and the line,  $y = |x|$  can be represented as



The given area is symmetrical about y-axis.

$$\therefore \text{Area OACO} = \text{Area ODBO}$$

The point of intersection of parabola,  $x^2 = y$ , and line,  $y = x$ , is A (1, 1). Area of OACO = Area  $\Delta$ OAB – Area OBACO

$$\therefore \text{Area of } \Delta\text{OAB} = \frac{1}{2} \times \text{OB} \times \text{AB} = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$$

$$\text{Area of OBACO} = \int_0^1 y \, dx = \int_0^1 x^2 \, dx = \left[ \frac{x^3}{3} \right]_0^1 = \frac{1}{3}$$

$$\Rightarrow \text{Area of OACO} = \text{Area of } \Delta\text{OAB} - \text{Area of OBACO}$$

$$\begin{aligned} &= \frac{1}{2} - \frac{1}{3} \\ &= \frac{1}{6} \end{aligned}$$

Therefore, required area =  $2 \left[ \frac{1}{6} \right] = \frac{1}{3}$  units

Question 10:

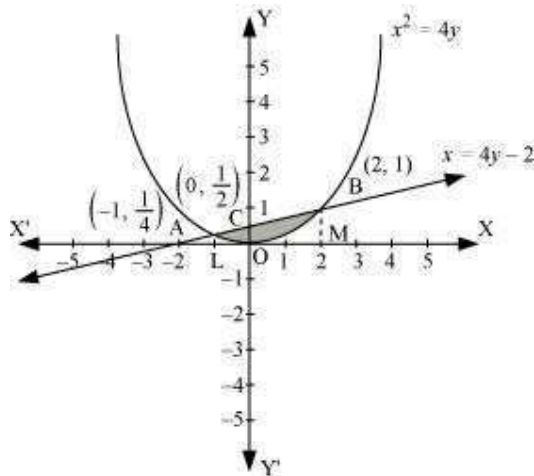
Find the area bounded by the curve  $x^2 = 4y$  and the line  $x = 4y - 2$

Answer

The area bounded by the curve,  $x^2 = 4y$ , and line,  $x = 4y - 2$ , is represented by the



shaded area OBAO.



Let A and B be the points of intersection of the line and parabola.

Coordinates of point A are  $\left(-1, \frac{1}{4}\right)$ .

Coordinates of point B are (2, 1).

We draw AL and BM perpendicular to x-axis.

It can be observed that,

Area OBAO = Area OBCO + Area OACO ... (1)

Then, Area OBCO = Area OMBC – Area OMBO

$$\begin{aligned} &= \int_0^2 \frac{x+2}{4} dx - \int_0^2 \frac{x^2}{4} dx \\ &= \frac{1}{4} \left[ \frac{x^2}{2} + 2x \right]_0^2 - \frac{1}{4} \left[ \frac{x^3}{3} \right]_0^2 \\ &= \frac{1}{4} [2+4] - \frac{1}{4} \left[ \frac{8}{3} \right] \\ &= \frac{3}{2} - \frac{2}{3} \\ &= \frac{5}{6} \end{aligned}$$

Similarly, Area OACO = Area OLAC – Area OLAO

$$\begin{aligned} &= \int_{-1}^0 \frac{x+2}{4} dx - \int_{-1}^0 \frac{x^2}{4} dx \\ &= \frac{1}{4} \left[ \frac{x^2}{2} + 2x \right]_{-1}^0 - \frac{1}{4} \left[ \frac{x^3}{3} \right]_{-1}^0 \\ &= -\frac{1}{4} \left[ \frac{(-1)^2}{2} + 2(-1) \right] - \left[ -\frac{1}{4} \left( \frac{(-1)^3}{3} \right) \right] \\ &= -\frac{1}{4} \left[ \frac{1}{2} - 2 \right] - \frac{1}{12} \\ &= \frac{1}{2} - \frac{1}{8} - \frac{1}{12} \\ &= \frac{7}{24} \end{aligned}$$

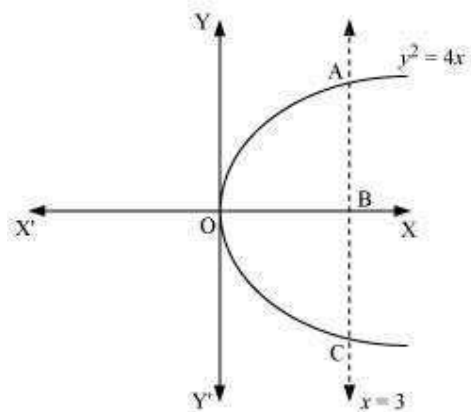
Therefore, required area =  $\left( \frac{5}{6} + \frac{7}{24} \right) = \frac{9}{8}$  units

Question 11:

Find the area of the region bounded by the curve  $y^2 = 4x$  and the line  $x = 3$

Answer

The region bounded by the parabola,  $y^2 = 4x$ , and the line,  $x = 3$ , is the area OACO.



The area OACO is symmetrical about x-axis.

$\therefore$  Area of OACO = 2 (Area of OAB)

$$\begin{aligned}\text{Area OACO} &= 2 \left[ \int_0^3 y \, dx \right] \\ &= 2 \int_0^3 2\sqrt{x} \, dx \\ &= 4 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^3 \\ &= \frac{8}{3} \left[ (3)^{\frac{3}{2}} \right] \\ &= 8\sqrt{3}\end{aligned}$$

Therefore, the required area is  $8\sqrt{3}$  units.

Question 12:

Area lying in the first quadrant and bounded by the circle  $x^2 + y^2 = 4$  and the lines  $x = 0$  and  $x = 2$  is

A.  $\pi$

$\frac{\pi}{2}$

$\frac{\pi}{3}$

$\frac{\pi}{4}$

B.

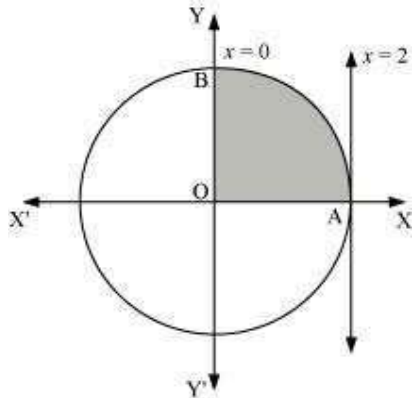
C.

D.

Answer

The area bounded by the circle and the lines,  $x = 0$  and  $x = 2$ , in the first quadrant is

represented as



$$\begin{aligned}\therefore \text{Area OAB} &= \int_0^2 y \, dx \\ &= \int_0^2 \sqrt{4-x^2} \, dx \\ &= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\ &= 2 \left( \frac{\pi}{2} \right) \\ &= \pi \text{ units}\end{aligned}$$

Thus, the correct answer is A.

Question 13:

Area of the region bounded by the curve  $y^2 = 4x$ , y-axis and the line  $y = 3$  is

A. 2

B.

$$\frac{9}{4}$$

$$\frac{9}{3}$$

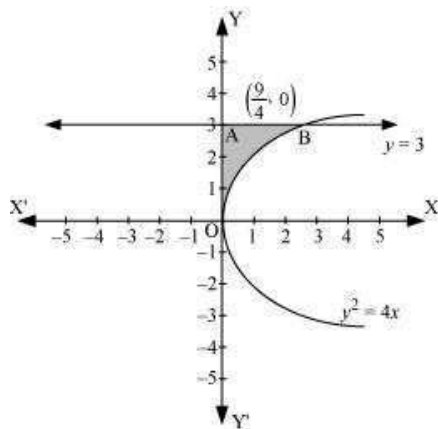
$$\frac{9}{2}$$

C.

D.

