Geometrical Constructions

13

We have learnt to draw line segment with a scale and angles with compass and protector. Now we shall learn to draw some closed shapes.

Let us Construct

If the length of three line segments are given, then is it always possible to construct a triangle with the given measurement? Discuss among yourselves.

If the three sides of a triangle are of 5 *cm*, 6 *cm* and 7 *cm*, then can we construct a triangle? Let us try:-

1. Draw a line segment QR of 6 *cm* (*Fig.*1(i)).
2. Spread the arms of the compass upto 5 *cm*, place one arm on point Q and make an arc. (*Fig.*1(ii)).
3. Spread the arms of compass upto 7 *cm* and place it on point R and then make an arc which intersects the first arc at point P. (*Fig.*1(iii))
4. P is the intersection point.
5. Join P with R and Q.

Fig. 1 (i)

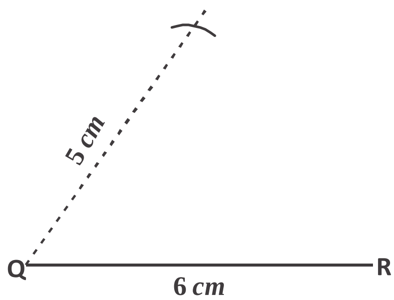


Fig. 1 (ii)

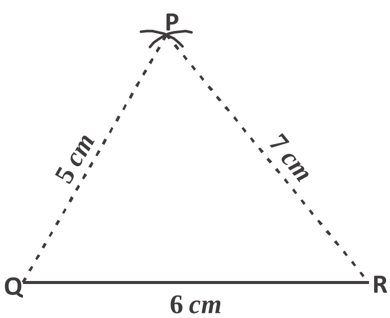


Fig. 1 (iii)

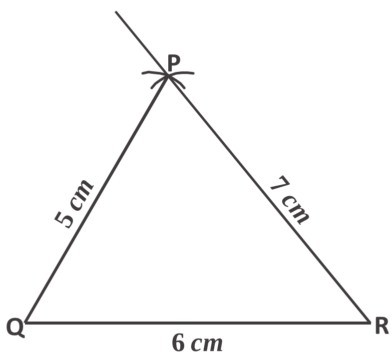
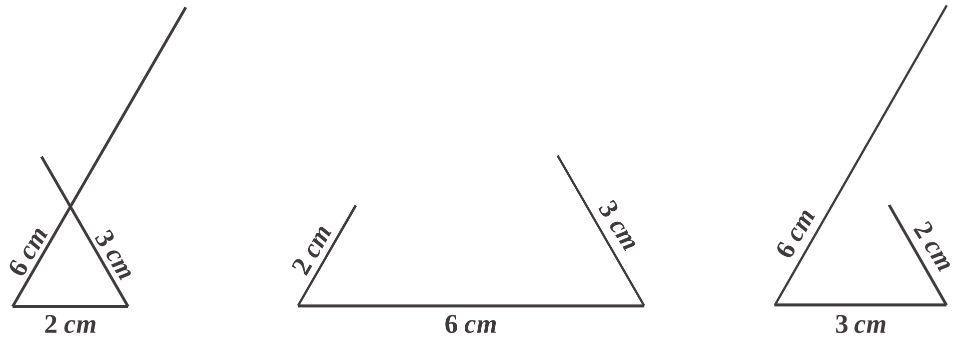


Fig. 1 (iv)

6. Thus the triangle PQR is constructed. (*Fig.*1(iv)).

Jayant tried to construct a triangle with sides 2 *cm*, 3 *cm* and 6 *cm*.



Can a triangle be constructed with these measurements? Why?

Try This

Is it possible to construct triangles with the following measurements?

(i) (2 *cm*, 3*cm*, 4 *cm*) (ii) (3 *cm*, 4 *cm*, 5 *cm*)

(iii) (2*cm*, 4 *cm*, 8 *cm*) (iv) (4 *cm*, 5 *cm*, 6 *cm*)

Of the given measurements, triangles can be formed only if the sum of two small sides is bigger than the measurement of the longest side.

Some More Constructions

Construction-1 : Construct a triangle when the measurement of two sides and the angle formed by them is given.

Example-1. Construct a triangle ABC where AB = 5 *cm*, AC = 4 *cm* and $\angle A = 45°$.

Steps of Constructions

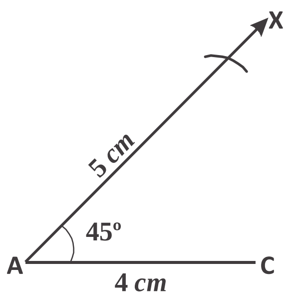


Fig. 2 (i)

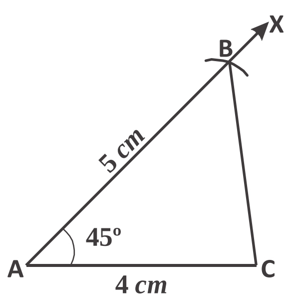


Fig. 2 (ii)

1. Draw a line segment AC of 4 *cm.*
2. Draw a ray AX on A which forms an angle of 45° with AC.
3. Draw an arc of 5 *cm* from point A, which cuts AX at point B. (*Fig*.2(i))
4. Draw a line segment joining points B and C. This way ΔABC is constructed. (*Fig*.2(ii)).

