Chapter 1
Numbers

Ex 1.1

Question 1.
Fill in the blanks:
(i) $\frac{-19}{5}$ lies between the integers _____ and _____ .
Answer:
-4 and -3

(ii) The decimal form of the rational number $\frac{15}{-4}$ is _____ .
Answer:
-3.75

(iii) The rational numbers $\frac{3}{-8}$ and $\frac{8}{3}$ are equidistant from _____.
Answer:
0

(iv) The next rational number in the sequence $\frac{-15}{24}, \frac{20}{-32}, \frac{-25}{40}$ is _____.
Answer:
$\frac{30}{-48}$

(v) The standard form of $\frac{58}{-78}$ is _____.
Answer:
$\frac{-29}{39}$

Question 2.
Say True or False
(i) 0 is the smallest rational number.
Answer:
False

(ii) $\frac{-4}{5}$ lies to the left of $\frac{-3}{4}$.
Answer:
True

(iii) $\frac{-19}{5}$ is greater than $\frac{15}{-4}$.

Answer:

False

(iv) The average of two rational numbers lies between them.

Answer:

True

(v) There are an unlimited number of rational numbers between 10 and 11.

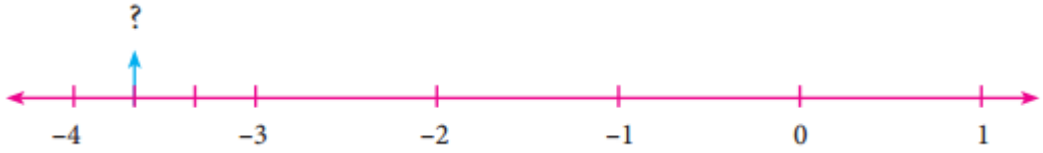
Answer:

True

Question 3.

Find the rational numbers represented by each of the question marks marked on the following number lines.

(i)

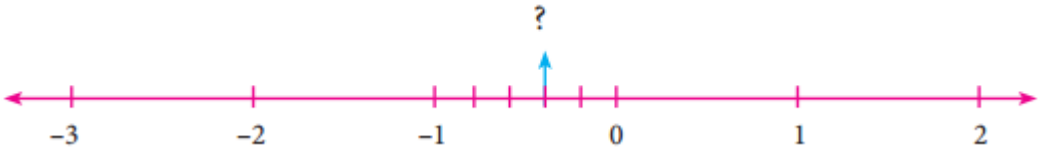


Answer:

The number lies between —3 and 4. The unit part between -3 and -4 is divided into 3 equal parts and the second part is asked.

∴ The required number is $-3\frac{2}{3}=\frac{-11}{3}$

(ii)



Answer:

The required number lies between 0 and -1. The unit part between 0 and -1 is divided into 5 equal parts, and the second part is taken.

∴ The required number is $\frac{-2}{5}$

(iii)

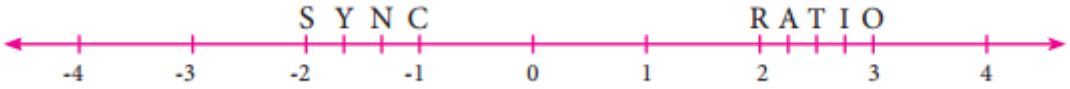
Answer:

The required number lies between 1 and 2. The unit part between 1 and 2 is divided into 4 equal parts and the third part is taken.

∴ The required number is $1\frac{3}{4}=\frac{7}{4}$

Question 4.

The points S, Y, N, C, R, A, T, I and O on the number line are such that CN=NY=YS and RA=AT=TI=IO. Find the rational numbers represented by the letters Y, N, A, T and I.



Answer:

$$Y = -2 + \frac{1}{3} = \frac{-6+1}{3} = \frac{-5}{3}$$

$$N = \frac{-5}{3} + \frac{1}{3} = \frac{-5+1}{3} = \frac{-4}{3}$$

$$RA = AT = TI = IO = \frac{1}{4}$$

$$A = 2 + \frac{1}{4} = \frac{8+1}{4} = \frac{9}{4}$$

$$T = \frac{9}{4} + \frac{1}{4} = \frac{9+1}{4} = \frac{10}{4}$$

$$I = \frac{10}{4} + \frac{1}{4} = \frac{10+1}{4} = \frac{11}{4}$$

Question 5.

Draw a number line and represent the following rational numbers on it.

(i) $\frac{9}{4}$

(ii) $\frac{-8}{3}$

(iii) $\frac{-17}{-5}$

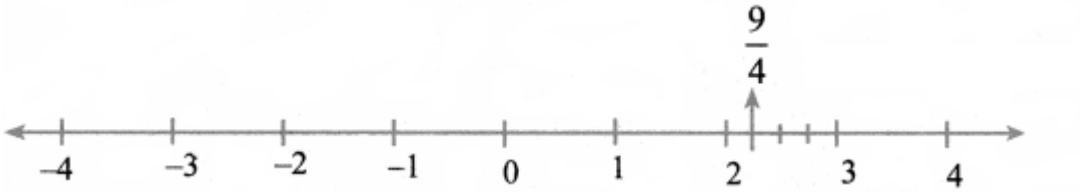
(iv) $\frac{15}{-4}$

Answer:

(i) $\frac{9}{4}$

$\frac{9}{4} = 2\frac{1}{4}$

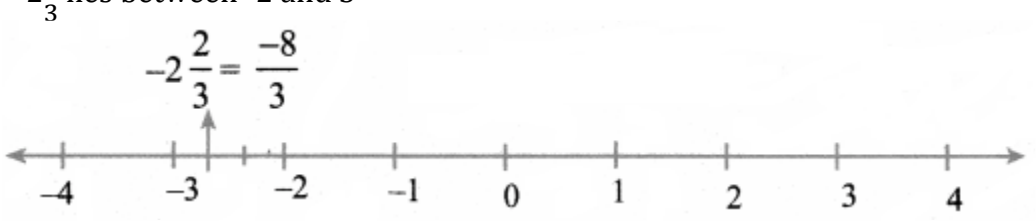
$\therefore \frac{9}{4}$ lies between 2 and 3



(ii) $\frac{-8}{3}$

$\frac{-8}{3}=-2\frac{2}{3}$

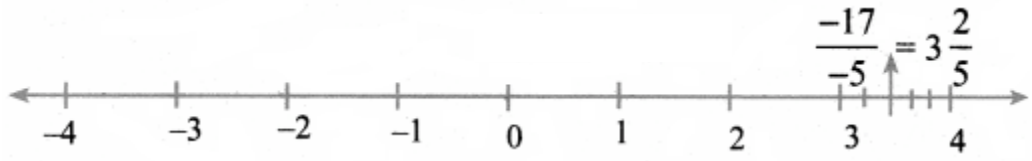
$-2\frac{2}{3}$ lies between -2 and 3



(iii) $\frac{-17}{-5}$

$\frac{-17}{-5}=3\frac{2}{5}$

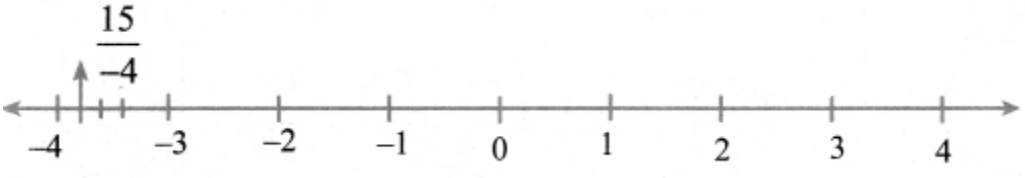
$3\frac{2}{5}$ lies between 3 and 4 in the number line.



(iv) $\frac{15}{-4}$

$\frac{15}{-4}=-3\frac{3}{4}$

$-3\frac{3}{4}$ lies between -3 and -4



Question 6.

Write the decimal form of the following rational numbers.

(i) $\frac{1}{11}$

(ii) $\frac{13}{4}$

(iii) $\frac{-18}{7}$

(iv) $1\frac{2}{5}$

(v) $-3\frac{1}{2}$

Answer:

(i) $\frac{1}{11}$

$\frac{1}{11}$ = 0.0909....

$$
\begin{array}{r}
0.0909 \\
11\overline{)\ 100} \\
\underline{\ 99} \\
100 \\
\underline{\ 99} \\
1
\end{array}
$$

(ii) $\frac{13}{4}$

$\frac{13}{4} = 3.25$

$$
\begin{array}{r}
3.25 \\
4\overline{)\ 13} \\
\underline{\ 12} \\
10 \\
\underline{\ 8} \\
20 \\
\underline{\ 20} \\
0
\end{array}
$$

(iii) $\frac{-18}{7}$

$\frac{-18}{7}$ = -2.571428571428....

$$
\begin{array}{r}
2.571428 \\
7\overline{)\ 18} \\
\underline{\ 14} \\
40 \\
\underline{\ 35} \\
50 \\
\underline{\ 49} \\
10 \\
\underline{\ 7} \\
30 \\
\underline{\ 28} \\
20 \\
\underline{\ 14} \\
60 \\
\underline{\ 56} \\
4
\end{array}
$$

(iv) $1\frac{2}{5}$

$1\frac{2}{5}=\frac{7}{5}=1.4$

$$\begin{array}{r} 1.4 \\ 5\overline{)\ 7} \\ \underline{5} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

(v) $-3\frac{1}{2}$

$-3\frac{1}{2}=\frac{-7}{2}=-3.5$

$$\begin{array}{r} 3.5 \\ 2\overline{)\ 7} \\ \underline{6} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

Question 7.

List any five rational numbers between the given rational numbers.

(i) 2 and 0

(ii) $\frac{-1}{2}$ and $\frac{3}{5}$

(iii) $\frac{1}{4}$ and $\frac{7}{20}$

(iv) $\frac{-6}{4}$ and $\frac{-23}{10}$

Answer:

(i) 2 and 0

i.e., $\frac{-2}{1}$ and $\frac{0}{1}$

$$\frac{-2}{1} = \frac{-2\times10}{1\times10} = \frac{-20}{10}$$

$$\frac{0}{1} = \frac{0\times10}{1\times10} = \frac{0}{10}$$

∴ Five rational number between −2010 (= -2) and 010 (= 0) are

$$\frac{-20}{10}, \frac{-19}{10}, \frac{-18}{10}, \frac{-7}{10}, \frac{-6}{10}, \frac{-5}{10}, \frac{0}{10} (= 0).$$

(ii) $\frac{-1}{2}$ and $\frac{3}{5}$

LCM of 2 and $5 = 2 \times 5 = 10$

$$-\frac{1}{2} = \frac{-1 \times 5}{2 \times 5} = \frac{-5}{10}$$

$$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$

∴ Five rational number between

(iii) $\frac{1}{4}$ and $\frac{7}{20}$

$$\frac{-1}{2}\left(= \frac{-5}{10}\right) \text{and } \frac{3}{5}\left(= \frac{6}{10}\right) \text{are } \frac{-3}{10}, \frac{-1}{10}, 0, \frac{1}{10}, \frac{2}{10}, \frac{5}{10}$$

∴ Five rational number between

$$\frac{1}{4}\left(= \frac{15}{60}\right) \text{ and } \frac{7}{20}\left(= \frac{21}{60}\right) \text{are}$$

$$\frac{16}{60}, \frac{17}{60}, \frac{18}{60}, \frac{19}{60}, \frac{20}{60}$$

(iv) $\frac{-6}{4}$ and $\frac{-23}{10}$

$$\frac{-6}{4} = \frac{-6 \times 5}{4 \times 5} = \frac{-30}{20}$$

$$\frac{-23}{10} = \frac{23 \times 2}{10 \times 2} = \frac{-46}{20}$$

∴ Five rational number between

$$\frac{-6}{4}\left(= \frac{-30}{20}\right) \text{ and } \frac{-23}{10}\left(= \frac{-46}{20}\right) \text{ are}$$

$$\frac{-31}{20}, \frac{-32}{20}, \frac{-33}{20}, \frac{-34}{20}, \frac{-35}{20}.$$

Question 8.