Answer

The area bounded by the curve,  $y^2 = 4x$ , y-axis, and  $y = 3$  is represented as



$$\begin{aligned}
 \therefore \text{Area OAB} &= \int_0^3 x \, dy \\
 &= \int_0^3 \frac{y^2}{4} \, dy \\
 &= \frac{1}{4} \left[ \frac{y^3}{3} \right]_0^3 \\
 &= \frac{1}{12} (27) \\
 &= \frac{9}{4} \text{ units}
 \end{aligned}$$

Thus, the correct answer is B.

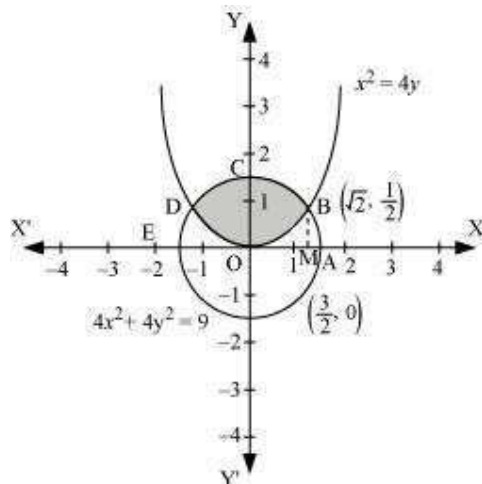
#### Exercise 8.2

Question 1:

Find the area of the circle  $4x^2 + 4y^2 = 9$  which is interior to the parabola  $x^2 = 4y$

Answer

The required area is represented by the shaded area OBCDO.



Solving the given equation of circle,  $4x^2 + 4y^2 = 9$ , and parabola,  $x^2 = 4y$ , we obtain the point of intersection as  $B\left(\sqrt{2}, \frac{1}{2}\right)$  and  $D\left(-\sqrt{2}, \frac{1}{2}\right)$ .

It can be observed that the required area is symmetrical about y-axis.

$$\therefore \text{Area OBCDO} = 2 \times \text{Area OBCO}$$

We draw BM perpendicular to OA.

Therefore, the coordinates of M are  $(\sqrt{2}, 0)$ .

$$\text{Therefore, Area OBCO} = \text{Area OMBCO} - \text{Area OMBO}$$



$$\begin{aligned}
 &= \int_0^{\sqrt{2}} \sqrt{\frac{(9-4x^2)}{4}} dx - \int_0^{\sqrt{2}} \sqrt{\frac{x^2}{4}} dx \\
 &= \frac{1}{2} \int_0^{\sqrt{2}} \sqrt{9-4x^2} dx - \frac{1}{4} \int_0^{\sqrt{2}} x^2 dx \\
 &= \frac{1}{4} \left[ x\sqrt{9-4x^2} + \frac{9}{2} \sin^{-1} \frac{2x}{3} \right]_0^{\sqrt{2}} - \frac{1}{4} \left[ \frac{x^3}{3} \right]_0^{\sqrt{2}} \\
 &= \frac{1}{4} \left[ \sqrt{2}\sqrt{9-8} + \frac{9}{2} \sin^{-1} \frac{2\sqrt{2}}{3} \right] - \frac{1}{12} (\sqrt{2})^3 \\
 &= \frac{\sqrt{2}}{4} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6} \\
 &= \frac{\sqrt{2}}{12} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} \\
 &= \frac{1}{2} \left( \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right)
 \end{aligned}$$

Therefore, the required area OBCDO is

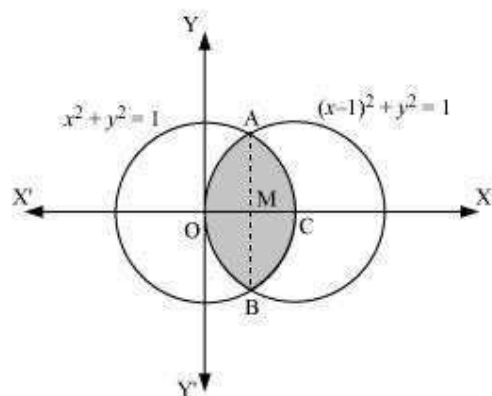
$$\left( 2 \times \frac{1}{2} \left[ \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \right) = \left[ \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \text{ units}$$

Question 2:

Find the area bounded by curves  $(x - 1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$

Answer

The area bounded by the curves,  $(x - 1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$ , is represented by the shaded area as



intersection as  $A\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  and  $B\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$  On solving the equations,  $(x - 1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$ , we obtain the point of

It can be observed that the required area is symmetrical about x-axis.

$$\therefore \text{Area OBCAO} = 2 \times \text{Area OCAO}$$

We join AB, which intersects OC at M, such that AM is perpendicular to OC.

The coordinates of M are  $\left(\frac{1}{2}, 0\right)$ .

$$\begin{aligned}
 \Rightarrow \text{Area } OCAO &= \text{Area } OMAO + \text{Area } MCAM \\
 &= \left[ \int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2} dx + \int_{\frac{1}{2}}^1 \sqrt{1-x^2} dx \right] \\
 &= \left[ \frac{x-1}{2} \sqrt{1-(x-1)^2} + \frac{1}{2} \sin^{-1}(x-1) \right]_0^{\frac{1}{2}} + \left[ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right]_{\frac{1}{2}}^1 \\
 &= \left[ -\frac{1}{4} \sqrt{1-\left(-\frac{1}{2}\right)^2} + \frac{1}{2} \sin^{-1}\left(\frac{1}{2}-1\right) - \frac{1}{2} \sin^{-1}(-1) \right] + \\
 &\quad \left[ \frac{1}{2} \sin^{-1}(1) - \frac{1}{4} \sqrt{1-\left(\frac{1}{2}\right)^2} - \frac{1}{2} \sin^{-1}\left(\frac{1}{2}\right) \right] \\
 &= \left[ -\frac{\sqrt{3}}{8} + \frac{1}{2} \left(-\frac{\pi}{6}\right) - \frac{1}{2} \left(-\frac{\pi}{2}\right) \right] + \left[ \frac{1}{2} \left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{8} - \frac{1}{2} \left(\frac{\pi}{6}\right) \right] \\
 &= \left[ -\frac{\sqrt{3}}{4} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12} \right] \\
 &= \left[ -\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2} \right] \\
 &= \left[ \frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right]
 \end{aligned}$$

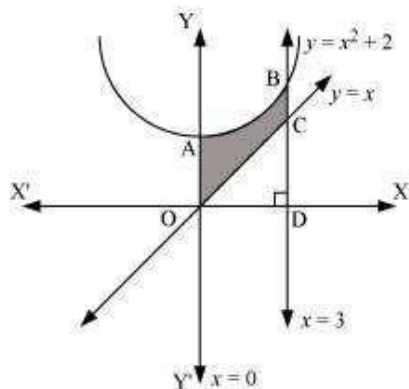
Therefore, required area OBCAO =  $2 \times \left( \frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right) = \left( \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right)$  units

Question 3:

Find the area of the region bounded by the curves  $y = x^2 + 2$ ,  $y = x$ ,  $x = 0$  and  $x = 3$

Answer

The area bounded by the curves,  $y = x^2 + 2$ ,  $y = x$ ,  $x = 0$ , and  $x = 3$ , is represented by the shaded area OCBAO as



Then, Area OCBAO = Area ODBAO – Area ODCO

$$\begin{aligned}
 &= \int_0^3 (x^2 + 2) dx - \int_0^3 x dx \\
 &= \left[ \frac{x^3}{3} + 2x \right]_0^3 - \left[ \frac{x^2}{2} \right]_0^3 \\
 &= [9 + 6] - \left[ \frac{9}{2} \right] \\
 &= 15 - \frac{9}{2} \\
 &= \frac{21}{2} \text{ units}
 \end{aligned}$$

Question 4:

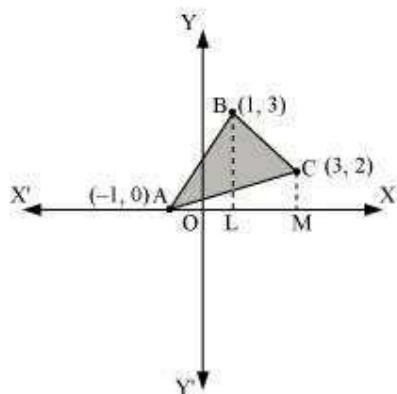
Using integration finds the area of the region bounded by the triangle whose vertices are  $(-1, 0)$ ,  $(1, 3)$  and  $(3, 2)$ .

