# 10. Locus

## Exercise 10.1

### 1. Question

Write true or false out of the following statements and give the justification of your answer.

(i) The set of points situated at equal distances from a line is a line.

(ii) A circle is the locus of those points which lie at a fixed distance from a given point.

(iii) Three given points will be collinear when they are not the members of the set of points of a line.

(iv) The locus of points equidistant from two lines will be a line parallel to both the lines.

(v) The locus of point equidistant from the given points is the perpendicular bisector of the line joining both the points.

**Answer**

(i) False

The locus of the points situated at equal distances from a line is the lines parallel to the line on its both sides.

(ii) True

(iii) False

Three given points will be collinear only when they belong to the same set of numbers.

(iv) False

It is not always true as if the lines are parallel then it will be a line parallel to them while in case of intersecting lines the line will be the bisector of the angle formed at the point of intersection.

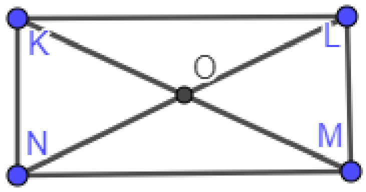
(v) True

### 2. Question

The diagonals of a quadrilateral bisect each other.

Prove that this quadrilateral as a parallelogram.

**Answer**




Given here a quadrilateral of sides MN , NK, KL and ML

Also it is given that the diagonals of the quadrilateral bisect each other.

To prove: Quad KLMN is a parallelogram

Proof: As the diagonals bisect each other,

So, NO=OL and KO=OM…………….(1)

Now, in ΔKOL and ΔNOM,

∠KOL=∠NOM (vertically opposite angles)

KO=OM (from (1))

NO=OL (from (1))

∴ ΔKOL ≅ ΔNOM (by SAS rule)

So, KL=NM (by cpct)

∠OMN=∠OKL (by cpct)……….(2)

∠OLK=∠ONM (by cpct)…………(3)

Similarly, ΔKON ≅ ΔLOM (by SAS rule)

So, KN=LM (by cpct)

∠ONK=∠OLM (by cpct)……….(4)

∠OKN=∠OML (by cpct)…………(5)

From (2) and (3) ,

We can say they are alternate interior angles.

So, KL||NM

Similarly, From (4) and (5) ,

We can say they are alternate interior angles.

So, KN||ML

Hence, KLMN is a parallelogram.

**3. Question**

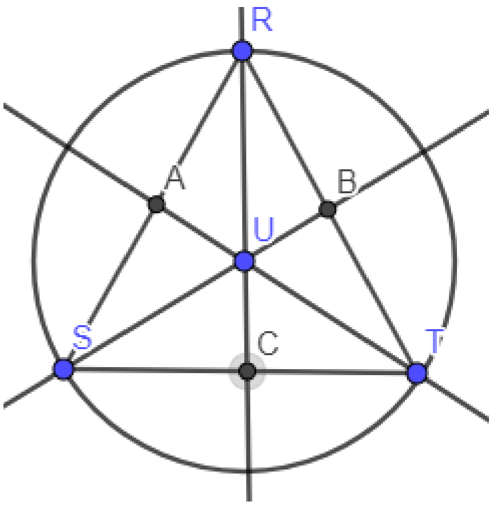
What will the locus of a point equidistant from three non-collinear points A, B and C? Explain the reason of your answer.

**Answer**

The locus of a point equidistant from three non-collinear points A, B and C is the circumcentre of the triangle.

The locus of the points is the point of intersection of the perpendicular bisectors of each of the two points. The circumcentre is equidistant from all the three points A, B and C.

Example:



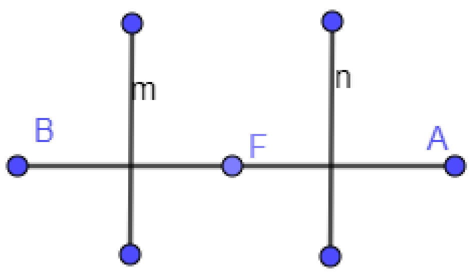

Here in ΔRST, U is the circumcentre of the triangle and RC ,SB and TA are the perpendicular bisectors of sides ST, RT and RS.

