1. $\int \frac{dx}{x^2 + 2x + 2} =$ ..... .

(A) $x \tan^{-1}(x + 1) + c$ (B) $\tan^{-1}(x + 1) + c$

(C) $(x + 1) \tan^{-1} x + c$ (D) $\tan^{-1} x + c$

જવાબ (B) $\tan^{-1}(x + 1) + c$

➥ $I = \int \frac{dx}{x^2 + 2x + 2}$

$= \int \frac{dx}{x^2 + 2x + 1 + 1}$ (∵ પૂર્ણવર્ગ બનાવતાં)

$= \int \frac{dx}{(x + 1)^2 + (1)^2}$

$I = \tan^{-1}(x + 1) + c$

∴ વિકલ્પ (B) આવે

2. $\int \frac{dx}{\sqrt{9x - 4x^2}} =$ ..... .

(A) $\frac{1}{9} \sin^{-1}\left(\frac{9x - 8}{8}\right) + c$ (B) $\frac{1}{2} \sin^{-1}\left(\frac{8x - 9}{9}\right) + c$

(C) $\frac{1}{3} \sin^{-1}\left(\frac{9x - 8}{8}\right) + c$ (D) $\frac{1}{2} \sin^{-1}\left(\frac{9x - 8}{9}\right) + c$

જવાબ (B) $\frac{1}{2} \sin^{-1}\left(\frac{8x - 9}{9}\right) + c$

➥ $I = \int \frac{dx}{\sqrt{9x - 4x^2}}$

$= \int \frac{dx}{2\sqrt{\frac{9}{4}x - x^2}}$

$= \frac{1}{2} \int \frac{dx}{\sqrt{\frac{81}{64} - \left(x^2 - \frac{9}{4}x + \frac{81}{64}\right)}}$

(∵ પૂર્ણવર્ગ બનાવવા માટે $\frac{81}{64}$ ઉમેરતાં અને બાદ કરતાં)

$$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{9}{8}\right)^2 - \left(x - \frac{9}{8}\right)^2}}$$

$$= \frac{1}{2}\sin^{-1}\frac{\left(x - \frac{9}{8}\right)}{\frac{9}{8}} + c$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{8x - 9}{9}\right) + c$$

∴ વિકલ્પ (B) આવે.

3. **સંકલન મેળવો :** $\int \frac{dx}{9x^2 - 7}$

➠ $\frac{1}{6\sqrt{7}}\log\left|\frac{3x - \sqrt{7}}{3x + \sqrt{7}}\right| + c$

4. **સંકલન મેળવો :** $\int \frac{1}{4x^2 + 9}\,dx$

➠ $\frac{1}{6}\tan^{-1}\frac{2x}{3} + c$

5. **સંકલન મેળવો :** $\int \frac{3x^2}{\sqrt{9 - x^6}}\,dx$

➠ $\sin^{-1}\left(\frac{x^3}{3}\right) + c$

6. **સંકલન મેળવો :** $\int \frac{e^x}{\sqrt{4 - e^{2x}}}\,dx$

➠ $\sin^{-1}\left(\frac{e^x}{2}\right) + c$

7. **સંકલન મેળવો :** $\int \frac{dx}{\sqrt{x^2 + 16}}\,dx$

➠ $\log\left|x + \sqrt{x^2 + 16}\right| + c$

8. **સંકલન મેળવો :** $\int \frac{x^2}{\sqrt{x^6 - 1}}\,dx$

➠ $\frac{1}{3}\log\left|x^3 + \sqrt{x^6 - 1}\right| + c$

9. **સંકલન મેળવો :** $\int \frac{\text{cosec}^2 x}{1 - \cot^2 x}\,dx$

➠ $-\frac{1}{2}\log\left|\frac{1 + \cot x}{1 - \cot x}\right| + c$

10. સંકલન મેળવો : $\int \frac{\sin\theta}{1 - 4\cos^2\theta}\, d\theta$

➠ $-\frac{1}{4}\log\left|\frac{1 + 2\cos\theta}{1 - 2\cos\theta}\right| + c$

11. સંકલન મેળવો : $\int \frac{x}{\sqrt{16x^4 + 9}}\, dx$

➠ $\frac{1}{8}\log\left|4x^2 + \sqrt{16x^2 + 9}\right| + c$

12. સંકલન મેળવો : $\int \frac{x\ dx}{x^4 + x^2 + 1}$

➠ $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2 + 1}{\sqrt{3}}\right) + c$

13. સંકલન મેળવો : $\int \frac{dx}{1 - 6x - 9x^2}$

➠ $\frac{-1}{6\sqrt{2}}\log\left|\frac{3x + 1 - \sqrt{2}}{3x + 1 + \sqrt{2}}\right| + c$

14. સંકલન મેળવો : $\int \frac{dx}{7x^2 + 2x + 10}$

➠ $\frac{1}{\sqrt{69}}\tan^{-1}\left(\frac{7x + 1}{\sqrt{69}}\right) + c$

15. સંકલન મેળવો : $\int \frac{dx}{\sqrt{(x + 5)\ (x + 1)}}$

➠ $\log\left|(x + 3) + \sqrt{x^2 + 6x + 5}\right| + c$

16. સંકલન મેળવો : $\int \frac{dx}{\sqrt{7 - 3x - 2x^2}}\, dx$

➠ $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4x + 3}{\sqrt{65}}\right) + c$