Answer

BL and CM are drawn perpendicular to x-axis.

It can be observed in the following figure that,

$$\text{Area}(\Delta ACB) = \text{Area}(\text{ALBA}) + \text{Area}(\text{BLMCB}) - \text{Area}(\text{AMCA}) \dots (1)$$



Equation of line segment AB is

$$y - 0 = \frac{3 - 0}{1 - (-1)}(x + 1)$$

$$y = \frac{3}{2}(x + 1)$$

$$\therefore \text{Area}(\text{ALBA}) = \int_{-1}^1 \frac{3}{2}(x + 1) dx = \frac{3}{2} \left[ \frac{x^2}{2} + x \right]_{-1}^1 = \frac{3}{2} \left[ \frac{1}{2} + 1 - \frac{1}{2} + 1 \right] = 3 \text{ units}$$

Equation of line segment BC is

$$y - 3 = \frac{2 - 3}{3 - 1}(x - 1)$$

$$y = \frac{1}{2}(-x + 7)$$

$$\therefore \text{Area}(\text{BLMCB}) = \int_1^3 \frac{1}{2}(-x + 7) dx = \frac{1}{2} \left[ -\frac{x^2}{2} + 7x \right]_1^3 = \frac{1}{2} \left[ -\frac{9}{2} + 21 + \frac{1}{2} - 7 \right] = 5 \text{ units}$$

Equation of line segment AC is

$$y - 0 = \frac{2 - 0}{3 + 1}(x + 1)$$

$$y = \frac{1}{2}(x + 1)$$

$$\therefore \text{Area}(\text{AMCA}) = \frac{1}{2} \int_{-1}^3 (x + 1) dx = \frac{1}{2} \left[ \frac{x^2}{2} + x \right]_{-1}^3 = \frac{1}{2} \left[ \frac{9}{2} + 3 - \frac{1}{2} + 1 \right] = 4 \text{ units}$$

Therefore, from equation (1), we obtain

$$\text{Area}(\Delta ABC) = (3 + 5 - 4) = 4 \text{ units}$$

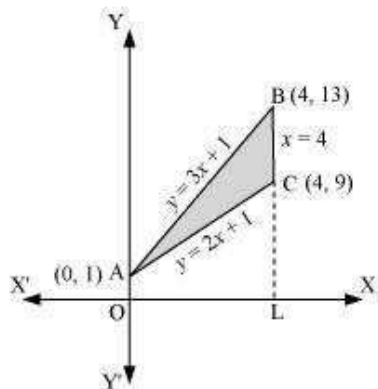
Question 5:

Using integration find the area of the triangular region whose sides have the equations  $y = 2x + 1$ ,  $y = 3x + 1$  and  $x = 4$ .

Answer

The equations of sides of the triangle are  $y = 2x + 1$ ,  $y = 3x + 1$ , and  $x = 4$ .

On solving these equations, we obtain the vertices of triangle as  $A(0, 1)$ ,  $B(4, 13)$ , and  $C(4, 9)$ .



It can be observed that,

$$\text{Area}(\Delta ACB) = \text{Area}(\text{OLBAO}) - \text{Area}(\text{OLCAO})$$

$$\begin{aligned}
 &= \int_0^1 (3x+1) dx - \int_0^1 (2x+1) dx \\
 &= \left[ \frac{3x^2}{2} + x \right]_0^1 - \left[ \frac{2x^2}{2} + x \right]_0^1 \\
 &= (24+4) - (16+4) \\
 &= 28 - 20 \\
 &= 8 \text{ units}
 \end{aligned}$$

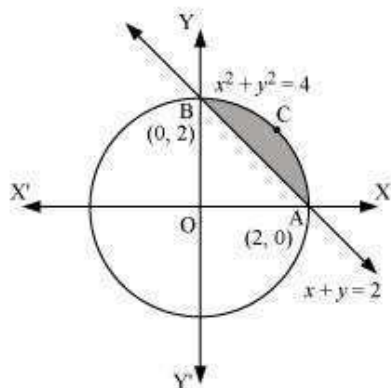
Question 6:

Smaller area enclosed by the circle  $x^2 + y^2 = 4$  and the line  $x + y = 2$  is

- A.  $2(\pi - 2)$
- B.  $\pi - 2$
- C.  $2\pi - 1$
- D.  $2(\pi + 2)$

Answer

The smaller area enclosed by the circle,  $x^2 + y^2 = 4$ , and the line,  $x + y = 2$ , is represented by the shaded area ACBA as



It can be observed that,

$$\text{Area ACBA} = \text{Area OACBO} - \text{Area } (\Delta OAB)$$

$$\begin{aligned}&= \int_0^2 \sqrt{4-x^2} \, dx - \int_0^2 (2-x) \, dx \\&= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[ 2x - \frac{x^2}{2} \right]_0^2 \\&= \left[ 2 \cdot \frac{\pi}{2} \right] - [4-2] \\&= (\pi-2) \text{ units}\end{aligned}$$

Thus, the correct answer is B.

Question 7:

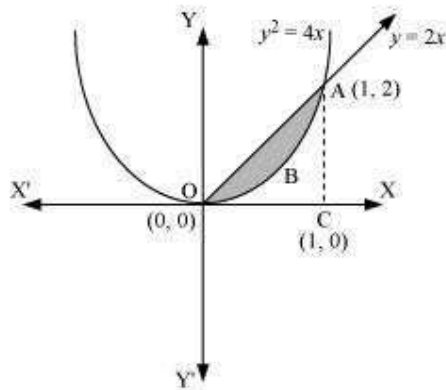
Area lying between the curve  $y^2 = 4x$  and  $y = 2x$  is

- A.  $\frac{2}{3}$
- $\frac{1}{3}$
- $\frac{1}{4}$
- $\frac{3}{4}$
- B.
- C.
- D.

Answer

The area lying between the curve,  $y^2 = 4x$  and  $y = 2x$ , is represented by the shaded area OBAO as





The points of intersection of these curves are O (0, 0) and A (1, 2).

We draw AC perpendicular to x-axis such that the coordinates of C are (1, 0).

∴ Area OBAO = Area (ΔOCA) – Area (OCABO)

$$= \int_0^1 2x \, dx - \int_0^1 2\sqrt{x} \, dx$$

$$= 2 \left[ \frac{x^2}{2} \right]_0^1 - 2 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1$$

$$= \left| 1 - \frac{4}{3} \right|$$

$$= \left| -\frac{1}{3} \right|$$

$$= \frac{1}{3} \text{ units}$$

Thus, the correct answer is B.