Try This

In this triangle AB = 5 *cm*, AC = 4 *cm* and $\angle$A = 45°. If we want we can draw a line segment AB of 5 *cm* and then a ray AY making on angle of 45° on AB.

Now from A make an arc of 4 *cm* on AC.

Is triangle ACB like the first triangle?

Do This Also

Construct a triangle ABC where

AB = 7 *cm*, AC = 6 *cm* and $\angle$B = 40°

Steps of construction:-

1. First of all draw a line segment AB = 7 *cm*
2. At point B draw a ray BX such that $\angle$ABX = 40º (*Fig*.3 (i))
3. From point A draw an arc of radius 6 *cm* intersecting ray BX at points C and D. (*Fig*.3(ii)).

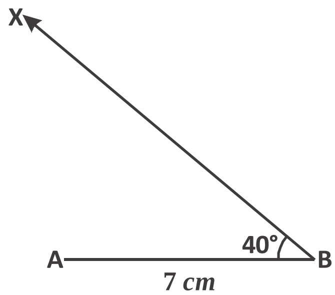


Fig. 3 (i)

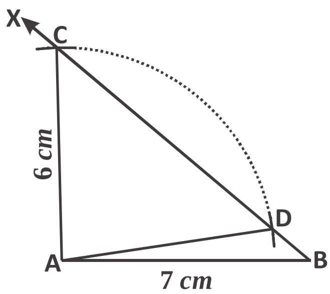


Fig. 3 (ii)

You can see that with the given conditions we get two points C and D on the ray BX. Therefore, it can be said that with the given measurements of the triangle two points A and B can be definitely determined; but the third point can be either C or D. As the third point can be either C or D, therefore the measurements given are not sufficient to construct a unique triangle.

Try This

Discuss with friends the measurements given to construct triangle:-

(i) AB = 7 *cm*, AC = 6 *cm*, $\angle$C = 40°

(ii) AB = 3 *cm*, BC = 4 *cm*, $\angle$A = 60°

(iii) PR = 6 *cm*, PQ = 5 *cm*, $\angle$Q = 75°

(iv) AB = 4.5 *cm*, AC = 6.3 *cm*, $\angle$A = 55°

You have seen that a unique triangle can be constructed only when the measurements of two sides and the angle formed by them is given.

Construction-2 : Construct a triangle when the measurement of one side and the two angles on it's two end points are given.

EXAMPLE-2. To construct a triangle ABC where AB = 6 *cm*; ∠BAC = 30°, ∠ABC = 100°.

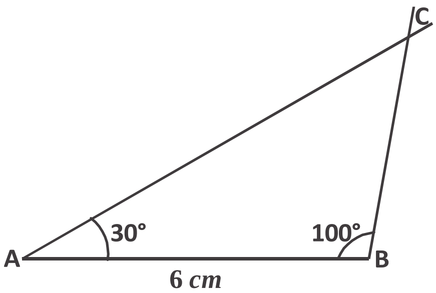


Fig. 4 (ii)

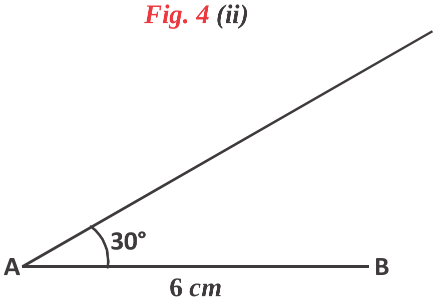


Fig. 4 (i)

Steps of construction:-

1. Draw a line segement AB = 6 *cm.*
2. On line segment AB draw an angle of 30° at point A with the help of the protractor. (*Fig.*4 (i))
3. Similarly at point B draw an angle of 100°.
4. Extent the arms of both the angles. Let the point of intersection be 'C'.
5. Then ABC is the required triangle (*Fig.*4 (ii)).

Try This

1. Construct a triangle with the given measurements and discuss which type of triangle are these:-
   (i) In ΔPQR, PQ = 5 *cm*, ∠P = 90°, ∠Q = 30°
   (ii) In ΔMNP, MN = 6 *cm*, ∠M = 90°, ∠N = 30°
2. Draw and see if it is possible to construct triangle of given measurement:-
   (i) PQ = 3.5 *cm*, ∠Q = 45°, ∠R = 50°
   (ii) XY = 7.5 *cm*, ∠Z = 70°, ∠Y = 40°

Special Type of Triangles

Construction-3 : To construct such a triangle where the base, angle formed on the base and the sum of remaining two sides are given.

EXAMPLE-3. Construct a triangle PQR where QR = 4 *cm,* PQ + PR = 7.5 *cm* and ∠PQR = 60°.

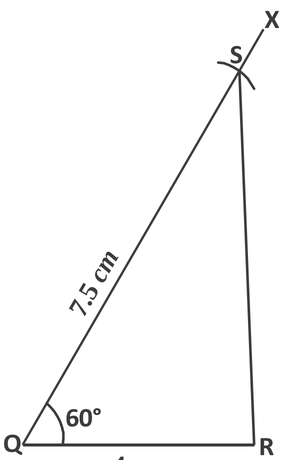


Fig. 5 (i)

Steps of Construction:-

1. Draw line segment QR = 4 *cm* and at point Q draw an ∠XQR = 60°.
2. With Q as centre, draw an arc of radius 7.5 cm intersecting QX at point S. Join RS. (*Fig.*5(i)).

