24. Kinetic Theory of Gases

Short Answer

1. Question

When we place a gas cylinder on a van and the van moves, does the kinetic energy of the molecules increase? Does the temperature increase?

Answer

No, the kinetic energy of the molecule does not increase.

Explanation

As we know that kinetic energy of ideal gas is given by

$K.E = \frac{3}{2} k_B T$ ……(i)

Where K. E=kinetic energy of gas

k_B = Boltzmann constant

T = temperature of gas

The Boltzmann constant is a physical constant that relates the average

kinetic energy of particles in a gas with the temperature of the gas.

Since k_B is a constant, kinetic energy of gas is proportional to

temperature of gas. Also, we know

$K.E = \frac{1}{2} mv^2$ …….. (ii)

Where v= velocity of gas

From equation (i) and equation (ii), we can say that the Kinetic energy of a gas is dependent on two factors-

1. Temperature (K.E. $\propto$ T)

2. Velocity (K.E. $\propto$ velocity^2)

We can conclude that kinetic energy of gas will change only when there is a change in velocity of gas which in turn will change the temperature of gas. So, if van moves with uniform velocity (i.e. constant speed) no change in kinetic energy or temperature will be observed. Kinetic energy and temperature of the gas will only change when the van either decelerate or accelerate (rate of change of velocity).

2. Question

While gas from a cooking gas cylinder is used, the pressure does not fall appreciably till the last few minutes. Why?

Answer

1. Cooking gas inside cylinder has approximately 85% of liquid and rest vapors of that liquid.

2. This liquid and vapor is in equilibrium with each other. The pressure in such a system is dependent only on temperature.

3. As the gas is released from the cylinder, the pressure and temperature inside cylinder decreases. To compensate this fall in pressure, phase transition from liquid to gas takes place and some of liquid changes into vapors.

4. For this to happen, system takes heat from surrounding and hence temperature of the system remains constant. And temperature constant means pressure also becomes constant.

5. If phase transition takes place, pressure inside the cylinder will remain same.

6. Hence because of continuous phase transition inside the cylinder, pressure of cooking gas cylinder does not fall appreciably.

3. Question

Do you expect the gas in a cooking gas cylinder to obey the ideal gas equation?

Answer

No, we cannot except gas inside cooking cylinder to obey ideal gas equation.

<u>Explanation</u>

1. Cooking gas is in liquid form inside cylinder. This means it is under high pressure and low temperature.

2. Ideal gas equation is valid only for gases at low pressure and high temperature. At low pressure and high temperature, the molecules of gas are far apart, and molecular interactions are negligible. Without interaction the gas behaves like ideal gas. So, cooking gas is not an ideal gas as it in liquified form.

4. Question

Can we define the temperature of (a) vacuum, (b) a single molecule?

Answer

Temperature is defined as the average kinetic energy of molecules.

A) No, we cannot define temperature of vacuum.

<u>Explanation</u>

Vacuum is defined as space which contains no matter or space where pressure is so low that no interaction can takes place between any entities or matter. When there are no interactions, no molecule can form as formation of molecules requires Vander Waal force of attraction. So, temperature of a vacuum cannot be defined.

B) No, temperature of single molecules cannot be defined.

Explanation

Temperature for single molecule cannot be defined as we define temperature as the average kinetic energy of all molecules of a gas. And a gas cannot just comprise of single molecule. If that were the case, then there should be no need to do average. So, the answer is no.

5. Question

Comment on the following statement: the temperature of all the molecules in a sample of a gas is the same.

Answer

1. All the molecules in sample of gas moves with different velocities. Therefore, we average out their velocities to go by our calculation of temperature.

2. The sole reason for averaging is to find that one value of velocity that we can assign to all the molecules and find the temperature of gas as per the definition which is – **the average kinetic energy of molecules in a gas is directly proportional to temperature of the gas.**

$$K.E = \frac{3}{2} k_B T$$

Where K. E=kinetic energy of gas

k_B = Boltzmann constant

T = temperature of gas

3. So yes, temperature of all molecules in a sample of a gas is the same.

6. Question

Consider a gas of neutrons. Do you expect it to behave much better as an ideal gas as compared to hydrogen gas at the same pressure and temperature?

Answer

Yes, neutron gas is expected to behave much better ideal gas than hydrogen gas.

Explanation

A gas can be called an ideal gas if it follows certain properties.

1) The volume of particles of ideal gas should negligible.

2) There should be no interaction between particles of ideal gas.

Now according to above properties, a neutron gas is expected to behave more like an ideal gas than hydrogen gas because of following reasons:

(i) Neutron is a neutral particle (no charge). So, there will be no interaction between neutron particles. Whereas hydrogen have electron and proton due to which there is possibility of interaction among hydrogen molecules.

(ii) Secondly neutrons are very small in size as compared to hydrogen molecule. So, neutron gas also fulfills the second condition of ideal gas more precisely than hydrogen gas.

7. Question

A gas is kept in a rigid cubical container. If a load of 10 kg is put on the top of the container, does the pressure increase?

Answer

No, pressure will not increase.

Explanation

1. Rigid solid is defined as the solids having definite shape and size. **A rigid body does not deform under any force or pressure**.

2. Pressure of gas inside a container will only change, if the container gets compressed (deform) under the action of force. In our case, the container is **rigid** cubical container.

3. So, a load of 10kg will not be able to deform the shape and size of container and hence the pressure of the gas will not increase.

8. Question

If it were possible for a gas in a container to reach the temperature 0 K, its pressure would be zero. Would the molecules not collide with the walls? Would they not transfer momentum to the walls?

Answer

No, the molecules of gas will neither collide nor transfer momentum.

First explanation

Kinetic energy of gas is given as

$$K.E = \frac{3}{2}k_B T$$

Where K. E=kinetic energy of gas

k_B = Boltzmann constant

T = temperature of gas

So, if T=0K then kinetic energy will be zero. Which also means that molecules of gas will not move at all. Hence, they will neither collide or transfer momentum to walls of container.

<u>Second explanation</u>

One can also understand this from zero pressure point of view.

Pressure of an ideal gas given as

$P= nmv^2$ ……... (i)

Where n=number of molecules of gas per unit volume

m=mass of molecule of gas

v= velocity of gas molecule

And momentum is given as

Momentum= mass $\times$ velocity …...(ii)

From equation (i) and (ii), we can conclude that if pressure is zero,

velocity of molecules will be zero and hence momentum will be zero.

9. Question

It is said that the assumptions of kinetic theory are good for gases having low densities. Suppose a container is so evacuated

that only one molecule is left in it. Which of the assumptions of kinetic theory will not be valid for such a situation? Can we assign a temperature to this gas?

Answer

No, we cannot assign temperature for this gas as temperature is not defined for single molecule.

<u>Explanation</u>

The following assumption of kinetic theory will not be valid for single molecule gas:

•A given amount of gas is **a collection of large number of molecules** that are in random motion colliding with each other and the walls of container.

10. Question

A gas is kept in an enclosure. The pressure of the gas is reduced by pumping out some gas. Will the temperature of the gas decrease by Charles's law?

Answer

No, temperature will not decrease.

Explanation

Charles's law states that for **fixed pressure,** volume of a gas is proportional to its absolute temperature.

$V \propto T$ …… (pressure=constant)

So, Charles's law is not even applicable because the pressure of the gas is being reduced in question.

11. Question

Explain why cooking is faster in a pressure cooker.

Answer

1. Boiling point of a substance increases with increase in pressure. Inside the pressure cooker, pressure is more than atmospheric pressure.

2. We know that boiling point of water at atmospheric pressure is 100°C.

3. So, boiling point of water is also above 100°C inside pressure cooker. This means that food will now cook at higher temperature than 100°C.

4. So, by increasing the pressure and the boiling point we are reducing the time taken for food to cook. Hence, cooking is faster in pressure cooker than in open vessel.

12. Question

If the molecules were not allowed to collide among themselves, would you expect more evaporation or less evaporation?

Answer

One should expect more evaporation.

Explanation

1. Evaporation is the transition from liquid state to gaseous state. To do so heat is provided so that forces holding molecule together in liquid state are weakened and vapor state can be achieved.

2. If molecules are not allowed collide among themselves this means that interaction between them is very weak. Hence, more evaporation will take place.

13. Question

Is it possible to boil water at room temperature, say 30°C? If we touch a flask containing water boiling at this temperature, will it be hot?

Answer

Yes, it is possible to boil water at $30^\circ C$.

Explanation

1. Boiling point decrease with decreases in pressure.

2. So, if the pressure of the flask containing water is reduced up to value so that boiling point is reduced from $100^\circ C$ to $30^\circ C$ then, the water will start boiling.

3. But since now the temperature at which the water has started boiling is very low, flask will not be hot.

14. Question

When you come out of a river after a dip, you feel cold. Explain.

Answer

After we come out of river water sticks to our body. That water evaporates by taking heat from our body. Thus, there is a transfer of heat from our body to water droplets. Therefore, temperature of our body reduces, and we feel cold.

Objective I

1. Question

Which of the following parameters is the same for molecules of all gases at a given temperature?

A. Mass

B. Speed

C. Momentum

D. Kinetic energy

Answer

Kinetic energy of gas is dependent on temperature of gas. Mathematically it is given as

$$K.E = \frac{3}{2} k_B T$$

Where K. E=kinetic energy of gas

k_B = Boltzmann constant

T = temperature of gas

From above formula kinetic energy is directly proportional to temperature. Since k_B is constant therefore, kinetic energy is constant and same for all gases at a given temperature.

2. Question

A gas behaves more closely as an ideal gas at

A. low pressure and low temperature

B. low pressure and high temperature

C. high pressure and low temperature

D. high pressure and high temperature.

Answer

1. According to kinetic theory of ideal gas, molecules of ideal gas should be in incessant random motion and constantly colliding with each other and with the wall of the container in which they are kept.

2. This can only happen when they have large velocities or kinetic energy. And we know that temperature and kinetic energy are directly proportional to each other.

$$K.E = \frac{3}{2}k_BT$$

Where K. E=kinetic energy of gas

k_B = Boltzmann constant

T = temperature of gas

So, large kinetic energy means high temperature.

3. Another postulate of kinetic theory states that size of molecule should be very small as compared to the volume of gas.

4. This can be achieved at low pressure because at low pressure concentration of gas is very low (less number of molecule in large volume or space). So, gas behaves as ideal gas at low pressure and at high temperature.

3. Question

The pressure of an ideal gas is written as $p = \frac{2E}{3V}$. Here E refers to

A. translational kinetic energy

B. rotational kinetic energy

C. vibrational kinetic energy

D. total kinetic energy.

Answer

1. Molecules of ideal gas have negligible interaction between them. This means that force acting due to each other while they are in motion is almost zero.

2. In such cases where force acting on particle is zero, particle moves in uniform motion and in straight line according to newton's first law of motion.

3. So, in kinetic theory of ideal gas, molecules of gas are moving in straight line. Hence, they will only have translational kinetic energy.

4. Question

The energy of a given sample of an ideal gas depends only on its

A. volume

B. pressure

C. density

D. temperature

Answer

As we know that kinetic energy of ideal gas is given by

$$K.E = \frac{3}{2}k_B T$$

Where K. E=kinetic energy of gas

k_B = Boltzmann constant

T = temperature of gas

Since k_B is a constant therefore, kinetic energy of gas is proportional to temperature of gas.

5. Question

Which of the following gases has maximum rms speed at a given temperature?

A. hydrogen

B. nitrogen

C. oxygen

D. carbon dioxide

Answer

Rms speed at given temperature is given as

$$v_{rms} = \frac{\sqrt{3RT}}{\sqrt{M}}$$

where R=gas constant whose value is 8.31 J/mol K.

T=temperature of gas.

M= molar mass of molecule of gas

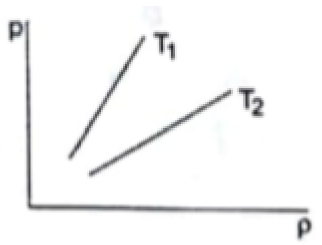
Therefore, at a given temperature rms speed of gas is inversely proportional to square root of molar mass of molecule of that gas.

Out of the four-option molar mass of hydrogen molecule is least

i.e. 2 amu. So rms speed of hydrogen will be maximum.

6. Question

Figure shows graphs of pressure vs density for an ideal gas at two temperatures T_1 and T_2.



A. $T_1 > T_2$

B. $T_1 = T_2$

C. $T_1 < T_2$

D. Any of three is possible

Answer

From the graph, we can see that slope of straight line with temperature

T_1 is greater than slope of straight line with temperature T_2.

Slope of straight line is given as:

Slope = $\frac{dy}{dx}$

In our graph, slope is $\frac{P}{\rho}$

where P=pressure

ρ= density of gas

So, ratio $\frac{P}{\rho}$ for T_1 is more than T_2.

Rms speed of gas is given as

$$v_{rms} = \sqrt{\frac{3RT}{M}} \ldots \ldots (i)$$

where R=gas constant whose value is 8.31 J/mol K.

T=temperature of gas.

M= molar mass of molecule of gas

We also know that ideal gas equation is

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

N=number of moles of gas

So, we can write $RT = \frac{PV}{n}$.

Putting the value of RT in equation (i),we get

$$v_{rms} = \sqrt{\frac{3PV}{nM}} \ldots \ldots (ii)$$

nM= total mass of gas 'm'

and density $\rho = \frac{total\ mass}{volume} = \frac{m}{V} = \frac{nM}{V}$

Putting the value of density ρ in equation (ii), we get

$$v_{rms} = \sqrt{\frac{3P}{\rho}}$$

Since ratio $\frac{P}{\rho}$ for T_1 is more than T_2 therefore rms speed for ideal gas at temperature T_1 is more than temperature T_2. And if rms speed is more for temperature T_1 then $T_1 > T_2$ from formula

$v_{rms} = \frac{\sqrt{3RT}}{\sqrt{M}}$

7. Question

The mean square speed of the molecules of a gas at absolute temperature T is proportional to

A. 1/T

B. √T

C. T

D. T^2

Answer

Root mean square velocity of gas is given as

$v_{rms} = \frac{\sqrt{3RT}}{\sqrt{M}}$ ……. (I)

where R=gas constant whose value is 8.31 J/mol K.

T=temperature of gas.

M= molar mass of molecule of gas

Squaring both side of equation (I) we are removing the square root and

we will get mean square velocity which is,

$$v^2 = \frac{3RT}{M}$$

So, mean square speed is directly proportional to temperature as for a

given gas its molar mass will not change.

8. Question

Suppose a container is evacuated to leave just one molecule of a gas in it. Let v_α and v_{rms} represent the average speed and the rms speed of the gas.

A. $v_\alpha > v_{rms}$

B. $v_\alpha < v_{rms}$

C. $v_\alpha = v_{rms}$

D. v_{rms} is undefined.

Answer

For a single molecule speed will be same whether it is rms speed or average speed.

9. Question

The rms speed of oxygen at room temperature is about 500 m/s. The rms speed of hydrogen at the same temperature is about

A. 125 ms^{-1}

B. 2000 ms^{-1}

C. 8000 ms^{-1}

D. 31 ms^{-1}

Answer

Root mean square velocity of gas is given as

$v_{rms} = \frac{\sqrt{3RT}}{\sqrt{M}}$

where R=gas constant whose value is 8.31 J/mol K

T=temperature of gas

M= molar mass of molecule of gas

rms speed of oxygen = $v_{rms}(o) = \sqrt{\frac{3RT}{M(o)}}$ ...(i)

rms speed of hydrogen=$v_{rms}(H) = \sqrt{\frac{3RT}{M(H)}}$ ...(ii)

Given:

$v_{rms}(o)=500m/s$

and we know molar mass of oxygen =M(o)=32

molar mass of hydrogen=M(H)=2

Diving equation (i) and (ii) we get

$$\frac{v_{rms}(O)}{v_{rms}(H)} = \frac{\sqrt{\frac{3RT}{M(O)}}}{\sqrt{\frac{3RT}{M(H)}}}$$

Since speed of both gases must be calculated at same temperature this equation will reduce to

$$\frac{v_{rms}(o)}{v_{rms}(H)} = \sqrt{\frac{M(H)}{M(o)}}$$

$$\frac{500}{v_{rms}(H)} = \sqrt{\frac{2}{32}}$$

$$v_{rms}(H) = 500 \times \sqrt{\frac{32}{2}}$$

$v_{rms}(H) = 500 \times 4 = 2000$m/s ($\because \sqrt{16} = 4$)

$\therefore$ Root mean square velocity of hydrogen is 2000m/s.