Miscellaneous Solutions

Question 1:

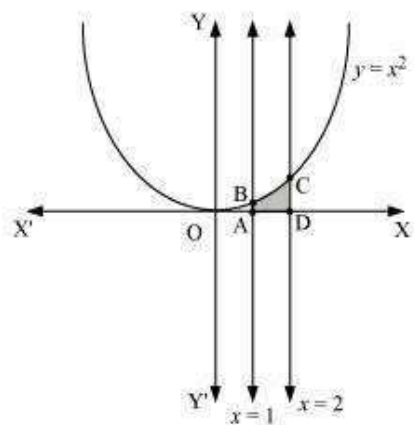
Find the area under the given curves and given lines:

(i)  $y = x^2$ ,  $x = 1$ ,  $x = 2$  and x-axis

(ii)  $y = x^4$ ,  $x = 1$ ,  $x = 5$  and x-axis

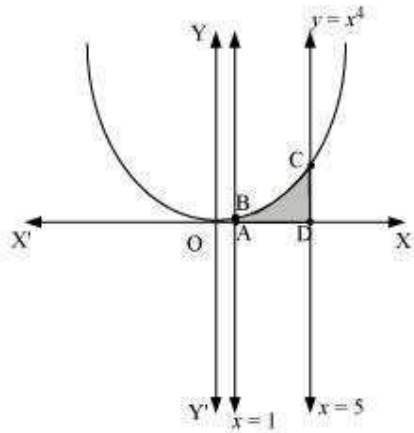
Answer

- i. The required area is represented by the shaded area ADCBA as



$$\begin{aligned}\text{Area ADCBA} &= \int_1^2 y dx \\ &= \int_1^2 x^2 dx \\ &= \left[ \frac{x^3}{3} \right]_1^2 \\ &= \frac{8}{3} - \frac{1}{3} \\ &= \frac{7}{3} \text{ units}\end{aligned}$$

- ii. The required area is represented by the shaded area ADCBA as



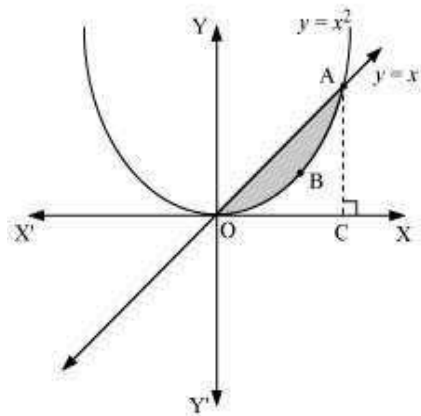
$$\begin{aligned}\text{Area ADCBA} &= \int_1^5 x^4 dx \\&= \left[ \frac{x^5}{5} \right]_1^5 \\&= \frac{(5)^5}{5} - \frac{1}{5} \\&= (5)^4 - \frac{1}{5} \\&= 625 - \frac{1}{5} \\&= 624.8 \text{ units}\end{aligned}$$

Question 2:

Find the area between the curves  $y = x$  and  $y = x^2$

Answer

The required area is represented by the shaded area OBAO as



The points of intersection of the curves,  $y = x$  and  $y = x^2$ , is A (1, 1).

We draw AC perpendicular to x-axis.

$$\therefore \text{Area (OBAO)} = \text{Area } (\triangle OCA) - \text{Area (OCABO)} \dots (1)$$

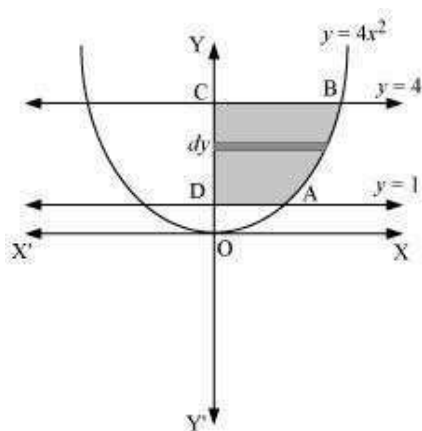
$$\begin{aligned} &= \int_0^1 x \, dx - \int_0^1 x^2 \, dx \\ &= \left[ \frac{x^2}{2} \right]_0^1 - \left[ \frac{x^3}{3} \right]_0^1 \\ &= \frac{1}{2} - \frac{1}{3} \\ &= \frac{1}{6} \text{ units} \end{aligned}$$

Question 3:

Find the area of the region lying in the first quadrant and bounded by  $y = 4x^2$ ,  $x = 0$ ,  $y = 1$  and  $y = 4$

Answer

The area in the first quadrant bounded by  $y = 4x^2$ ,  $x = 0$ ,  $y = 1$ , and  $y = 4$  is represented by the shaded area ABCDA as



$$\begin{aligned}
 \therefore \text{Area ABCD} &= \int_1^4 x \, dx \\
 &= \int_1^4 \frac{\sqrt{y}}{2} \, dy \\
 &= \frac{1}{2} \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \\
 &= \frac{1}{3} \left[ (4)^{\frac{3}{2}} - 1 \right] \\
 &= \frac{1}{3} [8 - 1] \\
 &= \frac{7}{3} \text{ units}
 \end{aligned}$$

Question 4:

Sketch the graph of  $y = |x + 3|$  and evaluate  $\int_{-6}^0 |x + 3| \, dx$

Answer

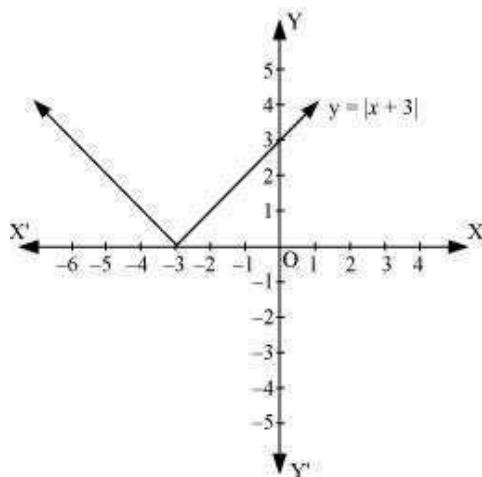
The given equation is  $y = |x + 3|$

The corresponding values of x and y are given in the following table.

x	-6	-5	-4	-3	-2	-1	0
y	3	2	1	0	1	2	3



On plotting these points, we obtain the graph of as follows.



It is known that,  $(x+3) \leq 0$  for  $-6 \leq x \leq -3$  and  $(x+3) \geq 0$  for  $-3 \leq x \leq 0$

$$\begin{aligned} \therefore \int_{-6}^0 |(x+3)| dx &= -\int_{-6}^{-3} (x+3) dx + \int_{-3}^0 (x+3) dx \\ &= -\left[ \frac{x^2}{2} + 3x \right]_{-6}^{-3} + \left[ \frac{x^2}{2} + 3x \right]_{-3}^0 \\ &= -\left[ \left( \frac{(-3)^2}{2} + 3(-3) \right) - \left( \frac{(-6)^2}{2} + 3(-6) \right) \right] + \left[ 0 - \left( \frac{(-3)^2}{2} + 3(-3) \right) \right] \\ &= -\left[ -\frac{9}{2} \right] - \left[ -\frac{9}{2} \right] \\ &= 9 \end{aligned}$$

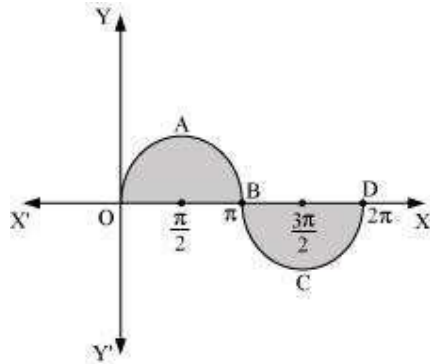
$$y = |x + 3|$$

Question 5:

Find the area bounded by the curve  $y = \sin x$  between  $x = 0$  and  $x = 2\pi$