17. સંકલન મેળવો : $\int \frac{dx}{\sqrt{x^2 - 6x + 10}}$

➠ $\log\left|(x - 3) + \sqrt{x^2 - 6x + 10}\right| + c$

18. સંકલન મેળવો : $\int \frac{2x - 3}{3x^2 + 4x + 5}\, dx$

➠ $\frac{1}{3}\log\left|3x^2 + 4x + 5\right| - \frac{13}{\sqrt[3]{11}}\tan^{-1}\left(\frac{3x + 2}{\sqrt{11}}\right) + c$

19. સંકલન મેળવો : $\int \frac{x}{x^2 + 3x + 2}\, dx$

➠ $\frac{1}{2}\log\left|x^2 + 3x + 2\right| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| + c$

20. સંકલન મેળવો : $\int \frac{2x}{\sqrt{1 - x^2 - x^4}}\, dx$

➠ $\sin^{-1}\left(\frac{2x^2 + 1}{\sqrt{2}}\right) + c$

21. સંકલન મેળવો : $\int \frac{4x + 5}{\sqrt{2x^2 + x - 3}}\, dx$

➠ $2\sqrt{2x^2+x-3}+2\sqrt{2}\log\left|\left(x+\frac{1}{4}\right)+\sqrt{\left(x+\frac{1}{4}\right)^2-\left(\frac{5}{4}\right)^2}\right|+c$

22. સંકલન મેળવો : $\int \sqrt{\frac{1 - x}{1 + x}}\, dx$

➠ $\sin^{-1} x + \sqrt{1 - x^2} + c$

23. સંકલન મેળવો : $\int \frac{x - 1}{\sqrt{(x + 1)\,(x - 2)}}\, dx$

➠ $\sqrt{x^2-x-2}-\frac{1}{2}\log\left|\left(x-\frac{1}{2}\right)+\sqrt{x^2-x-2}+c\right|$

24. સંકલન મેળવો : $\int \frac{x + 3}{\sqrt{5 - 4x + x^2}}$

➠ $\sqrt{x^2-4x+5}+5\log\left|(x-2)+\sqrt{x^2-4x+5}\right|+c$

25. વિધેયના સંકલિત મેળવો : $\frac{3x^2}{x^6 + 1}$

➥ $I = \int \frac{3x^2}{x^6 + 1}\, dx$

ધારો કે $x^3 = t \Rightarrow 3x^2\, dx = dt$

$\therefore I = \int \frac{3x^2}{\left(x^3\right)^2 + 1}\, dx$

$= \int \frac{1}{t^2 + 1}\, dt$

$= \tan^{-1}(t) + c$
$= \tan^{-1}(x^3) + c \qquad (\because\ t = x^3)$

26. વિધેયના સંકલિત મેળવો : $\frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}$

➥ $I = \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}\, dx$

ધારો કે $\tan x = \theta \Rightarrow \sec^2 x\, dx = d\theta$

$I = \int \frac{d\theta}{\sqrt{\theta^2 + (2)^2}}$

$= \log \left| \theta + \sqrt{\theta^2 + 2^2} \right| + c$ (સૂત્ર 6નો ઉપયોગ)

$I = \log \left| \tan x + \sqrt{\tan x + 4} \right| + c \quad (\because \theta = \tan x)$