Use the method of averages to write 2 rational numbers between $\frac{14}{5}$ and $\frac{16}{3}$

Answer:

The average of a and b is $\frac{1}{2}(a+b)$

The average of $\frac{14}{5}$ and $\frac{16}{3}$ is $C_1 = \frac{1}{2}\left(\frac{14}{5}+\frac{16}{3}\right)$

$$C_1 = \frac{1}{2}\left(\frac{42+80}{15}\right)$$

$$C_1 = \frac{122}{30}$$

$$C_1 = \frac{61}{15}$$

$$\frac{14}{5} < \frac{61}{15} < \frac{16}{3} \quad \text{.....(1)}$$

The average of $\frac{14}{5}$ and $\frac{61}{15}$ is $C_2 = \frac{1}{2}\left(\frac{14}{5}+\frac{61}{15}\right)$

$$C_2 = \frac{1}{2} \times \left(\frac{42+61}{15}\right)$$

$$C_2 = \frac{1}{2} \times \frac{103}{15} = \frac{103}{30}$$

$$\therefore \frac{14}{5} < \frac{103}{30} < \frac{61}{15} \quad \text{....(2)}$$

From (1), (2) we get, $\frac{14}{5} < \frac{103}{30} < \frac{61}{15} < \frac{16}{3}$

Question 9.

Compare the following pairs of rational numbers.

(i) $\frac{-11}{5}, \frac{-21}{8}$

(ii) $\frac{3}{-4}, \frac{-1}{2}$

(iii) $\frac{2}{3}, \frac{4}{5}$

Answer:

(i) $\frac{-11}{5}, \frac{-21}{8}$

LCM of 5, 8 is 40

$$\frac{-11}{5} = \frac{-11\times8}{5\times8} = \frac{-88}{40}$$
$$\frac{-21}{8} = \frac{-21\times5}{8\times5} = \frac{-105}{40}$$
$$\frac{-105}{40} < \frac{-88}{40}$$
$$\therefore \quad \frac{-21}{8} < \frac{-11}{5}$$

(ii) $\frac{3}{-4}, \frac{-1}{2}$

LCM of 4 and 2 = 4

$$\frac{3}{-4} = \frac{-3}{4}$$
$$\frac{-1}{2} = \frac{-1\times2}{2\times2} = \frac{-2}{4}$$
$$\frac{3}{-4} < \frac{-2}{4}$$
$$-\frac{3}{4} < \frac{-1}{2}$$

(iii) $\frac{2}{3}, \frac{4}{5}$

LCM of 3 and 5 is 15.

$$\frac{2}{3} = \frac{2\times5}{3\times5} = \frac{10}{15}$$
$$\frac{4}{5} = \frac{4\times3}{5\times3} = \frac{12}{15}$$
$$\frac{10}{15} < \frac{12}{15}$$
$$\therefore \quad \frac{2}{3} < \frac{4}{5}$$

Question 10.

Arrange the following rational numbers in ascending and descending order.

(i) $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{-7}{-9}, \frac{12}{36}$

(ii) $\frac{-17}{10}, \frac{-7}{5}, 0, \frac{-2}{4}, \frac{-19}{20}$

Answer:

(i) $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{-7}{-9}, \frac{12}{36}$

LCM of 12, 8, 24, 9, 36 is 4 × 3 × 2 × 3 = 72

Hint:

4	12, 8, 24, 9, 36
3	3, 2, 6, –9, 9
2	1, 2, 2, 3, 3
3	1, 1, 1, 3, 3
	1, 1, 1, 1, 1

$$\frac{-5}{12} = \frac{-5\times 6}{12\times 6} = \frac{-30}{72}$$

$$\frac{-11}{8} = \frac{-11\times 9}{8\times 9} = \frac{-99}{72}$$

$$\frac{-15}{24} = \frac{-15\times 3}{24\times 3} = \frac{-45}{72}$$

$$\frac{-7}{-9} = \frac{7\times 8}{9\times 8} = \frac{56}{72}$$

$$\frac{12}{36} = \frac{12\times 2}{36\times 2} = \frac{24}{72}$$

Now comparing the numerators – 30, – 99, -45, 56, 24 we get 56 > 24 > – 30 > – 45 > – 99

i.e $\frac{56}{72} > \frac{24}{72} > \frac{-30}{72} > \frac{-45}{72} > \frac{-99}{72}$ and so $\frac{-7}{-9} > \frac{12}{36} > \frac{-5}{12} > \frac{-15}{24} > \frac{-11}{8}$

$\therefore$ Descending order $\frac{-7}{-9} > \frac{12}{36} > \frac{-5}{12} > \frac{-15}{24} > \frac{-11}{8}$

Ascending order $\frac{-11}{8} < \frac{-15}{24} < \frac{-5}{12} < \frac{12}{36} < \frac{-7}{-9}$

(ii) $\frac{-17}{10}, \frac{-7}{5}, 0, \frac{-2}{4}, \frac{-19}{20}$

LCM of 10, 5, 4, 20 is $5 \times 2 \times 2 = 20$

Hint:

5	10, 5, 4, 20
3	2, 1, 4, 4
2	1, 1, 2, 2
	1, 1, 1, 1

$$\frac{-17}{10} = \frac{-17\times 2}{10\times 2} = \frac{-34}{20}$$
$$\frac{-7}{5} = \frac{-7\times 4}{5\times 4} = \frac{-28}{20}$$
$$\frac{-2}{4} = \frac{-2\times 5}{4\times 5} = \frac{-10}{20}$$
$$\frac{-19}{20} = \frac{-19}{20}$$

Negative numbers are less than zero.

∴ Arranging the numerators we get

– 34 < – 28 < – 19 < – 10 < 0

$$\therefore \frac{-34}{20} < \frac{-28}{20} < \frac{-19}{20} < \frac{-10}{20} < 0$$

$$\text{Ascending order} = \frac{-17}{10} < \frac{-7}{5} < \frac{-19}{20} < \frac{-2}{4} < 0$$

$$\text{Descending order } 0 > \frac{-2}{4} > \frac{-19}{20} > \frac{-7}{5} > \frac{-17}{10}$$

Objective Type Questions:

Question 11.

The number which is subtracted from $\frac{-6}{11}$ to get $\frac{8}{9}$ is _____ .

(A) $\frac{34}{99}$

(B) $\frac{-142}{99}$

(C) $\frac{142}{99}$

(D) $\frac{-34}{99}$

Answer:

(B) $\frac{-142}{99}$

Hint:

Let x be the number to be subtracted

$\frac{-6}{11} - x = \frac{8}{9}$

$\frac{-6}{11} - \frac{8}{9} = x$

$$x = \frac{(-6\times 9)+(-8\times 11)}{11\times 9} = \frac{-54+(-88)}{99} = \frac{-142}{99}$$

Question 12.

Which of the following pairs is equivalent?

(A) $\frac{-20}{12}, \frac{5}{3}$

(B) $\frac{16}{-30}, \frac{-8}{15}$

(C) $\frac{-18}{36}, \frac{-20}{44}$

(D) $\frac{7}{-5}, \frac{-5}{7}$

Answer:

(B) $\frac{16}{-30}, \frac{-8}{15}$

Hint:

$$\frac{-20}{12} = \frac{-20 \div 4}{12 \div 4} = \frac{-5}{3} \neq \frac{5}{3}$$

$$\frac{16}{-30} = \frac{-16 \div 2}{30 \div 2} = \frac{-8}{15}$$

$$\frac{-18}{36} = \frac{-18 \div 9}{36 \div 9} = \frac{-2}{4} = \frac{-2 \times 11}{4 \times 11} = \frac{-22}{44} \neq \frac{-20}{44}$$

∴ 16−30 and −815

Question 13.

$\frac{-5}{4}$ is a rational number which lies between _____ .

(A) 0 and $\frac{-5}{4}$

(B) -1 and 0

(C) -1 and -2

(D) -4 and -5

Answer:

(C) -1 and -2

Hint:

$\frac{-5}{4} = -1\frac{1}{4}$

∴ $\frac{-5}{4}$ lies between -1 and -2.

Question 14.

Which of the following rational numbers is the greatest?

(A) $\frac{-17}{24}$

(B) $\frac{-13}{16}$

(C) $\frac{7}{-8}$

(D) $\frac{-31}{32}$

Answer:

(A) $\frac{-17}{24}$

Hint:

LCM of 24, 16, 8, 32 = 8 × 2 × 3 × 2 = 96

8	24, 16, 8, 32
2	3, 2, 1, 4
3	3, 1, 1, 2
2	1, 1, 1, 2
	1, 1, 1, 1

$$\frac{-17}{24} = \frac{-17\times4}{24\times4} = \frac{-68}{96}$$

$$\frac{-13}{16} = \frac{-13\times6}{16\times6} = \frac{-78}{96}$$

$$\frac{7}{-8} = \frac{-7\times12}{8\times12} = \frac{-84}{96}$$

$$\frac{-31}{32} = \frac{-31\times3}{32\times3} = \frac{-93}{96}$$

$$\frac{-93}{96} < \frac{-84}{96} < \frac{-78}{96} < \frac{-68}{96}$$

$$\frac{-31}{32} < \frac{7}{-8} < \frac{-13}{16} < \frac{-17}{24}$$

∴ −1724 is the greatest number

Question 15.

The sum of the digits of the denominator in the simplest form of is $\frac{112}{528}$ is _____ .

(A) 4
(B) 5
(C) 6
(D)7

Answer:

(C) 6

Hint:

$$\frac{112}{528} = \frac{112\div8}{528\div8} = \frac{14}{66} = \frac{14\div2}{66\div2} = \frac{7}{33}$$

Sum of digits in the denominator = 3 + 3 = 6

Ex 1.2

Question 1.
Fill in the blanks:

(i) The value of $\frac{-5}{12}+\frac{7}{15}$ = _____ .
Answer:
$\frac{1}{20}$

(ii) The value of $(\frac{-3}{6})\times(\frac{18}{-9})$ is = _____ .
Answer:
1

(iii) The value of $(\frac{-15}{23})\div(\frac{30}{-46})$ is _____ .
Answer:
1

(iv) The rational number _____ does not have a reciprocal.
Answer:
0

(v) The multiplicative inverse of -1 is _____ .
Answer:
-1

Question 2.
Say True or False
(i) All rational numbers have an additive inverse.
Answer:
True

(ii) The rational numbers that are equal to their additive inverses are 0 and -1.
Answer:
False

(iii) The additive inverse of $\frac{-11}{-17}$ is $\frac{11}{17}$
Answer:
False

(iv) The rational number which is its own reciprocal is -1.
Answer:
True

(v) The multiplicative inverse exists for all rational numbers.