Answer

The graph of  $y = \sin x$  can be drawn as



∴ Required area = Area OABO + Area BCDB

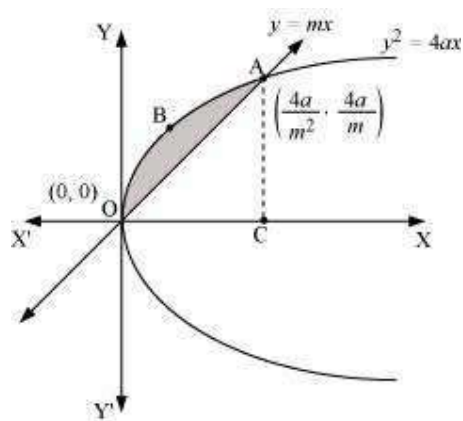
$$\begin{aligned} &= \int_0^{\pi} \sin x \, dx + \left| \int_{\pi}^{2\pi} \sin x \, dx \right| \\ &= \left[ -\cos x \right]_0^{\pi} + \left| \left[ -\cos x \right]_{\pi}^{2\pi} \right| \\ &= \left[ -\cos \pi + \cos 0 \right] + \left| -\cos 2\pi + \cos \pi \right| \\ &= 1 + 1 + \left| (-1 - 1) \right| \\ &= 2 + \left| -2 \right| \\ &= 2 + 2 = 4 \text{ units} \end{aligned}$$

Question 6:

Find the area enclosed between the parabola  $y^2 = 4ax$  and the line  $y = mx$

Answer

The area enclosed between the parabola,  $y^2 = 4ax$ , and the line,  $y = mx$ , is represented by the shaded area OABO as



The points of intersection of both the curves are  $(0, 0)$  and  $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$ .

We draw  $AC$  perpendicular to x-axis.

$$\therefore \text{Area } OABO = \text{Area } OCABO - \text{Area } (\triangle OCA)$$

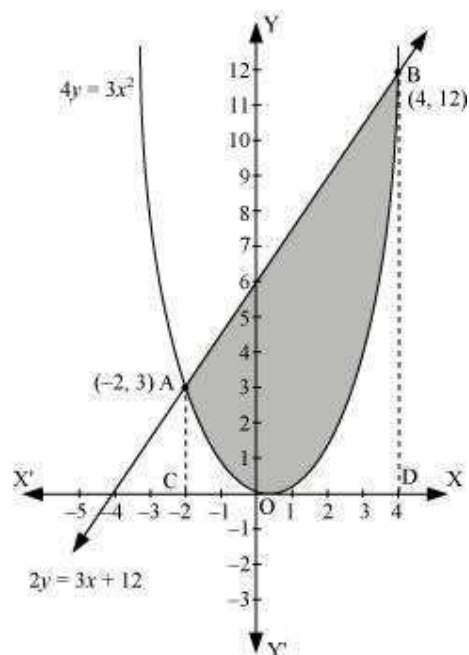
$$\begin{aligned}&= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax} \, dx - \int_0^{\frac{4a}{m^2}} mx \, dx \\&= 2\sqrt{a} \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{4a}{m^2}} - m \left[ \frac{x^2}{2} \right]_0^{\frac{4a}{m^2}} \\&= \frac{4}{3} \sqrt{a} \left( \frac{4a}{m^2} \right)^{\frac{3}{2}} - \frac{m}{2} \left[ \left( \frac{4a}{m^2} \right)^2 \right] \\&= \frac{32a^2}{3m^3} - \frac{m}{2} \left( \frac{16a^2}{m^4} \right) \\&= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3} \\&= \frac{8a^2}{3m^3} \text{ units}\end{aligned}$$

Question 7:

Find the area enclosed by the parabola  $4y = 3x^2$  and the line  $2y = 3x + 12$

Answer

The area enclosed between the parabola,  $4y = 3x^2$ , and the line,  $2y = 3x + 12$ , is represented by the shaded area OBAO as



The points of intersection of the given curves are A  $(-2, 3)$  and  $(4, 12)$ .

We draw AC and BD perpendicular to x-axis.

$$\therefore \text{Area OBAO} = \text{Area CDBA} - (\text{Area ODBO} + \text{Area OACO})$$

$$= \int_{-2}^4 \frac{1}{2}(3x+12) dx - \int_{-2}^4 \frac{3x^2}{4} dx$$

$$= \frac{1}{2} \left[ \frac{3x^2}{2} + 12x \right]_{-2}^4 - \frac{3}{4} \left[ \frac{x^3}{3} \right]_{-2}^4$$

$$= \frac{1}{2} [24 + 48 - 6 + 24] - \frac{1}{4} [64 + 8]$$

$$= \frac{1}{2} [90] - \frac{1}{4} [72]$$

$$= 45 - 18$$

$$= 27 \text{ units}$$

Question 8:

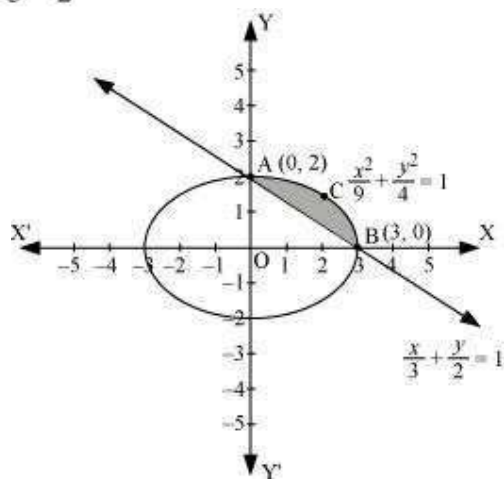
Find the area of the smaller region bounded by the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the line

$$\frac{x}{3} + \frac{y}{2} = 1$$

Answer

The area of the smaller region bounded by the ellipse,  $\frac{x^2}{9} + \frac{y^2}{4} = 1$ , and the line,

$\frac{x}{3} + \frac{y}{2} = 1$ , is represented by the shaded region BCAB as



$$\therefore \text{Area BCAB} = \text{Area (OBCAO)} - \text{Area (OBAO)}$$

$$\begin{aligned}
 &= \int_0^3 2\sqrt{1-\frac{x^2}{9}} dx - \int_0^3 2\left(1-\frac{x}{3}\right) dx \\
 &= \frac{2}{3} \left[ \int_0^3 \sqrt{9-x^2} dx \right] - \frac{2}{3} \int_0^3 (3-x) dx \\
 &= \frac{2}{3} \left[ \frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^3 - \frac{2}{3} \left[ 3x - \frac{x^2}{2} \right]_0^3 \\
 &= \frac{2}{3} \left[ \frac{9}{2} \left( \frac{\pi}{2} \right) \right] - \frac{2}{3} \left[ 9 - \frac{9}{2} \right] \\
 &= \frac{2}{3} \left[ \frac{9\pi}{4} - \frac{9}{2} \right] \\
 &= \frac{2}{3} \times \frac{9}{4} (\pi - 2) \\
 &= \frac{3}{2} (\pi - 2) \text{ units}
 \end{aligned}$$

Question 9:

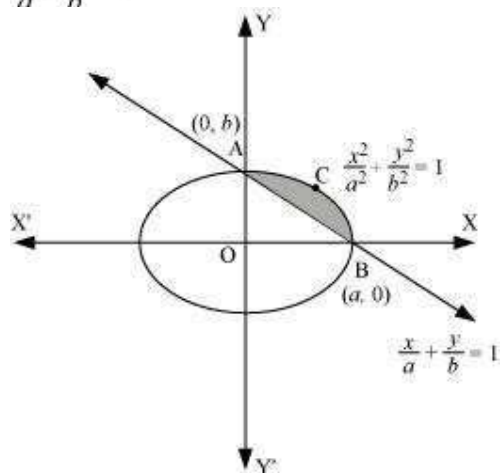
Find the area of the smaller region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the line

$$\frac{x}{a} + \frac{y}{b} = 1$$

Answer

The area of the smaller region bounded by the ellipse,  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , and the line,