10. Question

The pressure of a gas kept in an isothermal container is 200 kPa. If half the gas is removed from it, the pressure will be

A. 100 kPa

B. 200 kPa

C. 100 kPa

D. 800 kPa

Answer

Given:

Pressure P=200kPa

1kPa= 10^3Pa

We know that ideal gas equation is

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

So,

$P = \frac{nRT}{V}$ =200$\times 10^3$Pa ......(i)

Now according to question half of the gas is removed. That means number of moles left behind are also halved.

Therefore, number of moles left behind n'=$\frac{n}{2}$

So new pressure will be P'=$\frac{n'RT}{V}$

Putting the value of n' in above equation we get

$$P' = \frac{nRT}{2V}$$

From equation (i) we can write P' as

$$P' = \frac{P}{2} = \frac{200 \times 10^3}{2}$$

$P' = 100 \times 10^3$Pa=100kPa.

∴ New pressure will be 100kPa.

11. Question

The rms speed of oxygen molecules in a gas is *v*. If the temperature is doubled and the oxygen molecules dissociate into oxygen atoms, the rms speed will become

A. v

B. v√2

C. 2v

D. 4v.

Answer

Root mean square velocity of gas is given as

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

where R=gas constant whose value is 8.31 J/mol K.

T=temperature of gas.

M= molar mass of molecule of gas

Given,

Root mean square of oxygen

$$v_{rms} = \sqrt{\frac{3RT}{M}} = v$$

Molar mass of oxygen molecule M=32. So,

$$v = \sqrt{\frac{3RT}{32}} \dots\dots\dots (i)$$

When temperature is doubled, oxygen molecule dissociates into oxygen atom.

Molar mass of oxygen atom M'=16

New temperature T'=2T

Then rms speed of oxygen atom becomes

$$v_{rms} = \sqrt{\frac{3R(2T)}{16}}$$

Multiplying and diving above equation by 2 we get,

$$v_{rms} = \sqrt{\frac{3R(4T)}{32}} = \sqrt{4 \times \frac{3RT}{32}} = \sqrt{4}v \ldots\ldots\ldots (from(i))$$

$$v_{rms} = 2v$$

∴New rms speed of oxygen will ve 2v.

12. Question

The quantity $\frac{pV}{kT}$ represents

A. mass of the gas

B. kinetic energy of the gas

C. number of moles of the gas

D. number of molecules in the gas.

Answer

We know that ideal gas equation is

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

P=pressure of gas.

Gas constant $R=kN_A$

Where k=Boltzmann constant

N_A=Avogadro number

So, we can ideal gas equation as

$PV=nkN_AT$

$$nN_A = \frac{PV}{kT} \dots..(i)$$

Now, we know that

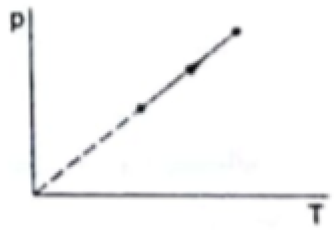
$n\times N_A$=number of molecules of gas ……(ii)

Therefore, from equation (i) and (ii)

Number of molecules of gas=$\frac{PV}{kT}$

13. Question

The process on ideal gas, shown in figure is



A. isothermal

B. isobaric

C. isochoric

D. none of these

Answer

From the graph, we can conclude that pressure is directly proportional to temperature because it is straight line graph between pressure and temperature.

We know that ideal gas equation is

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

P=pressure of gas.

$$P = \frac{nRT}{V}$$

From above equation, we can see that pressure will be directly proportional to temperature only when volume V is also a constant.

So, only for isochoric process (a process where volume is kept constant) pressure will be directly proportional to temperature.

14. Question

There is some liquid in a closed bottle. The amount of liquid is continuously decreasing. The vapor in the remaining part.

A. must be saturated

B. must be unsaturated

C. may be saturated

D. there will be no vapor

Answer

1. Saturated air means air having maximum humidity or moisture. If we keep on adding moisture to saturated air the extra moisture will condense. And saturated air will become unsaturated air at that temperature.

2. Since amount of liquid is continuously decreasing that means evaporation is taking place.

3. Now during vaporization, moisture is being continuously added to air above liquid. Temperature is changing continuously.

4. So, vapors in remaining part is unsaturated as air contains extra moisture.

15. Question

There is some liquid in a closed bottle. The amount of liquid remains constant as time passes. The vapor in the remaining part.

A. must be saturated

B. must be unsaturated

C. may be unsaturated

D. there will be no vapor

Answer

1. Since the amount of liquid remains constant that means no vapors or moisture is being added to air or vapors above it.

2. Saturated air means air having maximum humidity or moisture. If we keep on adding moisture to saturated air the extra moisture will condense. And saturated air will become unsaturated air at temperature.

3. Since no moisture is being added, remaining vapors must be saturated.

16. Question

Vapor is injected at a uniform rate in a closed vessel which was initially evacuated. The pressure in the vessel

A. increases continuously

B. decreases continuously

C. first increases and then decreases

D. first increase and then becomes constant.

Answer

1. Maximum pressure attainable by any liquid is saturation vapor pressure. Saturation vapor pressure is the pressure exerted by vapor above the surface of liquid when the air has become saturated.

2. Initially when vapor is being injected at a uniform rate the pressure inside the chamber will increase.

3. But after a certain limit air inside vessel will become saturated and then if we add more vapor to air it will condense.

4. At that point pressure will stop increasing and will become constant as pressure has reached the value of saturated vapor pressure.

So, pressure will increase first and then will become constant.

17. Question

A vessel A has volume V and a vessel B has volume 2V. Both contain some water which has a constant volume. The pressure in the space above water is p_a for vessel A and p_b for vessel B.

A. $p_a = p_b$

B. $p_a = 2p_b$

C. $p_b = 2p_a$

D. $p_b = 4p_a$

Answer

1. Maximum pressure attainable by any liquid is saturation vapor pressure. Saturation vapor pressure is the pressure exerted by vapor above the surface of

liquid when the air has become saturated.

2. When the air become saturated, pressure of the gas depends completely on temperature and not on volume of liquid.

3. So, in our case the water is same in both the vessel i.e. volume of water is same, and temperature of water is also constant.

4. Hence, pressure in both the vessel will be same.

Objective II

1. Question

Consider a collision between an oxygen molecule and a hydrogen molecule in a mixture of oxygen and hydrogen kept at room temperature. Which of the following are possible?

A. The kinetic energies of both the molecules increase.

B. The kinetic energies of both the molecules decrease.

C. Kinetic energy of the oxygen molecule increases and that of the hydrogen molecule decreases.

D. The kinetic energy of the hydrogen molecule increases and that of the oxygen molecule decreases.

Answer

1. According to kinetic theory of ideal gases, molecules of a gas are in incessant random motion, colliding against each other and with the walls of the container. All these collisions are perfectly elastic collision.

2. In perfectly elastic collision, total kinetic energy of gas is conserved.

3. Here, we have a mixture of oxygen and hydrogen gas. Therefore

K.E of oxygen + K.E of hydrogen=constant

4. So, if kinetic energy of oxygen molecules increases, then kinetic energy of hydrogen molecules decreases or vice-versa so that, sum of both kinetic energies remains constant.

5. Therefore, both option (c) and (d) are correct.

2. Question

Consider a mixture of oxygen and hydrogen kept at room temperature. As compared to a hydrogen molecule an oxygen molecule hits the wall

A. with greater average speed

B. with smaller average speed

C. with greater average kinetic energy

D. with smaller average kinetic energy

Answer

In kinetic theory of ideal gas, the average energy is given by

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

Where R=gas constant=8.31$Jmol^{-1}K^{-1}$

T=temperature of gas

M=molar mass of gas

1. From the formula for average speed, it can be seen that $v_{avg} \propto \frac{1}{M}$.

2. Molar mass of hydrogen molecule is 2 amu and molar mass of oxygen molecule is 16 amu.

3. So, molar mass of oxygen molecule is greater than molar mass of hydrogen molecule.

4. Therefore, average speed of oxygen molecule will be less than average speed of hydrogen molecule.

3. Question

Which of the following quantities is zero on an average for the molecules of an ideal gas in equilibrium?

A. Kinetic energy

B. Momentum

C. Density

D. Speed.

Answer

1. We know that momentum is a vector quantity defined as product of mass and velocity of particle in motion.

2. According to kinetic theory of ideal gas, molecules of gas are in random motion.

3. Due to this random motion, velocity on an average will be zero. This is because velocity is a vector quantity, having both direction and magnitude.

4. Because velocity have direction, in random motion all the components of velocity will cancel out each other and it will be zero on an average.

5. Thus, average momentum will also be zero.

4. Question

Keeping the number of moles, volume and temperature the same, which of the following are the same for all ideal gases?

A. rms speed of a molecule

B. Density

C. Pressure

D. Average magnitude of momentum

Answer

We know ideal gas equation is

PV=nRT

Where V= volume of gas

R=gas constant =8.3$JK^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

So, we can write

$$P = \frac{nRT}{V} \quad \dots\dots (i)$$

According to question, n, V and T are constant.

R= gas constant is universal constant.

So, from equation (i) Pressure will be same for all ideal gas.

5. Question

The average momentum of a molecule in a sample of an ideal gas depends on

A. temperature

B. number of moles

C. volume

D. none of these

Answer

1. Average momentum of a molecule in sample in ideal gas is zero.

2. We know that momentum is a vector quantity defined as product of mass and velocity of particle in motion.

3. According to kinetic theory of ideal gas, molecules of gas are in random motion.

4. Due to this random motion, velocity on an average will be zero. This is because velocity is a vector quantity, having both direction and magnitude.

5. Because velocity have direction, in random motion all the components of velocity will cancel out each other and it will be zero on an average.

6. Thus, average momentum will also be zero.

7. Therefore, average momentum is independent of all the quantities mentioned options.

6. Question

Which of the following quantities is the same for all ideal gases at the same temperature?

A. The kinetic energy of 1 mole

B. The kinetic energy of 1 g

C. volume

D. none of these

Answer

1. Avogadro's law states that, "equal volumes of all gases, at the same temperature and pressure, have the same number of molecules."

2. So, from Avogadro law, at same temperature all the ideal gases will have same volume.

3. As we know that kinetic energy of ideal gas is given by

$K.E = \frac{3}{2} k_B T$ ……(i)

Where K. E=kinetic energy of gas

k_B = Boltzmann constant

T = temperature of gas

4. Temperature is a quantity that is dependent on number of molecule in kinetic theory of gases.

5. Again, from Avogadro law, at same temperature all the gases will have same number of molecules which is equal to Avogadro number.

6. In 1 mole of any gas, number of molecules are 6.023×10^{23}.

7. Therefore, kinetic energy of 1 mole, will be same for all ideal gas at same temperature as all will contain same number of molecules.

7. Question

Consider the quantity $\frac{Mkt}{pV}$ of an ideal gas where M is the mass of the gas. It depends on the

A. temperature of the gas

B. volume of the gas

C. pressure of the gas

D. nature of the gas

Answer

In quantity $\frac{Mkt}{pV}$,

M=mass of the gas

k= Boltzmann constant

t= temperature

p=pressure

V=volume

We know ideal gas equation

pV=nRt

Where V= volume of gas

R=gas constant =8.3$JK^{-1}mol^{-1}$

t=temperature

n=number of moles of gas

p=pressure of gas.

Gas constant $R=kN_A$

Where k=Boltzmann constant

N_A=Avogadro number

So, we can ideal gas equation as

$pV=nkN_At$

$$nN_A=\frac{pV}{kt}\ \ldots\ldots(i)$$

Now, we know that

$n\times N_A$=number of molecules of gas ……(ii)

Therefore, from equation (i) and (ii)

Number of molecules of gas=$\frac{pV}{kt}$ .... (iii)

Using equation (iii) quantity $\frac{Mkt}{pV}$ becomes

$$\frac{Mkt}{pV}=M\times number\ of\ molecules=total\ mass\ of\ the\ gas$$

Hence, quantity $\frac{Mkt}{pV}$ depends on the nature of gas.

Exercises

1. Question

Calculate the volume of 1 mole of an ideal gas at STP.

Answer

STP means standard temperature-273.15K and pressure 101.325 kPa.

Given

Pressure P=1.01$\times 10^5$Pa

Number of moles n=1

Temperature T=273.15K

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant =8.3$JK^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

So, we can write

$$V = \frac{nRT}{P}$$

$$V = \frac{1 \times 8.31 \times 273.15}{1.01 \times 10^5} = 0.0224 m^3$$

∴The volume of 1 mole of an ideal gas at STP=$0.0224m^3$.

2. Question

Find the number of molecules of an ideal gas in a volume of 1.000 cm^3 at STP.

Answer

STP means standard temperature-273.15K and pressure

101.325 kPa.

Volume of ideal gas at STP= 22.4L

Number of molecule in 22.4L of ideal gas at STP=Avogadro number =6.022×10^{23}

Now we know that

1Litre=10^3cm^3

22.4L=$22.4\times10^3cm^3$

Number of molecule in $22.4\times10^3cm^3$ of ideal gas at STP=6.022×10^{23}

Therefore,

Number of molecules in 1 cm^3 of ideal gas at STP=$\frac{6.022\times10^{23}}{22.4\times10^3}$=$2.688 \times 10^{19}$.

3. Question

Find the number of molecules in 1 cm^3 of an ideal gas at 0°C and at a pressure of 10^{-5} mm of mercury.

Answer

Given

Volume of ideal gas V=$1cm^3$

1cm=10^{-2}m

V=$10^{-6}m^3$

Temperature of ideal gas=0°C

T(K)=T (°C)+273.15

T=T(K)=0+273.15=273.15K

Pressure of ideal gas P=10^{-5}mm of Hg

1mm of Hg= 133.32Pa

P=10^{-5}mm of Hg=133.32$\times 10^{-5}$Pa

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant=8.31J/molK

T=temperature

n=number of moles of gas

P=pressure of gas.

So,

$$n = \frac{PV}{RT} = \frac{133.32 \times 10^{-5} \times 10^{-6}}{8.31 \times 273.15}$$

$$n = 0.0587 \times 10^{-11}$$

$$n = 5.87 \times 10^{-13}$$

Number of molecules = Avogadro number$\times$number of moles

Number of molecules=$6.022 \times 10^{23} \times$n

=$6.022 \times 10^{23} \times 5.87 \times 10^{-13}$

=3.538$\times 10^{11}$

Therefore,

Number of molecules in 1 cm^3 of an ideal gas at 0°C and at a pressure of 10^{-5} mm of mercury is 3.538$\times 10^{11}$.

4. Question

Calculate the mass of 1 cm^3 of oxygen kept at STP.

Answer

STP means standard temperature-273.15K and pressure

101.325 kPa.

Given

Volume of oxygen gas=1 cm^3

We know that

Volume of oxygen gas at STP= 22.4L

$1Litre=10^3 cm^3$

$22.4L=22.4\times10^3 cm^3$

Therefore, volume of oxygen gas $=22.4\times10^3 cm^3$

We know that 22.4L of O_2 contains 1 mol O_2 at STP.

$22.4\times10^3 cm^3$ of O_2 =1 mol O_2

Therefore,

$1cm^3$ of O_2 $=\frac{1}{22.4\times10^3}$ mol of O_2

I mol of O_2 =32 grams of O_2

$\frac{1}{22.4\times10^3}$ mol of O_2 $=\frac{32}{22.4\times10^3}$ grams of $O_2=1.43\times10^{-3}$grams

$1g=10^{-3}mg$

$\therefore$The mass of 1 cm^3 of oxygen kept at STP=1.43mg.

5. Question

Equal masses of air are sealed in two vessels, one of volume V_0 and the other of volume $2V_0$. If the first vessel is maintained at a temperature 300 K and the other at 600 K, find the ratio of the pressures in the two vessels.

Answer

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

P=pressure of gas.

Given

Masses of both the gas is equal. Therefore, number of moles of both the gas is equal. So, we can write

$n_1 = n_2 = n$

Volume of first gas $V_1 = V_o$

Volume of second gas $V_2 = 2V_o$

Temperature of first gas $T_1 = 300K$

Temperature of second gas $T_2 = 600K$

Let pressure of first gas $= P_1$

Pressure of second gas $= P_2$

Applying ideal gas equation for both the gases

$P_1V_1 = n_1RT_1$

$$n_1 = \frac{P_1\ V_1}{RT_1} = \frac{P_1\,V_o}{RT_1} \text{ ...(I)}$$

$P_2V_2 = n_2RT_2$

$$n_2 = \frac{P_2V_2}{RT_2} = \frac{P_2\,2V_o}{RT_2} \text{ .... (II)}$$

Since $n_1 = n_2 = n$

Therefore

$$\frac{P_1\ V_o}{RT_1} = \frac{P_2\,2V_o}{RT_2}$$

$$\frac{P_1}{RT_1} = \frac{P_2\,2}{RT_2}$$

Rearranging the above equation

$$\frac{P_1}{P_2} = \frac{2T_1}{T_2} = 2 \times \frac{300}{600}$$

$$\frac{P_1}{P_2} = 1$$

$P_1 : P_2 = 1:1$

So, the ratio of pressure gas in two vessels is 1:1.