Answer:

False

Question 3.

Find the sum

(i) $\frac{7}{5}+\frac{3}{5}$

(ii) $\frac{7}{5}+\frac{5}{7}$

(iii) $\frac{6}{5}+\left(\frac{-14}{15}\right)$

(iv) $-4\frac{2}{3}+7\frac{5}{12}$

Answer:

(i) $\frac{7}{5}+\frac{3}{5}$

$$\frac{7}{5}+\frac{3}{5} = \frac{7+3}{5}=\frac{10}{5}=2$$

(ii) $\frac{7}{5}+\frac{5}{7}$

$$\frac{7}{5}+\frac{5}{7} = \frac{7\times7+5\times5}{35}=\frac{49+25}{35}=\frac{74}{35}$$

(iii) $\frac{6}{5}+(\frac{-14}{15})$

$$\frac{6}{5}+\left(\frac{-14}{15}\right) = \frac{6\times3+(14)}{15}=\frac{18+(-14)}{5}=\frac{4}{5}$$

(iv) $-4\frac{2}{3}+7\frac{5}{12}$

$$-4\frac{2}{3}+7\frac{5}{12} = \frac{14}{3}+\frac{18}{12}=\frac{-14\times4+89}{12}=\frac{-56+89}{12}=\frac{\cancel{33}^{11}}{\cancel{12}_{4}}=\frac{-11}{4}$$

Question 4.

Subtract $\frac{-8}{44}$ from $\frac{-17}{11}$

Answer:

$$\frac{-17}{11}-\left(\frac{-8}{44}\right) = \frac{-17}{11}+\frac{8}{44}=\frac{-17\times4+8}{44}=\frac{-68+8}{44}=\frac{\cancel{60}^{15}}{\cancel{44}_{11}}=\frac{-15}{11}$$

Question 5.

Evaluate

(i) $\frac{9}{132} \times \frac{-11}{3}$

(ii) $\frac{-7}{27} \times \frac{24}{-35}$

Answer:

(i) $\frac{9}{132} \times \frac{-11}{3}$

$$\frac{\cancel{9}^{3}}{\cancel{132}_{12}} \times \frac{-\cancel{11}^{1}}{\cancel{3}^{1}} = \frac{-1}{4}$$

(ii) $\frac{-7}{27} \times \frac{24}{-35}$

$$\frac{-\cancel{7}^{1}}{\cancel{27}_{9}} \times \frac{\cancel{24}^{8}}{-\cancel{35}_{5}} = \frac{8}{45}$$

Question 6.

Divide

(i) $\frac{-21}{5}$ by $\frac{-7}{-10}$

(ii) $\frac{-3}{13}$ by -3

(iii) -2 by $\frac{-6}{15}$

Answer:

(i) $\frac{-21}{5}$ by $\frac{-7}{-10}$

$$\frac{-21}{5} \div \frac{-7}{-10} = \frac{\cancel{-21}^{3}}{\cancel{5}} \times \frac{\cancel{10}^{2}}{\cancel{7}^{1}} = -6$$

(ii) −313 by -3

$$\frac{-3}{13} \div -3 = \frac{-3}{13} \times \frac{-1}{3} = \frac{-3 \times -1}{13 \times 3} = \frac{3}{39}$$

(iii) -2 by $\frac{-6}{15}$

$$-2 \div \frac{-6}{15} = -2 \times \frac{15}{-6} = \frac{-2 \times 15}{-6} = \frac{-\cancel{30}^{5}}{-\cancel{6}^{1}} = 5$$

Question 7.

Find (a + b) ÷ (a – b) if

Find (a + b) ÷ (a – b) if

(i) a = $\frac{1}{2}$, b = $\frac{2}{3}$

(ii) a = $\frac{-3}{5}$, b = $\frac{2}{15}$

Answer:

(i) a = $\frac{1}{2}$, b = $\frac{2}{3}$

$$a+b = \frac{1}{2}+\frac{2}{3}=\frac{1\times3+2\times2}{6}=\frac{3+4}{6}=\frac{7}{6}$$

$$a-b = \frac{1}{2}-\frac{2}{3}=\frac{1\times3-2\times2}{6}=\frac{3-4}{6}=\frac{-1}{6}$$

$$(a+b)\div(a-b) = \frac{7}{6}\div\frac{-1}{6}=\frac{7}{\cancel{6}}\times\frac{\cancel{6}}{-1}=-7$$

(ii) a = $\frac{-3}{5}$, b = $\frac{2}{15}$

$$a+b = \frac{-3}{5}+\frac{2}{15}=\frac{-3\times3+2}{15}=\frac{-9+2}{15}=\frac{-7}{15}$$

$$a-b = \frac{-3}{5}-\frac{2}{15}=\frac{-3\times3-2}{15}=\frac{-9-2}{15}=\frac{-11}{15}$$

$$(a+b)\div(a-b) = \frac{-7}{15}\div\frac{-11}{15}=\frac{-7}{\cancel{15}}\times\frac{\cancel{15}}{-11}=\frac{7}{11}$$

Question 8.

Simplify $\frac{1}{2}+\left(\frac{3}{2}-\frac{2}{5}\right)\div\frac{3}{10}\times3$ and show that it is a rational number between 11 and 12.

Answer:

$$\frac{1}{2}+\left(\frac{3}{7}-\frac{2}{5}\right)\div\frac{3}{10}\times3 = \frac{1}{2}+\left(\frac{15-4}{10}\right)\div\frac{3}{10}\times3=\frac{1}{2}+\frac{11}{\cancel{10}}\times\frac{\cancel{10}}{\cancel{3}}\times\cancel{3}$$

$$= \frac{1}{2}+11=11\frac{1}{2}=\frac{23}{2}$$

Question 9.

Simplify

(i) $\left[\frac{11}{8} \times \left(\frac{-6}{33}\right)\right] + \left[\frac{1}{3} + \left(\frac{3}{5} \div \frac{9}{20}\right)\right] - \left[\frac{4}{7} \times \frac{-7}{5}\right]$

(ii) $\left[\frac{4}{3} \div \left(\frac{8}{-7}\right)\right] - \left[\frac{3}{4} \times \frac{4}{3}\right] + \left[\frac{4}{3} \times \left(\frac{-1}{4}\right)\right]$

Answer:

(i) $\left[\frac{11}{8} \times \left(\frac{-6}{33}\right)\right] + \left[\frac{1}{3} + \left(\frac{3}{5} \div \frac{9}{20}\right)\right] - \left[\frac{4}{7} \times \frac{-7}{5}\right]$

$= \frac{11 \times (-6)}{8 \times 33} + \left[\frac{1}{3} + \left(\frac{3}{5} \times \frac{20}{9}\right)\right] - \left[\frac{4 \times -7}{7 \times 5}\right] = -\frac{1}{4} + \left[\frac{1}{3} + \frac{4}{3}\right] - \left(\frac{-4}{5}\right)$

$= -\frac{1}{4} + \frac{5}{3} + \frac{4}{5} = \frac{-15 + 100 + 48}{60} = \frac{133}{60}$

(ii) $\left[\frac{4}{3} \div \left(\frac{8}{-7}\right)\right] - \left[\frac{3}{4} \times \frac{4}{3}\right] + \left[\frac{4}{3} \times \left(\frac{-1}{4}\right)\right]$

$\left[\frac{4}{3} \div \left(\frac{8}{-7}\right)\right] - \left[\frac{3}{4} \times \frac{4}{3}\right] + \left[\frac{4}{3} \times \left(\frac{-1}{4}\right)\right] = \left[\frac{4}{3} \times \frac{-7}{8}\right] - \left[\frac{3}{4} \times \frac{4}{3}\right] + \left[\frac{4 \times (-1)}{3 \times 4}\right]$

$= \left(\frac{-7}{6}\right) - 1 + \left(\frac{-1}{3}\right)$

$= \frac{-7 - 6 + (-2)}{6} = \frac{-15}{6} = \frac{-5}{2}$

Question 10.

A student had multiplied a number by $\frac{4}{3}$ instead of dividing it by $\frac{4}{3}$ and got 70 more than the correct answer. Find the number.

Answer:

Let the number = a

$$a \times \frac{4}{3} - a \div \frac{4}{3} = 70$$
$$a \times \frac{4}{3} - a \times \frac{3}{4} = 70$$
$$a\left[\frac{4}{3} - \frac{3}{4}\right] = 70$$
$$a\left[\frac{4 \times 4 - 3 \times 3}{12}\right] = 70$$
$$a\left[\frac{16 - 9}{12}\right] = 70$$
$$a\left[\frac{7}{12}\right] = 70$$
$$a = \overset{10}{\cancel{70}} \times \frac{12}{\cancel{7}} = 120$$
$$a = 120$$

Objective Type Questions

Question 11.
The standard form of the sum is _____ .

(A) 1

(B) $\frac{-1}{2}$

(C) $\frac{1}{12}$

(D) $\frac{1}{22}$

Answer:

1

Hint:

$$\frac{3}{4} + \frac{5}{6} + \left(\frac{-7}{12}\right) = \frac{(3 \times 3) + (5 \times 2) + (-7)}{12} = \frac{9 + 10 + (-7)}{12} = \frac{19 - 7}{12} = \frac{12}{12} = 1$$

Question 12.

$\left(\frac{3}{4} - \frac{5}{8}\right) + \frac{1}{2}$ = _____ .

(A) $\frac{15}{64}$

(B) 1

(C) $\frac{5}{8}$

(D) $\frac{1}{16}$

Answer:

(C) $\frac{5}{8}$

Hint:

$$\left(\frac{3}{4}-\frac{5}{8}\right)+\frac{1}{2} = \left(\frac{3\times 2-5}{8}\right)+\frac{1}{2}=\frac{6-5}{8}+\frac{1}{2}=\frac{1}{8}+\frac{1}{2}$$

$$= \frac{1+1\times 4}{8}=\frac{1+4}{8}=\frac{5}{8}$$

Question 13.

$\frac{3}{4}+\left(\frac{5}{8}+\frac{1}{2}\right)=$

(A) $\frac{13}{10}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) $\frac{5}{8}$

Answer:

(B) $\frac{2}{3}$

Hint:

$$\frac{3}{4}\div\left(\frac{5}{8}+\frac{1}{2}\right) = \frac{3}{4}\div\left(\frac{5+(1\times 4)}{8}\right)=\frac{3}{4}\div\left(\frac{5+4}{8}\right)$$

$$= \frac{3}{4}\div\frac{9}{8}=\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{8}^{2}}{\cancel{9}_{3}}=\frac{2}{3}$$

Question 14.

= _____ .