27. વિધેયના સંકલિત મેળવો : $\dfrac{4x + 1}{\sqrt{2x^2 + x - 3}}$

➥ $I = \int \dfrac{4x + 1}{\sqrt{2x^2 + x - 3}}\, dx$

ધારો કે $2x^2 + x - 3 = t \Rightarrow (4x + 1)\, dx = dt$

$I = \int \dfrac{dt}{\sqrt{t}}$

$= \int t^{-\frac{1}{2}}\, dt$

$= \dfrac{t^{-\frac{1}{2}+1}}{\frac{1}{2}} + c$

$= 2\sqrt{t} + c$

$= 2\sqrt{2x^2 + x - 3} + c \quad (\because t = 2x^2 + x - 3)$

28. વિધેયના સંકલિત મેળવો : $\dfrac{1}{\sqrt{1 + 4x^2}}$

➥ $I = \int \dfrac{1}{\sqrt{1 + 4x^2}}\, dx$

$I = \int \dfrac{dx}{\sqrt{1 + (2x)^2}}$

ધારોકે $2x = \tan\theta \Rightarrow 2\, dx = \sec^2\theta\, d\theta$

$\Rightarrow dx = \dfrac{1}{2} \sec^2\theta\, d\theta$

$I = \dfrac{1}{2} \int \dfrac{\sec^2\theta}{\sqrt{1+\tan^2\theta}}\, d\theta$

$= \dfrac{1}{2} \int \dfrac{\sec^2\theta}{\sec\theta}\, d\theta$

$= \dfrac{1}{2} \int \sec\theta\, d\theta$

$= \dfrac{1}{2} \log|\sec\theta + \tan\theta| + c$

$= \frac{1}{2} \log\left| \tan\theta + \sqrt{1 + \tan^2\theta} \right| + c$

$= \frac{1}{2} \log\left| 2x + \sqrt{1 + 4x^2} \right| + c$

અથવા

➥ $I = \int \frac{1}{\sqrt{1 + 4x^2}}\, dx$

$I = \int \frac{dx}{\sqrt{1 + (2x)^2}}$

ધારોકે $2x = \tan\theta \Rightarrow 2\, dx = \sec^2\theta\, d\theta$

$\Rightarrow dx = \frac{1}{2} \sec^2\theta\, d\theta$

$I = \frac{1}{2} \int \frac{\sec^2\theta}{\sqrt{1+\tan^2\theta}}\, d\theta$

$= \frac{1}{2} \int \frac{\sec^2\theta}{\sec\theta}\, d\theta$

$= \frac{1}{2} \int \sec\theta\, d\theta$

$= \frac{1}{2} \log|\sec\theta + \tan\theta| + c$

$= \frac{1}{2} \log\left| \tan\theta + \sqrt{1 + \tan^2\theta} \right| + c$

$= \frac{1}{2} \log\left| 2x + \sqrt{1 + 4x^2} \right| + c$

અથવા

29. **વિધેયના સંકલિત મેળવો :** $\frac{1}{\sqrt{(2 - x)^2 + 1}}$

➥ $I = \int \frac{dx}{\sqrt{(2 - x)^2 + 1}}$

ધારો કે $2 - x = t \Rightarrow -dx = dt \Rightarrow dx = -dt$

$I = \int \frac{-dt}{\sqrt{t^2 + 1}}$

$= -\log|t + \sqrt{t^2 + 1}| + c \qquad \left(\because \int \frac{1}{\sqrt{x^2+a^2}}\, dx = \log\left|x+\sqrt{x^2 + a^2}\right| + c\right)$

$= -\log\left|(2 - x) + \sqrt{(2 - x)^2 + 1}\right| + c$

$= \log\left|\frac{1}{(2 - x) + \sqrt{(2-x)^2 + 1}}\right| + c$

$$= \log \left| \frac{1}{(2 - x) + \sqrt{x^2 - 4x + 5}} \right| + c$$

(નોંધ : આ દાખલો $2 - x = \tan \theta$ આદેશ લઈને પણ કરી શકાય.)

30. વિધેયના સંકલિત મેળવો : $\dfrac{1}{\sqrt{9 - 25x^2}}$

➥ $I = \int \dfrac{dx}{\sqrt{9 - 25x^2}}$

$$= \int \frac{dx}{5\sqrt{\frac{9}{25} - x^2}}$$

$$= \frac{1}{5} \int \frac{dx}{\sqrt{\left(\frac{3}{5}\right)^2 - x^2}}$$

$$I = \frac{1}{5} \sin^{-1} \frac{x}{\frac{3}{5}} + c$$

$$\left(\because \int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1} \frac{x}{a} + c \right)$$

$$\therefore I = \frac{1}{5} \sin^{-1}\left(\frac{5x}{3}\right) + c$$

31. વિધેયના સંકલિત મેળવો : $\dfrac{3x}{1 + 2x^4}$

➥ $I = \int \dfrac{3x}{1 + 2x^4}\, dx$

$$= \frac{3}{2} \int \frac{xdx}{\frac{1}{2} + (x^2)^2}$$

ધારો કે $x^2 = t \Rightarrow 2x\, dx = dt \Rightarrow xdx = \dfrac{dt}{2}$

$$I = \frac{3}{4} \int \frac{dt}{\left(\frac{1}{\sqrt{2}}\right)^2 + t^2}$$

$$= \frac{3}{4} \cdot \frac{1}{\frac{1}{\sqrt{2}}} \tan^{-1} \left(\frac{t}{\frac{1}{\sqrt{2}}} \right) + c$$

$$\left(\because \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} \right) + c$$

$= \frac{3}{2\sqrt{2}} \tan^{-1}\left(\sqrt{2}x^2\right) + c \qquad (\because\ t = x^2)$

32. વિધેયના સંકલિત મેળવો : $\frac{x^2}{1 - x^6}$

➥ $I = \int \frac{x^2}{1-x^6}\, dx$

$= \int \frac{x^2}{1 - (x^3)^2}\, dx$

ધારો કે $x^3 = t \Rightarrow 3x^2dx = dt \Rightarrow x^2dx = \frac{dt}{3}$

$\therefore\ I = \frac{1}{3}\int \frac{dt}{1^2 - t^2}$

$= \frac{1}{3} \cdot \frac{1}{2(1)} \log\left|\frac{1+t}{1-t}\right| + c$

$\left(\because \int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c\right)$

$I = \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + c \qquad (\because\ t = x^{3)}$

33. વિધેયના સંકલિત મેળવો : $\frac{x^2}{\sqrt{x^6 + a^6}}$

➥ $I = \int \frac{x^2}{\sqrt{x^6 + a^6}}\, dx$

$= \int \frac{x^2}{\sqrt{(x^3)^2 + (a^3)^2}}\, dx$

ધારો કે $x^3 = t \Rightarrow 3x^2\, dx = dt \Rightarrow x^2\, dx = \frac{dt}{3}$

$I = \frac{1}{3}\int \frac{dt}{\sqrt{t^2 + (a^3)^2}}$

$= \frac{1}{3}\log\left|t + \sqrt{t^2 + a^6}\right| + c$ (સૂત્ર : (6) નો ઉપયોગ)