**4. Question**

What will be the locus of a point equidistant from three collinear points? Explain the reason of your answer.

**Answer**

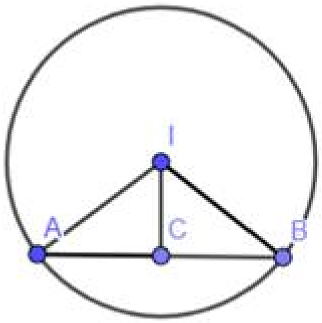
The locus of a point equidistant from three collinear points A, F and B is a null set that means there is no such locus of points.




**5. Question**

Prove that the locus of the centres of circles passing through points A and B is the perpendicular bisector of line segment AB.

**Answer**




Given that a circle with centre I and passing through points A and B

To prove: IC is the perpendicular bisector of AB

Proof: Here, IA=IB (radius)

So, ΔIAB is isosceles triangle.

So, ∠IAC=∠IBC………………..(1)

As we know that the altitude IC on AB is perpendicular bisector of AB.

Hence, ∠ICA=∠ICB=90°

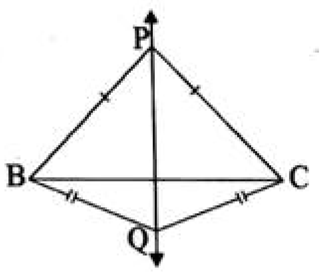
And AC=BC

As ΔICA and ΔICB are congruent triangles.

Proved.

**6. Question**

In the given figure on the common base BC there are two isosceles triangles ΔPBC and ΔQBC on the opposite sides of BC. Prove that the line joining the points P and Q bisects line BC at right angles.




**Answer**

In the given figure ΔPBC and ΔQBC are isosceles triangles.

So, PB=PC and BQ=QC

To prove: line joining the points P and Q bisects line BC at right angles.

Proof: Now, in $\Delta PBQ$ and $\Delta PCQ$

PB=PC (given)

BQ=QC (given)

PQ=PQ (common)

$\therefore$ $\Delta PBQ \cong \Delta PCQ$ (by SSS rule)

$\angle BPO = \angle CPO$ (by cpct)………(1)

And $\angle BQO = \angle CQO$ (by cpct)…………(2)

Now in $\Delta BOP$ and $\Delta COP$,

BP=CP (given)

OP=OP (common)

$\angle BPO = \angle CPO$ (from (1))

$\therefore$ $\Delta BOP \cong \Delta COP$ (by SAS rule)

So, BO=CO (by cpct)

$\angle BOP = \angle COP$ (by cpct)………(3)

Now, $\angle BOP + \angle COP = 180°$

$\Rightarrow \angle BOP + \angle BOP = 180°$ (from (3))

$\Rightarrow 2\angle BOP = 180°$

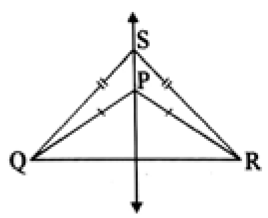
$\Rightarrow \angle BOP = 90°$

Hence , $\angle BPO = \angle CPO = 90°$

$\therefore$ PQ bisects BC at right angles.

**7. Question**

In the given figure two isosceles triangle PQR and SQR lie on the same side of the common base QR. Prove that line SP is the perpendicular bisector of line QR.




**Answer**

Given that $\Delta PQR$ and $\Delta SQR$ lie on the same side of the common base QR

Also, $\Delta PQR$ and $\Delta SQR$ are isosceles triangles .

So, QS=RS and QP=RP…………(1)

To prove : SP is the perpendicular bisector of line QR

Proof : In $\Delta PQS$ and $\Delta PRS$,

QS=RS (given)

QP=RP (given)

SP=SP (common)

$\therefore \Delta PQS \cong \Delta PRS$ (by SSS rule)

So, $\angle PSQ = \angle PSR$ (by cpct)……………..(2)

Now, in $\Delta QSO$ and $\Delta RSO$,

QS=RS (given)

SO=SO (common)

$\angle PSQ = \angle PSR$ (from (2))

$\therefore \Delta QSO \cong \Delta RSO$ (by SAS rule)

