

Mathematical Reasoning

Short Answer Type Questions

Q. 1 Which of the following sentences are statements? Justify

- (i) A triangle has three sides.
- (ii) 0 is a complex number.
- (iii) Sky is red.
- (iv) Every set is an infinite set.
- (v) $15 + 8 > 23$.
- (vi) $y + 9 = 7$
- (vii) Where is your bag?
- (viii) Every square is a rectangle.
- (ix) Sum of opposite angles of a cyclic quadrilateral is 180° .
- (x) $\sin^2 x + \cos^2 x = 0$

Sol. As we know, a statement is a sentence which is either true or false but not both simultaneously.

- (i) It is true statement.
- (ii) It is true statement.
- (iii) It is false statement.
- (iv) It is false statement.
- (v) It is false statement.
- (vi) $y + 9 = 7$

It is not considered as a statement, since the value of y is not given.

- (vii) It is a question, so it is not a statement.
- (viii) It is a true statement.
- (ix) It is true statement.
- (x) It is false statement.

Q. 2 Find the component statements of the following compound statements.

- (i) Number 7 is prime and odd.
- (ii) Chennai is in India and is the capital of Tamil Nadu.
- (iii) The number 100 is divisible by 3, 11 and 5.
- (iv) Chandigarh is the capital of Haryana and UP.
- (v) $\sqrt{7}$ is a rational number or an irrational number.
- (vi) 0 is less than every positive integer and every negative integer.
- (vii) Plants use sunlight, water and carbon dioxide for photosynthesis.
- (viii) Two lines in a plane either intersect at one point or they are parallel.
- (ix) A rectangle is a quadrilateral or a 5 sided polygon.

Sol. (i) p : Number 7 is prime.

q : Number 7 is odd.

(ii) P : Chennai is in India.

q : Chennai is capital of Tamil Nadu.

(iii) p : 100 is divisible by 3.

q : 100 is divisible by 11.

r : 100 is divisible by 5.

(iv) p : Chandigarh is capital of Haryana.

q : Chandigarh is capital of UP.

(v) p : $\sqrt{7}$ is a rational number.

q : $\sqrt{7}$ is an irrational number.

(vi) p : 0 is less than every positive integer.

q : 0 is less than every negative integer.

(vii) p : Plants use sunlight for photosynthesis.

q : Plants use water for photosynthesis.

r : Plants use carbon dioxide for photosynthesis.

(viii) p : Two lines in a plane intersect at one point.

q : Two lines in a plane are parallel.

(ix) p : A rectangle, is a quadrilateral.

q : A rectangle is a 5-sided polygon.

Q. 3 Write the component statements of the following compound statements and check whether the compound statement is true or false.

- (i) 57 is divisible by 2 or 3.
- (ii) 24 is a multiple of 4 and 6.
- (iii) All living things have two eyes and two legs.
- (iv) 2 is an even number and a prime number.

Sol. (i) Given compound statement is of the form ' $p \vee q$ '. Since, the statement ' $p \vee q$ ' has the truth value T whenever either p or q or both have the truth value T .
So, it is true statement.

Its component statements are

p : 57 is divisible by 2. [false]

q : 57 is divisible by 3. [true]

- (ii) Given compound statement is of the form ' $p \wedge q$ '. Since, the statement ' $p \wedge q$ ' have the truth value T whenever both p and q have the truth value T .

So, it is a true statement.

Its component statements are

p : 24 is multiple of 4 [true]

q : 24 is multiple of 6. [true]

- (iii) It is a false statement. Since ' $p \wedge q$ ' has truth value F whenever either p or q or both have the truth value F .

Its component statements are

p : All living things have two eyes. [false]

q : All living things have two legs. [false]

- (iv) It is a true statement.

Its component statements are

p : 2 is an even number. [true]

q : 2 is a prime number. [true]

Q. 4 Write the negative on the following simple statements.

- (i) The number 17 is prime.
- (ii) $2 + 7 = 6$.
- (iii) Violets are blue.
- (iv) $\sqrt{5}$ is a rational number.
- (v) 2 is not a prime number.
- (vi) Every real number is an irrational number.
- (vii) Cow has four legs.
- (viii) A leap year has 366 days.
- (ix) All similar triangles are congruent.
- (x) Area of a circle is same as the perimeter of the circle.

Sol. (i) The number 17 is not prime.

(ii) $2 + 7 \neq 6$.

(iii) Violets are not blue.