$= \frac{1}{3}\log\left|x^3 + \sqrt{x^6 + a^6}\right| + c$

34. વિધેયના સંકલિત મેળવો : $\frac{1}{\sqrt{x^2 + 2x + 2}}$

➥ $I = \int \frac{1}{\sqrt{x^2 + 2x + 2}}\, dx$

$= \int \frac{1}{\sqrt{x^2 + 2x + 1 + 1}}\, dx$

$$= \int \frac{1}{\sqrt{(x+1)^2 + (1)^2}}\, dx$$

$$= \log \left|(x+1) + \sqrt{(x+1)^2 + (1)^2}\right| + c \quad (\because \text{ સૂત્ર 6 નો ઉપયોગ})$$

$$= \log \left|(x+1) + \sqrt{x^2 + 2x + 2}\right| + c$$

35. વિધેયના સંકલિત મેળવો : $\frac{1}{\sqrt{7 - 6x - x^2}}$

➥ $$I = \int \frac{1}{\sqrt{7 - 6x - x^2}}\, dx$$

$$= \int \frac{1}{\sqrt{7 - (x^2 + 6x)}}\, dx$$

$$= \int \frac{1}{\sqrt{7 + 9 - (x^2 + 6x + 9)}}\, dx \quad \left(\because \text{ અંતિમ પદ } = \frac{(\text{મધ્યમ પદ})^2}{4 \times \text{પ્રથમ પદ}} \right.$$
$$\left. \text{પૂર્ણ વર્ગ બનાવતાં} \right)$$

$$= \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}}\, dx$$

$$= \sin^{-1}\left(\frac{x+3}{4}\right) + c \left(\because \int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\frac{x}{a} + c \right)$$

36. વિધેયના સંકલિત મેળવો : $\frac{x-1}{\sqrt{x^2 - 1}}$

➥ $$I = \int \frac{x-1}{\sqrt{x^2 - 1}}\, dx$$

$$= \int \frac{x}{\sqrt{x^2 - 1}}\, dx - \int \frac{1}{\sqrt{x^2 - 1}}\, dx$$

$$= I_1 - I_2 \qquad \text{....(i)}$$

હવે $I_1 = \int \frac{x}{\sqrt{x^2 - 1}}\, dx$

ધારો કે $x^2 - 1 = t \Rightarrow 2x\, dx = dt \Rightarrow x dx = \frac{dt}{2}$

$$I_1 = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$= \frac{1}{2}\int t^{-\frac{1}{2}}\,dt$

$= \frac{1}{2}\,\frac{t^{-\frac{1}{2}+1}}{\frac{1}{2}} + c_1 = \sqrt{t} + a = \sqrt{x^2-1} + a \qquad \left(\because t = \sqrt{x^2-1}\right)$

અને $I_2 = \int \frac{1}{\sqrt{x^2-1}}\,dx$

$= \log\left|x+\sqrt{x^2-1}\right| + c_2 \quad \left(\because \int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + c_2\right)$

સમીકરણ (i) માં I_1 તથા I_2 ની કિંમત મૂકતાં,

$I = \sqrt{x^2-1} + c_1 - \log\left|x+\sqrt{x^2-1}\right| - c_2$

$= \sqrt{x^2-1} - \log\left|x+\sqrt{x^2-1}\right| + c$

જ્યાં $c = c_1 - c_2$

37. વિધેયના સંકલિત મેળવો : $\frac{1}{9x^2+6x+5}$

➥ $I = \int \frac{1}{9x^2+6x+5}\,dx$

$= \frac{1}{9}\int \frac{1}{x^2+\frac{6}{9}x+\frac{5}{9}}\,dx$

$= \frac{1}{9}\int \frac{dx}{x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}+\frac{5}{9}}$ $\left[\because \text{અંતિમ પદ} = \frac{(\text{મધ્યમ પદ})^2}{4\times \text{પ્રથમ પદ}}\right.$ $\left.\text{સૂત્ર વડે અંતિમ પદ મેળવો}\right]$

$= \frac{1}{9}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2}$

$= \frac{1}{9}\times\frac{1}{\frac{2}{3}}\tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{2}{3}}\right) + c \qquad \left(\because \int \frac{1}{x^2+a^2}\,a^2 = \frac{1}{a}\tan^{-1}\frac{x}{a} + c\right)$

$I = \frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right) + c$

38. વિધેયના સંકલિત મેળવો : $\frac{1}{\sqrt{(x-1)(x-2)}}$

➥ $I = \int \frac{1}{\sqrt{(x-1)(x-2)}}\,dx$

$$= \int \frac{1}{\sqrt{x^2 - 3x + 2}}\, dx$$

$$= \int \frac{1}{\sqrt{x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 2}}\, dx \qquad \left(\because \text{અંતિમ પદ} = \frac{(\text{મધ્યમ પદ})^2}{4 \times \text{પ્રથમ પદ}} \right)$$

પૂર્ણ વર્ગ બનાવતાં

$$= \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}}}\, dx$$

$$= \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}\, dx$$

$$I = \log \left| x - \frac{3}{2} + \sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \right| + c \qquad \left(\because \int \frac{1}{x^2 - a^2}\, dx = \log \left| x + \sqrt{x^2 - a^2} \right| + c \right)$$

$$I = \log \left| x - \frac{3}{2} + \sqrt{x^2 - 3x + \frac{9}{4} - \frac{1}{4}} \right| + c$$

$$= \log \left| x - \frac{3}{2} + \sqrt{x^2 - 3x + 2} \right| + c$$

39. **વિધેયના સંકલિત મેળવો :** $\dfrac{1}{\sqrt{8 + 3x - x^2}}$

➥ $$I = \int \frac{1}{\sqrt{8 + 3x - x^2}}\, dx$$

$$= \int \frac{1}{\sqrt{8 - (x^2 - 3x)}}\, dx$$

$$= \int \frac{1}{\sqrt{8 + \frac{9}{4} - \left(x^2 - 3x + \frac{9}{4}\right)}}\, dx \qquad (\because \text{ પૂર્ણ વર્ગ બનાવતાં})$$