6. Question

An electric bulb of volume 250 cc was sealed during manufacturing at a pressure of 10^{-3} mm of mercury at 27°C. Compute the number of air molecules contained in the bulb. Avogadro constant = 6×10^{23} mol^{-1}, density of mercury = 13600 kg m^{-3} and g = 10 m s^{-2}.

Answer

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

P=pressure of gas.

Given

Volume of gas=250cc

$1cc=1cm^3 = 10^{-6}m^3$

$V=250 \times 10^{-6}m^3$

Pressure $P=10^{-3}$ mm of mercury

1mm of Hg= 133.32Pa

$P= 10^{-3}$mm of Hg=133.32×10^{-3}Pa

Temperature T=27°C

T(K)=T (°C)+273.15

T=T(K)=27+273.15=300.15K

From ideal gas equation, we can write

$$n = \frac{PV}{RT} = \frac{133.32 \times 10^{-3} \times 250 \times 10^{-6}}{8.31 \times 300.15}$$

$$n = 13.36 \times 10^{-9}$$

Number of molecules = Avogadro number $\times$ number of moles

Number of molecules=$6 \times 10^{23} \times$n

=$6 \times 10^{23} \times 13.36 \times 10^{-9} = 80.17 \times 10^{14}$

Number of molecules in electric bulb=8.01×10^{15}.

7. Question

A gas cylinder has walls that can bear a maximum pressure of 1.0×10^6 Pa. It contains a gas at 8.0×10^5 Pa and 300 K. The cylinder is steadily heated. Neglecting any change in the volume, calculate the temperature at which the cylinder will break.

Answer

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

P=pressure of gas.

Given

Maximum pressure $P_{max}=P_2=1.0 \times 10^6$ Pa

Pressure of gas $P_1=8.0 \times 10^5$ Pa

Temperature of gas T_1=300K

Since the volume has not been changed therefore,

$V_1=V_2=V$

Hence number of moles will also be same $n_1=n_2=n$

Temperature at which the cylinder will break=T_2

$$n_1 = \frac{P_1\ V_1}{RT_1}$$

$$n_2 = \frac{P_2 V_2}{RT_2}$$

Since $n_1 = n_2 = n$

Therefore,

$$\frac{P_1 V}{RT_1} = \frac{P_2 V}{RT_2}$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$T_2 = \frac{P_2 \times T_1}{P_1} = \frac{1.0 \times 10^6 \times 300}{8.0 \times 10^5}$$

T_2 =375K

∴The temperature at which the cylinder will break=375K.

8. Question

2g of hydrogen is sealed in a vessel of volume 0.02 m^3 and is maintained at 300K. Calculate the pressure in the vessel.

Answer

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

P=pressure of gas.

Given

Volume V=0.02 m^3

Temperature T=300K

Mass of hydrogen gas = 2g

Number of moles n =$\frac{given\ mass}{molar\ mass}$

Molar mass of hydrogen=2amu

$n = \frac{2}{2} = 1$mol

From ideal gas equation, we can write

$$P = \frac{nRT}{V} = \frac{1\times8.31\times300}{0.02} = 1.24\times10^5\text{Pa}$$

∴Pressure in the vessel is 1.24×10^5Pa.

9. Question

The density of an ideal gas is 1.25×10^{-3} g cm^{-3} at STP. Calculate the molecular weight of the gas.

Answer

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

P=pressure of gas

STP means standard temperature-273.15K and pressure 101.325 kPa.

Given:

T=273.15K

P=101.325$\times 10^3$Pa

Density of ideal gas ρ =1.25×10^{-3} g cm^{-3}

1gcm^{-3}=10^3kgm^3

$\rho = 1.25kgm^3$

Number of moles n =$\frac{given\ mass}{molar\ mass}=\frac{m}{M}$

Density $\rho = \frac{given\ mass}{voulme} = \frac{m}{V}$

From ideal gas equation, we can write

$$PV = nRT = \frac{m}{M}RT$$

$$M = \frac{mRT}{VP} = \frac{\rho RT}{P} = 1.25\times8.31\times\frac{300}{10^5} = 2.38\times10^{-2}gmol^{-1}$$

∴ Molecular weight of gas is $2.38 \times 10^{-2} gmol^{-1}$

10. Question

The temperature and pressure at Shimla are 15.0°C and 72.0 cm of mercury and at Kalka these are 35.0°C and 76.0 cm of mercury. Find the ratio of air density at Kalka to the air density at Shimla.

Answer

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

P=pressure of gas

Number of moles n $=\frac{given\ mass}{molar\ mass}=\frac{m}{M}$

Density $\rho = \frac{given\ mass}{voulme} = \frac{m}{V}$

Given:

Temperature of Shimla T_1=15.0°C

T(K)=T (°C)+273.15

T_1=T(K)=15+273.15=288.15K

Pressure of Shimla P_1 = 72.0 cm of mercury

Temperature of Kalka T_2 = 35.0°C

T(K)=T (°C)+273.15

T_2=T(K)=35+273.15=308.15K

Pressure of Kalka P_2=76.0 cm of mercury

Substituting the value of n and ρ in ideal gas equation, we get

$$PV = nRT = \frac{m}{M}RT = \frac{\rho V}{M}RT$$

So,

$$\rho = \frac{PM}{RT}$$

$$\rho_1 = \frac{P_1 M}{RT_1} \dots\dots(i)$$

$$\rho_2 = \frac{P_2 M}{RT_2} \dots\dots(ii)$$

Taking the ratio of equations (i) and (ii),

$$\frac{\rho_1}{\rho_2} = \frac{\frac{P_1}{T_1}}{\frac{P_2}{T_2}} = \frac{P_1}{P_2} \times \frac{T_2}{T_1}$$

$$\frac{\rho_1}{\rho_2} = \frac{0.72}{0.76} \times \frac{308.15}{288.15} = 1.02$$

$$\frac{\rho_2}{\rho_1} = \frac{1}{1.02} = 0.98$$

∴The ratio of air density at Kalka to the air density at Shimla is 0.98.

11. Question

Figure (24-EI) shows a cylindrical tube with adiabatic walls and fitted with a diathermic separator. The separator can be slid in the tube by an external mechanism. An ideal gas is injected into the two sides at equal pressures and equal temperatures. The separator remains in equilibrium at the middle. It is now slid to a position where it divides the tube in the ratio of 1: 3. Find the ratio of the pressures in the two parts of the vessel.

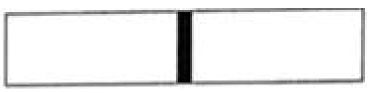


Figure 24-E1

Answer

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

P=pressure of gas

Given

Volume of first part=V

Volume of second part=3V

Initially separator had divided cylinder in two equal parts so, number of moles in both the parts will be same.

$n_1=n_2=n$

Since the walls of separator is diathermic, the temperature of both the parts will always be same.

$T_1=T_2=T$

Pressure of part 1

$P_1=\frac{nRT_1}{V}=\frac{nRT}{V}$

Pressure of part 2

$P_2=\frac{nRT_2}{3V}=\frac{nRT}{3V}$

Diving P_1 and P_2, we get

$$\frac{P_1}{P_2}=\frac{\frac{nRT}{V}}{\frac{nRT}{3V}}=\frac{3}{1}$$

$P_1:P_2=3:1$

∴The ratio of the pressures in the two parts of the vessel is 3:1.

12. Question

Find the rms speed of hydrogen molecules in a sample of hydrogen gas at 300 K. Find the temperature at which the rms speed is double the speed calculated in the previous part.

Answer

We know that rms speed of gas is given by

$$v_{rms}=\sqrt{\frac{3RT}{M}}$$

Where R=gas constant 8.31J/molK

T=temperature of gas

M=molar mass of gas

Given

Temperature T=300K

Molar mass of hydrogen=2g/mol

Therefore,

$$v_{rms} = \sqrt{\frac{3 \times 8.31 \times 300}{0.002}} = 1932.6ms^{-1}$$

Now in second part of question speed is doubled i.e. $2 \times 1932.6 ms^{-1}$

Let temperature at this speed be T_1

So, using the same formula of rms speed

$$\sqrt{\frac{3 \times 8.31 \times T_1}{0.002}} = 2 \times 1932.6ms^{-1}$$

Squaring both sides of above equation

$$3 \times 8.31 \times T_1 = 0.002 \times 2 \times 1932.6$$

$$T_1 = \frac{0.002 \times 2 \times 1932.6}{3 \times 8.31} = 1200K$$

∴Temperature of the gas when speed is doubled is 1200K which is 4 times the pervious temperature.

13. Question

A sample of 0.177 g of an ideal gas occupies 1000 cm^3 at STP. Calculate the rms speed of the gas molecules.

Answer

STP means standard temperature-273.15K and pressure 101.325 kPa.

We know that rms speed of gas is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Where R=gas constant 8.31J/molK

T=temperature of gas

M=molar mass of gas

Given

Temperature T=273.15K

Pressure P=101.325$\times 10^3$Pa

Mass =0.177g =0.177$\times 10^{-3}$kg

Volume = 1000cm^3

1cm=10^{-2}m

1cm^3=$10^{-6}m^3$

1000cm^3=$10^{-3}m^3$

Density $\rho = \frac{mass}{volume} = \frac{0.177\times10^{-3}kg}{10^{-3}m^3} = 0.177kgm^{-3}$

We know that ideal gas equation is

PV=nRT

Where V= volume of gas

R=gas constant=8.31J/molK

T=temperature

N=number of moles of gas

So, we can write $RT = \frac{PV}{n}$.

Putting the value of RT in v_{rms} we get

$v_{rms} = \sqrt{\frac{3PV}{nM}}$

nM= total mass of gas 'm'

and density $\rho = \frac{total\ mass}{volume} = \frac{m}{V}$

Putting the value of density ρ and m in v_{rms}, we get

$v_{rms} = \sqrt{\frac{3P}{\rho}}$

Putting the value of P and ρ in above equation we get

$$v_{rms} = \sqrt{\frac{3 \times 101.325 \times 10^3}{0.177}} = 1310.4ms^{-1}$$

∴The rms speed of the gas molecules at STP is 1310.4ms^{-1}.

14. Question

The average translational kinetic energy of air molecules is 0.040 eV (1 eV = 1.6 × 10^{-19}J). Calculate the temperature of the air. Boltzmann constant κ = 1.38 × 10^{-23} J K^{-1}.

Answer

We know that kinetic energy of ideal gas is given by

$K.E = \frac{3}{2} k_B T$ .... (I)

Where K. E=kinetic energy of gas

k_B = Boltzmann constant=1.38 × 10^{-23} J K^{-1}.

T = temperature of gas

In kinetic theory of ideal gas, molecule of gas is moving in straight line.

Hence, they will only have translational kinetic energy.

Given

K. E=0.040 eV

We know that

1 eV = 1.6 × 10^{-19}J

0.040eV=0.040$\times 1.6 \times 10^{-19}$J

From equation (I) we can write

$$T = \frac{2K.E}{3k_B}$$

$$T = \frac{2 \times 0.04 \times 1.6 \times 10^{-19}}{3 \times 1.38 \times 10^{-23}} = 309.2K$$

Therefore, the temperature of gas is 309.2K.

15. Question

Consider a sample of oxygen at 300 K. Find the average time taken by a molecule of travel a distance equal to the diameter of the earth.

Answer

In kinetic theory of ideal gas, the average energy is given by

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

Where R=gas constant=8.31$Jmol^{-1}K^{-1}$

T=temperature of gas

M=molar mass of gas

Given

Temperature T=300K

Molar mass of oxygen=32amu=32g/mol=32$\times 10^{-3}$ kg/mol

Therefore

$$v_{avg} = \sqrt{\frac{8 \times 8.31 \times 300}{3.14 \times 0.032}}$$

$$v_{avg} = 445.25ms^{-1}$$

We know that

$$time = \frac{distance}{speed}$$

Distance here is given as diameter of earth

Now radius of earth = 6400km=6400000m

Diameter=2$\times$radius

So, diameter of earth =2$\times$6400000m

So, time taken

$$Time = \frac{2 \times 6400000}{445.25} = 28747.83s$$

1 hour=60$\times$60 seconds=3600seconds

So, 28747.83s =$\frac{28747.83}{3600}$h = 7.98h $\approx$ 8h

So, average time taken by oxygen molecule to travel a distance equal to the diameter of the earth is 7.98h$\approx$8h.

16. Question

Find the average magnitude of linear momentum of a helium molecule in a sample of helium gas at 0°C. Mass of a helium molecule = 6.64×10^{-27} kg and Boltzmann constant = 1.38×10^{-23} J K^{-1}.

Answer

In kinetic theory of ideal gas, the average energy is given by

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

Where R=gas constant=$8.31 Jmol^{-1}K^{-1}$

T-temperature of gas

M=molar mass of gas

Given

Temperature T=0°C

T(K)=T (°C)+273.15

T=T(K)=0+273.15=273.15K

Mass of helium molecule m =6.64×10^{-27} kg

We know that,

Gas constant $R=k_BN_A$

Where k_B= Boltzmann constant = 1.38×10^{-23} J K^{-1}.

And N_A=Avogadro number=6.023×10^{-23} mol^{-1}

$$v_{avg} = \sqrt{\frac{8k_BN_AT}{\pi M}}$$

Molar mass of gas molecule M= Avogadro number$\times$mass of gas molecule

$M=N_A\times m \rightarrow \frac{M}{N_A} = m$

So average velocity becomes

$$v_{avg} = \sqrt{\frac{8k_BT}{\pi m}} = \sqrt{\frac{8 \times 1.38 \times 10^{-23} \times 273.15}{3.14 \times 6.64 \times 10^{-27}}}$$

$$v_{avg} = 1202.31 ms^{-1}$$

We know that

Momentum=mass$\times$velocity

Momentum=$6.64 \times 10^{-27} \times 1202.31 = 8 \times 10^{-24}$ kgms^{-1}

∴Average magnitude of momentum of helium at 0°C is 8×10^{-24} kgms^{-1}.

17. Question

The mean speed of the molecules of a hydrogen sample equals the mean speed of the molecules of a helium sample. Calculate the ratio of the temperature of the hydrogen sample to the temperature of the helium sample.

Answer

In kinetic theory of ideal gas mean speed also known as average

speed is given as

$$v_{mean} = \sqrt{\frac{8RT}{\pi M}}$$

Where R=gas constant=8.31Jmol^{-1}K^{-1}

T=temperature of gas

M=molar mass of gas

Given

$$v_{mean}(H) = v_{mean}(He)$$

Let temperature of hydrogen gas =T(H)

Temperature of helium gas= T(He)

Molar mass of hydrogen gas =2amu

Molar mass of helium gas =4amu

$$\sqrt{\frac{8RT(H)}{\pi M(H)}} = \sqrt{\frac{8RT(He)}{\pi M(He)}}$$

$$\sqrt{\frac{T(H)}{T(He)}} = \sqrt{\frac{M(H)}{M(He)}} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{2}}$$

Squaring both sides

$$\frac{T(H)}{T(He)} = \frac{1}{2}$$

the ratio of the temperature of the hydrogen sample to the temperature of the helium sample is 1:2.

18. Question

At what temperature the mean sped of the molecules of hydrogen gas equals the escape speed from the earth?