3. With the help of compass draw a perpendicular bisector l of RS which cuts QS at point P and SR at point T. (*Fig*.5(ii)).

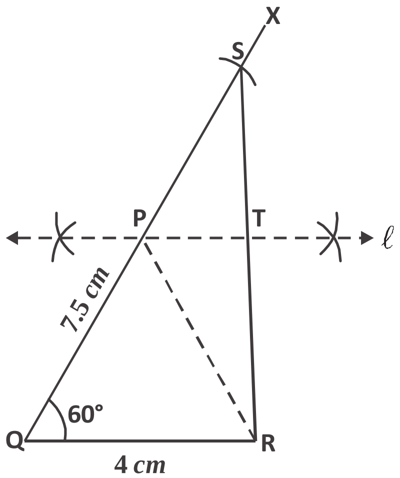
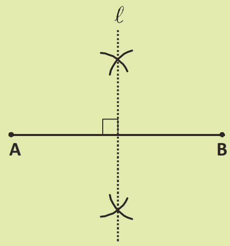


Fig. 5 (ii)

> **Perpendicular bisector :** Perpendicular bisector is that line which divides any line segment into two equal parts by forming right angle.
>
> **Construction of Perpendicular Bisector:**
>
> 1. Distance between two arms of the compass should be more than half of the line segment.
> 2. Now from point A cut an arc on both the sides of the line segment. Then from point B repeat the same process.
> 3. Join the cut points of both the arcs with a scale.
>
> This line l is the perpendicular bisector of AB.



4. Joint PR (*Fig*.5(iii)).

$\Delta PTS \cong \Delta PTR$. (Why?)

$\therefore$ PS = PR (CPCT)

QP + PS = QP + PR (=7.5 *cm*)

Therefore, ΔPQR is the required triangle.

Why is step 3 constructed like this?

We should locate point P on the side QS such that PS = PR

This could be done if both line segments could be seen as corresponding sides of two congruent triangles.

The perpendicular bisector of SR gives two such points P and T which divide ΔPSR into two congruent triangle by line segment PT.

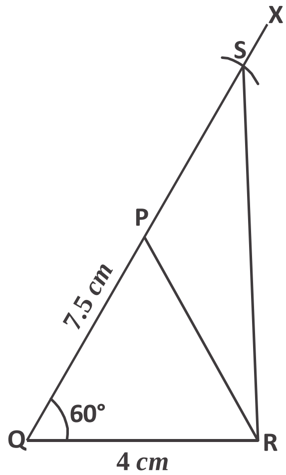


Fig. 5 (iii)

Alternate Method

Now we shall construct the same triangle in a different way.

Steps of Construction:-

1. Repeat steps 1 and 2 like. (*Fig*.6(i)).
2. Construct an $\angle SRY$ equal to $\angle QSR$. Intersecting QX at point P. (*Fig*.6(ii))

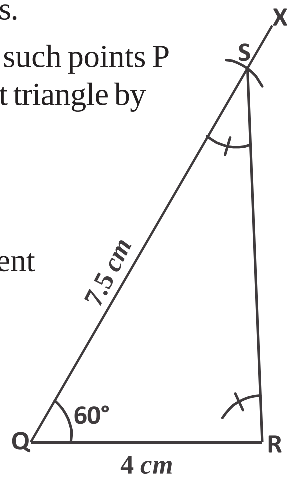


Fig. 6 (i)

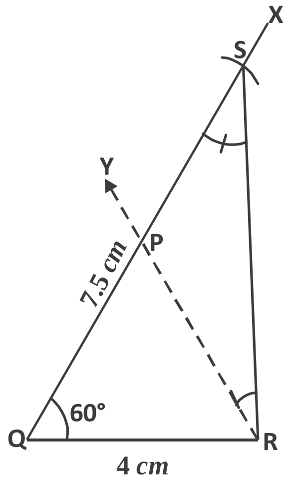
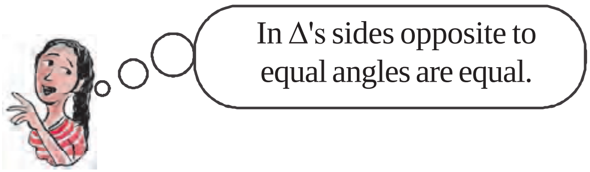


Fig. 6 (ii)

PS = PR (Why?)

QP + PS = QP + PR = 7.5 *cm*.

$\therefore$ PQR is the required triangle.



Try This

Construct a triangle ABC where BC = 6 *cm*, ∠B = 60° and AB+AC = 11 *cm*.

<u>Construction-4</u> : To construct such a triangle where base, an angle on base and difference between two remaining sides are given.