So, QO=RO (by cpct)

And $\angle QOS = \angle ROS$ (by cpct)………..(3)

Now, $\angle QOS + \angle ROS = 180°$ (straight angle)

$\Rightarrow \angle QOS + \angle QOS = 180°$ (from (3))

$\Rightarrow 2\angle QOS = 180°$

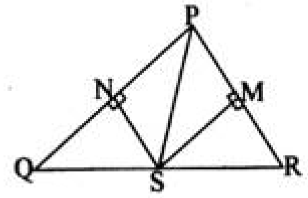
$\Rightarrow \angle QOS = 90°$

Hence, SO is the perpendicular bisector of QR

So, SP is also the perpendicular bisector of QR

**8. Question**

In the given figure bisector PS of $\angle P$ intersects side QR at S. SN $\perp$ PQ and SM $\perp$PR have been drawn. Prove that SN = SM.




**Answer**

Given that PS is the bisector of ∠P intersects side QR at S.

Also, SN ⊥ PQ and SM ⊥PR

So, ∠NPS=∠MPS…………(1)

And ∠PNS=∠PMS ……………..(2)

In ΔPNS and ΔPMS

∠NPS+∠PNS+∠PSN=180°

And ∠MPS+∠PMS+∠PSM=180°

From (1) and (2)

180°-∠PSN=180°-∠PSM

⇒ ∠PSN=∠PSM…………..(3)

Now, in ΔPNS and ΔPMS,

PS=PS (common)

∠NPS=∠MPS (given)

∠PSN=∠PSM (from (3))

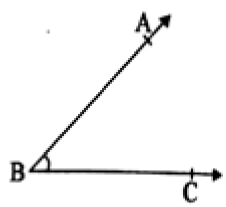
∴ ΔPNS ≅ ΔPMS (by ASA rule)

So, NS=MS (by cpct)

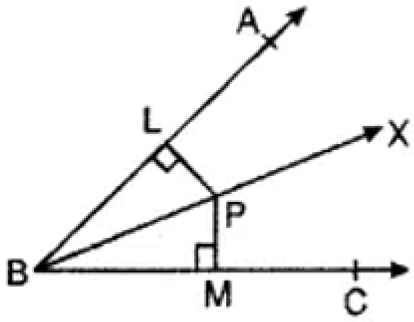
Hence, proved.

**9. Question**

In the given figure ∠ABC is given. Find the locus of points equidistant from BA and BC and lying in the interior part of ∠ABC.




**Answer**




Draw angle bisector BX of ∠ABC.

Take any point P on BX. Now draw ⊥ on AB and BC.

PL ⊥ AB and PM ⊥ BC

∴ ∠PLB = ∠PMB = 90°

In Δ PLB and Δ PMB,

∠PLB = ∠PMB (by construction)

∠LBP = ∠PBM (BP is bisector of ∠B)

BP = BP (common)

∴ Δ PLB ≅ Δ PMB (By AAS congruency)

⇒ PL = PM (By CPCT)

Thus, point P is the interior of ∠ABC and equidistant from AB and BC. So P is the locus.

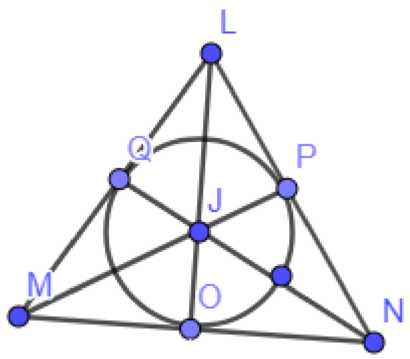
## Exercise 10.2

### 1. Question

Find the locus of point ℓ equidistant from three vertices and three sides of a triangle.

### Answer

The locus of point ℓ equidistant from three vertices is the circumcentre and three sides of a triangle is the incentre of the triangle.



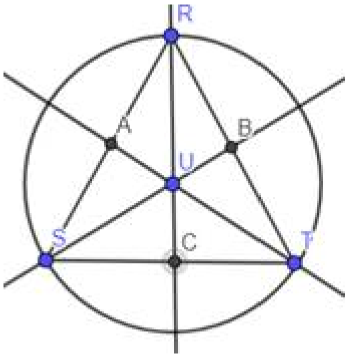
Here, J is the incentre of the ΔLMN.