$$= \int \frac{1}{\sqrt{\frac{\sqrt{41}}{4} - \left(x - \frac{3}{2}\right)^2}}\, dx$$

$$= \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}}\, dx$$

$$= \sin^{-1} \frac{\left(x - \frac{3}{2}\right)}{\frac{\sqrt{41}}{2}} + c \quad \left(\because \int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1} \frac{x}{a} + c \right)$$

$$= \sin^{-1} \left(\frac{2x-3}{\sqrt{41}} \right) + c$$

40. વિધેયના સંકલિત મેળવો : $\dfrac{1}{\sqrt{(x - a)\,(x - b)}}$

➥ $$I = \int \frac{1}{\sqrt{(x-a)\,(x - b)}}\, dx$$

$$= \int \frac{1}{\sqrt{x^2 - (a + b)x + ab}}\, dx$$

$$= \int \frac{dx}{\sqrt{x^2 - (a+b)x + \left(\frac{a+b}{2}\right)^2 - \left(\frac{a+b}{2}\right)^2 + ab}}$$ (∵ પૂર્ણ વર્ગ બનાવતાં અંતિમ પદ ઉમેરતાં અને બાદ કરતાં)

$$= \int \frac{dx}{\sqrt{\left(x - \frac{a + b}{2}\right)^2 - \left(\frac{(a + b)^2}{4} - ab\right)}}$$

$$= \int \frac{dx}{\sqrt{\left(x - \frac{a + b}{2}\right)^2 - \left(\frac{a - b}{2}\right)^2}}$$

$$= \log \left| \left(x - \frac{a+b}{2}\right) + \sqrt{\left(x - \frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2} \right| + c \quad \left(\because \int \frac{dx}{\sqrt{x^2 - a^2}} \log \left| x + \sqrt{x^2 - a^2} \right| + c \right)$$

$$= \log \left| \left(x - \frac{a + b}{2}\right) + \sqrt{(x-a)\,(x-b)} \right| + c$$

41. વિધેયના સંકલિત મેળવો : $\dfrac{x + 2}{\sqrt{x^2 - 1}}$

➥ $$I = \int \frac{x + 2}{\sqrt{x^2 - 1}}\, dx$$

$$= \int \left(\frac{x}{\sqrt{x^2 - 1}} + \frac{2}{\sqrt{x^2 - 1}} \right) dx$$

$$= \int \frac{x}{\sqrt{x^2 - 1}}\, dx + \int \frac{2}{\sqrt{x^2 - 1}}\, dx$$

$$I = \frac{1}{2} \int \frac{2x}{\sqrt{x^2 - 1}}\, dx + 2 \int \frac{1}{\sqrt{x^2 - (1)^2}}\, dx$$

$$= \frac{1}{2}\int \frac{\frac{d}{dx}(x^2-1)}{\sqrt{x^2-1}}\,dx + 2\int \frac{1}{\sqrt{x^2-(1)^2}}\,dx$$

હવે $\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + c$ થાય છે તથા

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x + \sqrt{x^2-a^2}\right| + c$ થાય છે.

આ બંને સૂત્રોનો ઉપયોગ કરતાં,

$$I = \frac{1}{2}\cdot 2\sqrt{x^2-1} + 2\log\left|x + \sqrt{x^2-1}\right| + c$$

$$= \sqrt{x^2-1} + 2\log\left|x + \sqrt{x^2-1}\right| + c$$

42. વિધેયના સંકલિત મેળવો : $\frac{5x-2}{1+2x+3x^2}$

➥ $I = \int \frac{5x-2}{1+2x+3x^2}\,dx$

ધારો કે $5x - 2 = A\,\frac{d}{dx}(1 + 2x + 3x^2) + B$

$\therefore 5x - 2 = A(2 + 6x) + B$ ....(i)

$\therefore 5x - 2 = 6Ax + 2A + B$

બંને બાજુ x નો સહગુણક સરખાવતાં, $5 = 6A \Rightarrow A = \frac{5}{6}$

બંને બાજું અચળ પદને સરખાવતાં

$2A + B = -2$ પરંતુ $A = \frac{5}{6}$ હોવાથી

$$2\left(\frac{5}{6}\right) + B = -2$$

$$\therefore B = -2 - \frac{5}{3} = -\frac{11}{3}$$

A અને B ની કિંમતો (i) માં મૂકતાં,

$$5x - 2 = \frac{5}{6}(2 + 6x) - \frac{11}{3}$$

$$\therefore I = \int \frac{5x-2}{1+2x+3x^2}\,dx$$

$$= \int \frac{\frac{5}{6}(2+6x) - \frac{11}{3}}{1+2x+3x^2}\,dx$$

$$= \frac{5}{6}\int \frac{2+6x}{1+2x+3x^2}\,dx - \frac{11}{3}\int \frac{1}{1+2x+3x^2}\,dx$$