(iv) $\sqrt{5}$ is not a rational number.

(v) 2 is a prime number.

(vi) Every real number is not an irrational number.

(vii) Cow has not four legs.

(viii) A leap year has not 366 days.

(ix) There exist similar triangles which are not congruent.

(x) Area of a circle is not same as the perimeter of the circle.

Q. 5 Translate the following statements into symbolic form

- (i) Rahul passed in Hindi and English.
- (ii) x and y are even integers.
- (iii) 2, 3 and 6 are factors of 12.
- (iv) Either x or $x + 1$ is an odd integer.
- (v) A number is either divisible by 2 or 3.
- (vi) Either $x = 2$ or $x = 3$ is a root of $3x^2 - x - 10 = 0$.
- (vii) Students can take Hindi or English as an optional paper.

Sol. (i) p : Rahul passed in Hindi.

q : Rahul passed in English.

$p \wedge q$: Rahul passed in Hindi and English.

(ii) p : x is even integers.

q : y is even integers.

$p \cap q$: x and y are even integers.

(iii) p : 2 is factor of 12.

q : 3 is factor of 12.

r : 6 is factor of 12.

$p \wedge q \wedge r$: 2, 3 and 6 are factor of 12.

(iv) p : x is an odd integer.

q : $(x + 1)$ is an odd integer.

$p \vee q$: Either x or $(x + 1)$ is an odd integer.

(v) p : A number is divisible by 2.

q : A number is divisible by 3.

$p \vee q$: A number is either divisible by 2 or 3.

(vi) p : $x = 2$ is a root of $3x^2 - x - 10 = 0$.

q : $x = 3$ is a root of $3x^2 - x - 10 = 0$.

$p \vee q$: Either $x = 2$ or $x = 3$ is a root of $3x^2 - x - 10 = 0$.

(vii) p : Students can take Hindi as an optional paper.

q : Students can take English as an optional subject.

$p \vee q$: Students can take Hindi or English as an optional paper.

Q. 6 Write down the negation of following compound statements.

- (i) All rational numbers are real and complex.
- (ii) All real numbers are rationals or irrationals.
- (iii) $x = 2$ and $x = 3$ are roots of the quadratic equation $x^2 - 5x + 6 = 0$.
- (iv) A triangle has either 3-sides or 4-sides.
- (v) 35 is a prime number or a composite number.
- (vi) All prime integers are either even or odd.
- (vii) $|x|$ is equal to either x or $-x$.
- (viii) 6 is divisible by 2 and 3.

💡 Thinking Process

Use (i) $\sim(p \wedge q) = \sim p \vee \sim q$

(ii) $\sim(p \vee q) = \sim p \wedge \sim q$

Sol. (i) Let p : All rational numbers are real.

q : All rational numbers are complex.

$\sim p$: All rational number are not real.

$\sim q$: All rational numbers are not complex.

$\sim(p \wedge q)$: All rational numbers are not real or not complex. $[\because \sim(p \wedge q) = \sim p \vee \sim q]$

(ii) Let p : All real numbers are rationals.

q : All real numbers are irrational.

Then, the negation of the above statement is given by

$\sim(p \vee q)$: All real numbers are not rational and all real numbers are not irrational.

$[\because \sim(p \vee q) = \sim p \wedge \sim q]$

(iii) Let p : $x = 2$ is root of quadratic equation $x^2 - 5x + 6 = 0$.

q : $x = 3$ is root of quadratic equation $x^2 - 5x + 6 = 0$.

Then, the negation of conjunction of above statement is given by

$\sim(p \wedge q)$: $x = 2$ is not a root of quadratic equation $x^2 - 5x + 6 = 0$ or $x = 3$ is not a root of the quadratic equation $x^2 - 5x + 6 = 0$.

(iv) Let p : A triangle has 3-sides.

q : A triangle has 4-sides.

Then, negation of disjunction of the above statement is given by

$\sim(p \vee q)$: A triangle has neither 3-sides nor 4-sides.

(v) Let p : 35 is a prime number.

q : 35 is a composite number.

Then, negation of disjunction of the above statement is given by

$\sim(p \vee q)$: 35 is not a prime number and it is not a composite number.

(vi) Let p : All prime integers are even.

q : All prime integers are odd.

Then negation of disjunction of the above statement is given by

$\sim(p \vee q)$: All prime integers are not even and all prime integers are not odd.

(vii) Let p : $|x|$ is equal to x .

q : $|x|$ is equal to $-x$.