At first draw the angle bisectors of all the angles i.e, ∠L , ∠M and ∠N.

LO, MP and NQ are the angle bisectors respectively.

Let LO, MP and NQ intersect at J,

∴ J is the incentre.




Here, U is the circumcenter of the ΔRST.

At first , draw the perpendicular bisectors of the sides ST, RT and RS of the triangle.

Here, RC, SB and AT are the required perpendicular bisectors.

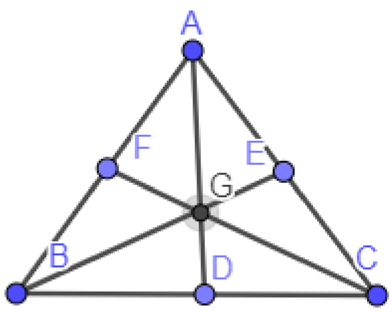
Lat them intersect at U

Hence, U is the circumcenter of the triangle.

**2. Question**

In a ΔABC the median AD, BE and CF intersect at a point G. If AG = 6 cm, BE = 9 cm and GF = 4.5 cm, then find GD, BG and CF.

**Answer**




Given that in ΔABC , medians AD, BE and CF intersects at a point G.

AG = 6 cm, BE = 9 cm and GF = 4.5 cm

As we know that if the medians in a triangle intersect at a point then the point is the centroid of the triangle and the centroid divides the median in 2:1 ratio.

Here, G is the centroid of the ΔABC.

And AG:GD = 2:1,

BG:GE = 2:1 ,

CG:GF = 2:1

Now, $\frac{AG}{GD} = \frac{2}{1}$

$\Rightarrow \frac{6}{GD} = \frac{2}{1}$

$\Rightarrow GD = \frac{6}{2} = 3$

GD=3 cm

Again, $\frac{CG}{GF} = \frac{2}{1}$

$\Rightarrow \frac{CG}{4.5} = \frac{2}{1}$

⇒ CG=4.5×2=9

CG=9 cm

BE = BG+GE………………(1)

$\frac{BG}{GE} = \frac{2}{1}$

⇒BG=2GE

Putting in equation (1)

9 = 2GE+GE

⇒ 3GE=9

⇒ GE = 3

GE = 3 cm

Hence, GD=3 cm, CG=9 cm and GE = 3 cm

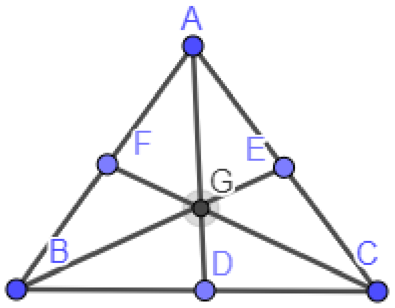
**3. Question**

The median AD, BE and CF in a ΔABC intersect at point G. Prove that

$$AD + BE > \frac{3}{2}AB$$

[**Hint :**AG + BG > AB]

**Answer**




As we know that in a triangle, sum of any two sides is always greater than the third side.

So, in ΔAGB,

AG+GB>AB…………….(1)

Also, AD=AG+GD

And BE=BG+GE

As we know that if the medians in a triangle intersect at a point then the point is the centroid of the triangle and the centroid divides the median in 2:1 ratio.

Here, G is the centroid of the ΔABC.

So, AG:GD=2:1 and BG:GE=2:1

$$\frac{AG}{GD} = \frac{2}{1}$$

$$\Rightarrow \frac{AG}{GD} + 1 = \frac{2}{1} + 1$$

$$\Rightarrow \frac{AG + GD}{GD} = \frac{3}{1}$$

$$\Rightarrow \frac{AD}{GD} = \frac{3}{1}$$

⇒ AD=3GD…………………..(2)

And AG=2GD…………………(3)

Dividing equation (2) by (3)

$$\Rightarrow \frac{AD}{AG} = \frac{3GD}{2GD}$$

$$\Rightarrow \frac{AD}{AG} = \frac{3}{2}$$

$\Rightarrow AG = \frac{2}{3} AD$……………….(4)