$$= \frac{5}{6} I_1 - \frac{11}{3} I_2 \quad \text{....(ii)}$$

➥ $I = \int \frac{5x - 2}{1 + 2x + 3x^2}\, dx$

ધારો કે $5x - 2 = A \frac{d}{dx}(1 + 2x + 3x^2) + B$

$\therefore 5x - 2 = A(2 + 6x) + B \quad \text{....(i)}$

$\therefore 5x - 2 = 6Ax + 2A + B$

બંને બાજુ x નો સહગુણક સરખાવતાં, $5 = 6A \Rightarrow A = \frac{5}{6}$

બંને બાજું અચળ પદને સરખાવતાં

$2A + B = -2$ પરંતુ $A = \frac{5}{6}$ હોવાથી

$$2\left(\frac{5}{6}\right) + B = -2$$

$\therefore B = -2 - \frac{5}{3} = -\frac{11}{3}$

A અને B ની કિંમતો (i) માં મૂકતાં,

$$5x - 2 = \frac{5}{6}(2 + 6x) - \frac{11}{3}$$

$\therefore I = \int \frac{5x - 2}{1 + 2x + 3x^2}\, dx$

$$= \int \frac{\frac{5}{6}(2 + 6x) - \frac{11}{3}}{1 + 2x + 3x^2}\, dx$$

$$= \frac{5}{6} \int \frac{2 + 6x}{1 + 2x + 3x^2}\, dx - \frac{11}{3} \int \frac{1}{1 + 2x + 3x^2}\, dx$$

$$= \frac{5}{6} I_1 - \frac{11}{3} I_2 \quad \text{....(ii)}$$

➥ $I = \int \frac{5x - 2}{1 + 2x + 3x^2}\, dx$

ધારો કે $5x - 2 = A \frac{d}{dx}(1 + 2x + 3x^2) + B$

$\therefore 5x - 2 = A(2 + 6x) + B \quad \text{....(i)}$

$\therefore 5x - 2 = 6Ax + 2A + B$

બંને બાજુ x નો સહગુણક સરખાવતાં, $5 = 6A \Rightarrow A = \frac{5}{6}$

બંને બાજું અચળ પદને સરખાવતાં

$2A + B = -2$ પરંતુ $A = \frac{5}{6}$ હોવાથી

$$2\left(\frac{5}{6}\right) + B = -2$$

$$\therefore \quad B = -2 - \frac{5}{3} = -\frac{11}{3}$$

A અને B ની કિંમતો (i) માં મૂકતાં,

$$5x - 2 = \frac{5}{6}(2 + 6x) - \frac{11}{3}$$

$$\therefore I = \int \frac{5x - 2}{1 + 2x + 3x^2}\, dx$$

$$= \int \frac{\frac{5}{6}(2 + 6x) - \frac{11}{3}}{1 + 2x + 3x^2}\, dx$$

$$= \frac{5}{6}\int \frac{2 + 6x}{1 + 2x + 3x^2}\, dx - \frac{11}{3}\int \frac{1}{1 + 2x + 3x^2}\, dx$$

$$= \frac{5}{6} I_1 - \frac{11}{3} I_2 \qquad \text{....(ii)}$$

43. વિધેયના સંકલિત મેળવો : $\frac{6x + 7}{\sqrt{(x - 5)(x - 4)}}$

➥ $$I = \int \frac{6x + 7}{\sqrt{(x - 5)(x - 4)}}\, dx$$

$$= \int \frac{6x + 7}{\sqrt{x^2 - 9x + 20}}\, dx$$

ધારો કે $6x + 7 = A \cdot \frac{d}{dx}(x^2 - 9x + 20) + B$

$\therefore 6x + 7 = A(2x - 9) + B \qquad$ ....(i)

$= 2Ax - 9A + B$

બંને બાજુ x નાં સહગુણકો સરખાવતાં,

$2A = 6 \Rightarrow A = 3$

બંને બાજુ અચળ પદોને સરખાવતાં

$-9A + B = 7$ પરંતુ $A = 3$ હોવાથી

$-27 + B = 7$

$\therefore B = 7 + 27 = 34$

A અને B ની કિંમતો (i) માં મૂકતાં

$6x + 7 = 3(2x - 9) + 34$

$$I = \int \frac{6x + 7}{\sqrt{x^2 - 9x + 20}}\, dx$$

$$= \int \frac{3(2x - 9) + 34}{\sqrt{x^2 - 9x + 20}}\, dx$$

$$= 3\int\frac{2x-9}{\sqrt{x^2-9x+20}}\,dx + 34\int\frac{1}{\sqrt{x^2-9x+20}}\,dx$$

$= 3I_1 + 34I_2$ ....(ii)

$$I_1 = \int\frac{2x-9}{\sqrt{x^2-9x+20}}\,dx$$

➥ $$I = \int\frac{6x+7}{\sqrt{(x-5)(x-4)}}\,dx$$

$$= \int\frac{6x+7}{\sqrt{x^2-9x+20}}\,dx$$

ધારો કે $6x + 7 = A\cdot\frac{d}{dx}(x^2 - 9x + 20) + B$

$\therefore 6x + 7 = A(2x - 9) + B$ ....(i)

$= 2Ax - 9A + B$

બંને બાજુ x નાં સહગુણકો સરખાવતાં,

$2A = 6 \Rightarrow A = 3$

બંને બાજુ અચળ પદોને સરખાવતાં

$-9A + B = 7$ પરંતુ $A = 3$ હોવાથી

$-27 + B = 7$

$\therefore B = 7 + 27 = 34$

A અને B ની કિંમતો (i) માં મૂકતાં

$6x + 7 = 3(2x - 9) + 34$

$$I = \int\frac{6x+7}{\sqrt{x^2-9x+20}}\,dx$$

$$= \int\frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}\,dx$$

$$= 3\int\frac{2x-9}{\sqrt{x^2-9x+20}}\,dx + 34\int\frac{1}{\sqrt{x^2-9x+20}}\,dx$$