(A) $\frac{5}{8}$

(B) $\frac{2}{3}$

(C) $\frac{15}{32}$

(D) $\frac{15}{16}$

Answer:

(D) $\frac{15}{16}$

Hint:

$$\frac{3}{4}\times\left(\frac{5}{8}\div\frac{1}{2}\right) = \frac{3}{4}\times\left(\frac{5}{\cancel{8}_{4}}\times\frac{\cancel{2}}{1}\right) = \frac{3}{4}\times\frac{5}{4} = \frac{3\times5}{4\times4} = \frac{15}{16}$$

Question 15.
Which of these rational number which have additive inverse?
(A) 7
(B) $\frac{-5}{7}$
(C) 0
(D) all of these
Answer:
(D) all of these
Hint:
Additive inverse of 7 is -7
Additive inverse of $\frac{-5}{7}$ is $\frac{5}{7}$
Additive inverse of 0 is 0

Ex 1.3

Question 1.
Verify the closure property for addition and multiplication for the rational numbers $\frac{-5}{7}$ and $\frac{8}{9}$.
Answer:
closure property for addition
Let a $= \frac{-5}{7}$ and b $= \frac{8}{9}$

$$a+b = \frac{-5}{7}+\frac{8}{9}$$

$$= \frac{(-5\times 9)+(8\times 7)}{7\times 9}$$

$$= \frac{-45+56}{63} = \frac{11}{63} \text{ is in Q.}$$

i.e $$a+b = \frac{-5}{7}+\frac{8}{9} = \frac{11}{63} \text{ is in Q.}$$

∴ Closure property is true for addition of rational numbers.
Closure property for multiplication

Let $$a = \frac{-5}{7} \text{ and b} = \frac{8}{9}$$

$$a\times b = \frac{-5}{7}\times\frac{8}{9} = \frac{-40}{63} \text{ is in Q.}$$

∴ Closure property is true for rnultiplìcation of rational numbers.

Question 2.
Verify the commutative property for addition and multiplication for the rational numbers $\frac{-10}{11}$ and \(\frac{-8{33}\).
Answer:
Let a $= \frac{-10}{11}$ and \(\frac{-8{33}\) be the given rational numbers.

Now $$a+b = \frac{-10}{11}+\left(\frac{-8}{33}\right) = \frac{(-10\times 3)+(-8\times 1)}{33} = \frac{-30+(-8)}{33}$$

$$a+b = \frac{-38}{33} \quad \text{.....(1)}$$

$$b+a = \frac{-8}{33}+\left(\frac{-10}{11}\right) = \frac{(-8\times 1)+((-10)\times 3)}{33} = \frac{-8+(-30)}{33}$$

$$b+a = \frac{-38}{33} \quad \text{....(2)}$$

From (1) and (2)
a + b = b + a and hence additionis commutative for rational numbers

$$\text{Further } a \times b = \frac{-10}{11} \times \left(\frac{-8}{33}\right) = \frac{80}{363}$$

$$a \times b = \frac{80}{363} \quad \text{.....(3)}$$

$$b \times a = \frac{-8}{33} \times \left(\frac{-10}{11}\right) = \frac{80}{363}$$

$$b \times a = \frac{80}{363} \quad \text{.... (4)}$$

From (3) and (4) a × b = b × a
Hence multiplication is commutative for rational numbers.

Question 3.

Verify the associative property for addition and multiplication for the rational numbers $\frac{-7}{9}, \frac{5}{6}$ and $\frac{-4}{3}$.

Answer:

$$\text{Let } a = \frac{-7}{9}, b = \frac{5}{6}, c = \frac{-4}{3} \text{ be the given rational numbers.}$$

$$(a+b)+c = \left(\frac{-7}{9}+\frac{5}{6}\right)+\left(\frac{-4}{3}\right) = \left(\frac{-7\times2+5\times3}{18}\right)+\left(\frac{-4}{3}\right)$$

$$= \left(\frac{-14+15}{18}\right)+\left(\frac{-4}{3}\right) = \frac{1}{18}+\left(\frac{-4}{3}\right)$$

$$= \frac{1+(-4)\times6}{18} = \frac{1+(-24)}{18} = \frac{-23}{18} \quad \text{...(1)}$$

$$a+(b+c) = -\frac{7}{9}+\left(\frac{5}{6}+\frac{(-4)}{3}\right) = \frac{-7}{9}+\left(\frac{5+(-4)2}{6}\right)$$

$$= \frac{-7}{9}+\left(\frac{5+(-8)}{6}\right) = -\frac{7}{9}+\left(\frac{-3}{6}\right) = -\frac{7}{9}+\left(\frac{-1}{2}\right)$$

$$= \frac{-7\times2+(-1)\times9}{18} = \frac{-14+(-9)}{18} = \frac{-23}{18} \quad \text{... (2)}$$

From (1) and (2), (a + b) + c = a + (b + c) is true for rational numbers.

Given the rational number $a = \frac{-1}{2}$; $b = \frac{2}{3}$ and $c = \frac{-5}{6}$

$$a \times (b + c) = \frac{-1}{2} \times \left(\frac{2}{3} + \left(\frac{-5}{6}\right)\right) = \frac{-1}{2} \times \left(\frac{(2\times 2) + (-5\times 1)}{6}\right)$$

$$= \frac{-1}{2} \times \left(\frac{4 + (-5)}{6}\right) = \frac{-1}{2} \times \left(\frac{-1}{6}\right)$$

$$a \times (b + c) = \frac{1}{12} \quad \text{.....(1)}$$

$$(a \times b) + (a \times c) = \left(\frac{-1}{2} \times \frac{2}{3}\right) + \left(\frac{-1}{2} \times \left(\frac{-5}{6}\right)\right)$$

$$= \frac{-2}{6} + \frac{5}{12} = \frac{(-2\times 2) + 5\times 1}{12} = \frac{-4 + 5}{12}$$

$$(a \times b) + (a \times c) = \frac{1}{12} \quad \text{....(2)}$$

From (1) and (2) (a × b) × c = (a × b) × c is true for rational numbers.
Thus associative property.

Question 4.

Verify the distributive property a × (b + c) = (a × b) + (a + c) for the rational numbers a = $\frac{-1}{2}$, b = $\frac{2}{3}$ and c = $\frac{-5}{6}$.

Answer:

Given the rational number $a = \frac{-1}{2}$; $b = \frac{2}{3}$ and $c = \frac{-5}{6}$

$$a \times (b + c) = \frac{-1}{2} \times \left(\frac{2}{3} + \left(\frac{-5}{6}\right)\right) = \frac{-1}{2} \times \left(\frac{(2\times 2) + (-5\times 1)}{6}\right)$$

$$= \frac{-1}{2} \times \left(\frac{4 + (-5)}{6}\right) = \frac{-1}{2} \times \left(\frac{-1}{6}\right)$$

$$a \times (b + c) = \frac{1}{12} \quad \text{.....(1)}$$

$$(a \times b) + (a \times c) = \left(\frac{-1}{2} \times \frac{2}{3}\right) + \left(\frac{-1}{2} \times \left(\frac{-5}{6}\right)\right)$$

$$= \frac{-2}{6} + \frac{5}{12} = \frac{(-2\times 2) + 5\times 1}{12} = \frac{-4 + 5}{12}$$

$$(a \times b) + (a \times c) = \frac{1}{12} \quad \text{....(2)}$$

From (1) and (2) we have a × (b + c) = (a × b) + (a × c) is true
Hence multiplication is distributive over addition for rational numbers Q.

Question 5.

Verify the identity property for addition and multiplication for the rational numbers $\frac{15}{19}$ and $\frac{-18}{25}$.

Answer:

$$\frac{15}{19}+0 = \frac{15}{19}+\frac{0}{19} = \frac{15+0}{19} = \frac{15}{19}$$

$$\frac{-18}{25}+0 = \frac{-18}{25}+\frac{0}{25} = \frac{-18+0}{25} = \frac{-18}{25}$$

Identify property for addition verified.

$$\frac{15}{19}\times 1 = \frac{15\times 1}{19} = \frac{15}{19}$$

$$\frac{-18}{25}\times 1 = \frac{-18\times 1}{25} = \frac{-18}{25}$$

Identify property for multiplication verified.

Question 6.

Verify the additive and multiplicative inverse property for the rational numbers $\frac{-7}{17}$ and $\frac{17}{27}$.

Answer:

$$\frac{-7}{17}+\frac{7}{17} = \frac{-7+7}{17} = \frac{0}{17} = 0$$

$$\frac{17}{27}+\left(-\frac{17}{27}\right) = \frac{17+(-17)}{27} = \frac{0}{27} = 0$$

Additive inverse for rational numbers verified.

$$\frac{-7}{17}\times\frac{17}{-7} = \frac{\cancel{-7}\times\cancel{17}}{\cancel{17}\times\cancel{(-7)}} = 1$$

$$\frac{17}{27}\times\frac{27}{17} = \frac{\cancel{17}\times\cancel{27}}{\cancel{27}\times\cancel{17}} = 1$$

Mulplicative inverse for rational numbers verified.

Objective Type Questions

Question 7.

Closure property is not true for division of rational numbers because of the number

(A) 1
(B) 1
(C) 0
(D) $\frac{1}{2}$

Answer:
(C) 0

Question 8.
$\frac{1}{2} - \left(\frac{3}{4} - \frac{5}{6}\right) \neq \left(\frac{1}{2} - \frac{3}{4}\right) - \frac{5}{6}$ illustrates that subtraction does not satisfy the _____ property for rational numbers.
(A) commutative
(B) closure
(C) distributive
(D) associative
Answer:
(D) associative

Question 9.
Which of the following illustrates the inverse property for addition?
(A) $\frac{1}{8} - \frac{1}{8} = 0$
(B) $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$
(C) $\frac{1}{8} + 0 = \frac{1}{8}$
(D) $\frac{1}{8} - 0 = \frac{1}{8}$
Answer:
(A) $\frac{1}{8} - \frac{1}{8} = 0$

Question 10.
$\frac{3}{4} \times \left(\frac{1}{2} - \frac{1}{4}\right) = \frac{3}{4} \times \frac{1}{2} - \frac{3}{4} \times \frac{1}{4}$ illustrates that multiplication is distributive over
(A) addition
(B) subtraction
(C) multiplication
(D) division
Answer:
(B) subtraction

Ex 1.4

Question 1.
Fill in the blanks:
(i) The ones digit in the square of 77 is _____ .
Answer:
9

(ii) The number of non-square numbers between 242 and 252 is _____ .
Answer:
48

(iii) The number of perfect square numbers between 300 and 500 is _____ .
Answer:
5

(iv) If a number has 5 or 6 digits in it, then its square root will have _____ digits.
Answer:
3

(v) The value of Jii lies between integers _____ and _____ .
Answer:
13, 14

Question 2.
Say True or False:
(i) When a square number ends in 6, its square root will have 6 in the unit's place.
Answer:
True

(ii) A square number will not have odd number of zeros at the end.
Answer:
True

(iii) The number of zeros in the square of 91000 is 9.
Answer:
False

(iv) The square of 75 is 4925.
Answer:
False

(v) The square root of 225 is 15.
Answer:
True

Question 3.
Find the square of the following numbers.
(i) 17
(ii) 203
(iii) 1098
Answer:
(i) 17

17 × 17
119
17
289

(ii) 203

203 × 203
609
000
406
41209

(iii) 1098

1098 × 1098
8784
9882
10980
1205604

Question 4.
Examine if each of the following is a perfect square.
(i) 725
(ii) 190
(iii) 841
(iv) 1089
Answer:
(i) 725
$725 = 5 \times 5 \times 29 = 5^2 \times 29$
Here the second prime factor 29 does not have a pair.
Hence 725 is not a perfect square number.

5	725
5	145
29	29
	1

(ii) 190
$190 = 2 \times 5 \times 19$
Here the factors 2, 5 and 9 does not have pairs.
Hence 190 is not a perfect square number.