$\frac{x}{a} + \frac{y}{b} = 1$ , is represented by the shaded region BCAB as



$$\therefore \text{Area BCAB} = \text{Area (OBCAO)} - \text{Area (OBAO)}$$



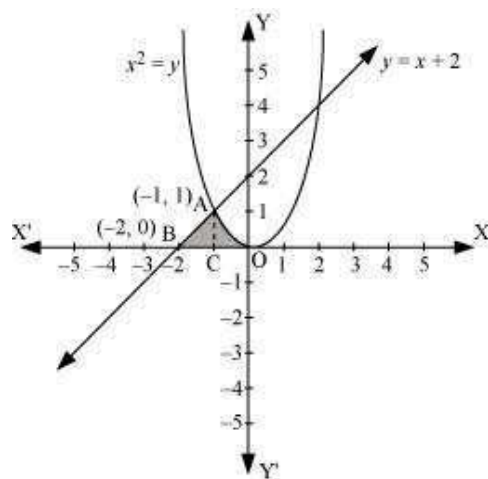
$$\begin{aligned}
 &= \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx - \int_0^a b \left(1 - \frac{x}{a}\right) dx \\
 &= \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} dx - \frac{b}{a} \int_0^a (a - x) dx \\
 &= \frac{b}{a} \left[ \left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\}_0^a - \left\{ ax - \frac{x^2}{2} \right\}_0^a \right] \\
 &= \frac{b}{a} \left[ \left\{ \frac{a^2}{2} \left( \frac{\pi}{2} \right) \right\} - \left\{ a^2 - \frac{a^2}{2} \right\} \right] \\
 &= \frac{b}{a} \left[ \frac{a^2 \pi}{4} - \frac{a^2}{2} \right] \\
 &= \frac{ba^2}{2a} \left[ \frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{2} \left[ \frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{4} (\pi - 2)
 \end{aligned}$$

Question 10:

Find the area of the region enclosed by the parabola  $x^2 = y$ , the line  $y = x + 2$  and x-axis

**Answer**

The area of the region enclosed by the parabola,  $x^2 = y$ , the line,  $y = x + 2$ , and x-axis is represented by the shaded region OABCO as



The point of intersection of the parabola,  $x^2 = y$ , and the line,  $y = x + 2$ , is  $A(-1, 1)$ .  $\therefore$

$$\text{Area OABCO} = \text{Area (BCA)} + \text{Area COAC}$$

$$\begin{aligned}&= \int_{-2}^{-1} (x+2) dx + \int_{-1}^0 x^2 dx \\&= \left[ \frac{x^2}{2} + 2x \right]_{-2}^{-1} + \left[ \frac{x^3}{3} \right]_{-1}^0 \\&= \left[ \frac{(-1)^2}{2} + 2(-1) - \frac{(-2)^2}{2} - 2(-2) \right] + \left[ -\frac{(-1)^3}{3} \right] \\&= \left[ \frac{1}{2} - 2 - 2 + 4 + \frac{1}{3} \right] \\&= \frac{5}{6} \text{ units}\end{aligned}$$

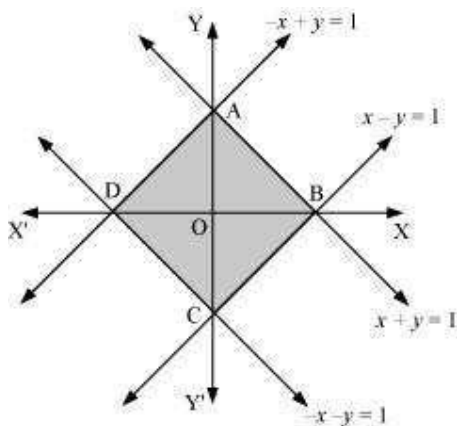
Question 11:

Using the method of integration find the area bounded by the curve  $|x| + |y| = 1$

[Hint: the required region is bounded by lines  $x + y = 1$ ,  $x - y = 1$ ,  $-x + y = 1$  and  $-x - y = 1$ ]

Answer

The area bounded by the curve,  $|x| + |y| = 1$   
as



, is represented by the shaded region ADCB  
The curve intersects the axes at points A (0, 1), B (1, 0), C (0, -1), and D (-1, 0).  
It can be observed that the given curve is symmetrical about x-axis and y-axis.

$$\therefore \text{Area ADCB} = 4 \times \text{Area OBAO}$$

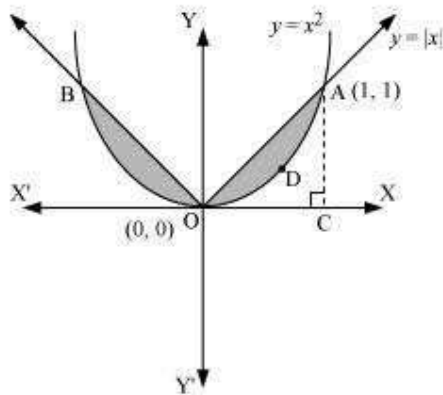
$$\begin{aligned} &= 4 \int_0^1 (1-x) dx \\ &= 4 \left( x - \frac{x^2}{2} \right)_0^1 \\ &= 4 \left[ 1 - \frac{1}{2} \right] \\ &= 4 \left( \frac{1}{2} \right) \\ &= 2 \text{ units} \end{aligned}$$

Question 12:

Find the area bounded by curves  $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$

Answer

The area bounded by the curves,  $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$ , is represented by the shaded region as



It can be observed that the required area is symmetrical about y-axis.

$$\text{Required area} = 2 \left[ \text{Area}(\text{OCAO}) - \text{Area}(\text{OCADO}) \right]$$

$$= 2 \left[ \int_0^1 x \, dx - \int_0^1 x^2 \, dx \right]$$

$$= 2 \left[ \left[ \frac{x^2}{2} \right]_0^1 - \left[ \frac{x^3}{3} \right]_0^1 \right]$$

$$= 2 \left[ \frac{1}{2} - \frac{1}{3} \right]$$

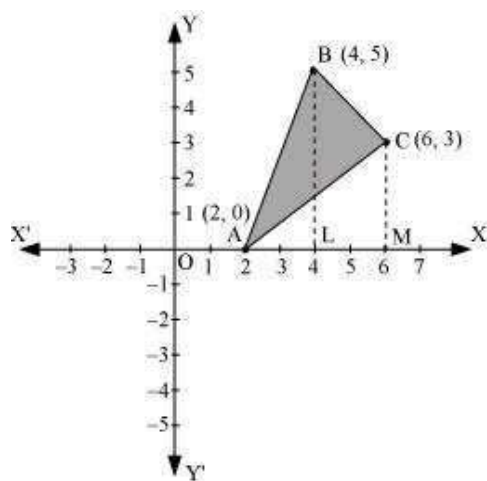
$$= 2 \left[ \frac{1}{6} \right] = \frac{1}{3} \text{ units}$$

Question 13:

Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A (2, 0), B (4, 5) and C (6, 3)

Answer

The vertices of  $\Delta ABC$  are A (2, 0), B (4, 5), and C (6, 3).