Answer

In kinetic theory of ideal gas mean speed also known as average speed is given as

$$v_{mean} = \sqrt{\frac{8RT}{\pi M}} \dots \dots . . (I)$$

Where R=gas constant=8.31$Jmol^{-1}K^{-1}$

T-temperature of gas

M=molar mass of gas

Molar mass of hydrogen=2amu=2g/mol=2$\times 10^{-3}$kg/mol

Escape speed of earth is the speed given to projectile so that it escapes the gravitational field of earth. It is given by formula

$$v_e = \sqrt{\frac{2GM_E}{R}} \dots . (II)$$

Where G=universal gravitational constant

M_E=mass of earth

R=radius of earth

We also know that acceleration due to gravity g is

$$g = \frac{GM_E}{R^2} \dots . . (III)$$

Multiplying and dividing equation (II) by R

$$v_e = \sqrt{\frac{2GM_E}{R} \times \frac{R}{R}} = \sqrt{\frac{2GM_E}{R^2}R}$$

Putting the value of g from equation (III) to above equation we get

$$v_e = \sqrt{2gR} \;\ldots\ldots(IV)$$

According to question v_{mean} is equal to v_e

So, from equation (I) and (IV) we get

$$\sqrt{2gR} = \sqrt{\frac{8RT}{\pi M}}$$

Radius of earth R=6400km=6400000m

g=9.8m/s

$$\sqrt{2 \times 9.8 \times 64 \times 10^5} = \sqrt{\frac{8 \times 8.31 \times T}{3.14 \times 2 \times 10^{-3}}}$$

Squaring both sides

$$2 \times 9.8 \times 64 \times 10^5 = \frac{8 \times 8.31 \times T}{3.14 \times 2 \times 10^{-3}}$$

$$T \approx 118 \times 10^2 K$$

Temperature at which the mean sped of the molecules of hydrogen gas equals the escape speed from the earth is 11800K.

19. Question

Find the ratio of the mean speed of hydrogen molecules to the mean speed of nitrogen molecules in a sample containing a mixture of the two gases.

Answer

In kinetic theory of ideal gas, mean speed also known as average

speed is given as

$$v_{mean} = \sqrt{\frac{8RT}{\pi M}} \;\ldots\ldots(I)$$

Where R=gas constant=8.31$Jmol^{-1}K^{-1}$

T=temperature of gas

M=molar mass of gas

Molar mass of hydrogen molecule M(H)=2 amu

Molar mass of nitrogen molecule M(N)=28 amu

Mean speed of hydrogen molecule=

$$v_{mean}(H) = \sqrt{\frac{8RT}{\pi M(H)}} \dots\dots(II)$$

Mean speed of nitrogen molecule=

$$v_{mean}(N) = \sqrt{\frac{8RT}{\pi M(N)}} \dots\dots(III)$$

Temperature of both the gases is same.

Dividing equation (II) and (III) we get

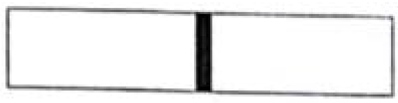
$$\frac{v_{mean}(H)}{v_{mean}(N)} = \frac{\sqrt{\frac{8RT}{\pi M(H)}}}{\sqrt{\frac{8RT}{\pi M(N)}}}$$

$$\frac{v_{mean}(H)}{v_{mean}(N)} = \sqrt{\frac{M(N)}{M(H)}} = \sqrt{\frac{28}{2}} = \sqrt{14} = 3.74$$

∴The ratio of the mean speed of hydrogen molecules to the mean speed of nitrogen molecules in a sample containing a mixture of the two gases is 3.74.

20. Question

Figure shows a vessel partitioned by a fixed diathermic separator. Different ideal gases are filled in the two parts. The rms speed of the molecules in the left part equals the mean speed of the molecules in the right part. Calculate the ratio of the mass of a molecule in the left part to the mass of a molecule in the right part.



Answer

In kinetic theory of ideal gas, the average energy is given by

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

Where R=gas constant=8.31$Jmol^{-1}K^{-1}$

T=temperature of gas

M=molar mass of gas

We know that,

Gas constant $R=k_BN_A$

Where k_B= Boltzmann constant = 1.38×10^{-23} J K^{-1}.

N_A=Avogadro number=6.023×10^{-23} mol^{-1}

$$v_{avg} = \sqrt{\frac{8k_BN_AT}{\pi M}}$$

Molar mass of gas molecule M= Avogadro number $\times$ mass of gas molecule

$M=N_A \times m \rightarrow \frac{M}{N_A} = m$

So average velocity becomes

$$v_{avg} = \sqrt{\frac{8k_BT}{\pi m}} \dots \dots \dots (I)$$

Rms speed of gas molecule is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Where R=gas constant 8.31J/molK

T=temperature of gas

M=molar mass of gas

Putting the value gas constant $R=k_BN_A$

So rms speed becomes

$$v_{rms} = \sqrt{\frac{3k_BN_AT}{M}}$$

$M=N_A \times m \rightarrow \frac{M}{N_A} = m$

Therefore,

$$v_{rms} = \sqrt{\frac{3k_BT}{m}} \;\ldots\ldots(II)$$

Let the mass of molecule in left part=m_1

Mass of molecule in right part=m_2

According to question, the rms speed of the molecules in the left part equals the mean speed of the molecules in the right part.

So, from equation (I) and(II) we get

$$\sqrt{\frac{8k_BT}{\pi m_2}} = \sqrt{\frac{3k_BT}{m_1}}$$

Since the walls of separator is diathermic therefore temperature of both the parts will be same.

Squaring the above equation, we get

$$3 \times m_2 = \frac{8 \times m_1}{3.14}$$

$$\frac{m_1}{m_2} = \frac{3.14 \times 3}{8} = 1.17$$

∴The ratio of the mass of a molecule in the left part to the mass of a molecule in the right part is 1.17.

21. Question

Estimate the number of collisions per second suffered by a molecule in a sample of hydrogen at STP. The mean free path (average distance covered by a molecule between successive collisions) = 1.38×10^{-5} cm.

Answer

Number of collision per second means frequency of collision.

In kinetic theory of ideal gas, the average energy is given by

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

Where R=gas constant=$8.31 J mol^{-1} K^{-1}$

T-temperature of gas

M=molar mass of gas

Molar mass of hydrogen =2 amu=2$\times 10^{-3}$kg/mol

$$v_{avg} = \sqrt{\frac{8 \times 8.31 \times 273.15}{3.14 \times 2 \times 10^{-3}}}$$

$$v_{avg} = 1700ms^{-1}$$

Given

Distance between successive collision λ =1.38 × 10^{-5} cm

λ =1.38$\times 10^{-8}$m

Time between two collisions

$$time = \frac{distance}{velocity}$$

$$t = \frac{\lambda}{v_{\ avg}} = \frac{1.38 \times 10^{-8}}{1700} = 8 \times 10^{-12} s$$

Frequency of collision =$\frac{1}{t} = \frac{1}{8 \times 10^{-12}} = 1.23 \times 10^{11}$

$\therefore$Number of collision per second means frequency of collision which is equal to 1.23$\times 10^{11}$.

22. Question

Hydrogen gas is contained in a closed vessel at 1 atm (100 kPa) and 300 K

(a) Calculate the means speed of the molecules.

(b) Suppose the molecules strike the wall with this speed making an average angle of 45° with it.

How many molecules strike each square meter of the wall per second?

Answer

(a)

In kinetic theory of ideal gas, mean speed also known as average speed is given as

$$v_{mean} = \sqrt{\frac{8RT}{\pi M}} \dots \dots . . (I)$$

Where R=gas constant=8.31$Jmol^{-1}K^{-1}$

T=temperature of gas

M=molar mass of gas

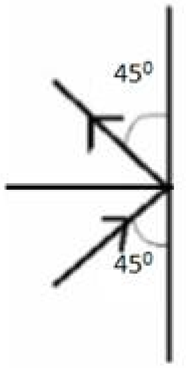
Given

T=300k

Molar mass of hydrogen gas =2amu=2g/mol=2$\times$ 10^{-3}kg/mol

$$v_{mean} = \sqrt{\frac{8 \times 8.31 \times 300}{3.14 \times 2 \times 10^{-3}}} = 1780m/s$$

(b)



Let velocity be u=v_{mean} from part (a)

From figure we can see that

Total Momentum in vertical direction

mucos45∘-mucos45∘=0

Total momentum in horizontal direction

musin45∘-(-musin45∘)=2musin45∘=2mu$\times \frac{1}{\sqrt{2}} = \sqrt{2}$mu

Total change momentum of 1 molecules =$\sqrt{2}$mu

Total change momentum of n molecules =$\sqrt{2}$nmu

We know that,

$$force \times time = change\ in\ momentum$$

Let ‘t’ be the time taken to changing the momentum.

So, force per unit second due to 1 molecule=$\sqrt{2}$mu

Force per unit second due to n molecule=$\sqrt{2}$nmu

Given

Pressure by n molecule=10^5 Pa

Area=$1m^2$

Pressure =$\frac{force}{area}$

$$\frac{\sqrt{2}nmu}{1} = 10^5$$

$$number\ of\ molecule\ striking\ per\ second = n = \frac{10^5}{\sqrt{2}mu}$$

We know that

Mass of 6.023$\times 10^{23}$ of hydrogen molecule=2$\times 10^{-3}$kg

Mass of 1 hydrogen molecule m =$\frac{2\times10^{-3}}{6.023\times10^{23}} = 3.3 \times 10^{-27}$kg

Therefore

$$n = \frac{10^5}{\sqrt{2} \times 3.3 \times 10^{-27} \times 1780} = 1.2 \times 10^{28}$$

∴Number of molecules strike each square meter of the wall per second=1.2$\times 10^{28}$.

23. Question

Air is pumped into an automobile tyre's tube up to a pressure of 200 kPa in the morning when the air temperature is 20°C. During the day the temperature rises to 40°C and the tube expands by 2%. Calculate the pressure of the air in the tube at this temperature.

Answer

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

P=pressure of gas

Given

Pressure at temperature 20°C P_1=200$\times 10^3$Pa

Volume at temperature $20^\circ C = V_1$

Increase in volume $= 2\% V_1$

Volume at temperature $40^\circ C$ $V_2 = V_1 + 2\% V_1$

$V_2 = V_1 + 0.02V_1 = 1.02V_1$

$20^\circ C = 293.15K$

$40^\circ C = 313.15K$

From ideal gas equation, we can write

$$nR = \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Since, tube got expanded by the end of the day, only volume will change. Number of moles remains the same, before and after expansion as no new gas has been added. So, the product of nR will be same before and after expansion.

$$\frac{200 \times 10^3 \times V_1}{293.15} = \frac{P_2 \times 1.02V_1}{313.15}$$

$$P_2 = \frac{313.15 \times 200 \times 10^3}{293.15 \times 1.02} = 209.45kPa$$

$\therefore$ Pressure of the tube at 40^0C is 209.45kPa.

24. Question

Oxygen is filled in a closed metal jar of volume 1.0×10^{-3} m^3 at a pressure of 1.5×10^5 Pa and temperature 400 K. The jar has a small leak in it. The atmospheric temperature is 300 K. Find the mass of the gas that leaks out by the time the pressure and the temperature inside the jar equalize with the surrounding.

Answer

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant

T=temperature

n=number of moles of gas

P=pressure of gas

Given

Volume inside jar V_1= 1.0×10^{-3} m^3

Pressure inside jar P_1 =1.5×10^5 Pa

Temperature inside jar T_1=400K

Pressure of surrounding P_2 =1atm=1.0×10^5Pa

Temperature of surrounding T_2=300K

Let volume of oxygen at T_2 and P_2 =V_2

When jar is in equilibrium with surrounding, temperature and pressure of oxygen gas inside jar will T_2 and P_2.

Number of moles will be same inside the jar, before and after equilibrium as no new oxygen gas has been added. Just temperature and pressure has been changed. Due to which volume will change.

Assuming there is no leak in jar, applying ideal gas equation before and after equilibrium, we get

$$nR = \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_2 = \frac{1.5 \times 10^5 \times 1.0 \times 10^{-3} \times 300}{1.0 \times 10^5 \times 400} = 1.125 \times 10^{-3} m^3$$

Now if we consider leak,

Volume of gas leaked =V_2-V_1

= $(1.125-1)\times 10^{-3}m^3$

= $1.25\times 10^{-4}m^3$

If n_2 are number of moles leaked out, then

Mass of the gas leaked out =$n_2\times$molar mass of oxygen molecule

$$n_2 = \frac{P_2 \times volume\ of\ leaked\ gas}{T_2}$$

$$n_2 = \frac{1 \times 10^5 \times 1.25 \times 10^{-4}}{300} = 0.005 mol$$

Molar mass of oxygen molecule=32g/mol

Mass of gas leaked out=0.005×32=0.16g

$\therefore$The mass of the gas that leaked out =0.16g.

25. Question

An air bubble of radius 2.0 mm is formed at the bottom of a 3.3 m deep river. Calculate the radius of the bubble as it comes to the surface. Atmospheric pressure = 1.0×10^5 Pa and density of water = 1000 kg m^{-3}.

Answer

Given

Radius of bubble at the bottom of deep river R_1=2.0mm=2.0×10^{-3}m

Depth of the river h=3.3m

Density of water ρ= 1000 kg m^{-3}

We know that

Pressure at depth inside a fluid is related to atmospheric pressure by relation

$P_1=P_a + h\rho g$

Where P_1 =pressure at depth h

P_a=atmospheric pressure=1.0×10^5 Pa

g=acceleration due to gravity=9.8ms^{-2}

ρ=density of fluid.

So,

$P_1=1.0\times10^5+3.3\times1000\times9.8=1.32\times10^5$Pa

Temperature is same for both water at the bottom and the water at the surface. So, we can apply Boyle's law which says that PV=constant, when temperature is constant.

Let V_1 be the volume of air bubble at bottom of deep river

$$V_1 = \frac{4}{3} \times \pi \times R_1^3 \dots\dots..(I)$$

V_a be the volume of air bubble at surface of river

$$V_a = \frac{4}{3} \times \pi \times R_a^3 \dots\dots..(II)$$

Where R_a = radius of bubble at surface of river

So, according to Boyle's law

$P_1 V_1 = P_a V_a$

$$1.32 \times 10^5 \times \frac{4}{3} \times \pi \times R_1^3 = 1.0 \times 10^5 \times \frac{4}{3} \times \pi \times R_a^3 \; (from(I)and(II))$$

$$1.32 \times 10^5 \times (2.0 \times 10^{-3})^3 = 1.0 \times 10^5 \times R_a^3$$

$$R_a^3 = \frac{1.32 \times 10^5 \times (2.0 \times 10^{-3})^3}{1.0 \times 10^5}$$

$$R_a = \sqrt[3]{\frac{1.32 \times 10^5 \times (2.0 \times 10^{-3})^3}{1.0 \times 10^5}}$$

$R_a = 2.2 \times 10^{-3}$m

∴Radius of the air bubble at the surface of river is 2.2×10^{-3}m.

26. Question

Air is pumped into the tubes of a cycle rickshaw at a pressure of 2 atm. The volume of each tube at this pressure is 0.002 m^3. One of the tubes gets punctured and the volume of the tube reduces to 0.0005 m^3. How many moles of air have leaked out? Assume that the temperature remains constant at 300K and that the air behaves as an ideal gas.

Answer

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant=8.31$Jmol^{-1}K^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas

Given

Pressure inside the tyre P_1=2atm=2×10^5Pa

Volume at P_1, V_1=0.002m^3

Reduced volume V_2=0.0005m^3

Temperature remains constant so $T_1=T_2=300K$

Let when the gas is leaked out the pressure P_2 becomes equal to atmospheric pressure. So $P_2=1.0\times 10^5$Pa.

Number of moles initially n_1

$$n_1 = \frac{P_1V_1}{RT_1} = \frac{2\times 10^5 \times 0.002}{8.31\times 300} = 0.16$$

Similarly

Final number of moles n_2

$$n_2 = \frac{P_2\,V_2}{RT_2} = \frac{1.0\times 10^5 \times 0.0005}{8.31\times 300} = 0.02$$

So, number of moles leaked out will be $n_1-n_2=0.16-0.02=0.14$.

27. Question

0.040 g of He is kept in a closed container initially at 100.0°C. The container is now heated. Neglecting the expansion of the container, calculate the temperature at which the internal energy is increased by 12 J.

Answer

Given

Mass of helium =0.040g

Molar mass of helium=4g/mol

Number of moles n=$\frac{given\ mass}{molar\ mass}$

Number of moles for helium n=$\frac{0.040}{4} = 0.01$

Temperature $T_1=100°C$

T(K)=T (°C)+273.15

T=T(K)=100+273.15=373.15K

Internal energy U in kinetic theory is given as

$$U = \frac{3}{2}nC_vT$$

Where C_v= molar specific heat capacity

N= number of moles

T=temperature of gas

Also, internal energy depends only the temperature of the gas.

Helium is a monoatomic gas and for monoatomic gas

$$C_v = \frac{3}{2} \times R$$

So, C_V for helium is $\frac{3}{2} \times 8.31 = 12.45 Jmol^{-1}K^{-1}$

Increase in internal energy is given in question as 12J. That means

$U_2 - U_1 = nC_v(T_2 - T_1)$

Since the gas has not been changed, just expanded no change in molar specific heat capacity and number of moles of gas.