2	190
5	95
	19

(iii) 841
$841 = 29 \times 29$
Hence 841 is a perfect square

(vi) 1089
$1089 = 3 \times 3 \times 11 \times 11 = 33 \times 33$
Hence 1089 is a perfect square

The factors of 144 are 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144.

Question 5.
Find the square root by prime factorisation method.
(i) 144
(ii) 256
(iii) 784
(iv) 1156
(v) 4761
(vi) 9025
Answer:
(i) 144
$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$\sqrt{144} = 2 \times 2 \times 3 = 12$

2	144
2	72
2	36
2	18
3	9
	3

(ii) 256
$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$\sqrt{256} = 2 \times 2 \times 2 \times 2 = 16$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

(iii) 784

$784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$

$\sqrt{784} = 2 \times 2 \times 2 \times 2 \times 7 \times 7 = 28$

2	784
2	392
2	196
2	98
7	49
	7

(iv) 1156

$1156 = 2 \times 2 \times 17 \times 17$

$1156 = 2^2 \times 17^2$

$1156 = (2 \times 17)^2$

$\therefore \sqrt{1156} = (2 \times 17)2 = 2 \times 17 = 34$

$\therefore \sqrt{1156} = 34$

2	1156
2	578
17	289
17	17
	1

(v) 4761

$4761 = 3 \times 3 \times 23 \times 23$

$4761 = 3^2 \times 23^2$

$4761 = (3 \times 23)^2$

$\sqrt{4761} = \sqrt{(3 \times 23)^2}$

$\sqrt{4761} = 3 \times 23$

$\sqrt{4761} = 69$

3	4761
3	1587
23	529
23	23
	1

(vi) 9025

$9025 = 5 \times 5 \times 19 \times 19$

$9025 = 5^2 \times 19^2$

$9025 = (5 \times 19)^2$

$\sqrt{925} = \sqrt{(5 \times 19)^2} = 5 \times 19 = 95$

5	9025
5	1805
19	361
19	19
	1

Question 6.

Find the square root by long division method.

(i) 1764

(ii) 6889

(iii) 11025

(iv) 17956

(v) 418609

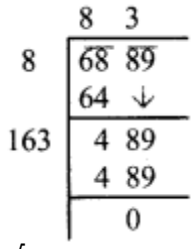
Answer:

(i) 1764

	4	2
4	17	64
	16	↓
82	1	64
	1	64
	0	

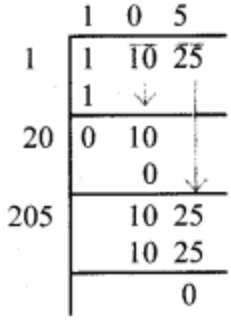
$\sqrt{1764} = 42$

(ii) 6889



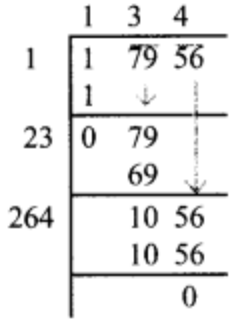
$\sqrt{6889} = 83$

(iii) 11025



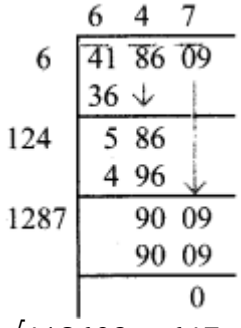
$\sqrt{11025} = 105$

(iv) 17956



$\sqrt{17956} = 134$

(v) 418609



$\sqrt{418609} = 647$

Roots Calculator is a free online tool that displays the roots of the given quadratic equation.

Question 7.
Estimate the value of the following square roots to the nearest whole number:
(i) $\sqrt{440}$
(ii) $\sqrt{800}$
(iii) $\sqrt{1020}$
Answer:
(i) $\sqrt{440}$
we have $20^2 = 400$
$21^2 = 441$
$\therefore \sqrt{440} \simeq 21$

(ii) $\sqrt{800}$
we have $28^2 = 784$
$29^2 = 841$
$\therefore \sqrt{800} \simeq 28$

(iii) $\sqrt{1020}$
we have $31^2 = 961$
$32^2 = 1024$
$\therefore \sqrt{1020} \simeq 32$

Question 8.
Find the square root of the following decimal numbers and fractions.
(i) 2.89
(ii) 67.24
(iii) 2.0164
(iv) $\frac{144}{225}$
(v) $7\frac{18}{49}$
Answer:
(i) 2.89

	1 . 7	
1	2 . 89	
	1	
27	1 89	
	1 89	
	0	

$\sqrt{2.89} = 1.7$

(ii) 67.24

```
      8 . 2
    ------
 8 | 67.24
   | 64
    ------
162 | 3 24
    | 3 24
    ------
    |   0
```

$\sqrt{67.24} = 8.2$

(iii) 2.0164

```
      1 . 4  2
    ---------
 1 | 2.  01 64
   | 1
    ---------
24 | 1  01
   |    96
    ---------
282|    5 64
   |    5 64
    ---------
   |      0
```

$\sqrt{2.0164} = 1.42$

(iv) $\frac{144}{225}$

$$\frac{144}{225} = \sqrt{\frac{144}{225}} = \sqrt{\frac{144}{225}} = \frac{12}{15}$$

(v) $7\frac{18}{49}$

$$7\frac{18}{49} = \sqrt{\frac{361}{49}} = \frac{\sqrt{361}}{\sqrt{49}} = \frac{\sqrt{19^2}}{\sqrt{7^2}} = \frac{19}{7} \qquad \left[\because \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} (b \neq 0)\right]$$

$$= \frac{19}{7} = 2\frac{5}{7}$$

$$\sqrt{7\frac{18}{49}} = 2\frac{5}{7}$$

49 ×
7
343 +
18
361

$\sqrt{7\frac{18}{49}} = 2\frac{5}{7}$

Question 9.

Find the least number that must be subtracted to 6666 so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.

Let us work out the process of finding the square root of 6666 by long division method.

```
    8   1
  ------------
8 | 66  66
    64
  ------------
161 | 2  66
    | 1  61
  ------------
    | 1  05
```

The remainder in the last step is 105. Is if 105 be subtracted from the given number the remainder will be zero and the new number will be a perfect square.
∴ The required number is 105. The square number is 6666 – 105 = 6561.

Question 10.
Find the least number by which 1800 should be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.
Answer:
We find $1800 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 2$
$= 2^2 \times 3^2 \times 5^2 \times 2$
Here the last factor 2 has no pair. So if we multiply 1800 by 2, then the number becomes a perfect square.

2	1800
2	900
3	450
3	150
5	50
5	10
2	2
	1

∴ $1800 \times 2 = 3600$ is the required perfect square number.
∴ $3600 = 1800 \times 2$
$3600 = 2^2 \times 3^2 \times 5^2 \times 2 \times 2$
$3600 = 2^2 \times 3^2 \times 5^2 \times 2^2$
$= (2 \times 3 \times 5 \times 2)^2$
$\sqrt{3600} = \sqrt{(2 \times 3 \times 5 \times 2)^2}$
$= 2 \times 3 \times 5 \times 2 = 60$
$\therefore \sqrt{3600} = 60$

Objective Type Questions

Question 11.
The square of 43 ends with the digit .
(A) 9

(B) 6
(C) 4
(D) 3
Answer:
(A) 9
Hint:
Ones digit $= 3 \times 3 = 9$

Question 12.
_____ is added to 24^2 to get 25^2.
(A) 4^2
(B) 5^2
(C) 6^2
(D) 7^2
Answer:
(D) 7^2
Hint:
$25^2 = 25 \times 25 = 625$
$24^2 = 24 \times 24 = 576$

$$\begin{array}{r} 625 \\ -\,576 \\ \hline 49 \end{array}$$

Question 13.
$\sqrt{48}$ is approximately equal to .
(A) 5
(B) 6
(C) 7
(D) 8
Answer:
(C) 7
Hint:
$\sqrt{49} = 7$

Question 14.
$\sqrt{128}-\sqrt{98}+\sqrt{18}$
(A) $\sqrt{2}$
(B) $\sqrt{8}$
(C) $\sqrt{48}$
(D) $\sqrt{32}$
Answer:
(D) $\sqrt{32}$

Question 15.
The number of digits in the square root of 123454321 is _____.

(A) 4
(B) 5
(C) 6
(D) 7

Answer:

(B) 5

Hint:

$\frac{n+1}{2}=\frac{10}{2}=5$

Ex 1.5

Question 1.
Fill in the blanks
(i) The ones digits in the cube of 73 is _____ .
Answer:
7

(ii) The maximum number of digits in the cube of a two digit number is _____ .
Answer:
6

(iii) The smallest number to be added to 3333 to make it a perfect cube is _____ .
Answer:
42

(iv) The cube root of 540×50 is _____ .
Answer:
30

(v) The cube root of 0.000004913 is _____ .
Answer:
0.017

Question 2.
Say True or False.
(i) The cube of 24 ends with the digit 4.
Answer:
True

(ii) Subtracting 103 from 1729 gives 93.
Answer:
True

(iii) The cube of 0.0012 is 0.000001728.
Answer:
False

(iv) 79570 is not a perfect cube.
Answer:
True

(v) The cube root of 250047 is 63.
Answer:
True

Question 3.
Show that 1944 is not a perfect cube.
Answer:

2	1944
2	972
2	486
3	243
3	81
3	27
3	9
3	3
	1

$1944 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$

$= \overline{2 \times 2 \times 2} \times \overline{3 \times 3 \times 3} \times 3 \times 3$

$= 2^3 \times 3^3 \times 3 \times 3$

There are two triplets to make further triplets we need one more 3.
∴ 1944 is not a perfect cube.

Question 4.
Find the smallest number by which 10985 should be divided so that the quotient is a perfect cube.
Answer:

5	10985
13	2197
13	169
13	13
	1

We have $10985 = 5 \times 13 \times 13 \times 13$

$= 5 \times 13 \times 13 \times 13$

Here we have a triplet of 13 and we are left over with 5.
If we divide 10985 by 5, the new number will be a perfect cube.
∴ The required number is 5.

Question 5.
Find the smallest number by which 200 should be multiplied to make it a perfect cube.
Answer:

2	200
2	100
2	50
5	25
5	5
	1

We find $200 = \overline{2 \times 2 \times 2} \times 5 \times 5$

Grouping the prime factors of 200 as triplets, we are left with 5×5

We need one more 5 to make it a perfect cube.

So to make 200 a perfect cube multiply both sides by 5.

$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5$

Now 1000 is a perfect cube.

∴ The required number is 5.

Question 6.

Find the cube root $24 \times 36 \times 80 \times 25$.

Answer:

2	24
2	12
2	6
3	3
	1

2	36
2	18
3	9
3	3
	1

2	80
2	40
2	20
2	10
5	5
	1

$$24 = \overline{2 \times 2 \times 2} \times 3$$

$$36 = \overline{2 \times 2} \times \overline{3 \times 3}$$

$$80 = \overline{2 \times 2 \times 2} \times 2 \times 5$$

$$25 = \overline{5 \times 5}$$

$$24 \times 36 \times 80 \times 25 = \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times \overline{3 \times 3 \times 3} \times \overline{2 \times 2 \times 2} \times \overline{5 \times 5 \times 5}$$

$$\sqrt[3]{24 \times 36 \times 80 \times 25} = 2 \times 2 \times 3 \times 2 \times 5 = 120$$

Question 7.