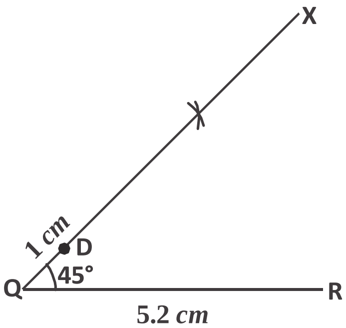
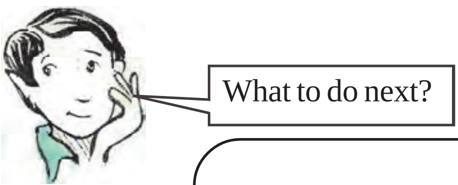


Fig. 7 (i)

EXAMPLE-4. Construct a triangle PQR where the base QR = 5.2 *cm*, ∠PQR = 45° and PQ – PR = 1 *cm*.

Steps of construction

1. Draw a line segment QR = 5.2 *cm*.
2. At point Q make an ∠XQR = 45°. On ray QX take a point D such that QD = 1 *cm* (PQ – PR = 1 *cm*) (*Fig.*7(i)).



I'll tell you!

On QX we need a point P such that PD = PR (then the difference between PQ and PR will be 1 *cm*.)

If PD and PR could be seen as corresponding sides of two congruent triangles then PD will be equal to PR. If we draw perpendicular bisector of side DR then we get two congruent triangles PDT and PRT.

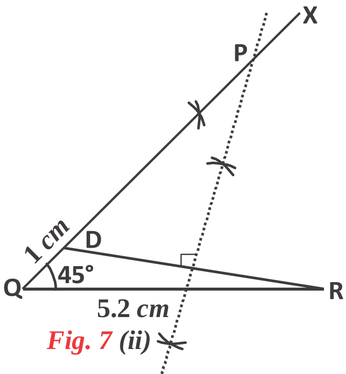


Fig. 7 (ii)

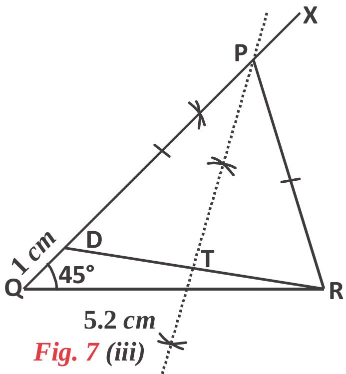


Fig. 7 (iii)

3. Join the points R and D. Draw a perpendicular bisector of the arm RD which cuts the ray QX at P. (*Fig.*7(ii)).
4. Join points P and R. Thus the required ΔPQR is constructed. (*Fig.*7(iii))

Construction-5 : To construct a triangle where the sum of three sides (perimeter) and the two base angles are given.

Example-5. Construct a triangle ABC where ∠B = 30°, ∠C = 60° and AB + BC + CA = 10.5 *cm.*

Steps of Construction

1. Draw a line segment PQ = 10.5 *cm* (PQ = BC + CA + AB)
2. Construct angles of measure 30° (equivalent to ∠B) and 60° (equivalent to ∠C) at point P & Q respectively. Now construct angle bisector of both the above angles & name their point of intersection as A. (*Fig.*8(i)).

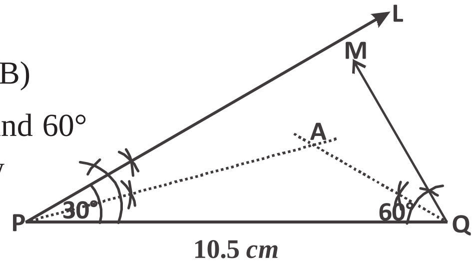


Fig. 8 (i)

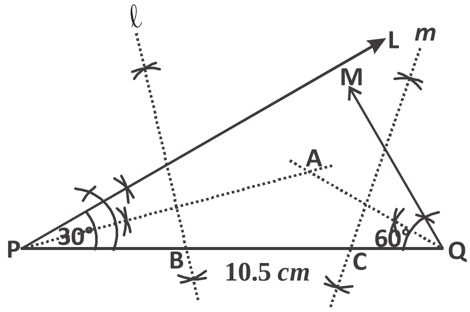


Fig. 8 (ii)

3. Now draw perpendicular bisector *l* and *m* of side PA and QA respectively which bisect the line segment PQ at the points B and C. (*Fig.*8(iii)).

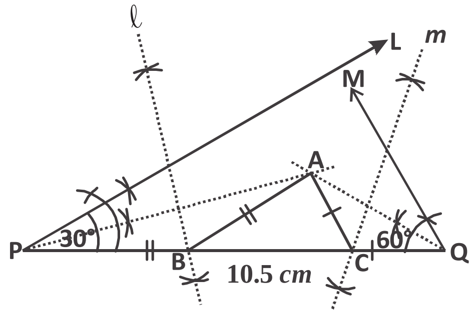


Fig. 8 (iii)

4. Join point B and point C with A. (*Fig.*8(iii))

   ΔABC is the required triangle.

Try This

Measure the three sides of the triangle which you have constructed and add them. Is AB + BC + CA = 10.5 *cm?*

If one side of the triangle is extended, the external angle formed on the extended side is equal to the sum of the two internally opposite angles.