$= 3I_1 + 34I_2$ ....(ii)

$$I_1 = \int\frac{2x-9}{\sqrt{x^2-9x+20}}\,dx$$

➥ $$I = \int\frac{6x+7}{\sqrt{(x-5)(x-4)}}\,dx$$

$$= \int\frac{6x+7}{\sqrt{x^2-9x+20}}\,dx$$

ધારો કે $6x + 7 = A\cdot\frac{d}{dx}(x^2 - 9x + 20) + B$

$\therefore 6x + 7 = A(2x - 9) + B$ ....(i)

$= 2Ax - 9A + B$

બંને બાજુ x નાં સહગુણકો સરખાવતાં,

$2A = 6 \Rightarrow A = 3$

બંને બાજુ અચળ પદોને સરખાવતાં

$-9A + B = 7$ પરંતુ $A = 3$ હોવાથી

$-27 + B = 7$

$\therefore\ B = 7 + 27 = 34$

A અને B ની કિંમતો (i) માં મૂકતાં

$6x + 7 = 3(2x - 9) + 34$

$$I = \int \frac{6x + 7}{\sqrt{x^2 - 9x + 20}}\, dx$$

$$= \int \frac{3(2x - 9) + 34}{\sqrt{x^2 - 9x + 20}}\, dx$$

$$= 3\int \frac{2x - 9}{\sqrt{x^2 - 9x + 20}}\, dx + 34\int \frac{1}{\sqrt{x^2 - 9x + 20}}\, dx$$

$= 3I_1 + 34I_2$ ....(ii)

$$I_1 = \int \frac{2x - 9}{\sqrt{x^2 - 9x + 20}}\, dx$$

44. **વિધેયના સંકલિત મેળવો :** $\dfrac{x + 2}{\sqrt{4x - x^2}}$

➥ $$I = \int \frac{x + 2}{\sqrt{4x - x^2}}\, dx$$

$$x + 2 = A\,\frac{d}{dx}(4x - x^2) + B$$

$\therefore\ x + 2 = A(4 - 2x) + B$ ....(i)

$= 4A - 2Ax + B$

બંને બાજુ x નાં સહગુણકો સરખાવતાં,

$$-2A = 1 \Rightarrow A = -\frac{1}{2}$$

બંને બાજુનાં અચળ પદો સરખાવતાં,

$4A + B = 2$ પરંતુ $A = -\dfrac{1}{2}$ હોવાથી

$-2 + B = 2 \Rightarrow B = 4$

A અને B ની આ કિંમત (i) માં મૂકતાં,

$$x + 2 = -\frac{1}{2}(4 - 2x) + 4$$

$$\therefore I = \int \frac{x + 2}{\sqrt{4x - x^2}}\, dx$$

$$= \int \frac{-\frac{1}{2}(4 - 2x) + 4}{\sqrt{4x - x^2}}\, dx$$

$$= -\frac{1}{2}\int \frac{4 - 2x}{\sqrt{4x - x^2}}\, dx + 4\int \frac{1}{\sqrt{4x - x^2}}\, dx$$

$= -\dfrac{1}{2}\, I_1 + 4I_2$ ....(ii)

$$I_1 = \int \frac{4 - 2x}{\sqrt{4x - x^2}}\, dx$$

➥ $$I = \int \frac{x + 2}{\sqrt{4x - x^2}}\, dx$$

$$x + 2 = A\, \frac{d}{dx}\, (4x - x^2) + B$$

$$\therefore\ x + 2 = A(4 - 2x) + B \qquad \text{....(i)}$$
$$= 4A - 2Ax + B$$

બંને બાજુ x નાં સહગુણકો સરખાવતાં,

$$-2A = 1 \Rightarrow A = -\frac{1}{2}$$

બંને બાજુનાં અચળ પદો સરખાવતાં,

$4A + B = 2$ પરંતુ $A = -\frac{1}{2}$ હોવાથી

$-2 + B = 2 \Rightarrow B = 4$

A અને B ની આ કિંમત (i) માં મૂકતાં,

$$x + 2 = -\frac{1}{2}\, (4 - 2x) + 4$$

$$\therefore I = \int \frac{x + 2}{\sqrt{4x - x^2}}\, dx$$

$$= \int \frac{-\frac{1}{2}(4 - 2x) + 4}{\sqrt{4x - x^2}}\, dx$$

$$= -\frac{1}{2} \int \frac{4 - 2x}{\sqrt{4x - x^2}}\, dx + 4 \int \frac{1}{\sqrt{4x - x^2}}\, dx$$

$$= -\frac{1}{2}\, I_1 + 4I_2 \qquad \text{....(ii)}$$

$$I_1 = \int \frac{4 - 2x}{\sqrt{4x - x^2}}\, dx$$

45. વિધેયના સંકલિત મેળવો : $\dfrac{x + 2}{\sqrt{x^2 + 2x + 3}}$

➥ $$I = \int \frac{x + 2}{\sqrt{x^2 + 2x + 3}}\, dx$$

$$I = \int \frac{\frac{1}{2}\, (2x + 4)}{\sqrt{x^2 + 2x + 3}}\, dx$$

$$= \int \frac{\frac{1}{2}\, (2x + 2 + 2)}{\sqrt{x^2 + 2x + 3}}\, dx$$

$$= \int \frac{\frac{1}{2}(2x + 2) + \frac{1}{2}(2)}{\sqrt{x^2 + 2x + 3}}\, dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}\, dx + \int \frac{1}{\sqrt{x^2+2x+3}}\, dx$$

$$= \frac{1}{2}\int \frac{\frac{d}{dx}(x^2+2x+3)}{\sqrt{x^2+2x+3}}\, dx + \int \frac{1}{\sqrt{x^2+2x+1+2}}\, dx \; (\because \text{પૂર્ણ વર્ગ બનાવતાં})$$

$$= \frac{1}{2}\int \frac{\frac{d}{dx}(x^2+2x+3)}{\sqrt{x^2+2x+3}}\, dx + \int \frac{1}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)2}}\, dx$$