Again, $\frac{BG}{GE} = \frac{2}{1}$

$$\Rightarrow \frac{BG}{GE} + 1 = \frac{2}{1} + 1$$

$$\Rightarrow \frac{BG + GE}{GE} = \frac{3}{1}$$

$$\Rightarrow \frac{BE}{GE} = \frac{3}{1}$$

⇒ BE=3GE…………………..(2)

And BG=2GE…………………(3)

Dividing equation (2) by (3)

$$\Rightarrow \frac{BE}{BG} = \frac{3GE}{2GE}$$

$$\Rightarrow \frac{BE}{BG} = \frac{3}{2}$$

$\Rightarrow BG = \frac{2}{3} BE$……………….(5)

Putting the value of AG and BG in equation (1)

AG+BG>AB

$$\Rightarrow \frac{2}{3}AD + \frac{2}{3}BE > AB$$

$$\Rightarrow \frac{2}{3}(AD + BE) > AB$$

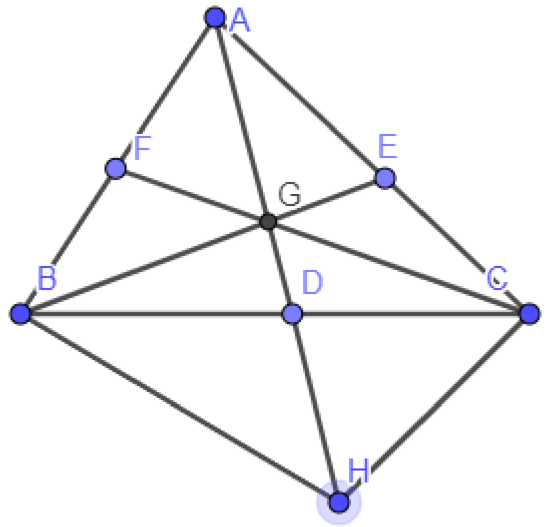
$$\Rightarrow AD + BE > \frac{3}{2}AB$$

Proved.

**4. Question**

Prove that the sum of two medians of a triangle is greater than the third median.

**Answer**




Given a ΔABC in which AD, BE and CF are the medians on the sides BC , AC and AB.

To prove: AD+BE>CF

BE+CF>AD

AD+CF>BE

Proof:

We will extend AD to H to form ΔBHC

Also, AG=GH………………(1)

F is the midpoint of AB and G is the midpoint of AH.

So, by midpoint theorem,

FG||BH

And FG= 1/2 BH

Similarly, GC||BH and BG||CH

So, we can see from the above that

BGCH is a parallelogram.

So, BH=GC …………(2)

And BG=HC…………..(3)

Now, in ΔBGH,

BG+GH>BH

⇒ BG+AG>GC (from (1),(2))

So, BE+AD>CF

Similarly, BE+CF>AD

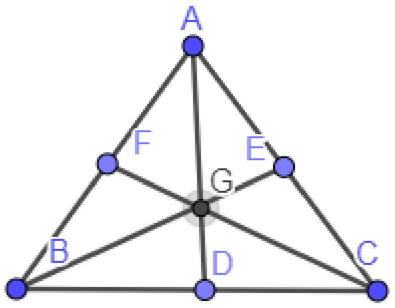
AD+CF>BE

Proved.

**5. Question**

The medians AD, BE and CF in a triangle ABC intersect at the point G. Prove that 4(AD + BE + CF) > 3(AB + BC + CA).

**Answer**




Given that in ΔABC , medians AD, BE and CF intersects at a point G.

As we know that the sum of any two sides is always greater than the third side in a triangle.