Find the cube root of 729 and 6859 prime factorisation.

Answer:

(i)

3	729
3	243
3	81
3	27
3	9
3	3
	1

$$\sqrt[3]{729} = \sqrt[3]{\overline{3 \times 3 \times 3} \times \overline{3 \times 3 \times 3}}$$

$= 3 \times 3$

$\sqrt[3]{729} = 9$

19	6859
19	361
19	19
	1

(ii) $\sqrt[3]{6859} = \sqrt[3]{19 \times 19 \times 19}$

$\sqrt[3]{6859} = 19$

Question 8.

What is the square root of cube root of 46656?

Answer:

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

We have to find out $\sqrt{(\sqrt[3]{46656})}$

First we will find $\sqrt[3]{46656}$

$$\sqrt[3]{46656} = (2\times2\times2\times2\times2\times2\times3\times3\times3\times3\times3\times3)^{\frac{1}{3}}$$

$$\sqrt[3]{46656} = 2 \times 2 \times 3 \times 3$$

$$\sqrt[3]{46656} = 2^2 \times 3^2 = 36$$

Now $\sqrt{(\sqrt[3]{46656})} = \sqrt{36} = \sqrt{2^2 \times 3^2} = 2 \times 3 = 6$

∴ The required number is 6.

Question 9.

If the cube of a squared number is 729, find the square root of that number.

Answer:

$729 = \overline{3 \times 3 \times 3} \times \overline{3 \times 3 \times 3}$
$(729)^{1/3} = 3 \times 3 = 9$
∴ The cube of 9 is 729.
$9 = 3 \times 3$ [ie 3 is squared to get 9]

	1 . 7	3	2	0
1	$\overline{3}$. $\overline{00}$	$\overline{00}$	$\overline{00}$	$\overline{00}$
	1	↓	↓	↓
27	2 00	↓	↓	
	1 89	↓		
343	11 00	↓		
	10 29	↓		
3462	0 71	00		
	69	24	↓	
34640	1	76	00	
			0	
	1	76	00	

We have to find out $\sqrt{3}$,
$\sqrt{3} = 1.732$

Question 10.
Find two smallest perfect square numbers which when multiplied together gives a perfect cube number.
Answer:
Consider the numbers 2^2 and 4^2
The numbers are 4 and 16.
Their proclucl $4 \times 16 = 64$
$64 = 4 \times 4 \times 4$
∴ The required square numbers are 4 and 16

Ex 1.6

Question 1.
Fill in the blanks.
(i) $(-1)^{\text{even integer}}$ is _____ .
Answer:
1

(ii) For $a \neq 0$, a^0 is _____ .
Answer:
1

(iii) $4^{-3} \times 5^{-3} =$ _____ .
Answer:
20^{-3}

(iv) $(-2)^{-7}$ is = _____ .
Answer:
$\frac{-1}{128}$

(v) $(\frac{-1}{3})^{-5} =$ _____ .
Answer:
– 243

Question 2.
Say True or False:
(i) If $8^x = \frac{1}{64}$, the value of x is -2.
Answer:
True

(ii) The simplified form of $(256)^{\frac{-1}{4}} \times 4^2$ is $\frac{1}{4}$.
Answer:
True

(iii) Using the power rule, $(3^7)^{-2} = 3^5$
Answer:
True

(iv) The standard form of 2×10^{-4} is 0.0002.
Answer:
False

(v) The scientific form of 123.456 is 1.23456×10^{-2}.

Answer:

True

Question 3.

Evaluate

(i) $\left(\frac{1}{2}\right)^3$

(ii) $\left(\frac{1}{2}\right)^{-5}$

(iii) $\left(\frac{-5}{6}\right)^{-3}$

(iv) $(2^{-5} \times 2^7) \div 2^{-2}$

(v) $(2^{-1} \times 3^{-1}) \div 6^{-2}$

Answer:

(i) $\left(\frac{1}{2}\right)^3$

$$\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{2\times2\times2} = \frac{1}{8}$$

(ii) $(\frac{1}{2})^{-5}$

$$\left(\frac{1}{2}\right)^{-5} = \frac{1^{-5}}{2^{-5}} = \frac{1}{2^{-5}} = 2^5 = 2\times2\times2\times2\times2 = 32$$

(iii) $(\frac{-5}{6})^{-3}$

$$\left(\frac{-5}{6}\right)^{-3} = \frac{(-5)^{-3}}{6^{-3}} = \frac{6^3}{(-5)^3} = \left(\frac{(-5\times-5\times-5)}{(6\times6\times6)}\right) = -\frac{216}{125}$$

(iv) $(2^{-5} \times 2^7) \div 2^{-2}$

$(2^{-5} \times 2^7) \div 2^{-2} = (2^{-5+7}) \div 2^{-2}$

$= 2^2 \div 2^{-2}$

$= 2^{2+2}$

$= 2^4$

$= 16$

(v) $(2^{-1} \times 3^{-1}) \div 6^{-2}$

$(2^{-1} \times 3^{-1}) \div 6^{-2} = (2 \times 3)^{-1} \div 6^{-2}$

$= (6^{-1}) \div 6^{-2}$

$= 6^{(-1)-(-2)}$

$= 6^1$

$= 6$

Question 4.
Evaluate

(i) $\left(\frac{2}{5}\right)^4 \times \left(\frac{5}{2}\right)^{-2}$

(ii) $\left(\frac{4}{5}\right)^{-2} \div \left(\frac{4}{5}\right)^{-3}$

(iii) $2^7 \times \left(\frac{1}{2}\right)^{-3}$

Answer:

(i) $\left(\frac{2}{5}\right)^4 \times \left(\frac{5}{2}\right)^{-2}$

$$\left(\frac{2}{5}\right)^4 \times \left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)^{4+2} = \left(\frac{2}{5}\right)^6$$

(ii) $\left(\frac{4}{5}\right)^{-2} \div \left(\frac{4}{5}\right)^{-3}$

$$= \left(\frac{4}{5}\right)^{-2} \times \left(\frac{4}{5}\right)^3 = \left(\frac{4}{5}\right)^{-2+3} = \left(\frac{4}{5}\right)^{-2+3} = \left(\frac{4}{5}\right)^1 = \frac{4}{5}$$

(iii) $2^7 \times \left(\frac{1}{2}\right)^{-3}$

$= 2^7 \times 2^3$

$= 2^{7+3}$

$= 2^{10}$

Question 5.
Evaluate:

(i) $(5^0 + 6^{-1}) \times 3^2$

(ii) $(2^{-1} + 3^{-1}) \div 6^{-1}$

(iii) $(3^{-1} + 4^{-2} + 5^{-3})^0$

Answer:

(i) $(5^0 + 6^{-1}) \times 3^2$

$$(5^0 + 6^{-1}) \times 3^3 = (5^0 \times 3^3) + (6^{-1} \times 3^3) = (1 \times 27) + \left(\frac{1}{2 \times 3} \times 3^3\right)$$

$$= 27 + \left(\frac{1}{2} \times 3^{3-1}\right) = 27 + \left(\frac{1}{2} \times 3^2\right)$$

$$= 27 + \frac{9}{2} = \frac{54+9}{2} = \frac{63}{2}$$

(ii) $(2^{-1} + 3^{-1}) \div 6^{-1}$

Answer:

$(2^{-1} + 3^{-1}) \div 6^{-1} = \left(\frac{1}{2} + \frac{1}{3}\right) + 6^{-1}$

$= \left(\frac{3+2}{6}\right) + 6^{-1} = \left(\frac{5}{6}\right) + 6^{-1} = \frac{5}{6} \times 6 = 5$

(iii) $(3^{-1} + 4^{-2} + 5^{-3})^0$

Answer:

$(3^{-1} + 4^{-2} + 5^{-3})^0 = 1$

[$\because a^0 = 1$ where $a \neq 0$]

Question 6.

Simplify

(i) $(3^2)^3 \times (2 \times 3^5)^{-2} \times (18)^2$

(ii) $\frac{9^2 \times 7^3 \times 2^5}{84^3}$

(iii) $\frac{2^8 \times 2187}{3^5 \times 3^2}$

Answer:

(i) $(3^2)^3 \times (2 \times 3^5)^{-2} \times (18)^2$

$$(3^2)^3 \times (2 \times 3^5)^{-2} \times (18)^2 = 3^{2\times3} \times \frac{1}{(2\times3^5)^2} \times 18^2$$

$$= 3^6 \times \frac{1}{2^2 \times (3^5)^2} \times 18^2 = 3^6 \times \frac{1}{2^2 \times 3^{10}} \times (2\times3^2)^2$$

$$= 3^6 \times \frac{1}{2^2 \times 3^{10}} \times 2^2 \times 3^{2\times2} = 3^6 \times \frac{1}{2^2 \times 3^{10}} \times 2^2 \times 3^4$$

$$= \frac{3^{6+4} \times 2^2}{2^2 \times 3^{10}} = \frac{3^{10} \times 2^2}{2^2 \times 3^{10}} = 1$$

(ii) $\frac{9^2 \times 7^3 \times 2^5}{84^3}$

$$\frac{9^2 \times 7^3 \times 2^5}{84^3} = \frac{(3^2)^2 \times 7^3 \times 2^5}{(2^2 \times 3 \times 7)^3} = \frac{3^{2\times2} \times 7^3 \times 2^5}{2^{2\times3} \times 3^3 \times 7^3} = \frac{3^4 \times 7^3 \times 2^5}{2^6 \times 3^3 \times 7^3}$$

$$= 3^{4-3} \times 7^{3-3} \times 2^{5-6} = 3^1 \times 7^0 \times 2^{-1}$$

$$= 3 \times 1 \times 2^{-1} = 3 \times \frac{1}{2} = \frac{3}{2}$$

(iii) $\frac{2^8 \times 2187}{3^5 \times 3^2}$

3	2187
3	729
3	243
3	81
3	27
3	9
	3

$$\frac{2^8 \times 2187}{3^5 \times 3^2} = \frac{2^8 \times 3^7}{3^5 \times 2^5}$$

$= 2^{8-5} \times 3^{7-5}$
$= 2^3 \times 3^2$
$= 8 \times 9$
$= 72$

Question 7.
Solve for x:

(i) $\frac{2^{2x-1}}{2^{x+2}} = 4$

(ii) $\frac{5^5 \times 5^{-4} \times 5^x}{5^{12}} = 5^{-5}$

Answer:

(i) $\frac{2^{2x-1}}{2^{x+2}} = 4$

$2^{2x-1-(x+2)} = 2^2$
$2^{2x-1-x-2)} = 2^2$
$2^{2x-3} = 2^2$
Equating the powers of the same base 2.
$x - 3 = 2$
$x - 3 + 3 = 2 + 3$
$x = 5$

(ii) $\frac{5^5 \times 5^{-4} \times 5^x}{5^{12}} = 5^{-5}$

$$\frac{5^5 \times 5^{-4} \times 5^x}{5^{12}} = 5^{-5} \Rightarrow \frac{5^{5-4+x}}{5^{12}} = 5^{-5}$$

$$\Rightarrow \frac{5^{1+x}}{5^{12}} = 5^{-5}$$

$\Rightarrow 5^{1+x-12} = 5^{-5}$

$\Rightarrow 5^{x-11} = 5^{-5}$
Equating the powers of same base 5.
$x - 11 = -5$
$x - 11 + 11 = -5 + 11$
$x = 6$

