

Surface Areas and Volumes of Solids

- Space occupied by a solid shape is its volume while the maximum quantity of liquid that it can hold shows its capacity.
- When each of the length, breadth and height of a cube measures 1 cm, its volume is said to be 1 cubic centimeter. It is written as c.c. or cm^3 , which is the fundamental unit of volume. This cube is called the unit cube of side 1 cm.



Example:

By using the small cube in figure (a), find the volume of the solid in figure (b).

Figure (a)

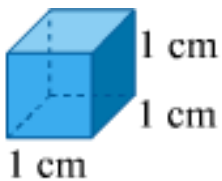
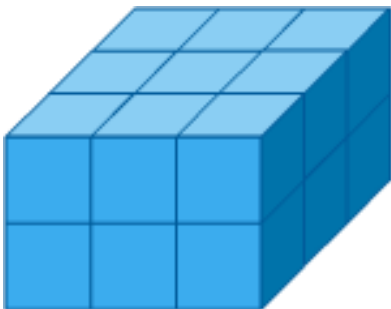


Figure (b)



Solution:

Volume of smaller cube in figure (a) = 1 cm^3

It can be observed that the solid in figure (b) consists of 18 cubes like figure (a).

$$\therefore \text{Volume of solid} = (18 \times 1) \text{ cm}^3 = 18 \text{ cm}^3$$

- **Volume of cube and cuboid**

- Volume of cube = a^3 , where a is the side of the cube
- Volume of cuboid = $l \times b \times h$, where l , b and h are respectively the length, breadth and height of the cuboid.

Example:

What is the side of a cube of volume 512 cm^3 ?

Solution:

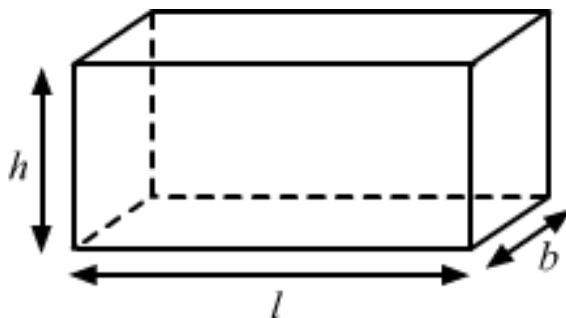
$$\text{Volume of cube} = 512 \text{ cm}^3$$

$$\Rightarrow a^3 = 512 \text{ cm}^3$$

$$\Rightarrow a = \sqrt[3]{512 \text{ cm}^3}$$

$$\Rightarrow a = 8 \text{ cm}$$

- **Surface areas of cuboid:**



$$\text{Lateral surface area of the cuboid} = 2h(l + b)$$

$$\text{Total surface area of the cuboid} = 2(lb + bh + hl)$$

Note: Length of the diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

Example:

Find the edge of a cube whose surface area is 294 m^2 .

Solution:

Let the edge of the given cube be a .

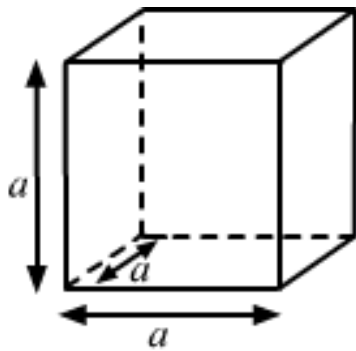
$$\therefore \text{Surface area of the cube} = 6a^2$$

$$\text{Given, } 6a^2 = 294$$

$$\Rightarrow a^2 = 49 \text{ m}^2$$

$$\therefore a = \sqrt{49} \text{ m} = 7 \text{ m}$$

- **Surface areas of cube:**



$$\text{Lateral surface area of the cube} = 4a^2$$

$$\text{Total surface area of the cube} = 6a^2$$

Note: Length of the diagonal of a cube $= \sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \sqrt{3}a$