Here, G is the centroid of the ΔABC

Now, in ΔADB and ΔADC,

AD+BD>AB………..(1)

AD+DC>AC…………(2)

In ΔBEC and ΔBEA,

BE+EC>BC………..(3)

BE+AE>AB………..(4)

In ΔCFA and ΔCFB,

CF+AF>AC………..(5)

CF+FB>BC………..(6)

Adding equation (1),(2),(3),(4),(5)and (6)

2AD+2BE+2CF+(BD+DC)+(EC+AE)+(AF+FB)>2AB+2BC+2AC

⇒ 2(AD+BE+CF)+BC+AC+AB>2(AB+BC+AC)

⇒ 2(AD+BE+CF)> 2(AB+BC+AC)-(BC+AC+AB)

⇒ 2(AD+BE+CF)> AB+BC+AC

Multiplying by 2 in the above equation

⇒ 4(AD+BE+CF)> 2(AB+BC+AC)

Hence, Proved.

**6. Question**

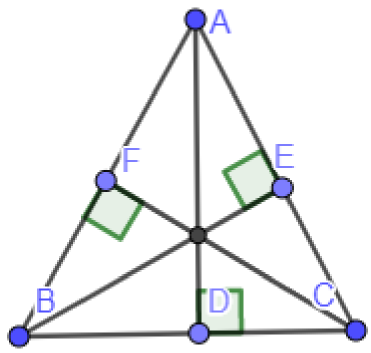
The orthocenter of ΔABC is P. Prove that the orthocenter of ΔABC is the point A.

**Answer**

Given that P is the orthocenter of ΔOBC.

To prove: the orthocenter of ΔABC is the point A

Proof:




P is the orthocenter of ΔABC

As we know that orthocenter is the point of all the perpendicular bisectors of the sides of a triangle.

Let AO is extended to D , BO is extended to E and CO is extended to F respectively.

So, AD=AO+OD

BE=BO+OE

And CF=CO+OF

As AD, BE and CF are the perpendicular bisectors

So, AD⊥BC , BE⊥AC and CF⊥AB

We can say that AD⊥BC,

AB⊥CO and

AC⊥BO

So, A is the orthocenter of ΔOBC.

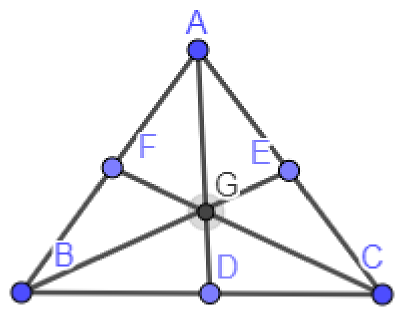
**7. Question**

The medians AD, BE and CF in ΔABC pass through point G.

(a) If GF = 4 cm then find the value of GC.

(b) If AD = 7.5 cm then find the value of GD.

**Answer**




Given that in ΔABC, medians AD, BE and CF intersects at a point G.

GF = 4 cm and AD=7.5 cm

As we know that if the medians in a triangle intersect at a point then the point is the centroid of the triangle and the centroid divides the median in 2:1 ratio.

Here, G is the centroid of the ΔABC.

And AG:GD = 2:1,

BG:GE = 2:1 ,

CG:GF = 2:1

Now, AD=AG+GD

$$\frac{AG}{GD} = \frac{2}{1}$$

$$\Rightarrow \frac{AG}{GD} = \frac{2}{1}$$

⇒AG=2GD…………(1)

AD = AG+GD………………(1)

⇒AG=2GD

Putting in equation (1)

7.5 = 2GD+GD

$\Rightarrow 3GD=7.5$

$\Rightarrow GD = 2.5$

GD = 2.5 cm

Again, $\frac{CG}{GF} = \frac{2}{1}$

$$\Rightarrow \frac{CG}{4} = \frac{2}{1}$$

$\Rightarrow CG=4\times2=8$

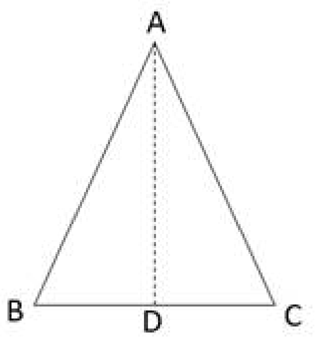
CG=8 cm

Hence, GD=2.5 cm, and CG=8 cm

**8. Question**

ΔABC is an isosceles triangle in which AB = AC, the mid-point of BC is D. Prove that circumcenters, incentre, orthocenter and centroid all lie on line AD.

**Answer**




For circumcentre we have to show AD is perpendicular bisector of BC.