Exercise - 13.1

1. Construct triangles by the given measurements of their sides and angles.

S.No.	Traingle	Given Measurements		
(i)	ΔDEF	DE = 4.5 *cm*	EF = 5.5 *cm*	DF = 4 *cm*
(ii)	ΔPQR	∠Q = 30°	∠R = 30°	QR = 4.7 *cm*
(ii)	ΔABC	∠B = 60°	BC = 5 *cm*	AB + AC = 8 *cm*

2. Construct a right angled triangle with a base of 4 *cm* and the sum of the other sides is 8 *cm*.
3. Construct a triangle PQR where QR = 7 *cm*, $\angle$Q = 45° and PQ – PR = 2 *cm*.
4. Construct a traingle XYZ where $\angle$XYZ = 50°, YZ = 5 *cm* and XZ – XY = 2.5 *cm*.
5. Construct a triangle ABC where AB + BC + CA = 13 *cm* and $\angle$B = 45°, $\angle$C = 70°.

Construction of Quadrilateral

So far you have constructed quadrilaterals in different situation. Now we shall construct these in some new situations.

Construction-6 : Construct a parallelogram where its two diagonals and the angle formed between them is given.

EXAMPLE-6. Construct a parallelogram ABCD where AC = 7 *cm* and BD = 6 *cm* and the angle formed between them is of 40°.

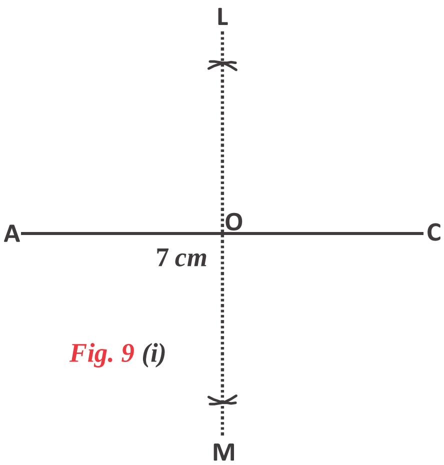


Fig. 9 (i)

Steps of construction:-

1. Draw a line segment AC = 7 *cm*.
2. Draw a perpendicular bisector LM of line segment AC which intersects AC at O. (*Fig*.9(i))
3. Draw a ray OX where $\angle$AOX = 40°. Now extend ray OX to XOX'. (*Fig*.9(ii)).
4. Taking O as the centre point, draw two arcs of 3 *cm* (half of the length of another diagonal i.e. half of 6*cm* = 3 *cm*) on XOX' namely B and D. Join both B and D with A and C. (*Fig*.9(iii)).

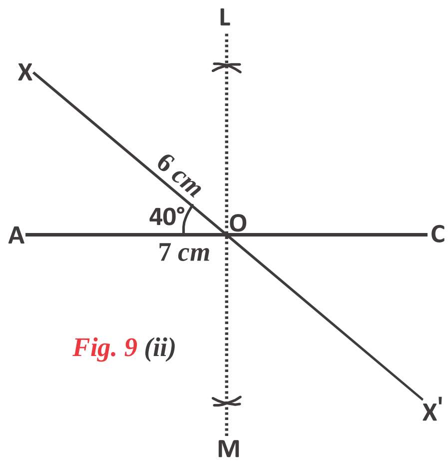


Fig. 9 (ii)

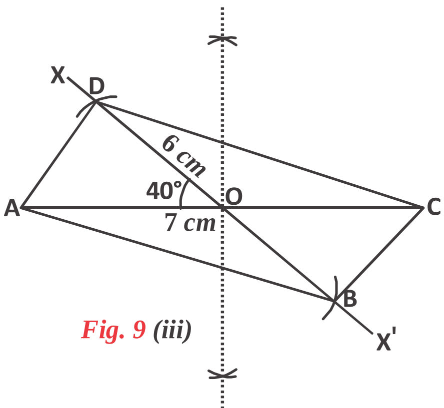


Fig. 9 (iii)

Thus, the required parallelogram ABCD is constructed.

Remember:- Diagonals in a parallelogram bisect each other. Therefore we draw perpendicular bisector of AC by which we got the central point O. On point O, we constructed an $\angle AOX = 40°$ and OB = OD = 3 *cm*.

Try This

Can you similarly construct a rectangle and a square?

Discuss with friends and form two questions based on it and construct it.

Construction-7 : Construct a trapezium when two adjacent sides, the angle formed by them and the parallel sides are known.

EXAMPLE-7. Construct a trapezium ABCD when AB = 5 *cm*, BC = 2.8 *cm*, AD = 3 *cm*, $\angle A = 60°$ and AB || CD.

Steps of construction:-

1. Draw a line segment AB = 5 *cm*.
2. Draw a ray AX by forming an angle of 60° at point A.

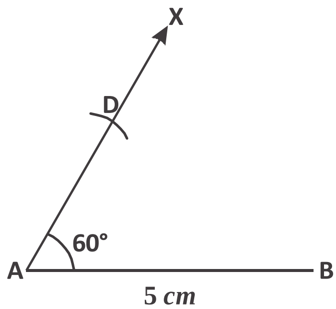


Fig. 10 (i)

3. Cut an arc AD of 3 *cm* on AX to get point D. (*Fig*.10(i)).

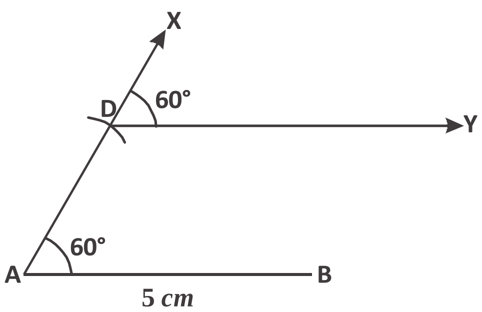


Fig. 10 (ii)