Cross Section

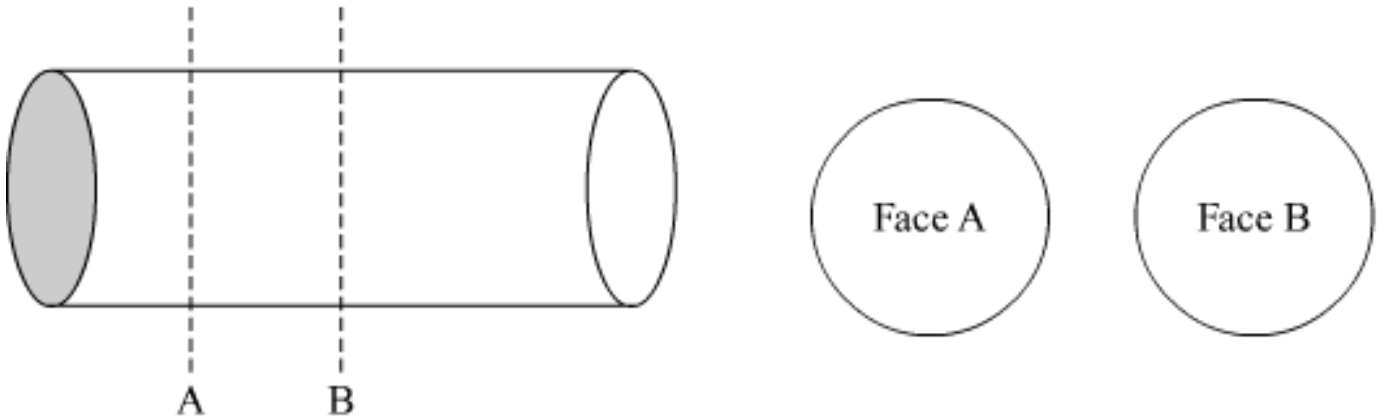
It is a cut which is made through a solid perpendicular to its length.
Cross Section is of two types:

(i) Uniform Cross Section:

A solid is said to have Uniform cross section, if the perpendicular cut is of the same shape and size at each point of its length.

Example:

When a cylinder is cut through the points A and B perpendicular to its length, the faces obtained as the cross section at both of the points are of same shape and size.



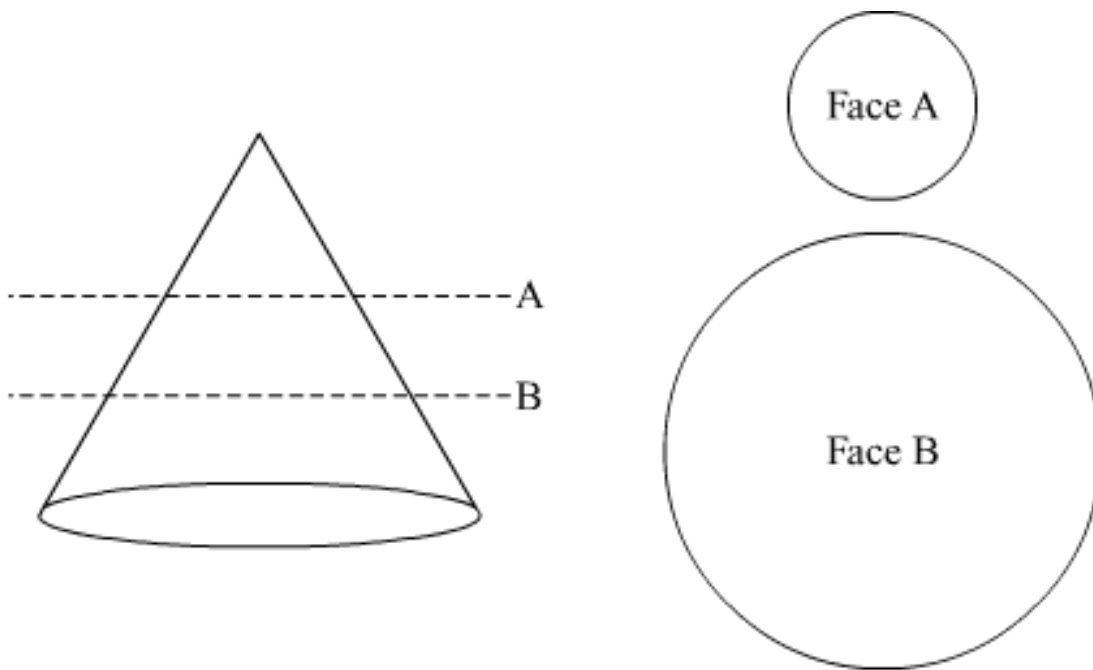
So, cylinder has a uniform cross section.

(ii) Non- uniform Cross Section:

A solid is said to have Non-Uniform cross section, if the perpendicular cut is not of the same shape and size at each point of its length.

Example:

When a cone is cut through the points A and B perpendicular to its length, the faces obtained as the cross section at both of the points are not of same shape and size.



Let us find out volume and surface area of **uniform** cross section body.

1) Volume = Area of cross section $\times$ length

2) Surface area (excluding cross-section) = Perimeter of cross section $\times$ length

Flow of Water (or any other liquid)

If water is flowing through a pipe of uniform cross section, then the volume of the water flowing out in unit time is Area of the cross section multiplied by speed of the flow.

i.e volume of the water flowing out = Area of the cross section $\times$ speed of the flow