Question 8.
Expand using exponents:
(i) 6054.321
(ii) 897.14
Answer:
(i) 6054.321
$6054.321 = (6 \times 1000) + (0 \times 100) + (5 \times 10) + (4 \times 10^0) + \frac{3}{10} + \frac{2}{100} + \frac{1}{1000}$
$= (6 \times 10^3) + (5 \times 10^1) + (4 \times 10^0) + \frac{3}{10} + \frac{2}{100} + \frac{1}{1000}$
$= (6 \times 10^3) + (5 \times 10^1) + (4 \times 10^0) + (3 \times 10^{-1}) + (2 \times 10^{-2}) + (1 \times 10^{-3})$

(ii) 897.14
$= (8 \times 100) + (9 \times 10) + (7 \times 10^0) + \frac{1}{10} + \frac{4}{100}$
$= (8 \times 1o^2) + (9 \times 10^1) + (7 \times 10^0) + (1 \times \frac{1}{10}) + (4 \times \frac{1}{100})$
$= (8 \times 10^3) + (9 \times 10^3) + (7 \times 10^0) + (1 \times 10^{-1}) + (4 \times 10^{-2})$

Question 9.
Find the number is standard form:
(i) $8 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 2 \times 1 + 4 \times 10^{-2} + 7 \times 10^{-4}$
(ii) $5 \times 10^3 + 5 \times 10^1 + 5 \times 10^{-1} + 5 \times 10^{-3}$
(iii) The radius of a hydrogen atom is 2.5×10^{-11} m
Answer:
(i) $8 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 2 \times 1 + 4 \times 10^{-2} + 7 \times 10^{-4}$
$= 8 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 2 \times 1 + 4 \times 10^{-2} + 7 \times 10^{-4}$
$= 8 \times 10000 + 7 \times 1000 + 6 \times 100 + 5 \times 10 + 2 \times 1 + 4 \times \frac{1}{100} + 7 \times \frac{1}{10000}$
$= 80000 + 7000 + 600 + 50 + 2 + \frac{4}{100} + \frac{7}{10000}$
$= 87652.0407$

(ii) $5 \times 10^3 + 5 \times 10^1 + 5 \times 10^{-1} + 5 \times 10^{-3}$
$= 5 \times 10^3 + 5 \times 10^1 + 5 \times 10^{-1} + 5 \times 10^{-3}$
$= 5 \times 1000 + 5 \times 10 + 5 \times \frac{1}{10} + 5 \times \frac{1}{1000}$
$= 5000 + 50 + \frac{5}{10} + \frac{5}{1000} = 5050.505$

(iii) The radius of a hydrogen atom is $2.5\ 10^{-11}$ m
Radiys of a hydrogen atom $= 2.5 \times 10^{-11}$ m

$= 2.5 \times \frac{1}{10^{11}} \text{m} = \frac{2.5}{10^{11}} \text{m}$

$= 0.000000000025$ m

Question 10.
Write the following numbers in scientific notation:
(i) 467800000000
Answer:
$467800000000 = 4.678 \times 10^{11}$

(ii) 0.000001972
Answer:
$0.000001972 = 1.972 \times 10^{-6}$

(iii) 1642.398
Answer:
$1642.398 = 1.642398 \times 10^{3}$

(iv) Earth's volume is about 1,083,000,000,000 cubic kilometres
Answer:
1,083,000,000,000
Earth's volume $= 1.083\ 110 \times 10^{2}$ cubic kilometres

(v) If you fill a bucket with dirt, the portion of the whole Earth that is in the bucket will be 0.00000000000000000000000 16 kg
Answer:
Portion of earth in the bucket = 0.00000000000000000000000 16 kg
$= 1.6\ 10 \times 10^{24}$ kg.

Objective Type **Questions**

Question 11.
By what number should $(-4)^{-1}$ be multiplied so that the product becomes 10^{-1}?
(A) $\frac{2}{3}$
(B) $\frac{-2}{5}$
(C) $\frac{5}{2}$
(D) $\frac{-5}{2}$
Answer:
(B) $\frac{-2}{5}$
Hint:
$(-4)^{-1} = \left(-\frac{1}{4}\right)^{1} = \frac{-1}{4}$

Question 12.

$(-2)^{-3} \times (-2)^{-2}$ = _____.

(A) $\frac{-1}{32}$

(B) $\frac{1}{32}$

(C) 32

(D) -32

Answer:

(A) $\frac{-1}{32}$

Question 13.

Which is not correct?

(A) $\left(\frac{-1}{4}\right)^2 = 4^{-2}$

(B) $\left(\frac{-1}{4}\right)^2 = \left(\frac{1}{2}\right)^4$

(C) $\left(\frac{-1}{4}\right)^2 = 16^{-1}$

(D) $-\left(\frac{1}{4}\right)^2 = 16^{-1}$

Answer:

$-\left(\frac{1}{4}\right)^2 = 16^{-1}$

Hint:

(-2) – 3 x (- 2) – 2 = (-2) – 3 – 2 = (-2) – 5 $(-\frac{1}{2})5 = -\frac{1}{32}$

Question 14.

If $\frac{10^x}{10^{-3}} = 10^9$, then x is _____ .

(A) 4
(B) 5
(C) 6
(D) 7

Question 15.

0.0000000002020 in scientific form is _____ .

(A) 2.02×10^9
(B) 2.02×10^{-9}
(C) 2.02×10^{-8}
(D) 2.02×10^{-10}

Answer:

(D) 2.02×10^{-10}

Hint:

0.0000000002020

Ex 1.7

Miscellaneous Practice Problems

Question 1.

If $\frac{3}{4}$ of a box of apples weighs 3kg and 225 gm, how much does a full box of apples weigh?

Answer:

Let the total weight of a box of apple = x kg.

Weight of $\frac{3}{4}$ of a box apples = 3 kg 225 gm.

= 3.225kg

$\frac{3}{4} \times x = 3225$

$x = \frac{3.225 \times 4}{3}$ kg

$= 1.075 \times 4\text{kg} = 4.3\text{kg}$

= 4 kg 300 gm

Weight of the box of apples = 4 kg 300 gm.

Question 2.

Mangalam buys a water jug of capacity $3\frac{4}{5}$ litre. If she buys another jug which is $2\frac{2}{3}$ times as large as the smaller jug, how many litre can the larger one hold?

Answer:

Capacity of the small waterug = $3\frac{4}{5}$ litres.

Capacity of the big jug = $2\frac{2}{3}$ times the small one.

$= 2\frac{2}{3} \times 3\frac{4}{5} = \frac{8}{3} \times \frac{19}{5} = \frac{152}{15}$

$= \frac{2}{15}$ litres

Capacity of the large jug = $\frac{2}{15}$ litres.

Question 3.

Ravi multiplied $\frac{25}{8}$ and $\frac{16}{5}$ to obtain $\frac{400}{120}$. He says that the simplest form of this product is $\frac{10}{3}$ and Chandru says the answer in the simplest form is $3\frac{1}{3}$. Who is correct? (or) Are they both correct? Explain.

Answer:

$$\text{Product of } \frac{25}{8} \text{ and } \frac{16}{15} = \frac{\cancel{25}^{5}}{\cancel{8}} \times \frac{\cancel{16}^{2}}{\cancel{15}_{3}}$$

$$= \frac{10}{3} = 3\frac{1}{3}$$

$$\text{Answer obtained} = \frac{400}{120}$$

$$= \frac{400 \div 40}{120 \div 40} = \frac{10}{3} = 3\frac{1}{3}$$

∴ The product is 400120 and its simplest form improper fraction is $\frac{10}{3}$

And mixed fraction is $3\frac{1}{3}$

∴ Both are correct

Question 4.

Find the length of a room whose area is $\frac{153}{10}$ sq.m and whose breadth is $2\frac{11}{20}$ m.

Answer:

Length of the room × Breadth = Area of the room

Breadth of the room = $2\frac{11}{20}$ m

Area of the room = $\frac{153}{10}$ sq.m

Length x $2\frac{11}{20}$ = $\frac{153}{10}$

$$\text{Length} = \frac{153}{10} \div 2\frac{11}{20} = \frac{153}{10} \div \frac{51}{20} = \frac{\cancel{153}^{3}}{\cancel{10}_{1}} \times \frac{\cancel{20}^{2}}{\cancel{51}_{1}} = 6 \text{ m}$$

Length of the room = 6 m

Question 5.

There is a large square portrait of a leader that covers an area of 4489 cm². 1f each side has a 2 cm liner, what would be its area?

Answer:

67	4489
67	67
	0

Area of the square = 4489 cm²

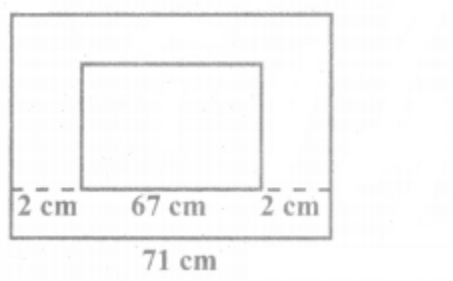
(side)2 = 4489 cm²

(side)2 = 67 × 67

side = 67²

Length of a side = 67

Length of a side with liner = 67 + 2 + 2 cm
= 71 cm



Area of the larger square = 71 × 71 cm^2
= 5041 cm^2
Area of the liner = Area of big square – Area of small square
= (5041 – 4489) cm^2
= 552 cm^2

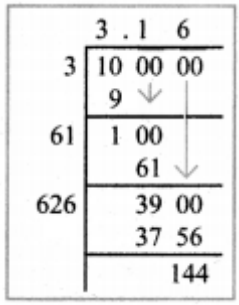
Question 6.
A greeting card has an area 90 cm^2. Between what two whole numbers is the length of its side?
Answer:

1	90
5	45
3	9
3	3
	1

Area of the greeting card = 90 cm^2
$(side)^2 = 90\ cm^2$
$(side)^2 = 2 \times 5 \times 3 \times 3 = 2 \times 5 \times 3^2$

$$\sqrt{(side)^2} = \sqrt{2 \times 5 \times 3^2}$$

Side = $3\sqrt{2 \times 5}$
side = $3\sqrt{10}$ cm
side = 3 × 3.2cm
side = 9.6 cm
∴ Side lies between the whole numbers 9 and 10.



Question 7.
225 square shaped mosaic tiles, each of area 1 square decimetre exactly cover a square shaped verandah. How long is each side of the square shaped verandah?
Answer:
Area of one tile = 1 sq.decimeter
Area of 225 tiles = 225 sq.decimeter
225 square tiles exactly covers the square shaped verandah.
∴ Area of 225 tiles = Area of the verandah
Area of the verandah = 225 sq.decimeter
side × side = 15 × 15 sq.decimeter
side = 15 decimeters
Length of each side of verandah = 15 decimeters.