4. From point D draw a ray DY such that $\angle XDY = 60°$ (*Fig*.10(ii))

5. From point B, draw an arc of 2.8 *cm* intersecting DY at points C and C' (*Fig*.10(iii)).

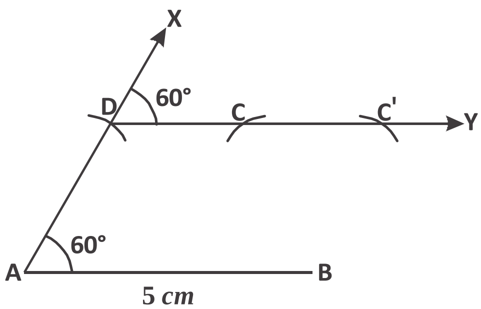


Fig. 10 (iii)

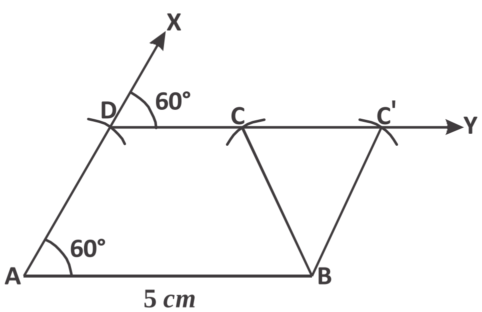


Fig. 10 (iv)

6. Join B with C and C'. (*Fig*.10 (iv)). This way the required trapezium ABCD and ABC'D is formed.

Exercise - 13.2

1. Construct a parallelogram ABCD where AD = 4 *cm*, AB = 6 *cm* and $\angle A = 65°$.
2. Construct a parallelogram where AB = 4 *cm*, AD = 3 *cm* and diagonal AC = 4.5 *cm*.
3. Construct a rectangle where one side is of 3 *cm* and the diagonal is of 5 *cm*.
4. Construct a rhombus where the two diagonals are of lengths 4.5 *cm* and 6 *cm* respectively.
5. Construct a trapezium ABCD where AB || CD, AB = 5 *cm*, BC = 3 *cm*, AD = 3.5 *cm* and the distance between the parallel lines is 2.5 *cm*.

Construction-8 : To construct a triangle which is equal in area to the area of a given quadrilateral.

EXAMPLE-8. Construct a quadrilateral where AB = 7 *cm*, CD = 6 *cm*, BC = 4 *cm*, AD = 5 *cm* and $\angle BAD = 60°$.

And taking AB as one side construct a triangle which is equal in area to the area of a quadrilateral.

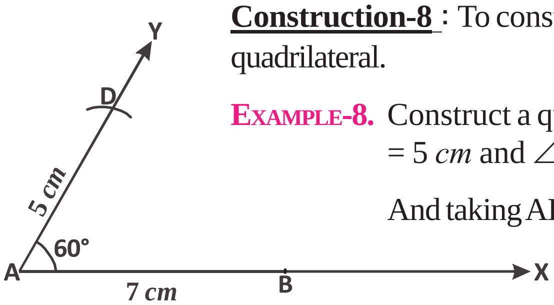


Fig. 11 (i)

Steps of construction:-

1. Draw a ray AX. On AX mark line segment AB of 7 *cm*.
2. On point A draw an $\angle BAY = 60°$ and cut an arc of 5 *cm* which cuts AY at D. (*Fig.*11(i))
3. Cut arc of length 4 *cm* & 6 *cm* from points B & D respectively, intersecting at point C.
4. Join BC, CD. Quadrilateral ABCD is the required quadrilateral. (*Fig.*11 (ii))

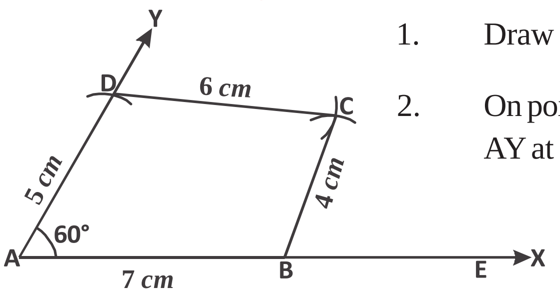


Fig. 11 (ii)

5. Now join BD. From point C draw CE || BD which cuts AX at E. (*Fig.*11(iii)). Join E and D. (*Fig.*11(iv))

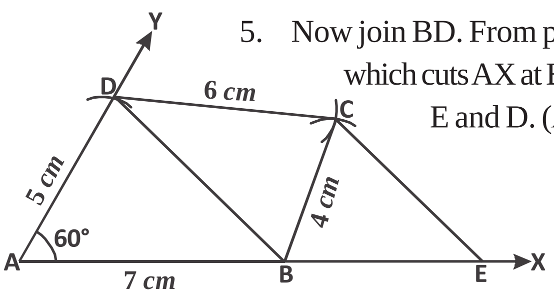


Fig. 11 (iii)

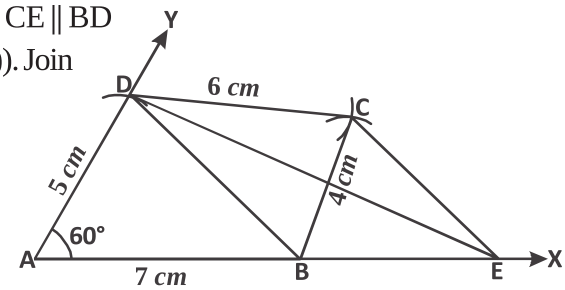


Fig. 11 (iv)

Thus, we get the required triangle which is equal in area to the area of quadrilateral ABCD.