Putting the value of change in internal energy and T_1,

$12 = 0.01 \times 12.45 \times (T_2 - 373.15)$

$$T_2 = \frac{12}{0.01 \times 12.45} + 373.15 = 469.53K$$

∴The temperature at which the internal energy is increased by 12J is 469.53J =196.38°C.

28. Question

During an experiment, an ideal gas is found to obey an additional law pV^2 = constant. The gas is initially at a temperature T and volume V. Find the temperature when it expands to a volume 2V.

Answer

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant=$8.31 Jmol^{-1}K^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas

So $P = \frac{nRT}{V}$ .... (I)

Now differentiating the ideal gas equation, we get

PdV + VdP=nRdT …………. (II) (we have applied product rule for

differentiation of PV)

Now as given in question the ideal here follows and additional law which is PV^2=constant.

So, differentiating this additional law as well we get

$2PVdV + V^2dP=0$

Taking V as common we get

2PdV + VdP=0 ……… (III)

Subtract equation (III) from (II)

2PdV + VdP - PdV - VdP = -nRdT

PdV=-nRdT

From equation (I), substitute the value of P in above equation we get

$$\frac{nRTdV}{V} = -nRdT$$

$$\frac{dV}{V} = \frac{dT}{T} \dots\dots\dots (IV)$$

Integrating equation (IV) from limits V to 2V and T1 to T2

$$\int_{V}^{2V} \frac{dV}{V} = -\int_{T_1}^{T_2} \frac{dT}{T}$$

We know $\int \frac{dx}{x} = lnx$. Applying this formula

$$\ln(2V) - \ln(V) = -\ln(T_2) + \ln(T_1)$$

$$\ln\left(\frac{2V}{V}\right) = \ln(\frac{T_1}{T_2})$$

Where we have applied the property of ln which is

ln(a)-ln(b)=ln(a/b)

$$\ln(2) = \ln(\frac{T_1}{T_2})$$

$$T_2 = \frac{T_1}{2}$$

So, the temperature at which the gas expands is half of the initially temperature.

29. Question

A vessel contains 1.60 g of oxygen and 2.80 g of nitrogen. The temperature is maintained at 300 K and the volume of the vessel is 0.166 m^3. Find the pressure of the mixture.

Answer

Given

Mass of oxygen gas =1.60g

Mass of nitrogen gas =2.80g

Temperature of vessel=300K

Volume of vessel=0.166m^3

We know that

Number of moles n=$\frac{given\ mass}{molar\ mass} = \frac{m}{M}$

Molar mass of oxygen= 32g/mol

Molar mass of nitrogen=28g/mol

Number of moles of oxygen $n_1 = \frac{1.60}{32} = 0.05$

Number of moles of nitrogen $n_2 = \frac{2.80}{28} = 0.1$

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant=8.31$Jmol^{-1}K^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas

In a mixture of non-interacting ideal gases, the pressure that a gas in a mixture of gases would exert if it occupied the same volume as the mixture at the same temperature is called the partial pressure of that gas.

Partial pressure of oxygen gas

$$P_o = \frac{n_1RT}{V} = \frac{0.05 \times 8.31 \times 300}{0.166} = 750Pa$$

Partial pressure of nitrogen gas

$$P_N = \frac{n_2RT}{V} = \frac{0.1 \times 8.31 \times 300}{0.166} = 1500Pa$$

According to Dalton's law of partial pressure, the total pressure of a mixture of ideal gas is the sum of partial pressures.

So, total pressure

$P = P_o + P_N = 750 + 1500 = 2250Pa$

∴The pressure of the mixture of oxygen and nitrogen gas is 2250Pa.

30. Question

A vertical cylinder of height 100 cm contains air at a constant temperature. The top is closed by a frictionless light piston. The atmospheric pressure is equal to 75 cm of mercury. Mercury is slowly poured over the piston. Find the maximum height of the mercury column that can be put on the piston.

Answer

Given

Height of vertical cylinder=100cm=1m

Pressure P_1=75cm of Hg=0.75m of Hg

1mm of Hg = hρg Pa

Where h= height of mercury column =1mm=0.001m

ρ= density of mercury

g= acceleration due to gravity

So,

P_1= 0.75m of Hg= 0.75 ρg Pa

Let h be height of mercury above the piston. When mercury is poured over piston the piston will move down and gas inside vessel will get compressed.

So, let the pressure of gas when mercury is poured be P_2

So,

$P_2=P_1+h\rho g=0.75\rho g+h\rho g$

Let the circular area of cylinder be A.

Then, volume of gas before mercury was poured $V_1=A\times$height of cylinder

$V_1=A\times1=A$

Height of cylinder when mercury was poured =(1-h) m

Volume of gas after mercury was poured $V_2=A\times(1-h)$

Since it is given in question that temperature has not being changed so we can apply Boyle's law which states that PV=constant, if temperature is constant.

$P_1V_1=P_2V_2$

$0.75\rho g\times A=0.75\rho g+h\rho g\times A\times(1-h)$

Taking ρgA common from both side of equation we get

$0.75=(0.75+h)(1-h)$

$0.75=0.75+h-0.75h-h^2$

$h^2-0.25h=0$

$h-0.25=0\rightarrow h=0.25m$

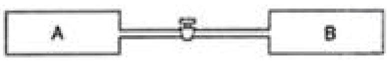
$\therefore$ The maximum height of the mercury column that can be put on the piston is 25cm.

31. Question

Figure shows two vessels A and B with rigid walls containing ideal gases. The pressure, temperature and the volume are p_A. T_A, V in the vessel A and p_B, T_B, V in the vessel B. The vessels are now connected through a small tube. Show that the pressure p and the temperature T satisfy.

$$\frac{p}{T}=\frac{1}{2}\left(\frac{p_A}{T_A}+\frac{p_B}{T_B}\right)$$

When equilibrium is achieved.



Answer

Let the partial pressure of gas A and B be P'_A and P'_B respectively.

Given:

Pressure, temperature and volume of gas A P_A T_A, V

Pressure, temperature and volume of gas B P_B, T_B, V

We know that ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant =8.3$JK^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

In a mixture of non-interacting ideal gases, the pressure that a gas in a mixture of gases would exert if it occupied the same volume as the mixture at the same temperature is called the partial pressure of that gas.

Using this definition volume of gas, A when gas B in not present, is 2V and temperature T.

So, from ideal gas equation

$P'_A 2V=nRT$

Also

$P_A V=nRT_A$

Equating nR from both the above equation

$$\frac{P_A V}{T_A} = \frac{P'_A 2V}{T}$$

$$P'_A = \frac{P_A T}{2T_A}$$

Doing the same above procedure for gas B

$P'_B 2V=nRT$

Also

$P_B V=nRT_B$

$$\frac{P_B V}{T_B} = \frac{P'_B 2V}{T}$$

$$P'_B = \frac{P_B T}{2T_B}$$

According to Dalton's law of partial pressure, the total pressure of a mixture of ideal gas is the sum of partial pressures.

$$P = P'_A + P'_B$$

$$P = \frac{P_A T}{2T_A} + \frac{P_B T}{2T_B}$$

$$\frac{P}{T} = \frac{1}{2}\left(\frac{P_A}{T_A} + \frac{P_B}{T_B}\right) \ldots\ldots hence\ proved.$$

32. Question

A container of volume 50 cc contains air (mean molecular weight = 28.8 g) and is open to atmosphere where the pressure is 100 kPa. The container is kept in a bath containing melting ice (0°C).

(a) Find the mass of the air in the container when thermal equilibrium is reached.

(b) The container is now placed in another bath containing boiling water (100°C).

Find the mass of air in the container.

(c) The container is now closed and placed in the melting-ice bath. Find the pressure of the air when thermal equilibrium is reached.

Answer

Given

Volume of container V_1=50cc=$50\times10^{-6}m^3$

Molecular mass of air in container M= 28.8g

Pressure of air P_1= 100kPa=10^5Pa

(a) We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant=$8.31Jmol^{-1}K^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas

In first case the air is kept in container having ice. So, temperature in case will be $T_1 = 0°C = 273.15K$

Number of moles $n = \frac{P_1 V_1}{RT_1}$ .... (1)

Number of moles $n = \frac{mass}{molar\,mass} = \frac{m}{M}$ .... (2)

Equating (1) and (2) we get

$$\frac{m}{M} = \frac{P_1 V_1}{RT_1}$$

$$m = \frac{MP_1 V_1}{RT_1} = \frac{10^5 \times 5 \times 10^{-5} \times 28.8}{8.31 \times 273.15} = 0.0635g$$

So, mass of air when temperature is 0°C is 0.0635g.

(b) Now in second case the container having air is kept in a bath having boiling water. So, temperature will be $T_2 = 100°C = 373.15K$.

Since, now temperature is 100°C therefore, some of the air will be expelled as air will expand but the volume of container is fixed. So, some of the air will go out of the container as container is open.

So, first we will calculate the mass of air expelled from container and then we will subtract it from the original volume V_1 to get the mass of remaining air.

Pressure will be same as before, as the air is still open to atmosphere. So $P_2 = P_1$.

Let the volume of expanded gas be V_2. Number of moles in volume V_2 be the same as before because no extra gas is added. It has just expanded.

$$nR = \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

As $P_2 = P_1$, therefore

$$V_2 = \frac{T_2 V_1}{T_1} = \frac{373.15 \times 5 \times 10^{-5}}{273.15} = 6.831 \times 10^{-5} m^3$$

Volume of gas expelled out the container

$V = V_2 - V_1 = 6.831 \times 10^{-5} - 5 \times 10^{-5} = 1.831 \times 10^{-5} m^3$

Number of moles of expelled gas

$$n' = \frac{m'}{M} = \frac{P_2 V}{RT_2}$$

$$m' = \frac{P_2VM}{RT_2} = \frac{10^5 \times 1.831 \times 10^{-5} \times 28.8}{8.31 \times 373.15} = 0.017g$$

So, the mass of gas remaining in the container

=m'-m=0.0635-0.017=0.0465g

So, the mass of gas when temperature is 100°C is 0.0456g.

(c) Now the container is kept in ice bath i.e. temperature 0°C and container is closed. So, now the pressure will change.

Number of moles left $= \frac{mass\ left}{molar\ mass} = \frac{0.0456}{28.8}$

Applying ideal gas equation

$$P = \frac{nRT}{V} = \frac{0.0456 \times 8.31 \times 273.15}{28.8 \times 5 \times 10^{-5}} 0.731 \times 10^5 kPa$$

∴Pressure of gas when lid is closed, and temperature is 0°C is 73.1kPa.

33. Question

A uniform tube closed at one end, contains a pellet of mercury 10 cm long. When the tube is kept vertically with the closed-end upward, the length of the air column trapped is 20 cm. Find the length of the air column trapped when the tube is inverted so that the closed-end goes down. Atmospheric pressure = 75 cm of mercury.

Answer

Let the curved surface area of tube be A.

Volume =area$\times$height

Given

Initial length of trapped air=20cm =0.2m

Length of mercury column=10cm=0.1m

So, Mercury column pressure =0.1ρg Pa

Initial volume of air trapped V_1=0.2$\times$A

Atmospheric pressure =75cm of Hg=0.75m of Hg

1mm of Hg = hρg Pa

Where h= height of mercury column =1mm=0.001m

ρ= density of mercury

g= acceleration dur to gravity

So, atmospheric pressure= 0.75m of Hg= 0.75 ρg Pa

Let the pressure of the trapped air when closed end of the tube is upward be P_1. Now, pressure of the mercury and trapped air will then be equal to atmospheric pressure.

$P_1+0.1\rho g=0.75\rho g$

$P_1=0.65\rho g$

When the tube is inverted such that closed end is downward then pressure of trapped air will be P_2.

P_2=atmospheric pressure + mercury column pressure

$P_2=0.75\rho g+0.1\rho g=0.85\rho g$

Let length of air column at P_2=x

Volume of air trapped will be $V_2=A\times x$

Now temperature in both cases will remain same, as no heat is being either added or abstracted from tube.

Applying boyle's law,

$P_1V_1=P_2V_2$

$0.65\rho g\times 0.2A=0.85\rho g\times Ax$

$$x = \frac{0.65 \times 0.2}{0.85} = 0.15m$$

$\therefore$The length of the air column trapped when the tube is inverted so that the closed-end goes down is 0.15m=15cm.

34. Question

A glass tube, sealed at both ends, is 100 cm long. It lies horizontally with the middle 10 cm containing mercury. The two ends of the tube contain air at 27°C and at a pressure 76 cm of mercury. The air column on one side is maintained at 0°C and the other side is maintained at 127°C. Calculate the length of the air column of the cooler side. Neglect the changes in the volume of mercury and of the glass.

Answer

Let the curved surface area of tube A

Given

Length of mercury column=10cm=0.1m

Length of tube =100cm=1m

Pressure of mercury column P_1=76cm of Hg=0.76m of Hg

Temperature of mercury column T_1=27∘C=300.15K

Temperature of air at cooler side T_2=0∘C=273.15K

Temperature of air at hotter side T'_2=127∘C=400.15K

Let the length of air column at cooler side be x

The length of air column at hotter end be y

Volume of cooler air =Ax

Volume of hotter air =Ay

Volume of mercury column = V

Pressure of cooler air =P_2

Pressure of hotter air =P'_2

We know that ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant =8.3$JK^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

Applying ideal gas equation between cooler air and mercury column

$$\frac{P_1V}{T_1} = \frac{P_2V_2}{T_2}$$

$$P_2 = \frac{P_1VT_2}{T_1Ax}$$

Applying ideal gas equation between hotter air and mercury column

$$\frac{P_1V}{T_1} = \frac{P'_2V'_2}{T'_2}$$

$$P'_2 = \frac{P_1 V T'_2}{T_1 A y}$$

Under equilibrium condition the pressure P2 and P’2 will be same

$$\frac{P_1 V T_2}{T_1 A x} = \frac{P_1 V T'_2}{T_1 A y}$$

$$\frac{T_2}{x} = \frac{T'_2}{y} \ldots\ldots(i)$$

Now length of entire tube

x+y+0.1=1

y=0.9-x

Substituting the value of y in equation (i)

$$\frac{T_2}{x} = \frac{T'_2}{0.9 - x}$$

$$\frac{273.15}{x} = \frac{400.15}{0.9 - x}$$

(0.9-x)$\times$273.15=400.15$\times$x

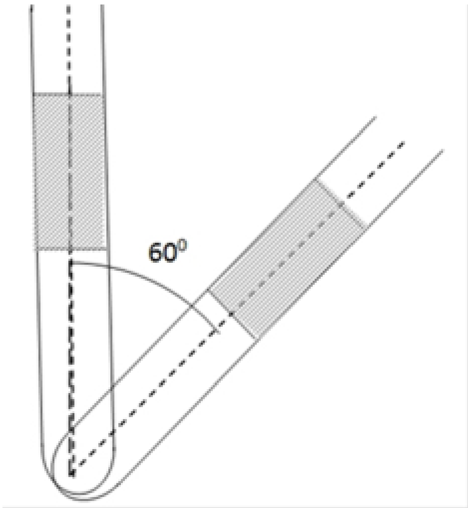
$$x = 0.365m$$

So, the length of air column on the cooler side is 0.365m=36.5cm.

35. Question

An ideal gas is trapped between a mercury column and the closed-end of a narrow vertical tube of uniform base containing the column. The upper end of the tube is open to the atmosphere. The atmospheric pressure equals 76 cm of mercury. The lengths of the mercury column and the trapped air column are 20 cm and 43 cm respectively. What will be the length of the air column when the tube is tilted slowly in a vertical plane through an angle of 60°? Assume the temperature to remains constant.

Answer



Let curved surface area of tube =A

Given

Length of air column=43cm=0.43m

Length of mercury column=20cm=0.20m

Pressure due to mercury column=P_H=0.2m of Hg

Atmospheric pressure=P_a=0.76m of Hg

Let the pressure of air column before titling =P_1

So $P_1=P_a+P_H$

P_1=0.76+0.2=0.96m of Hg

Volume =area$\times$height

Volume of trapped air V_1=A$\times$length of air column=0.43A

If the tube is titled through an angle 60^o only pressure of mercury column will get affected and not the atmospheric pressure.

So, change in P_H will be

$P'_H=P_H\cos60^o = 0.2\times0.5$ =0.1m of Hg

So now the pressure of air column will become P_2

$P_2=P_a+P'_H$=0.76+0.1=0.86m of Hg

Then volume will change. Let it now be V_2=lA where l is new length of air column.