Then negation of disjunction of the above statement is given by

$\sim(p \vee q)$: $|x|$ is not equal to x and it is not equal to $-x$.

(viii) Let p : 6 is divisible by 2.

q : 6 is divisible by 3.

Then, negation of conjunction of above statement is given by

$\sim(p \wedge q)$: 6 is not divisible by 2 or it is not divisible by 3

Q. 7 Rewrite each of the following statements in the form of conditional statements.

- (i) The square of an odd number is odd.
- (ii) You will get a sweet dish after the dinner.
- (iii) You will fail, if you will not study.
- (iv) The unit digit of an integer is 0 or 5, if it is divisible by 5.
- (v) The square of a prime number is not prime.
- (vi) $2b = a + c$, if a , b and c are in AP.

Sol. We know that, some of the common expressions of conditional statement $p \rightarrow q$ are

- (i) if p , then q
- (ii) q if p
- (iii) p only if q
- (iv) p is sufficient for q
- (v) q is necessary for p
- (vi) $\sim q$ implies $\sim p$

So, use above information to get the answer

- (i) If the number is odd number, then its square is odd number.
- (ii) If you take the dinner, then you will get sweet dish.
- (iii) If you will not study, then you will fail.
- (iv) If an integer is divisible by 5, then its unit digits are 0 or 5.
- (v) If the number is prime, then its square is not prime.
- (vi) If a , b and c are in AP, then $2b = a + c$.

Q. 8 Form the biconditional statement $p \leftrightarrow q$, where

- (i) p : The unit digits of an integer is zero.
 q : It is divisible by 5.
- (ii) p : A natural number n is odd.
 q : Natural number n is not divisible by 2.
- (iii) p : A triangle is an equilateral triangle.
 q : All three sides of a triangle are equal.

Sol. (i) $p \leftrightarrow q$: The unit digit of an integer is zero, if and only if it is divisible by 5.

(ii) $p \leftrightarrow q$: A natural number n is odd if and only if it is not divisible by 2.

(iii) $p \leftrightarrow q$: A triangle is an equilateral triangle if and only if all three sides of triangle are equal.

Q. 9 Write down the contrapositive of the following statements.

- (i) If $x = y$ and $y = 3$, then $x = 3$.
- (ii) If n is a natural number, then n is an integer.
- (iii) If all three sides of a triangle are equal, then the triangle is equilateral.
- (iv) If x and y are negative integers, then xy is positive.
- (v) If natural number n is divisible by 6, then n is divisible by 2 and 3.
- (vi) If it snows, then the weather will be cold.
- (vii) If x is a real number such that $0 < x < 1$, then $x^2 < 1$.

Thinking Process

We know that, the statement $(\sim q) \rightarrow (\sim p)$ is called contrapositive of the statement $p \rightarrow q$.

- Sol.**
- (i) If $x \neq 3$, then $x \neq y$ or $y \neq 3$.
 - (ii) If n is not an integer, then it is not a natural number.
 - (iii) If the triangle is not equilateral, then all three sides of the triangle are not equal.
 - (iv) If xy is not positive integer, then either x or y is not negative integer.
 - (v) If natural number n is not divisible by 2 or 3, then n is not divisible by 6.
 - (vi) The weather will not be cold, if it does not snow.
 - (vii) If $x^2 \not< 1$, then x is not a real number such that $0 < x < 1$.

Q. 10 Write down the converse of following statements.

- (i) If a rectangle ' R ' is a square, then R is a rhombus.
- (ii) If today is Monday, then tomorrow is Tuesday.
- (iii) If you go to Agra, then you must visit Taj Mahal.
- (iv) If sum of squares of two sides of a triangle is equal to the square of third side of a triangle, then the triangle is right angled.
- (v) If all three angles of a triangle are equal, then the triangle is equilateral.
- (vi) If $x : y = 3 : 2$, then $2x = 3y$.
- (vii) If S is a cyclic quadrilateral, then the opposite angles of S are supplementary.
- (viii) If x is zero, then x is neither positive nor negative.
- (ix) If two triangles are similar, then the ratio of their corresponding sides are equal.

Thinking Process

We know that, the converse of the statement " $p \rightarrow q$ " is " $(q) \rightarrow (p)$ ".

