Network Synthesis

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Before any network is synthesized, we have to check whether the given network is :

1. **Physically realisable:** For a physically realisable network, all the element values must be positive. Therefore, all the poles of the given network function must lie in the left half of the s-plane.
2. **Stable:** For a stable network, the output of the network must be finite for the finite input.

Condition for Physically Realisability & Stability of a Network

$$F(s) = \frac{\text{Numerator of polynomial}}{\text{Denominator of polynomial}}$$

(a) **Network is always physically realisable when**

Any network function F(s) with poles in left half of s-plane has inverse Laplace transform, which is zero for $t < 0$. Therefore, such a network will be a causal network and therefore, will always be a physically realisable network.

(b) **For stability of a network:**

1. The network function F(s) cannot have poles in the right half of the s-plane.
2. F(s) cannot have multiple poles on the $j\omega$ axis.
3. The degree of numerator of the network function F(s) cannot exceed the degree of denominator by more than unity.

Note:

Any stable network will always be a physically realisable network but the reverse may not be true.

Feature of Hurwitz Polynomial

1. The Hurwitz polynomial represents the denominator polynomial of a stable and physically realizable network function.
2. This polynomial represents whether all the poles of the network function lie in left half of s-plane or not.

3. No term in a polynomial must be missing unless all the even parts or all the odd parts are missing.
4. No coefficient in the polynomial must be negative.
5. The continue fraction expansion (C.F.E.) of even to odd parts or odd to even parts has all positive quotient terms. These quotient terms represent the numerical value of RLC component of the network.
6. If any polynomial have only even parts or odd parts, then the given polynomial P(s) is Hurwitz when the continued fraction expansion of $\frac{P(s)}{P'(s)}$ has all positive quotient term.
7. If $P(s) = P_1(s) \cdot P_2(s)$, then the polynomial P(s) is Hurwitz when the polynomial $P_1(s)$ and $P_2(s)$ are individually Hurwitz.

Positive Real Function (PRF)

The positive real functions represent physically realizable and stable passive driving point immittance function.

Features of PRF

1. If F(s) is positive real function, then $\frac{1}{F(s)}$ is also a PRF.
2. The sum of two PRF is also a PRF.
3. The poles and zeroes of a PRF cannot lie in right half of the s-plane.
4. Only simple poles with positive real residue can exist on jω axis.
5. The poles and zeroes of a PRF are real or occur in complex conjugate pair.
6. The highest and lowest power of the numerator and denominator polynomial may differ at most by unity.

The Necessary and Sufficient Condition for F(s) to be PRF

1. F(s) must have no poles and zeros in the right half of s-plane, therefore the numerator and denominator polynomial must be Hurwitz.
2. F(s) may have only simple poles on the jω axis with positive real residues. Therefore, the partial fraction expansion is found and is verify for the residues of the poles which lie on the jω axis.
3. $\text{Re}\left[F(s)\right]_{s=j\omega} \geq 0$, for all values of ω.

Synthesis of Different Network Function

Case-1: Synthesis of LC immittance function (Z_{LC}, Y_{LC})

Features

1. This function is a ratio of odd to even or even to odd polynomials.
2. The poles and zero are simple, lie on the jω axis and they alternate.
3. There must be either a zero or a pole at the origin and at ∞.
4. Highest power of numerator and denominator must differ by unity.
5. Lowest powers of numerator and denominator polynomial must also differ at most by unity.

Case-2: Synthesis of RC impedance function or RL admittance function (Z_{RC}, Y_{RL})

Features

1. All the poles and zeros lie on negative real axis and they alternate.
2. The singularity nearest to or at the origin must be a pole where as the singularity nearest to or at ∞ must be a zero.
3. The residues of the poles must be real and positive.

Case-3: Synthesis of RL impedance function or RC admittance function (Z_{RL}, Y_{RC})

Features

1. All the poles and zeroes lie on negative real axis and they alternate.
2. The singularity nearest to or at the origin must be a zero, where as the singularity nearest to or at ∞ must be a pole.
3. The residues of the poles must be real and negative.

Case-4: Driving point RLC immittance function (Z_{RLC}, Y_{RLC})

Features

No set rules are follow for the location of the poles and zeros. Therefore, the poles and zeros can lie anywhere in the left half of s-plane.

Synthesis of Network Function by Cauer's Form

1. In both Cauer's type-I and type-II forms, the element are connected in series-parallel-series-parallel form.
2. If an impedance function is given, then the first element represents impedance and therefore represents a series element in the ladder network.
3. If an admittance function is given, the first element represents admittance and therefore, represents a parallel element.
4. In a RLC network, the number of elements is equal to sum of total number of zeros and poles.
5. In Cauer's type-I form, inductor is always a series element where as a capacitor is a shunt element.
6. In Cauer's type-II form, the capacitor is always a series element where as inductor is a shunt element.
7. In Cauer's type-I form, continued fraction expansion (CFE) is found by rearranging the numerator and denominator polynomial in the descending power of s.
8. In Cauer's type-II form, continued fraction expansion is found by rearranging numerator and denominator polynomial in ascending power of s.