It is given in question that the temperature remains same. So, according to boyle's law which states that PV=constant when temperature is constant, we can write,

$P_1V_1=P_2V_2$

$$V_2 = \frac{P_1 V_1}{P_2}$$

$$Al = \frac{0.96 \times 0.43A}{0.86}$$

$$l = 0.48m$$

∴Length of the air column will become 0.48m=48cm

36. Question

Figure shows a cylindrical tube of length 30 cm which is partitioned by a tight-fitting separator. The separator is very weakly conducting and can freely slide along the tube. Ideal gases are filled in the two parts of the vessel. In the beginning, the temperatures in the parts A and B are 400 K and 100 K respectively. The separator slides to a momentary equilibrium position shown in the figure. Find the final equilibrium position of the separator, reached after a long time.

Answer

Let the initial pressure of the chamber A and B be P_a and P_b respectively.

Let the final pressure of the chamber A and B be P'_a and P'_b respectively.

Let the curved surface area of tube be A

Given:

Length of chamber A=20cm=0.2m

Length of chamber B=10cm=0.1m

Volume =area×height

Initial volume of chamber A=V_a=0.2A

Initial volume of chamber B=V_b=0.1A

Initial Temperature of chamber A=T_a=400K

Initial Temperature of chamber B=T_b=100K

For first (momentary) equilibrium, pressure of both chamber will be same.

$P_a = P_b$

Let the final temperature at equilibrium be T.

We know that ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant $=8.3JK^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

Then,

For chamber A, number of moles, before and after final equilibrium will be same as no new gas has been added. So, applying ideal gas equation, before and after final equilibrium and equation nR, we get,

$$\frac{P_a V_a}{T_a} = \frac{P'_a V'_a}{T}$$

$$P'_a = \frac{P_a \times 0.2A \times T}{400 \times V'_a} \dots\dots (i)$$

Similarly, for chamber B

$$\frac{P_b V_b}{T_b} = \frac{P'_b V'_b}{T}$$

$$P'_b = \frac{P_b \times 0.1A \times T}{100 \times V'_b} \dots\dots (ii)$$

At second equilibrium pressures on both sides will be same again.

$P'_a = P'_b$

$$\frac{P_a \times 0.2A \times T}{400 \times V'_a} = \frac{P_b \times 0.1A \times T}{100 \times V'_b}$$

$$\frac{P_a}{2 \times V'_a} = \frac{P_b}{V'_b}$$

Now $P_a=P_b$ so,

$V'_b = 2 \times V'_a$ ……(iii)

Volume of chamber A plus volume of chamber B will be equal to total volume. So,

$V'_b + V'_a = V = 0.3A$

$2V'_a + V'_a = 0.3A$ (from (iii))

$3V'_a = 0.3A$

$V'_a = 0.1A$

Now we know that volume=length$\times$area. So,

$V'_a = lA$

Where l=length of chamber A after equilibrium

$lA = 0.1A$

$l = 0.1m = 10cm$

$\therefore$ length of chamber A after equilibrium is 10 cm.

37. Question

A vessel of volume V_0 contains an ideal gas at pressure p_0 and temperature T. Gas is continuously pumped out of this vessel at a constant volume-rate dV/dt = r keeping the temperature constant. The pressure of the gas being taken out equals the pressure inside the vessel. Find (a) the pressure of the gas as a function of time, (b) the time taken before half the original gas is pumped out.

Answer

Let P be the pressure and n be the number of moles of gas inside the vessel at any given time.

As mentioned in question, pressure is decreasing continuously. So, suppose a small amount of gas 'dn' moles are pumped out and the decrease in pressure is 'dP'.

So, pressure of remaining gas =P-dP

Number of moles of remaining gas =n-dn

Given

The volume of gas $=V_o$

Temperature of gas=T

We know that ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant $=8.3JK^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

So, applying ideal gas equation for the remaining gas

$(P-dP)\ V_o=(n-dn)\ RT$

$P \times V_o - dP \times V_o = n \times RT - dn \times RT$ …… (1)

Applying ideal gas equation, before gas was taken out

$PV_o = nRT$ ……. (2)

Using equation (2) in (1) we get

$n \times RT - V_o \times dP = n \times RT - dn \times RT$

$V_o \times dP = dn \times RT$ …… (3)

According to question, pressure of gas being taken out is equal to inner pressure of gas always. So inner pressure is equal to P-dP

Let the volume of gas taken out dV.

Applying ideal gas equation to gas pumped out

$(P-dP)\ dV = dn \times RT$

$PdV = dn \times RT$ …… (4)

Where we have ignored dPdV as it is very small and can be neglected.

From equation (3) and (4)

$V_o \times dP = P \times dV$

$$\frac{dP}{P} = \frac{dV}{V_o} \;\ldots.(5)$$

Given $\frac{dV}{dt} = r \rightarrow dV = -rdt$. Since volume is decreasing so rate should be negative.

Putting this value of dV in equation (5)

$$\frac{dP}{P} = \frac{-rdt}{V_o} \;\ldots..(6)$$

(a) Integrating equation (6) from Po to P and t=0 to t

$$\int_{P_o}^{P} \frac{dP}{P} = -\frac{r}{V_o} \times \int_0^t dt$$

$$\ln(P) - \ln(P_o) = -\frac{rt}{V_o}$$

Where we have used the formula

$$\int \frac{1}{x} dx = \ln(x)$$

And

$$\int x^n dx = \frac{x^{n+1}}{n+1} \; here \; n = 0$$

$$\ln(P) - \ln(P_o) = -\frac{rt}{V_o}$$

$$ln\left(\frac{P}{P_o}\right) = -\frac{rt}{V_o}$$

Taking exponential on both sides,

$$P = P_o \times e^{\frac{-rt}{V_o}} \; \ldots \ldots (7)$$

∴The pressure of the gas as a function of time is given as $P = P_o \times e^{\frac{-rt}{V_o}}$

(b) In second part the final pressure becomes half of the initial pressure

$$P = \frac{P_o}{2}$$

Putting this value of P in equation (7)

$$\frac{P_o}{2} = P_o \times e^{\frac{-rt}{V_o}}$$

$$\frac{1}{2} = e^{\frac{-rt}{V_o}}$$

$$2 = e^{\frac{rt}{V_o}}$$

Taking natural logarithm on both side

$$ln2 = \frac{rt}{V_o}$$

$$t = \frac{V_o ln2}{r}$$

∴The time taken before half the original gas is pumped out is $t = \frac{V_o ln2}{r}$

38. Question

One mole of an ideal gas undergoes a process

$$p = \frac{p_0}{1 + (V/V_0)^2}$$

where p_0 and V_0 are constants. Find the temperature of the gas when $V = V_0$.

Answer

Given

$$P = \frac{P_o}{1 + \left(\frac{V}{V_o}\right)^2}$$

Multiplying both sides by V

$$PV = \frac{P_o V}{1 + \left(\frac{V}{V_o}\right)^2} \dots (1)$$

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant=8.31Jmol^{-1}K^{-1}

T=temperature

n=number of moles of gas

P=pressure of gas

Here, it is given that number of moles n=1

So, PV=RT. Putting this value of PV in equation 1

$$RT = \frac{P_o V}{1 + \left(\frac{V}{V_o}\right)^2}$$

$$T = \frac{1}{R}\left(\frac{P_o V}{1 + \left(\frac{V}{V_o}\right)^2}\right)$$

According to question $V=V_o$

$$T = \frac{1}{R}\left(\frac{P_o V}{1 + \left(\frac{V}{V}\right)^2}\right) = \frac{1}{R}\left(\frac{P_o V}{1 + 1}\right)$$

$$T = \frac{P_o V}{2R} = \frac{P_o V_o}{2R}$$

∴The temperature of the gas when $V = V_0$ is $\frac{P_O V_O}{2R}$.

39. Question

Show that the internal energy of the air (treated as an ideal gas) contained in a room remains constant as the temperature changes between day and night. Assume that the atmospheric pressure around remains constant and the air in the room maintains this pressure by communicating with the surrounding through the windows, doors, etc.

Answer

We know that internal energy at a temperature

$U=nC_vT$

Where U=internal energy

n=number of moles

C_v=molar specific heat at constant volume

T=temperature

Air in home is chiefly diatomic molecules, so

C_v for diatomic moles is given as

$$C_v = \frac{5}{2}R$$

So,

$$U = \frac{5}{2}nRT$$

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant=$8.31 Jmol^{-1}K^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas

Substituting the value of nRT from ideal gas equation to U

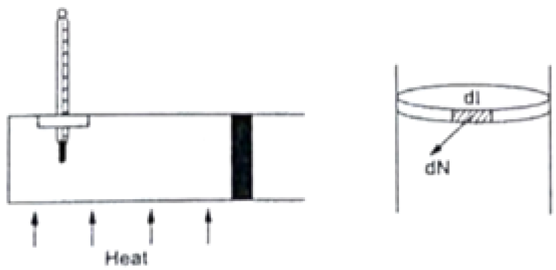
$$U = \frac{5}{2}PV$$

Now, it is given in question that pressure remains constant throughout the day and volume V of room is also constant. So, PV is a constant.

Hence U is also constant.

40. Question

Figure shows a cylindrical tube of radius 5 cm and length 20 cm. It is closed by a tight-fitting cork. The friction coefficient between the cork and the tube is 0.20. The tube contains an ideal gas at a pressure of 1 atm and a temperature of 300 K. The tube is slowly heated, and it is found that the cork pops out when the temperature reaches 600 K. Let dN denote the magnitude of the normal contact force exerted by a small length dl of the cork along the periphery (see the figure). Assuming that the temperature of the gas is uniform at any instant, calculate dN/dl.



Answer

Given

Pressure of gas P_1=1atm=10^5Pa

Radius of tube R=5cm=0.05m

So, area of tube = $\pi \times (0.05)^2$

Length of tube =20cm=0.2m

So, volume=area$\times$length

Volume of cylindrical tube = $\pi \times (0.05)^2 \times 0.2 = 0.0016 m^2$

Initial temperature T_1 =300K

Final temperature T_2=600K

Coefficient of friction μ=0.2

Let final pressure be P_2. So, volume of the gas remains same until pressure becomes P_2 and then corks pop out. Number of moles will also be same. So, we can apply ideal gas equation which is

PV=nRT

Where V= volume of gas

R=gas constant =$8.3JK^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

$$\frac{P_1 V}{T_1} = \frac{P_2 V}{T_2}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$P_2 = \frac{T_2 P_1}{T_1} = \frac{600}{300} \times 10^5 = 2 \times 10^5 Pa$$

Net pressure on cork = $P_2 - P_1 = 2 \times 10^5 - 10^5 = 10^5$Pa

We know that

$$pressure = \frac{force}{area}$$

So, force acting on the cork=pressure on cork $\times$ area of cork

$F = 10^5 \times \pi \times (0.05)^2$

According to law of friction,

$F = \mu \times N$

Where F=force of friction

N=normal to the surface of cork

μ=coffieicent of friction

Now, friction is always equal to the applied force until body starts to slide.

So,

$$N = \frac{F}{\mu} = \frac{10^5 \times \pi \times (0.05)^2}{0.2}$$

In question N is denoted as dN. So, N=dN

And dl=length of cork around periphery of cork i.e. dl=circumference of cork.

$$\frac{dN}{dl} = \frac{N}{2\pi R} = \frac{10^5 \times \pi \times (0.05)^2}{0.2 \times 3.14 \times 0.05} = 1.25 \times 10^5 N/m$$

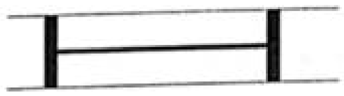
Thus, the value of $\frac{dN}{dl} = 1.25 \times 10^5 N/m$.

41. Question

Figure shows a cylindrical tube of cross-sectional area A fitted with two frictionless pistons. The pistons are connected to each other by a metallic wire. Initially, the temperature of the gas is T_0 and its pressure is p_0 which equals the atmospheric pressure.

(a) What is the tension in the wire?

(b) What will be the tension if the temperature is increased to $2T_0$?



Answer

(a) Initially, the pressure inside the cylinder is equal to atmospheric pressure as given in question. So, pressure (thus force) on piston will be same from outside the cylinder and inside the cylinder which is atmospheric pressure. Therefore, no net force will act on piston and tension in wire will be zero.

(b)

Given

Initial temperature $T_1=T_o$

Increased temperature $T_2=2T_o$

Initial pressure of gas $P_1=P_o=10^5$Pa

Let curved surface area of cylinder be A

Let the piston be L distance apart

Volume =area$\times$length=A$\times$L

Since, no new gas is added to cylinder so, number of moles, before and after temperature change will be same and the volume of cylinder is same. So, applying ideal gas equation, before and after the increase of temperature.

PV=nRT

Where V= volume of gas

R=gas constant =8.3$JK^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

$$\frac{P_1V}{T_1}=\frac{P_2V}{T_2}$$

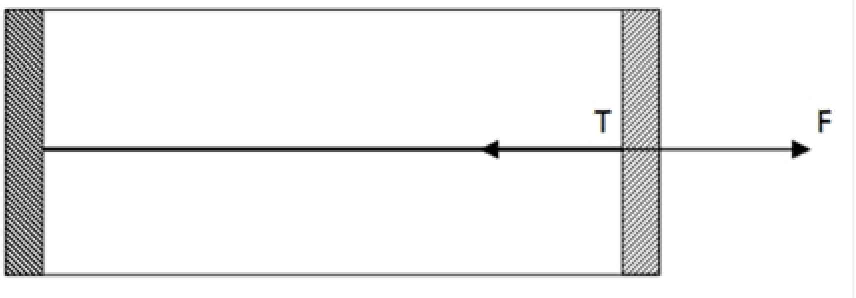
$$\frac{10^5}{T_o}=\frac{P_2}{2T_o}$$

$P_2=2\times10^5=2P_1=2P_o$

Net pressure P_2-P_1=$2P_o$-P_o=P_o

Net force acting outwards is force=pressure$\times$area

F=$P_o\times$A ......(i)



Since, temperature is increased, gas inside the cylinder will expand and piston will move outward freely without any acceleration, as the piston is frictionless. So, from diagram F>T

From newton second law of motion, which states that 'net external force on particle is equal to rate of change of momentum', we can write

F-T=0

Rate of change of momentum will be zero because piston moves without any acceleration. So, $m\times\frac{dv}{dt}=m\times a=0.$

Therefore,

F=T= $P_o\times$A (from (i))

∴The tension if the temperature is increased to $2T_0$ is P_oA.

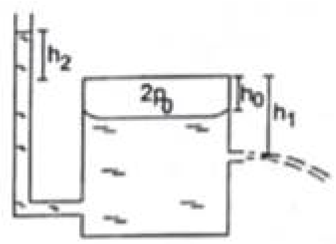
42. Question

Figure shows a large closed cylindrical tank containing water. Initially the air trapped above the water surface has a height h_0 and pressure $2p_0$ where p_0 is the atmospheric pressure. There is a hole in the wall of the tank at a depth h_1 below the top from which water comes out. A long vertical tube is connected as shown.

(a) Find the height h_2 of the water in the long tube above the top initially.

(b) Find the speed with which water comes out of the hole.

(c) Find the height of the water in the long tube above the top when the water stops coming out of the hole.



Answer

Given

Initial height of air trapped $=h_o$

Initial pressure$=2P_o$

P_o=atmospheric pressure = 10^5Pa

Depth at which there is hole in tank $=h_1$

Let the density of water be ρ.

From the diagram we can see that,

Pressure of water above the water level of the bigger tank is given by

$P=(h_2+h_o)\rho g$

Let the atmospheric pressure above the tube be P_o.

Total pressure above the tube$=P_o+P=(h_2+h_o)\rho g+P_o$

This pressure initially is balanced by pressure above the tank $2P_o$ (from diagram).

Therefore,

$2P_o=(h_2+h_o)\rho g+P_o$

$P_o=(h_2-h_0)\rho g$

$$\frac{P_o}{\rho g} - h_o = h_2$$

$\therefore$The height h_2 of the water in the long tube above the top initially is given by $\frac{P_o}{\rho g} - h_o$.

(b)

Efflux means water flowing out from tube or outlet.

Velocity of efflux out of the outlet depends upon the total pressure above the outlet.

Total pressure above the outlet=$2P_o+(h_1-h_o)\rho g$

Let the velocity of efflux be v_1 and the velocity with which the level of tank falls be v_2.

Pressure outside the outlet =P_o

Bernoulli's principle states that an increase in the speed of a fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy.