હવે, $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$ થાય તથા $\int \frac{1}{x^2+a^2}dx = \log\left|x+\sqrt{x^2+a^2}\right| + c$ થાય

આ બંને સૂત્રોનો ઉપયોગ કરતાં,

$$I = \frac{1}{2}\times 2\sqrt{x^2+2x+3} + \log\left|(x+1)+\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}\right| + c$$

$$= \sqrt{x^2+2x+3} + \log\left|(x+1)+\sqrt{x^2+2x+3}\right| + c$$

46. વિધેયના સંકલિત મેળવો : $\frac{x + 3}{x^2 - 2x - 5}$

➥ $$I = \int \frac{x + 3}{x^2 - 2x - 5}\, dx$$

$$= \int \frac{\frac{1}{2}(2x + 6)}{x^2 - 2x - 5}\, dx$$

$$= \int \frac{\frac{1}{2}(2x - 2 + 8)}{x^2 - 2x - 5}\, dx$$

$$= \int \frac{\frac{1}{2}(2x - 2) + \frac{1}{2}(8)}{x^2 - 2x - 5}\, dx$$

$$= \frac{1}{2}\int \frac{2x - 2}{x^2 - 2x - 5}\, dx + 4\int \frac{1}{x^2 - 2x - 5}\, dx$$

$$= \frac{1}{2}\int \frac{2x - 2}{x^2 - 2x - 5}\, dx + 4\int \frac{1}{x^2 - 2x + 1 - 6}\, dx$$

$$= \frac{1}{2}\int \frac{\frac{d}{dx}(x^2 - 2x - 5)}{x^2 - 2x - 5}\, dx + 4 \qquad \int \frac{1}{(x-1)^2 - \left(\sqrt{6}\right)^2}\, dx$$

હવે, $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$ થાય તથા

$\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$ થાય આ બંને સૂત્રોનો ઉપયોગ કરતાં,

$$I = \frac{1}{2}\log\left|x^2 - 2x - 5\right| + 4.\frac{1}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + c$$

$$= \frac{1}{2}\log\left|x^2 - 2x - 5\right| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + c$$

47. **વિધેયના સંકલિત મેળવો :** $\frac{5x+3}{\sqrt{x^2+4x+10}}$

➥ $I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$

$5x + 3 + A\,\frac{d}{dx}(x^2 + 4x + 10) + B$

$\therefore 5x + 3 = A(2x + 4) + B$ ....(i)

$\therefore 5x + 3 = 2A \cdot x + 4A + B$

બંને બાજુનાં x નાં સહગુણકો સરખાવતાં,

$2A = 5 \Rightarrow A = \frac{5}{2}$

બંને બાજુનાં અચળપદો સરખાવતાં,

$4A + B = 3$ પરંતુ $A = \frac{5}{2}$ હોવાથી $4\left(\frac{5}{2}\right) B = 3$

$\therefore B = 3 - 10 = -7$

A અને B ની આ કિંમતો સમીકરણ (i) માં મૂકતાં

$5x + 3 = \frac{5}{2}(2x + 4) - 7$

$I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$

$= \int \frac{\frac{5}{2}(2x+4) - 7}{\sqrt{x^2+4x+10}}\,dx$

$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}\,dx - 7\int \frac{1}{\sqrt{x^2+4x+10}}\,dx$

$= \frac{5}{2}\,I_1 - 7I_2$ ....(ii)

$I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}\,dx$

ધારો કે $x^2 + 4x + 10 = t \Rightarrow (2x + 4)\,dx = dt$

➥ $I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$

$5x + 3 + A \frac{d}{dx} (x^2 + 4x + 10) + B$

$\therefore 5x + 3 = A(2x + 4) + B$ ....(i)

$\therefore 5x + 3 = 2A \cdot x + 4A + B$

બંને બાજુનાં x નાં સહગુણકો સરખાવતાં,

$2A = 5 \Rightarrow A = \frac{5}{2}$

બંને બાજુનાં અચળપદો સરખાવતાં,

$4A + B = 3$ પરંતુ $A = \frac{5}{2}$ હોવાથી $4 \left(\frac{5}{2}\right) B = 3$

$\therefore B = 3 - 10 = -7$

A અને B ની આ કિંમતો સમીકરણ (i) માં મૂકતાં

$5x + 3 = \frac{5}{2} (2x + 4) - 7$

$$I = \int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}} dx$$

$$= \int \frac{\frac{5}{2}(2x + 4) - 7}{\sqrt{x^2 + 4x + 10}} dx$$

$$= \frac{5}{2} \int \frac{2x+4}{\sqrt{x^2+4x+10}} dx - 7 \int \frac{1}{\sqrt{x^2+4x+10}} dx$$

$= \frac{5}{2} I_1 - 7I_2$ ....(ii)

$$I_1 = \int \frac{2x + 4}{\sqrt{x^2 + 4x + 10}} dx$$

ધારો કે $x^2 + 4x + 10 = t \Rightarrow (2x + 4)\, dx = dt$