Think and Discuss

Areas of ΔADE and quadrilateral ABCD are equal. How?

Construction-9 : Construct a parallelogram and rectangle which is equal in area to the area of a given triangle.

EXAMPLE-9. Construct a parallelogram which is equal in area to that of a triangle ABC where AB = 9 *cm*, AC = 5 *cm* and ∠CAB = 60°.

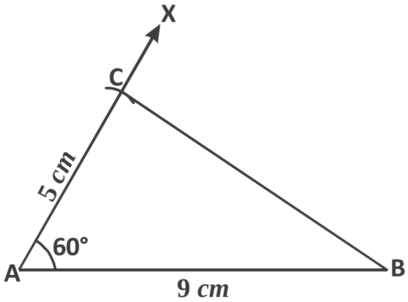


Fig. 12 (i)

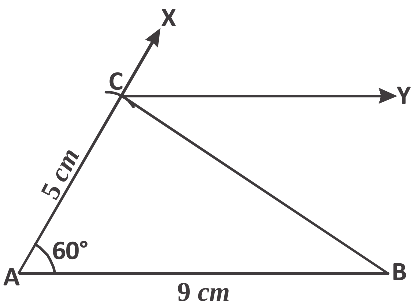


Fig. 12 (ii)

Steps of construction:-

1. Draw a line segment AB = 9 *cm* and an ∠BAX of 60° at A.
2. On ray AX cut an arc of 5 *cm* at point C. Join BC. Required triangle ABC is formed (*Fig*.12(i)).
3. From point C draw a ray CY parallel to AB. (*Fig*.12(ii)).
4. Draw a perpendicular bisector PQ of side AB which bisects AB at O. (*Fig*.12(iii)).
5. From point O draw OD|| AC (*Fig*.12(iv))

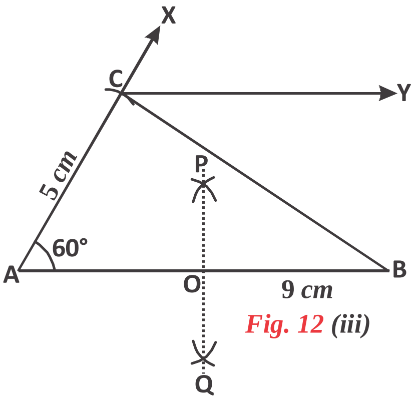


Fig. 12 (iii)

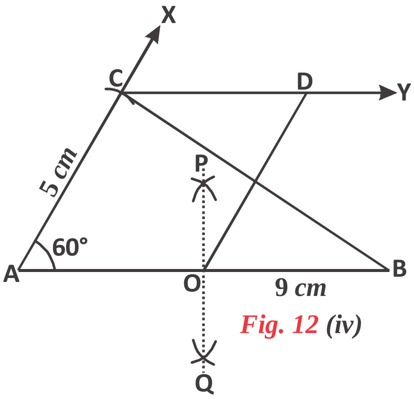


Fig. 12 (iv)

Thus, we get the required parallelogram which is equal in area to the area of ΔABC. Discuss why ?

Exercise - 13.3

1. Construct a quadrilateral ABCD where AB = 5 *cm*, BC= 6 *cm*, CD = 7 *cm* and ∠B = ∠C = 90°. Then on AB as base construct a triangle which is equal in area to that of the quadrilateral.
2. Construct a triangle whose area is equal to the area of the rhombus whose sides are of 6 *cm* and one angle of 60°.

3. Construct an isosceles triangle with a base of 6 *cm* and base angles of 70°, construct a parallelogram and rectangle which is equal in area to that of the triangle.
4. Construct a traingle PQR where PQ = 8 *cm*, PR = 6 *cm*, ∠QPR = 65°. Construct a parallelogram whose area is equal to the area of the triangle.

Constructing a Circumscribed Regular Polygon Around a Circle and Inscribed Regular Polygon in a Circle

Construction-10 : Construct a regular pentagon inscribed in a 3 *cm* radius circle.

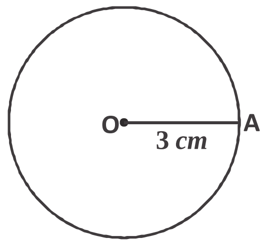


Fig. 13 (i)

Steps of construction:-

1. With centre O draw a circle of radius 3 *cm* Join O with point A on the circumference. (*Fig.*13(i)).
2. Since we have to construct a regular pentagon therefore divide the circle into 5 equal parts. The value of an angle subtended at the centre would be $\dfrac{360°}{5} = 72°$ (Why?).