Equation of line segment AB is

$$y - 0 = \frac{5 - 0}{4 - 2}(x - 2)$$

$$2y = 5x - 10$$

$$y = \frac{5}{2}(x - 2) \quad \dots(1)$$

Equation of line segment BC is

$$y - 5 = \frac{3 - 5}{6 - 4}(x - 4)$$

$$2y - 10 = -2x + 8$$

$$2y = -2x + 18$$

$$y = -x + 9 \quad \dots(2)$$

Equation of line segment CA is

$$y - 3 = \frac{0 - 3}{2 - 6}(x - 6)$$

$$-4y + 12 = -3x + 18$$

$$4y = 3x - 6$$

$$y = \frac{3}{4}(x - 2) \quad \dots(3)$$

$$\text{Area } (\Delta ABC) = \text{Area (ABLA)} + \text{Area (BLMCB)} - \text{Area (ACMA)}$$

$$\begin{aligned} &= \int_2^4 \frac{5}{2}(x-2)dx + \int_4^6 (-x+9)dx - \int_2^6 \frac{3}{4}(x-2)dx \\ &= \frac{5}{2} \left[ \frac{x^2}{2} - 2x \right]_2^4 + \left[ \frac{-x^2}{2} + 9x \right]_4^6 - \frac{3}{4} \left[ \frac{x^2}{2} - 2x \right]_2^6 \\ &= \frac{5}{2} [8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4} [18 - 12 - 2 + 4] \\ &= 5 + 8 - \frac{3}{4}(8) \\ &= 13 - 6 \\ &= 7 \text{ units} \end{aligned}$$

Question 14:

Using the method of integration find the area of the region bounded by lines:

$$2x + y = 4, 3x - 2y = 6 \text{ and } x - 3y + 5 = 0$$

Answer

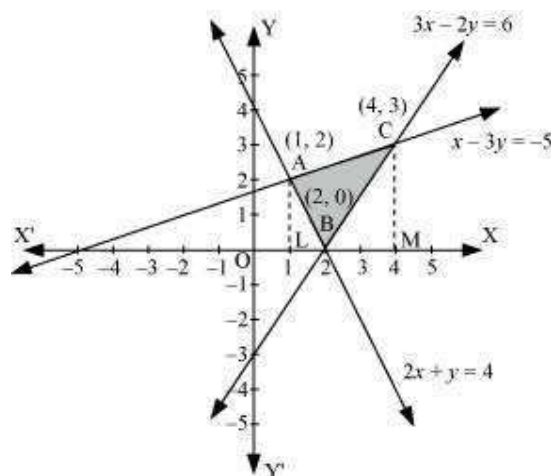
The given equations of lines are

$$2x + y = 4 \dots (1)$$

$$3x - 2y = 6 \dots (2)$$

$$\text{And, } x - 3y + 5 = 0 \dots (3)$$





The area of the region bounded by the lines is the area of  $\Delta ABC$ . AL and CM are the perpendiculars on x-axis.

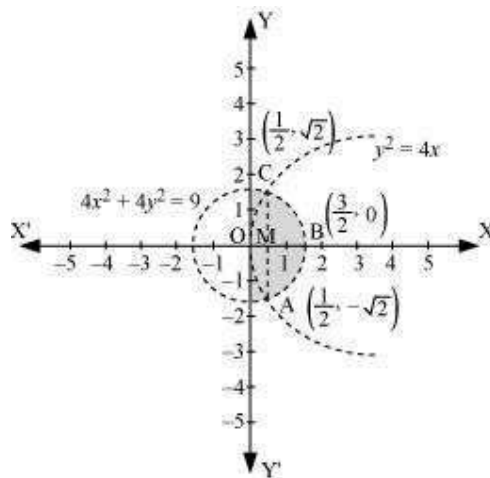
Area ( $\Delta ABC$ ) = Area (ALMCA) – Area (ALB) – Area (CMB)

$$\begin{aligned}
 &= \int_1^4 \left( \frac{x+5}{3} \right) dx - \int_1^2 (4-2x) dx - \int_2^4 \left( \frac{3x-6}{2} \right) dx \\
 &= \frac{1}{3} \left[ \frac{x^2}{2} + 5x \right]_1^4 - \left[ 4x - x^2 \right]_1^2 - \frac{1}{2} \left[ \frac{3x^2}{2} - 6x \right]_2^4 \\
 &= \frac{1}{3} \left[ 8 + 20 - \frac{1}{2} - 5 \right] - [8 - 4 - 4 + 1] - \frac{1}{2} [24 - 24 - 6 + 12] \\
 &= \left( \frac{1}{3} \times \frac{45}{2} \right) - (1) - \frac{1}{2} (6) \\
 &= \frac{15}{2} - 1 - 3 \\
 &= \frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2} \text{ units}
 \end{aligned}$$

Question 15:

Find the area of the  $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$  region Answer

The area bounded by the  $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$  curves, , is represented as



The points of intersection of both the curves are  $\left(\frac{1}{2}, \sqrt{2}\right)$  and  $\left(\frac{1}{2}, -\sqrt{2}\right)$ .

The required area is given by OABCO.

It can be observed that area OABCO is symmetrical about x-axis.

$$\therefore \text{Area OABCO} = 2 \times \text{Area OBC}$$

$$\text{Area OBCO} = \text{Area OMC} + \text{Area MBC}$$

$$\begin{aligned} &= \int_0^1 2\sqrt{x} \, dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{9-4x^2} \, dx \\ &= \int_0^1 2\sqrt{x} \, dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{(3)^2 - (2x)^2} \, dx \end{aligned}$$

Question 16:

Area bounded by the curve  $y = x^3$ , the x-axis and the ordinates  $x = -2$  and  $x = 1$  is A. -9

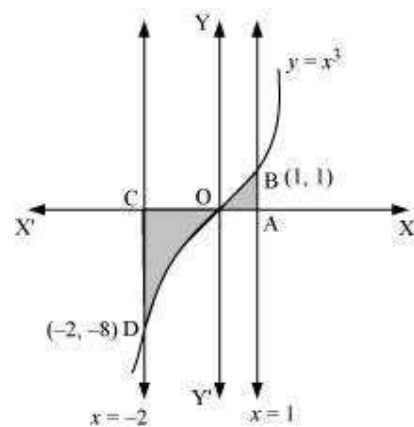
$$-\frac{15}{4}$$

B.

C.  $\frac{15}{4}$

D.  $\frac{17}{4}$

Answer



Required area =  $\int_{-2}^1 y dx$

Question 17:

The area bounded by the curve  $y = x|x|$ , x-axis and the ordinates  $x = -1$  and  $x = 1$  is given by

[Hint:  $y = x^2$  if  $x > 0$  and  $y = -x^2$  if  $x < 0$ ]

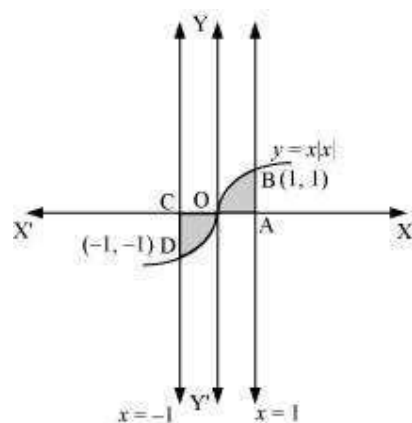
A. 0

B.  $\frac{1}{3}$

C.  $\frac{2}{3}$

D.  $\frac{4}{3}$

Answer



$$\text{Required area} = \int_{-1}^1 y dx$$

$$= \int_{-1}^1 x|x| dx$$

$$= \int_{-1}^0 x^2 dx + \int_0^1 x^2 dx$$

$$= \left[ \frac{x^3}{3} \right]_{-1}^0 + \left[ \frac{x^3}{3} \right]_0^1$$

$$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$$

$$= \frac{2}{3} \text{ units}$$

Thus, the correct answer is C.