- Sol.**
- (i) If the rectangle ' R ' is rhombus, then it is square.
 - (ii) If tomorrow is Tuesday, then today is Monday.
 - (iii) If you must visit Taj Mahal, you go to Agra.
 - (iv) If the triangle is right angle, then sum of squares of two sides of a triangle is equal to the square of third side.
 - (v) If the triangle is equilateral, then all three angles of triangle are equal.

- (vi) If $2x = 3y$, then $x:y = 3:2$
 (vii) If the opposite angles of a quadrilateral are supplementary, then S is cyclic.
 (viii) If x is neither positive nor negative, then x is 0.
 (ix) If the ratio of corresponding sides of two triangles are equal, then triangles are similar.

Q. 11 Identify the quantifiers in the following statements.

- (i) There exists a triangle which is not equilateral.
 (ii) For all real numbers x and y , $xy = yx$.
 (iii) There exists a real number which is not a rational number.
 (iv) For every natural number x , $x + 1$ is also a natural number.
 (v) For all real numbers x with $x > 3$, x^2 is greater than 9.
 (vi) There exists a triangle which is not an isosceles triangle.
 (vii) For all negative integers x , x^3 is also a negative integers.
 (viii) There exists a statement in above statements which is not true.
 (ix) There exists an even prime number other than 2.
 (x) There exists a real number x such that $x^2 + 1 = 0$.

Sol. Quantifier are the phrases like 'There exist' and 'For every', 'For all' etc.

- | | |
|--------------------|---------------------|
| (i) There exists | (ii) For all |
| (iii) There exists | (iv) For every |
| (v) For all | (vi) There exists |
| (vii) For all | (viii) There exists |
| (ix) There exists | (x) There exists |

Q. 12 Prove by direct method that for any integer ' n ', $n^3 - n$ is always even.

Thinking Process

We know that, in direct method to show a statement, if p then q is true, we assume p is true and show q is true i.e., $p \rightarrow q$.

Sol. Here, two cases arise

Case I When n is even,

$$\begin{aligned} \text{Let } n &= 2K, K \in N \\ \Rightarrow n^3 - n &= (2K)^3 - (2K) = 2K(4K^2 - 1) \\ &= 2\lambda, \text{ where } \lambda = K(4K^2 - 1) \end{aligned}$$

Thus, $(n^3 - n)$ is even when n is even.

Case II When n is odd,

$$\begin{aligned} \text{Let } n &= 2K + 1, K \in N \\ \Rightarrow n^3 - n &= (2K + 1)^3 - (2K + 1) \\ &= (2K + 1)[(2K + 1)^2 - 1] \\ &= (2K + 1)[4K^2 + 1 + 4K - 1] \\ &= (2K + 1)(4K^2 + 4K) \\ &= 4K(2K + 1)(K + 1) \\ &= 2\mu, \text{ when } \mu = 2K(K + 1)(2K + 1) \end{aligned}$$

Then, $n^3 - n$ is even when n is odd.

So, $n^3 - n$ is always even.

Q. 13 Check validity of the following statement.

(i) p : 125 is divisible by 5 and 7.

(ii) q : 131 is a multiple of 3 or 11.

Sol. (i) p : 125 is divisible by 5 and 7.

Let q : 125 is divisible by 5.

r : 125 is divisible by 7.

q is true, r is false.

$\Rightarrow q \wedge r$ is false.

[since, $p \wedge q$ has the truth value F (false) whenever either p or q or both have the truth value F.]

Hence, p is not valid.

(ii) p : 131 is a multiple of 3 or 11.

Let q : 131 is multiple of 3.

r : 131 is a multiple of 11.

p is true, r is false.

$\Rightarrow p \vee r$ is true.

[since, $p \vee q$ has the truth value T (true) whenever either p or q or both have the truth value T]

Hence, q is valid.

Q.14 Prove the following statement by contradiction method

p : The sum of an irrational number and a rational number is irrational.

Sol. Let p is false i.e., sum of an irrational and a rational number is rational.

Let $\sqrt{m}$ is irrational and n is rational number.

$\Rightarrow \sqrt{m} + n = r$

[rational]

$\Rightarrow \sqrt{m} = r - n$

$\sqrt{m}$ is irrational, where as $(r - n)$ is rational. This is contradiction.

Then, our supposition is wrong.

Hence, p is true.

Q. 15 Prove by direct method that for any real number x, y if $x = y$, then

$$x^2 = y^2.$$

💡 Thinking Process

In direct method assume p is true and show q is true i.e., $p \Rightarrow q$.