Mathematically Bernoulli's theorem can be written as

$$\frac{v^2}{2} + gz + \frac{P}{\rho} = constant$$

Where v=velocity of fluid at a point

z= elevation of that point from a reference level

P=pressure of fluid at that point

ρ=density of that fluid

G=acceleration due to gravity

Applying Bernoulli's theorem, at points outside the outlet and above the outlet, we get

$$\frac{2P_o + (h_1 - h_o)\rho g}{\rho} + gz + \frac{v_2^2}{2} = \frac{P_o}{\rho} + gz + \frac{v_1^2}{2}$$

Consider the difference in elevation of both the points very small, so that we can ignore 'gz' term on both the sides.

$$\frac{P_o + (h_1 - h_o)\rho g}{\rho} + \frac{v_2^2}{2} = \frac{v_1^2}{2}$$

Again, the speed with which the water level of the tank goes out is very less compared to the velocity of the efflux. Thus, $v_2=0$

$$\frac{P_o + (h_1 - h_o)\rho g}{\rho} = \frac{v_1^2}{2}$$

$$v_1^2 = \frac{2}{\rho}(P_o + (h_1 - h_o)\rho g)$$

$$v_1 = \left(\frac{2}{\rho}(P_o + (h_1 - h_o)\rho g)\right)^{\frac{1}{2}}$$

∴The speed with which water comes out of the hole is given by

$$\left(\frac{2}{\rho}(P_o + (h_1 - h_o)\rho g)\right)^{\frac{1}{2}}$$

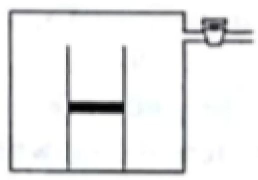
(c) Water maintains its own level, so height of the water of the tank will be h_1 when water will stop flowing out.

Thus, height of water in the tube below the tank height will be $=h_1$

Hence height if the water above the tank will be $=-h_1$

43. Question

An ideal gas is kept in a long cylindrical vessel fitted with a frictionless piston of cross-sectional area 10 cm^2 and weight 1 kg (figure). The vessel itself is kept in a big chamber containing air at atmospheric pressure 100 kPa. The length of the gas column is 20 cm. If the chamber is now completely evacuated by an exhaust pump, what will be the length of the gas column? Assume the temperature to remain constant throughout the process.



Answer

Given

Cross sectional area A $=10cm^2=10\times10^{-4}m^2$

mass of piston = 1kg

Pressure inside chamber = $100kPa=10^5Pa$

Pressure due to the weight of the piston $=P_p$

$$pressure = \frac{force}{area}$$

$$P_p = \frac{mg}{A} = \frac{1 \times 9.8}{10 \times 10^{-4}} = 9.8 \times 10^3 Pa$$

Pressure of vessel P_1=pressure of chamber+ pressure due to piston

$P1 = 10^5 + 9.8 \times 10^3$

Volume of gas inside the vessel V_1=length of gas column$\times$area.

Given length of gas column=20cm=0.2m

V_1=length $\times$A

$V_1 = 0.2 \times 10 \times 10^{-4} = 2 \times 10^{-4} m^3$

After evacuation, pressure of chamber will be zero. So, pressure inside vessel after evacuation P2 will just be pressure due to piston.

$P_2 = 0 + 9.8 \times 10^3$

Let L be the final length of the gas column

Final volume $V_2 = AL = 10 \times 10^{-4} \times L$

Since, it is given in question that temperature remains constant, we can apply boyle's law which states that 'PV=constant when temperature is constant'.

Applying boyle's law before and after evacuation,

$P_1V_1 = P_2V_2$

$(10^5 + 9.8 \times 10^3) \times 2 \times 10^{-4} = 9.8 \times 10^3 \times 10 \times 10^{-4} \times L$

$$L = \frac{(10^5 + 9.8 \times 10^3) \times 2 \times 10^{-4}}{9.8 \times 10^3 \times 10 \times 10^{-4}} = 2.2m$$

$\therefore$The length of the gas column after evacuation =2.2m

44. Question

An ideal gas is kept in a long cylindrical vessel fitted with a frictionless piston of cross-sectional area 10 cm^2 and weight 1 kg. The length of the gas column in the vessel is 20 cm. The atmospheric pressure is 100 kPa. The vessel is now taken into a spaceship revolving round the earth as a satellite. The air pressure in the spaceship is maintained at 100 kPa. Find the length of the gas column in the cylinder.

Answer

Given

Area of cross section A= $10cm^2 = 10 \times 10^{-4} m^2$

Mass of piston 'm' = 1kg

∴Weight of piston 'mg'=1×9.8N

Length of the gas column l=20cm=0.20m

Atmospheric pressure P_o=100kPa=10^5Pa

Air pressure in the spaceship =100kPa=P_o

Let the length of the gas column in the spaceship be l'.

Pressure on gas before taking to spaceship= P_1

P_1= pressure due to weight of piston + atmospheric pressure

$$pressure\ due\ to\ piston = \frac{weight}{area} = \frac{mg}{A}$$

$$\therefore P_1 = \frac{mg}{A} + P_o$$

Now,

Volume of the gas column before taking to spaceship V1=area$\times$length

$V_1 = A \times l$

Volume of the gas after taking to spaceship $V_2 = A \times l'$

The pressure of surroundings has been kept same as the atmospheric pressure. So, this means the temperature of surrounding, before and after taking to spaceship is same.

Therefore, we can apply boyle's law we state that 'PV=constant, when temperature is constant'.

We also know that in satellite or in spaceship the effect of gravity is negligible. So, pressure on gas due to weight of piston in spaceship will be zero.

So, pressure on the gas in spaceship P_2=air pressure of spaceship=P_o

Applying boyle's law before and after taking vessel to spaceship, we get

$P_1V_1 = P_2V_2$

$$\left(\frac{mg}{A} + P_o\right) \times A \times l = P_o \times A \times l'$$

$$\left(\frac{1 \times 9.8}{10 \times 10^{-4}} + 10^5\right) \times 0.20 = 10^5 \times l'$$

$(9.8 \times 10^3 + 10^5) \times 0.2 = 10^5 \times l'$

$109.8 \times 10^3 \times 0.2 = 10^5 \times l'$

$$l' = \frac{109.8 \times 0.2}{10^2} = 0.2196 \approx 0.22m = 22cm$$

∴The length of gas column in spaceship is 22cm.

45. Question

Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C at a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other is placed in a water bath maintained at 62°C. What is the new value of the pressure inside the bulbs? The volume of the connecting tube is negligible.

Answer

Given

Initial temperature of gas in both bulbs $T_1 = 0°C$

Initial pressure of gas in both bulbs $P_1 = P_2 = 76$cm of Hg=0.76m of Hg

Temperature of bulb placed in ice $T_2 = 0°C = 273.15K$

Temperature of other bulb $T'_2 = 62°C = 335.15K$

Let each of the bulbs have n_1 moles initially.

Now since the second bulb has kept at higher temperature gas in second bulb will expand and some of the gas will flow to first bulb and number of moles will change in second bulb.

Let the number of moles in second bulb after its pressure reached P be n_2.

Volume of both the bulbs is same so $V_1 = V_2 = V$.

Applying ideal gas equation in both bulbs

PV=nRT

Where V= volume of gas

R=gas constant $=8.3 JK^{-1} mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

and equating the value of constant R

$$\frac{P_1 V}{T_1 n_1} = \frac{PV}{T_2 n_2}$$

$$\frac{0.76}{273.15 n_1} = \frac{P}{335.15 n_2}$$

$$n_2 = \frac{P \times 273.15 \times n_1}{335.15 \times 0.76}$$

Number of moles left out of second bulb after temperature rose=n_1-n_2

$$= n_1 - \frac{P \times 273.15 \times n_1}{335.15 \times 0.76}$$

Let n_3 moles be left when pressure reached P in fist bulb. Applying ideal gas equation in first bulb, before and after temperature change.

$$\frac{P_1 V}{T_1 n_1} = \frac{PV}{T_1 n_3}$$

$$\frac{0.76}{n_1} = \frac{P}{n_3}$$

$$n_3 = \frac{P n_1}{0.76}$$

Also,

n_3=own moles of first bulb n_1+moles received from second bulb

n_3=n_1+n_1-n_2

Substituting the value of n_3 and n_2 in above equation

$$\frac{P n_1}{0.76} = n_1 + n_1 - \frac{P \times 273.15 \times n_1}{335.15 \times 0.76}$$

Taking n_1 common

$$\frac{P}{0.76} = 2 - \frac{P \times 273.15}{335.15 \times 0.76}$$

$$\frac{P}{0.76} + \frac{P \times 273.15}{335.15 \times 0.76} = 2$$

$$P = 2 \times \left(\frac{0.76 \times 335.15}{335.15 + 273.15}\right)$$

P=0.8375m of Hg

∴The new value of the pressure inside the bulbs is 83.75cm of Hg.

46. Question

The weather report reads, "Temperature 20°C: Relative humidity 100%". What is the dew point?

Answer

Dew point is 20°C.

Explanation

Formula for relative humidity is given as

$$relative\ humidity = \frac{actual\ vapor\ pressure}{saturation\ vapour\ pressure} \times 100\%$$

Where both actual vapor pressure and saturation vapor pressure are at same temperature.

Given

Temperature of air =20°C

Relative humidity=100%

So,

$$100\% = \frac{actual\ vapor\ pressure}{saturation\ vapour\ pressure} \times 100\%$$

$$\frac{actual\ vapor\ pressure}{saturation\ vapour\ pressure} = 1$$

∴ actual vapor pressure =saturation vapor pressure at the temperature 20°C

The temperature at which relative humidity is 100% i.e. air is completely saturated, is called dew point.

Here, the temperature at which the air is completely saturated is 20°C.

So, dew point is 20°C.

47. Question

The condition of air in a closed room is described as follows. Temperature 25°C, relative humidity = 60%, pressure = 104 kPa. If all the water vapor is removed from the room without changing the temperature, what will be the new pressure? The saturation vapor pressure at 25°C = 3.2 kPa.

Answer

Given

Temperature T = 25°C=298K

Relative humidity =60%

Initial pressure of room=104kPa=1.04$\times$ 10^5Pa

Saturation vapor pressure=3.2kPa=3.2$\times$ 10^3Pa

Formula for relative humidity is given as

$$relative\ humidity = \frac{actual\ vapor\ pressure}{saturation\ vapour\ pressure} \times 100\%$$

Where both actual vapor pressure and saturation vapor pressure are at same temperature.

So,

$$relative\ humidity = 60\% = \frac{actual\ vapor\ pressure}{saturation\ vapour\ pressure} \times 100\%$$

$$\frac{actual\ vapor\ pressure}{saturation\ vapour\ pressure} = \frac{6}{10} = 0.6$$

Therefore, actual vapor pressure of water vapor=0.6 $\times$saturation vapor pressure

Vapor pressure of water vapor=0.6$\times$3.2$\times$$10^3$=1.92$\times$$10^3$Pa

If all the water vapor is removed, then new pressure =pressure of room -pressure due to water vapours

=1.04$\times$$10^5-1.92\times$$10^5$

= 1.02$\times$$10^5$Pa

If all the water vapour is removed from the room without changing the temperature, the new pressure will be 102kPa.

48. Question

The temperature and the dew point in an open room are 20°C and 10°C. If the room temperature drops to 15 °C, what will be the new dew point?

Answer

Given

Temperature of open room = 20°C

Dew point =10°C

New temperature of room =15 °C

The temperature at which relative humidity is 100% i.e. air is completely saturated, is called dew point.

So, dew point will not change until the temperature of room is less than dew point.

In our question, room temperature drops to 15°C which greater than dew point.

So, the new dew point will be same as the old one.

49. Question

Pure water vapour is trapped in a vessel of volume 10 cm^3. The relative humidity is 40%. The vapour is compressed slowly and isothermally. Find the volume of the vapour at which it will start condensing.

Answer

Given

Relative humidity= 40%

Initial volume of vapor $V_1=10cm^3=10\times 10^{-6}m^3$.

$$relative\ humidity = \frac{actual\ vapor\ pressure}{saturation\ vapour\ pressure} \times 100\%$$

$$40\% = \frac{actual\ vapor\ pressure}{saturation\ vapour\ pressure} \times 100\%$$

$$\frac{actual\ vapor\ pressure}{saturation\ vapour\ pressure} = 0.4$$

Let saturation vapor pressure be P_o

So, initial pressure of vapours= $P_1=0.4P_o$

As mentioned in question, final stage comes when vapours start to condense.

When vapor starts to condense, relative humidity becomes 100% and vapor pressure attains the maximum pressure called saturation vapor pressure.

So final pressure of vapor $P_2=P_o$

Since the process is isothermal, temperature remains constant throughout. Applying Boyle's law which states that PV=constant, if temperature is constant.

$P_1V_1=P_2V_2$

$$V_2 = \frac{P_1V_1}{P_2} = \frac{0.4P_o \times 10 \times 10^{-6}}{P_o} = 4 \times 10^{-6}m^3$$

∴the volume of the vapour at which it will start condensing is $4 \times 10^{-6} m^3$

50. Question

A barometer tube is 80 cm long (above the mercury reservoir). It reads 76 cm on a day. A small amount of water is introduced in the tube and the reading drops to 75.4 cm. Find the relative humidity in the space above the mercury column if the saturation vapor pressure at the room temperature is 1.0 cm.

Answer

The reading of barometer gives the pressure in cm of Hg

Given

Reading of barometer on a day= atmospheric pressure of that day P= 76cm of Hg=0.76m of Hg

Drop in pressure when water is introduced P'=75.4cm of Hg

=0.754m of Hg

So, vapor pressure =P-P'=0.76-0.754=0.006m of Hg

Given saturation vapor pressure = 1cm of Hg=0.01m of Hg

So

$$relative\ humidity = \frac{actual\ vapor\ pressure}{saturation\ vapour\ pressure} \times 100\%$$

$$relative\ humidity = \frac{0.006}{0.01} \times 100\% = 60\%$$

∴the relative humidity in the space above the mercury column is 60%.

51. Question

Using figure of the text, find the boiling point of methyl alcohol at 1 atm (760 mm of mercury) and at 0.5 atm.

Answer

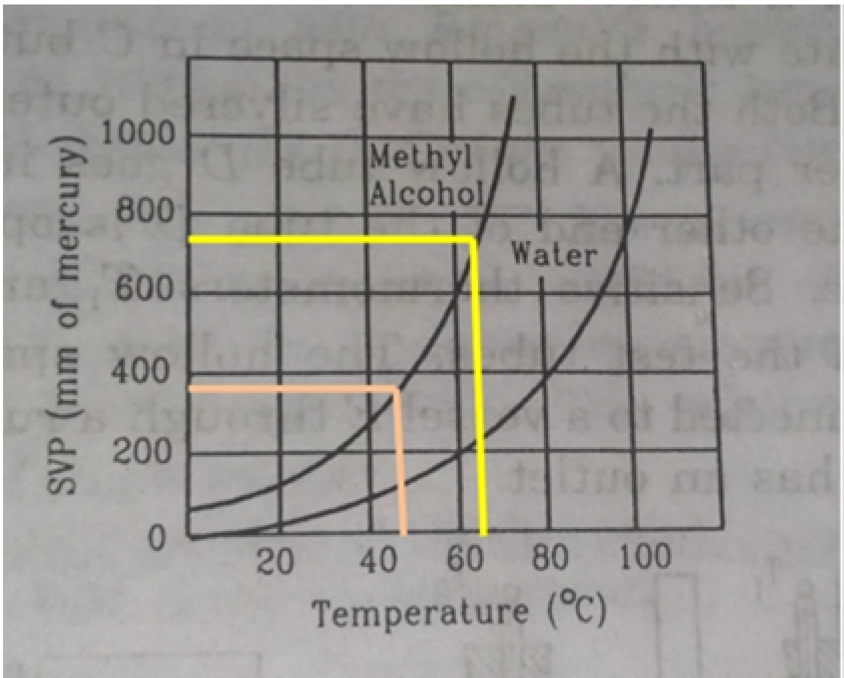


Figure 24.6

Boiling point of methyl alcohol at 1atm is 65°C and at 0.5 atm is 48°C.

Explanation

To find the boiling point from above figure, we must look for the temperature on x-axis corresponding to pressures given in question.

So, for first pressure i.e. 1atm=760mm of Hg, the corresponding temperature on x-axis is 65°C. This shown by yellow line in figure.

For the second pressure i.e. 0.5 atm

1atm=760mm of Hg

0.5atm =760$\times$0.5 mm of Hg= 375mm of Hg

For 375mm of Hg the corresponding temperature on x-axis is 48°C. This is shown by orange line in the figure.