Question 18:

The area of the circle  $x^2 + y^2 = 16$  exterior to the parabola  $y^2 = 6x$  is

A.  $\frac{4}{3}(4\pi - \sqrt{3})$

B.  $\frac{4}{3}(4\pi + \sqrt{3})$

C.  $\frac{4}{3}(8\pi - \sqrt{3})$

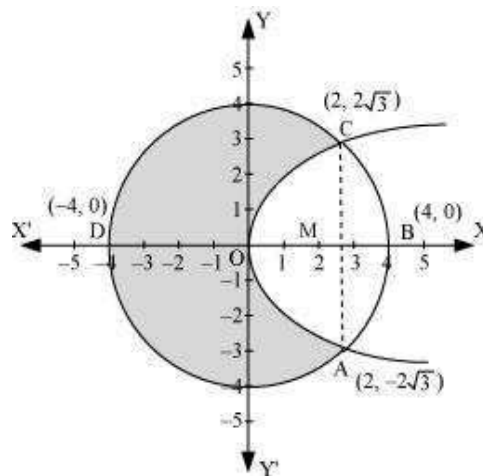
D.  $\frac{4}{3}(4\pi + \sqrt{3})$

Answer C.

The given equations are

$x^2 + y^2 = 16$  ... (1)  $y^2 =$

$6x$  ... (2)



Area bounded by the circle and parabola

$$\begin{aligned}
 &= 2 \left[ \text{Area}(\text{OADO}) + \text{Area}(\text{ADBA}) \right] \\
 &= 2 \left[ \int_0^2 \sqrt{16x} dx + \int_2^4 \sqrt{16-x^2} dx \right] \\
 &= 2 \left[ \sqrt{6} \left\{ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right\}_0^2 \right] + 2 \left[ \frac{x}{2} \sqrt{16-x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_2^4 \\
 &= 2\sqrt{6} \times \frac{2}{3} \left[ x^{\frac{3}{2}} \right]_0^2 + 2 \left[ 8 \cdot \frac{\pi}{2} - \sqrt{16-4} - 8 \sin^{-1} \left( \frac{1}{2} \right) \right] \\
 &= \frac{4\sqrt{6}}{3} (2\sqrt{2}) + 2 \left[ 4\pi - \sqrt{12} - 8 \frac{\pi}{6} \right] \\
 &= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8}{3}\pi \\
 &= \frac{4}{3} [4\sqrt{3} + 6\pi - 3\sqrt{3} - 2\pi] \\
 &= \frac{4}{3} [\sqrt{3} + 4\pi] \\
 &= \frac{4}{3} [4\pi + \sqrt{3}] \text{ units}
 \end{aligned}$$

Area of circle =  $\pi (r)^2$

$$= \pi (4)^2$$

$$= 16\pi \text{ units}$$

$$\begin{aligned}
 \therefore \text{Required area} &= 16\pi - \frac{4}{3} [4\pi + \sqrt{3}] \\
 &= \frac{4}{3} [4 \times 3\pi - 4\pi - \sqrt{3}] \\
 &= \frac{4}{3} (8\pi - \sqrt{3}) \text{ units}
 \end{aligned}$$

Thus, the correct answer is C.



Question 19:

The area bounded by the y-axis,  $y = \cos x$  and  $y = \sin x$  when  $0 \leq x \leq \frac{\pi}{2}$

$2(\sqrt{2}-1)$

$\sqrt{2}-1$

$\sqrt{2}+1$

$\sqrt{2}$

A.

B.

C.

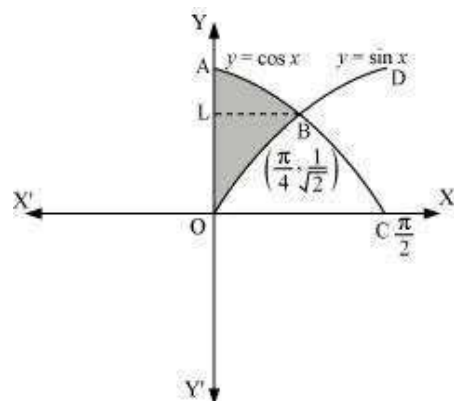
D.

Answer

The given equations are  $y =$

$\cos x \dots (1)$

And,  $y = \sin x \dots (2)$



Required area = Area (ABLA) + area (OBLO)

$$\begin{aligned}
 &= \int_{\frac{1}{\sqrt{2}}}^1 x dy + \int_0^{\frac{1}{\sqrt{2}}} x dy \\
 &= \int_{\frac{1}{\sqrt{2}}}^1 \cos^{-1} y dy + \int_0^{\frac{1}{\sqrt{2}}} \sin^{-1} x dy
 \end{aligned}$$

Integrating by parts, we obtain

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$$\begin{aligned}
 \text{Cl} &= \left[ y \cos^{-1} y - \sqrt{1-y^2} \right]_{\frac{1}{\sqrt{2}}}^1 + \left[ x \sin^{-1} x + \sqrt{1-x^2} \right]_0^{\frac{1}{\sqrt{2}}} \\
 &= \left[ \cos^{-1}(1) - \frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} \right] + \left[ \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} - 1 \right] \\
 &= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \\
 &= \frac{2}{\sqrt{2}} - 1 \\
 &= \sqrt{2} - 1 \text{ units}
 \end{aligned}$$

Thus, the correct answer is B.

$$\text{Put } 2x = t \Rightarrow dx = \frac{dt}{2}$$

$$\text{When } x = \frac{3}{2}, t = 3 \text{ and when } x = \frac{1}{2}, t = 1$$

$$\begin{aligned}
 &= \int_0^{\frac{1}{2}} 2\sqrt{x} \, dx + \frac{1}{4} \int_1^3 \sqrt{(3)^2 - (t)^2} \, dt \\
 &= 2 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}} + \frac{1}{4} \left[ \frac{t}{2} \sqrt{9-t^2} + \frac{9}{2} \sin^{-1}\left(\frac{t}{3}\right) \right]_1^3 \\
 &= 2 \left[ \frac{2}{3} \left(\frac{1}{2}\right)^{\frac{3}{2}} \right] + \frac{1}{4} \left[ \left\{ \frac{3}{2} \sqrt{9-(3)^2} + \frac{9}{2} \sin^{-1}\left(\frac{3}{3}\right) \right\} - \left\{ \frac{1}{2} \sqrt{9-(1)^2} + \frac{9}{2} \sin^{-1}\left(\frac{1}{3}\right) \right\} \right] \\
 &= \frac{2}{3\sqrt{2}} + \frac{1}{4} \left[ \left\{ 0 + \frac{9}{2} \sin^{-1}(1) \right\} - \left\{ \frac{1}{2} \sqrt{8} + \frac{9}{2} \sin^{-1}\left(\frac{1}{3}\right) \right\} \right] \\
 &= \frac{\sqrt{2}}{3} + \frac{1}{4} \left[ \frac{9\pi}{4} - \sqrt{2} - \frac{9}{2} \sin^{-1}\left(\frac{1}{3}\right) \right] \\
 &= \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8} \sin^{-1}\left(\frac{1}{3}\right) \\
 &= \frac{9\pi}{16} - \frac{9}{8} \sin^{-1}\left(\frac{1}{3}\right) + \frac{\sqrt{2}}{12}
 \end{aligned}$$

Therefore, the required area is  $\left[ 2 \times \left( \frac{9\pi}{16} - \frac{9}{8} \sin^{-1}\left(\frac{1}{3}\right) + \frac{\sqrt{2}}{12} \right) \right] = \frac{9\pi}{8} - \frac{9}{4} \sin^{-1}\left(\frac{1}{3}\right) + \frac{1}{3\sqrt{2}}$  units