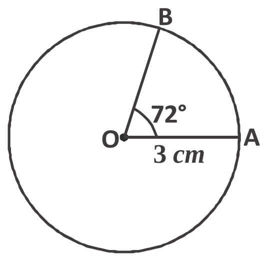


Fig. 13 (ii)

3. On OA, draw an angle of 72° at point O which cuts the circumference at B. (*Fig.*13(ii)).
4. Measure the arc AB with the compass and mark arcs on the circumference and we get points C, D and E. (*Fig.*13(iii)).

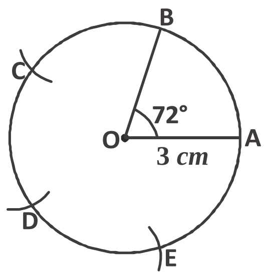


Fig. 13 (iii)

5. Joint A with B, B with C, C with D, D with E and E with A. (*Fig.*13(iv)).

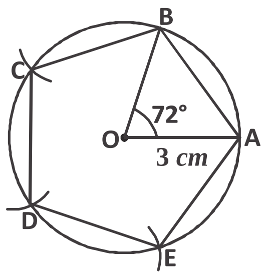


Fig. 13 (iv)

This way a required regular pentagon is obtained.

Similarly any regular polygon can be inscribed in a circle.

Think and Discuss

For pentagon the angle at the centre is $\dfrac{360°}{5}$, for hexagon it is $\dfrac{360°}{6}$, so would the angle at the centre of a polygon of n sides be $\dfrac{360°}{n}$?

<u>Construction-11</u> : To construct a regular hexagon circumscribed around a circle of radius 3.5 *cm*.

Steps of construction:-

1. With centre O draw a circle of radius 3.5 *cm* Take a point A on the circumference and join it with O. (*Fig.*14(i))

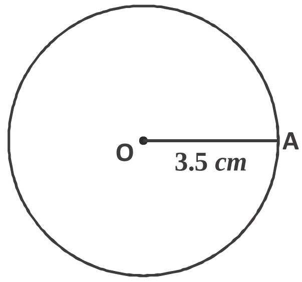


Fig. 14 (i)

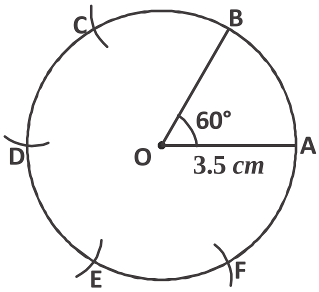


Fig. 14 (ii)

2. The value of the internal angle of regular hexagon at the circle will be $= \dfrac{360°}{6} = 60°$. On OA draw an angle of 60° at point O which cuts the circumference at B.

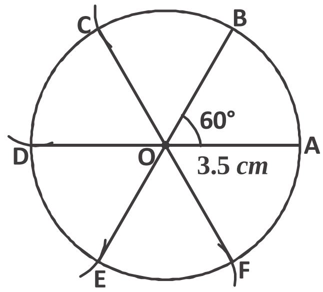


Fig. 14 (iii)

3. As in construction-10 with arc AB mark the points C, D, E and F (*Fig.*14(ii)).

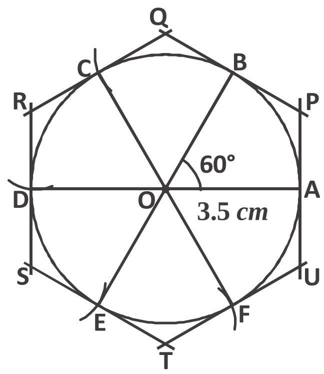


Fig. 14 (iv)

4. Joint points C, D, E and F with the centre O. (*Fig.*14(iii)).

5. On OA, OB, OC, OD, OE and OF draw perpendicular lines UAP, PBQ, QCR, RDS, SET and TFU. (*Fig.*14(iv)).

Thus, we get the required hexagon PQRSTU which is the circumscribing the circle.

Similarly in any circle we can construct a regular polygon inscribing or circumscribing it.

Exercise - 13.4

1. Construct a regular quadrilateral inscribed in a circle of a radius of 2 *cm*.
2. Construct a regular octagon inscribed in a circle of a radius of 3 *cm*.
3. Construct a regular pentagon circumscribed around a circle of radius 2.5 *cm*.
4. Construct a regular octagon circumscribed around a circle of radius 3 *cm*.

What Have We Learnt

1. A triangle can be constructed only when:-
   - (i) The sum of two small sides is bigger than the measurement of the longest side.
   - (ii) Measurement of two sides and angle formed by them is given.
   - (iii) Measurement of one side and angles on both its ends are given.
   - (iv) When the base of a triangle, any one angle on the base and sum of the remaining two sides given.
   - (v) When the base of a triangle, any one angle on the base and the difference between the remaining two sides is given.
   - (vi) When perimeter of a triangle and both angles on the base are given.
2. A parallelogram can be constructed when its two diagonals and angle between them is given.
3. Trapezium can be constructed when two adjacent sides, angle formed by them and parallel sides are given.
4. Area of two triangles formed on one base and between same two parallel lines, is equal.
5. Angles formed at the centre by each sides of polygon of n sides will be $\frac{360°}{n}$.
6. A regular polygon inscribed in a circle and circumscribed around a circle can be constructed.