Sol. Let $p : x = y, \quad x, y \in R$

On squaring both sides,

$$x^2 = y^2 : q$$

[say]

$$p \Rightarrow q$$

Hence, we have the result.

Q. 16 Using contrapositive method prove that, if n^2 is an even integer, then n is also an even integer.

💡 Thinking Process

In contrapositive method assume $\sim q$ is true and show $\sim p$ is true i.e., $q \Rightarrow \sim p$.

Sol. Let $p : n^2$ is an even integer.

$q : n$ is also an even integer.

Let $\sim p$ is true i.e., n is not an even integer.

$\Rightarrow n^2$ is not an even integer.

[since, square of an odd integer is odd]

$\Rightarrow \sim p$ is true.

Therefore, $\sim q$ is true $\Rightarrow \sim p$ is true.

Hence proved.

Objective Type Questions

Q. 17 Which of the following is a statement?

- (a) x is a real number
- (b) Switch off the fan
- (c) 6 is a natural number
- (d) Let me go

Sol. (c) As we know a statement is a sentence which is either true or false.
So, 6 is a natural number, which is true.
Hence, it is a statement.

Q. 18 Which of the following is not a statement.

- (a) Smoking is injurious to health
- (b) $2 + 2 = 4$
- (c) 2 is the only even prime number
- (d) Come here

Sol. (d) 'Come here' is not a statement. Since, no sentence can be called a statement, if it is an order.

Q. 19 The connective in the statement ' $2 + 7 > 9$ or $2 + 7 < 9$ ' is

- (a) and
- (b) or
- (c) $>$
- (d) $<$

Sol. (b) In ' $2 + 7 > 9$ or $2 + 7 < 9$ ', or is the connective.

Q. 20 The connective in the statement "Earth revolves round the Sun and Moon is a satellite of earth" is

- (a) or
- (b) Earth
- (c) Sun
- (d) and

Sol. (d) Connective word is 'and'.

- Q. 21** The negation of the statement "A circle is an ellipse" is
- (a) An ellipse is a circle
 - (b) An ellipse is not a circle
 - (c) A circle is not an ellipse
 - (d) A circle is an ellipse

Sol. (c) Let p : A circle is an ellipse.
 $\sim p$: A circle is not an ellipse.

- Q. 22** The negation of the statement "7 is greater than 8" is
- (a) 7 is equal to 8
 - (b) 7 is not greater than 8
 - (c) 8 is less than 7
 - (d) None of these

Sol. (b) Let p : 7 is greater than 8.
 $\sim p$: 7 is not greater than 8.

- Q. 23** The negation of the statement "72 is divisible by 2 and 3" is
- (a) 72 is not divisible by 2 or 72 is not divisible by 3
 - (b) 72 is not divisible by 2 and 72 is not divisible by 3
 - (c) 72 is divisible by 2 and 72 is not divisible by 3
 - (d) 72 is not divisible by 2 and 72 is divisible by 3

Sol. (b) Let p : 72 is divisible by 2 and 3.
Let q : 72 is divisible by 2.
 r : 72 is divisible by 3.
 $\sim q$: 72 is not divisible by 2.
 $\sim r$: 72 is not divisible by 3.
 $\sim(q \wedge r) : \sim q \vee \sim r$
 $\Rightarrow$ 72 is not divisible by 2 or 72 is not divisible by 3.

- Q. 24** The negation of the statement "Plants take in CO_2 and give out O_2 " is
- (a) Plants do not take in CO_2 and do not given out O_2
 - (b) Plants do not take in CO_2 or do not give out O_2
 - (c) Plants take is CO_2 and do not give out O_2
 - (d) Plants take in CO_2 or do not give out O_2

Sol. (b) Let p : Plants take in CO_2 and give out O_2 .
Let q : Plants take in CO_2 .
 r : Plants give out O_2 .
 $\sim q$: Plants do not take in CO_2 .
 $\sim r$: Plants do not give out O_2 .
 $\sim(q \wedge r) : \text{Plants do not take in } \text{CO}_2 \text{ or do not give out } \text{O}_2$.

- Q. 25** The negative of the statement "Rajesh or Rajni lived in Bangaluru" is
- (a) Rajesh did not live in Bengaluru or Rajni lives in Bengaluru
 - (b) Rajesh lives in Bengaluru and Rajni did not live in Bengaluru
 - (c) Rajesh did not live in Bengaluru and Rajni did not live in Bengaluru
 - (d) Rajesh did not live in Bengaluru or Rajni did not live in Bengaluru