Question 8.
If $\sqrt{1906624} \times \sqrt{x} = 3100$, find x.
Answer:

$$\sqrt[3]{1906624} \times \sqrt{x} = 3100$$

$$\sqrt[3]{2\times2\times2\times2\times2\times2\times31\times31\times31} \times \sqrt{x} = 3100$$

$$\sqrt[3]{2^3 \times 2^3 \times 31^3} \times \sqrt{x} = 3100$$

$$2 \times 2 \times 31 \times \sqrt{x} = 3100$$

$$\sqrt{x} = \frac{3100}{2\times2\times31} = \frac{3100}{2\times2\times31}$$

$$\sqrt{x} = 25$$

Squaring on both sides

$$(\sqrt{x})^2 = 25^2$$

$$x = 625$$

Question 9.
If $2^{m-1} + 2^{m+1} = 640$, then find 'm'.
Answer:
Given $2^{m-1} + 2^{m+1} = 640$
$2^{m-1} + 2^{m+1} = 128 + 512$
$2^{m-1} + 2^{m+1} - 2^7 + 2^9$
m – 1 = 7
m = 7 + 1
m = 8
[consecutive powers of 2]

Powers of 2:
2, 4, 8, 16, 32, 64, 128, 256, 512,....

Question 10.
Give the answer in scientific notation:
A human heart beats at an average of 80 beats per minute. How many times does it beat in
i) an hour?
ii) a day?
iii) a year?
iv) 100 years?
Answer:
Heart beat per minute = 80 beats
(i) an hour
One hour = 60 minutes
Heart beat in an hour = 60×80
$= 4800$
$= 4.8 \times 10^3$

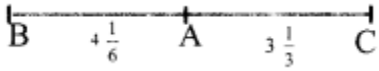
(ii) In a day
One day = 24 hours = 24×60 minutes
∴ Heart beat in one day = $24 \times 60 \times 80 = 24 \times 4800 = 115200$
$= 1.152 \times 10^5$

(iii) a year
One year = 365 days = 365×24 hours = $365 \times 24 \times 60$ minutes
∴ Heart beats in a year = $365 \times 24 \times 60 \times 80$
$= 42048000$
$= 4.2048 \times 10^7$

(iv) 100 years
Heart beats in one year = 4.2048×10^7
heart beats in 100 years = $4.2048 \times 10^7 \times 100 = 4.2048 \times 10^7 \times 10^2$
$= 4.2048 \times 10^9$

Challenging Problems:

Question 11.
In a map, if 1 inch refers to 120 km, then find the distance between two cities B and C which are $4\frac{1}{6}$ inches and $3\frac{1}{3}$ inches from the city A which lies between the cities B and C.
Answer:



1 inch = 120 km
Distance between A and B = $4\frac{1}{6}$

Distance between A and C $= 3\frac{1}{3}$

∴ Distance between B and C $= 4\frac{1}{6} + 3\frac{1}{3}$ inches

$$= \frac{25}{6} + \frac{10}{3} = \frac{25}{6} + \frac{20}{6} = \frac{25+20}{6} = \frac{45}{6} \text{ inches}$$

1 inch = 120km

∴ $\frac{45}{6}$ inches $= \frac{45}{6} \times 120$ km $= 900$ km

Distance between B and C = 900 km

Question 12.

Give an example and verify each of the following statements.

(i) The collection of all non-zero rational numbers is closed under division.

Answer:

let a $= \frac{5}{6}$ and b $= \frac{-4}{3}$ be two non zero rational numbers.

$$a \div b = \frac{5}{6} \div \frac{-4}{3} = \frac{5}{6} \times \frac{3}{-4} = \frac{5}{-8} \text{ is in Q}$$

∴ Collection of non-zero rational numbers are closed under division.

(ii) Subtraction is not commutative for rational numbers.

Answer:

let a $= \frac{1}{2}$ and b $= -\frac{5}{6}$ be two rational numbers.

$$a - b = \frac{1}{2} - \left(-\frac{5}{6}\right) = \frac{1}{2} + \left(+\frac{5}{6}\right) = \frac{3}{6} + \frac{5}{6} = \frac{3+5}{6} = \frac{8}{6}$$

$$= 1\frac{2}{6} = 1\frac{1}{3}$$

$$b - a = -\frac{5}{6} - \frac{1}{2} = -\frac{5}{6} - \frac{3}{6} = \frac{-5-3}{6} = \frac{-8}{6} = -1\frac{1}{3}$$

$$a - b \neq b - a$$

a – b ≠ b – a

∴ Subtraction is not commutative for rational numbers.

(iii) Division is not associative for rational numbers.

Answer:

Let a $= \frac{2}{5}$, b $= \frac{6}{5}$, c $= \frac{3}{5}$ be three rational numbers.

$$a \div (b \div c) = \frac{2}{5} \div \left(\frac{6}{5} \div \frac{3}{5}\right) = \frac{2}{5} \div \left(\frac{6}{5} \times \frac{5}{3}\right)$$

$$= \frac{2}{5} \div \frac{2}{1} = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5} \quad \text{....(1)}$$

$$(a \div b) \div c = \left(\frac{2}{5} \div \frac{6}{5}\right) \div \frac{3}{5}$$

$$= \left(\frac{2}{5} \times \frac{5}{6}\right) \div \frac{3}{5} = \frac{1}{3} \div \frac{3}{5} = \frac{1}{3} \times \frac{5}{3} = \frac{5}{9} \quad \text{....(2)}$$

From (1) and (2) $\frac{1}{5} \neq \frac{5}{9}$

a ÷ (b ÷ c) ≠ (a ÷ b) ÷ c

∴ Division is not associative for rational numbers.

(iv) Distributive property of multiplication over subtraction is true for rational numbers. That is, a (b – c) = ab – ac.

Answer:

Let a = $\frac{2}{9}$, b = $\frac{3}{6}$, c = $\frac{1}{3}$ be three rational numbers.

To prove a × (b – c) = ab – bc

$$a \times (b - c) = \frac{2}{9} \times \left(\frac{3}{6} - \frac{1}{3}\right)$$

$$= \frac{2}{9} \times \left(\frac{3 - (1 \times 2)}{6}\right) = \frac{2}{9} \times \frac{(3-2)}{6}$$

$$= \frac{2}{9} \times \frac{1}{6} = \frac{1}{27} \quad \text{....(1)}$$

$$ab - ac = \left(\frac{2}{9} \times \frac{3}{6}\right) - \left(\frac{2}{9} \times \frac{1}{3}\right) = \frac{1}{9} - \frac{2}{27}$$

$$= \frac{(1 \times 3) - 2}{27} = \frac{3-2}{27} = \frac{1}{27} \quad \text{....(2)}$$

∴ From (1) and (2)

a × (b – c) = ab – bc

∴ Distributivity of multiplication over subtraction is true for rational numbers.

(v) The mean of two rational numbers is rational and lies between them.

Answer:

Let a = $\frac{2}{11}$ and b = $\frac{5}{6}$ be two rational numbers

$$\text{Mean of } a \text{ and } b \text{ is } c = \frac{1}{2}(a+b) = \frac{1}{2}\left(\frac{2}{11}+\frac{5}{6}\right) = \frac{1}{2}\left(\frac{(2\times6)+(5\times11)}{66}\right)$$

$$= \frac{1}{2}\times\left(\frac{12+55}{66}\right) = \frac{1}{2}\times\frac{67}{66} = \frac{67}{132} \text{ is in Q.}$$

$$\text{Also } \frac{2}{11} = \frac{2\times12}{11\times12} = \frac{24}{132}$$

$$\frac{5}{6} = \frac{5\times22}{6\times22} = \frac{110}{132}$$

$$\therefore \frac{24}{132} < \frac{67}{132} < \frac{110}{132}$$

∴ The mean lies between the given rational numbers $\frac{2}{11}$ and $\frac{5}{6}$

Question 13.

If $\frac{1}{4}$ of a ragi adai weighs 120 grams, what will be the weight of 23 of the same ragi adai ?

Answer:

Let the weight of 1 ragi adai = x grams

given $\frac{1}{4}$ of x = 120gm

$\frac{1}{4} \times x = 120$

$x = 120 \times 4$

$x = 480$gm

$\therefore \frac{2}{3}$ of the adai $= \frac{2}{3} \times 480$ gm $= 2 \times 160$ gm $= 320$gm

$\frac{2}{3}$ of the weight of adai = 320gm

Question 14.

If p + 2q =18 and pq = 40, find $\frac{2}{p}+\frac{1}{q}$

Answer:

Given p + 2q = 18 ..... (1)

pq = 40 ..... (2)

$$\frac{2}{p}+\frac{1}{q} = \frac{(2\times q)+(1\times p)}{pq} = \frac{2q+p}{pq} = \frac{18}{40} \quad [\because \text{from (1) and (2)}]$$

$$\frac{2}{p}+\frac{1}{q} = \frac{9}{20}$$

Question 15.

Find x if $5\frac{x}{5} \times 3\frac{3}{4} = 21$.

Answer:

$$5\frac{x}{5} \times 3\frac{3}{4} = 21$$

$$\frac{25+x}{5} \times \frac{15}{4} = 21$$

$$\frac{25+x}{5} = 21 \div \frac{15}{4}$$

$$\frac{25+x}{5} = 21 \times \frac{4}{15}$$

$$\frac{25+x}{5} = \frac{28}{5}$$

$$25+x = \frac{28\times 5}{5}$$

25 + x = 28
x = 28 – 25
x = 3

Question 16.

A group of 1536 cadets wanted to have a parade forming a square design. Is it possible? If it is not possible, how many more cadets would be required?

Answer:

Number of cadets to form square design

$$1536 = \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2} \times 2 \times 3$$

2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
	3

The numbers 2 and 3 are unpaired
∴ It is impossible to have the parade forming square design with 1536 cadets.

```
    3  9
  ------
3 | 15 36
    9
  ------
61| 6 36
    6 21
  ------
      15
```

$39 \times 39 = 1521$
Also $40 \times 40 = 1600$
∴ We have to add $(1600 - 1536) = 64$ to make 1536 a perfect square.
∴ 64 more cadets would be required to form the square design.

Question 17.
Evaluate: $\sqrt{286225}$ and use it to compute $\sqrt{2862.25} + \sqrt{28.6225}$
Answer:

```
      5   3   5
    ------------
3 | 28  62  25
    25
    ------------
103 |  3  62
      3  09
      ---------
1065|     53  25
      53  25
      ---------
            0
```

$\therefore \sqrt{286225} = 535$

$$\sqrt{2862.25} = \sqrt{\frac{286225}{100}} = \frac{\sqrt{286225}}{100} = \frac{535}{10} = 53.5$$

$$\sqrt{28.6225} = \sqrt{\frac{286225}{1000}} = \frac{\sqrt{286225}}{10000} = \frac{535}{100} = 5.35$$

53.5
5.35
58.85

$\therefore \sqrt{2862.25} + \sqrt{28.6225} = 53.5 + 5.35 = 58.85$

Question 18.
Simplify: $(3.769 \times 10^5) + (4.21 \times 10^5)$
Answer:
$(3.769 \times 10^5) + (4.21 \times 10^5) = 3{,}76{,}900 + 4{,}21{,}000$
$= 7{,}97{,}000$
$= 7.979 \times 10^5$

Question 19.
Order the following from the least to the greatest: 16^{25}, 8^{100}, 3^{500}, 4^{400}, 2^{600}
Answer:
$16^{25} = (2^4)^{25} = 2^{100}$
$8^{100} = (2^3)^{100} = 2^{300}$

$4^{400} = (2^2)^{400} = 2^{800}$
$2^{600} = 2^{600}$
Comparing the powers we have.
$2^{100} < 2^{300} < 2^{600} < 2^{800}$
∴ The required order: 16^{25}, 8^{100}, 3^{500}, 4^{400}, 2^{600}