52. Question

The human body has an average temperature of 98°F. Assume that the vapor pressure of the blood in the veins behaves like that of pure water. Find the minimum atmospheric pressure which is necessary to prevent the blood from boiling. Use figure of the text for the vapor pressures.

Answer

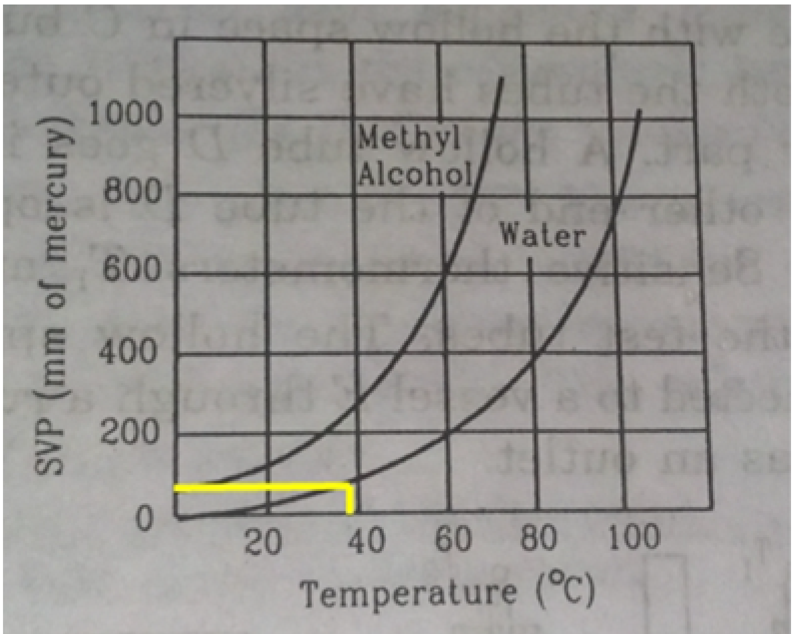


Figure 24.6

Minimum atmospheric pressure necessary to prevent blood from boiling is 50mm of Hg.

Explanation

Given

Average temperature of human body=98°F

T (°F)=98°F

T (°C)=$\frac{5}{9}(T(°F) - 32) = \frac{5}{9}(98 - 32) = \frac{5}{9} \times 66 = 36.7°C$

Now above 36.7°C, the human blood will start boiling. So, the minimum atmospheric pressure to prevent boiling of human blood will the pressure corresponding to this temperature.

So, from figure pressure corresponding to 36.7°C temperature on y-axis is 50mm of Hg which shown by yellow line in the figure.

53. Question

A glass contains some water at room temperature 20°C. Refrigerated water is added to it slowly. When the temperature of the glass reaches 10°C, small droplets condense on the outer surface. Calculate the relative humidity in the room. The boiling pointing of water at a pressure of 17.5 mm of mercury is 20°C and at 8.9 mm of mercury it is 10°C.

Answer

Given

Temperature of water=20°C

Droplets starts to form at 10°C.

Therefore, dew point is =10°C. This is because at dew point vapor pressure becomes saturated vapor pressure and after that air cannot hold more moisture and will start to condense.

At boiling point, saturation vapor pressure becomes equals to atmospheric pressure.

So, given that,

At temperature 20°C pressure is 17.5mm of Hg.

SVP (saturation vapor pressure) at 20°C = 17.5mm of Hg

SVP at dew point i.e. 10°C =8.9mm of Hg

Now relation between relative humidity and dew point is

$$relative\ humidity = \frac{SVP\ at\ dew\ point}{SVP\ at\ air\ temperature} \times 100\%$$

$$relative\ humidity = \frac{8.9}{17.5} \times 100\% = 51\%$$

∴ Relative humidity in the room is 51%.

54. Question

50 m^3 of saturated vapor is cooled down from 30°C to 20°C. Find the mass of the water condensed. The absolute humidity of saturated water vapor is 30 g m^{-3} at 30°C and 16 g m^{-3} at 20°C.

Answer

Given

Initial temperature =30°C

Final temperature =20°C

Absolute humidity of saturated water vapor at 30°C= 30 g m^{-3}

This means 1m^3 of air contains 30g of water vapor at 30°C.

So, amount of water vapor in 50m^3 of air at 30°C=50×30=1500g

Absolute humidity of saturated water vapor at 20°C= 16 g m^{-3}

So, amount of water vapor in 50m^3 of air at 20°C=50×16=800g

Amount water vapor condensed from 30 to 20°C = 1500-800

= 700g

∴ The mass of the water condensed is 700g.

55. Question

A barometer correctly reads the atmospheric pressure as 76 cm of mercury. Water droplets are slowly introduced into the barometer tube by a dropper. The height of the mercury column first decreases and then becomes constant. If the saturation vapor pressure at the atmospheric temperature is 0.80 cm of mercury, find the height of the mercury column when it reaches its minimum value.

Answer

Given:

Atmospheric pressure=76cm of Hg

Saturation vapor pressure=0.80 cm of Hg

When the water is introduced in barometer, water evaporates.

Thus, it exerts its vapor pressure over the mercury meniscus.

As more and more water evaporates, the vapor pressure increases that forces down the mercury level further down.

Finally, when the volume is saturated with the vapor at atmospheric temperature, the highest vapor pressure, i.e. saturation vapor pressure is observed, and the fall of mercury level reaches its minimum.

Thus,

Net pressure acting on the column = atmospheric pressure -saturation vapor pressure

= 76-0.80 =75.2 cm of Hg.

∴ The height of the mercury column when it reaches its minimum value is 75.2cm.

56. Question

50 cc of oxygen is collected in an inverted gas jar over water. The atmospheric pressure is 99.4 kPa and the room temperature is 27°C. The water level in the jar is same as the level outside. The saturation vapor pressure at 27°C is 3.4 kPa. Calculate the number of moles of oxygen collected in the jar.

Answer

Given

Volume of oxygen = 50cc=50cm^3=50$\times 10^{-6} m^3$

Atmospheric pressure P_o=99.4kPa=99.4$\times 10^3$Pa

Temperature =27°C=300.15K

Saturation vapor pressure P_s=3.4kPa=3.4$\times 10^3$Pa

According to question water level in the jar is same as level outside. So

Pressure inside the jar = pressure outside the jar

Pressure outside the jar =atmospheric pressure P_o

Pressure inside the jar=P_o .... (1)

But

Pressure inside the jar is also = vapor pressure of oxygen+ Saturation vapor pressure

Pressure inside the jar=P+P_s .... (2)

From equation (1) and (2), we can write

P_o=P+P_s

P=P_o-P_s=99.4$\times 10^3$-3.4$\times 10^3$=96$\times 10^3$Pa

Applying ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant =8.3J$K^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

$$n = \frac{PV}{RT}$$

$$n = \frac{96 \times 10^3 \times 50 \times 10^{-6}}{8.31 \times 300.15} = 1.93 \times 10^{-3}$$

∴The number of moles of oxygen collected in the jar is 1.93$\times 10^{-3}$.

57. Question

A faulty barometer contains certain amount of air and saturated water vapor. It reads 74.0 cm when the atmospheric pressure is 76.0 cm of mercury and reads

72.10 cm when the atmospheric pressure is 74.0 cm of mercury. Saturation vapor pressure at the air temperature = 1.0 cm of mercury. Find the length of the barometer tube above the mercury level in the reservoir.

Answer

Let the length of barometer be x cm.

Let the curved surface area of barometer tube be A

Given

Saturation vapor pressure (SVP) = 1.0 cm of mercury

In first case length of mercury column is 74cm.

Length of air above mercury = x-74

Volume of air column $V_1 = (x-74) \times A$

In first case atmospheric pressure is 76 cm of Hg.

Let pressure of air column be P_1 in first case.

Then,

Atmospheric pressure= SVP+ P_1+mercury column height

76=1+P+74

$P_1=1$ .... (1)

In second case,

Length of mercury column is 72.10cm.

Length of air above mercury = x-72.10

Volume of air column $V_2 = (x-72.1) \times A$

Atmospheric pressure is 74 cm of Hg.

Let pressure of air column be P_2.

Atmospheric pressure= SVP+ P_2 +mercury column height

$74=1+P_2+72.1$

$74=P_2+73.1$

$P_2=74-73.1=0.9$

$P_2=0.9$ .... (2)

Since temperature has not changed, we can apply Boyle's law which states that PV=constant, if temperature is constant, for both the cases

$P_1V_1=P_2V_2$

$1\times(x-74)\times A=0.9\times(x-72.1)\times A$

$(x-74)\times A=0.9\times(x-72.1)\times A$

$0.1x=9.11$

$x=91.1cm$

Therefore, length of barometer tube is 91.1cm.

58. Question

On a winter day, the outside temperature is 0°C and relative humidity 40%. The air from outside comes into a room and is heated to 20°C. What is the relative humidity in the room? The saturation vapor pressure at 0°C is 4.6 mm of mercury and at 20°C it is 18 mm of mercury.

Answer

Given:

Temperature of air outside the room=$0^\circ C$=273.15K

Temperature of air inside the room=$20^\circ C$=293.15K

Relative humidity at $0^\circ C$ =40%

$$relative\ humidity=\frac{vapor\ pressure}{saturation\ vapour\ pressure}\times 100\%$$

Let vapor pressure of air P_1.

$$40\%=\frac{P_1}{saturation\ vapour\ pressure}\times 100\%$$

$$\frac{P_1}{SVP}=0.4$$

SVP at $0^\circ C$=4.6mm of Hg

$P_1=0.4\times4.6=1.84$mm of Hg

Volume of room is not changed so, applying ideal gas equation at 0 and $20^\circ C$.

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant =8.3JK^{-1}mol^{-1}

T=temperature

n=number of moles of gas

P=pressure of gas.

We get,

$$\frac{P_1V}{T_1} = \frac{P_2V}{T_2}$$

$$P_2 = \frac{1.84 \times 293.15}{273.15} = 1.97mm\ of\ Hg$$

Relative humidity inside home i.e. at temperature 20°C

SVP at 20°C=18mm of Hg

$$relative\ humidity = \frac{P_2}{saturation\ vapour\ pressure} \times 100\%$$

$$relative\ humidity = \frac{1.97}{18} \times 100\% = 10.9\%$$

∴ The relative humidity in the room is 10.9%.

59. Question

The temperature and humidity of air are 27°C and 50% on a day. Calculate the amount of vapor that should be added to 1 cubic meter of air to saturate it. The saturation vapor pressure at 27°C = 3600 Pa.

Answer

Given

Temperature =27°C=300K

Relative humidity=50%

Volume =1m^3

saturation vapor pressure (SVP) at 27°C = 3600 Pa

$$relative\ humidity = \frac{vapor\ pressure}{saturation\ vapour\ pressure} \times 100\%$$

$$50\% = \frac{vapor\ pressure}{3600} \times 100\%$$

Vapor pressure P=0.5×3600=1800Pa

Molar mass of water H_2O M=16+2=18g

Let m_1 be the mass of water present in 50% humid air.

Applying ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant =8.3J$K^{-1}$$mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

Number of moles n= $\frac{given\ mass}{molar\ mass} = \frac{m}{M}$

$$PV = \frac{m_1}{M} RT$$

$$1800 \times 1 = \frac{m_1}{18} \times 8.31 \times 300$$

m_1=13g

Required pressure for saturation =3600Pa

Let m_2 be the amount of water required for saturation.

Again, applying ideal gas equation

$$3600 \times 1 = \frac{m_2}{M} RT = \frac{m_2}{18} \times 8.31 \times 300$$

$$m_2 = \frac{3600 \times 18}{8.31 \times 300} = 26g$$

∴Total excess water vapor that must be added=m_2-m_1=26-13=13g.

60. Question

The temperature and relative humidity in a room are 300 K and 20% respectively. The volume of the room is 50 m^3. The saturation vapor pressure at 300 K is 3.3 kPa. Calculate the mass of the water vapor present in the room.

Answer

Given

Temperature T=300K

Relative humidity=20%

Volume of room= $50m^3$

Saturation vapor pressure at 300 K is 3.3 kPa=3.3×10^3Pa.

$$relative\ humidity = \frac{vapor\ pressure}{saturation\ vapour\ pressure} \times 100\%$$

$$20\% = \frac{P}{3.3 \times 10^3} \times 100\%$$

$P=0.2\times 3.3\times 10^3=660Pa$

Molar mass of water H_2O =2+16=18g

Applying ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant =$8.3JK^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

Number of moles n=$\frac{given\ mass}{molar\ mass} = \frac{m}{M}$

$$PV = \frac{m}{M}RT$$

$$660 \times 50 = \frac{m}{18} \times 8.31 \times 300$$

$$m = \frac{660 \times 50 \times 18}{8.31 \times 300}$$

m=238.55g

∴ Mass of water vapor present in the room=238.55g.

61. Question

The temperature and the relative humidity are 300K and 20% in a room of volume 50 cm^3. The floor is washed with water, 500 g of water sticking on the floor. Assuming no communication with the surrounding, find the relative humidity when the floor dries. The changes in temperature and pressure may be neglected. Saturation vapor pressure at 300 K = 3.3 kPa.

Answer

Given

Temperature T=300K

Relative humidity=20%

Volume of room= $50m^3$

Mass of water=500g

Molar mass of H_2O M=2+16=18g

Saturation vapor pressure(SVP) at 300 K = 3.3 kPa=3300Pa

Let vapor pressure inside the room be P_1

$$relative\ humidity = \frac{vapor\ pressure}{saturation\ vapour\ pressure} \times 100\%$$

$$20\% = \frac{P_1}{3.3 \times 10^3} \times 100\%$$

$P_1 = 0.2 \times 3.3 \times 10^3 = 660Pa$

Since the floor has dried that means water on the floor has been evaporated.

Let P_2 be the partial pressure of evaporated water

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant =$8.3JK^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

And Number of moles n=$\frac{given\ mass}{molar\ mass} = \frac{m}{M}$

$$P_2V = \frac{m}{M}RT$$

$$P_2 = \frac{500 \times 8.31 \times 300}{18 \times 50}$$

P_2=1385Pa

Total pressure of room= partial pressure of evaporated water+ pressure of air inside the room

$P=P_2+P_1$

P=1385+660=2045Pa

$$relative\ humidity = \frac{2045}{660} \times 100\% = 61.9\%$$

∴ The relative humidity when the floor dries is 61.9%.

62. Question

A bucket full of water is paced in a room at 15°C with initial relative humidity 40%. The volume of the room is 50 cm^3.

(a) How much water will evaporate?

(b) If the room temperature is increased by 5°C, how much more water will evaporate? The saturation vapor pressure of water at 15°C and 20°C are 1.6 kPa and 2.4 kPa respectively.

Answer

Given

Temperature =15°C

Relative humidity=40%

Volume =$50cm^3$

Molar mass of H_2O M=2+16=18g.

We know ideal gas equation

PV=nRT

Where V= volume of gas

R=gas constant =$8.3JK^{-1}mol^{-1}$

T=temperature

n=number of moles of gas

P=pressure of gas.

The saturation vapor pressure (SVP)of water at 15°C=1.6kPa=1.6$\times 10^3$Pa.

$$relative\ humidity = \frac{vapor\ pressure}{saturation\ vapour\ pressure} \times 100\%$$

$$40\% = \frac{P}{1.6 \times 10^3} \times 100\%$$

P=0.4$\times$1.6$\times 10^3$Pa

Evaporation will occur if the atmosphere is not saturated.

Net pressure change= SVP-P

P' =1.6$\times$103-0.4$\times$1.6$\times 10^3$=0.96$\times 10^3$Pa

Number of moles n=$\frac{given\ mass}{molar\ mass} = \frac{m}{M}$

Applying equation of ideal gas.

$$P'V = \frac{m}{M}RT$$

$$m = \frac{P'VM}{RT}$$

$$m = \frac{0.96 \times 10^3 \times 50 \times 18}{8.31 \times 288.15} = 361g$$

∴The amount of water that will evaporate=361g

(b)

SVP at 20°C=2.4kPa

SVP at 15°C=1.6kPa

Net pressure changes in increasing the temperature,

Net pressure changes P''=2.4-1.6=0.8kPa=0.8$\times 10^3$Pa.

Applying equation of ideal gas.

$$P''V = \frac{m'}{M}RT$$

$$m' = \frac{P''VM}{RT}$$

$$m' = \frac{0.8 \times 10^3 \times 50 \times 18}{8.31 \times 293.15} = 296g$$

So, if the room temperature is increased by 5°C, the amount of water will evaporate is 296g.