In Δ ABD and Δ ADC,

AB = AC (given)

AD = AD (common)

BD = DC (D is midpoint of BC)

Δ ABD ≅ Δ ADC (BY SSS congruency)

$\Rightarrow \angle ADB + \angle ADC = 180°$

$\Rightarrow AD \perp BC$,

$\Rightarrow BD = DC$

So AD is perpendicular bisector of BC>

So, the circumcentre lie on AD.

For incentre we have to show AD is bisector of ∠BAC.

Since Δ ABD ≅ Δ ADC

⇒ ∠BAD = ∠CAD ( By CPCT)

⇒ AD is the bisector of ∠BAC.

Hence, incenter lies on AD.

For orthocenter we need to prove AD is altitude corresponding to side BC.

Since Δ ABD ≅ Δ ADC

⇒ ∠ ADB + ∠ ADC = 180°

⇒ AD ⊥ BC,

⇒ AD is altitude corresponding to side BC.

For centroid we have to prove that AD is median corresponding to BC.

Since, it is given that D is the midpoint of BC. Ad is the median.

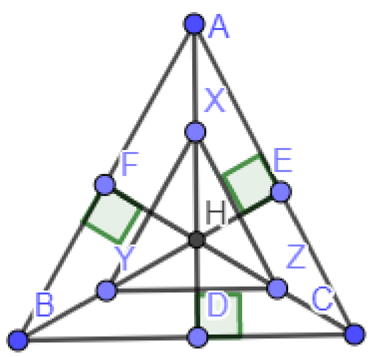
So, centroid lies on AD.

Hence Proved

**9. Question**

The orthocentre of ΔABC is H. The mid-points of AH, BH and CH are respectively x, y and z. Prove that the orthocenter of ΔXYZ is also H.

**Answer**




Since , H is the orthocenter of the ΔABC

So, AD⊥BC , BE⊥AC and CF⊥AB

X, Y and Z are the midpoints of AH, BH and CH respectively

∴ XYZ is a triangle.

$$\frac{HX}{AX} = \frac{HZ}{CZ}$$

Hence, XZ||AC

So, ∠HOX=∠HEA=90°

Similarly, XY||AB

And ∠HPX=∠HFA=90°

And YZ||BC

So, ∠HQZ=∠HDC=90°

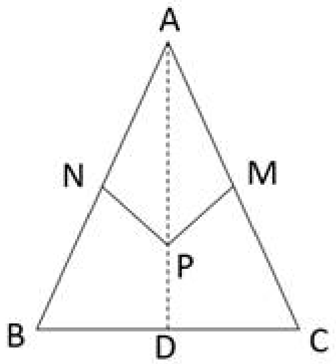
Hence, HO, HP and HQ are the altitudes of ΔXYZ

So, H is the orthocenter of ΔXYZ

**10. Question**

How will you find the point in side BC of ΔABC which is equidistant from sides AB and AC.

**Answer**




In Δ ABC, draw bisector of BC which cuts it at D.

Take any point P on AD. Draw PN⊥ AB and PM ⊥ AC.

In Δ APN and Δ APM

∠ PNA = ∠ PMA = 90° ( by construction)

∠PAN = ∠PAM ( AD is bisector of ∠A)

AP = AP )( common)

Δ APN ≅ Δ APM (By AAS rule)

⇒ PN = PM ( By CPCT)

So, P is equidistant from AB and AC.

∴ any point on AD will be equidistant from AB and AC.

## Miscellaneous Exercise 10

### 1. Question

The point equidistant from the vertices of a triangle is called

A. centroid

B. circumcentre

C. orthocenter

D. incentre

**Answer**

.

### 2. Question

The centroid of a triangle is

A. The point of concurrency of the perpendicular bisectors drawn through the mid-points of the sides of the triangle.

B. The point of concurrency of the bisectors of the angles of the triangle.

C. The point of concurrency of the medians of the triangle.

D. The point of concurrency of the orthocentre of the triangle.

**Answer**

.

### 3. Question

The locus of the centre of a circle rolling in a plane is—

A. circle

B. curve

C. line parallel to plane

D. line perpendicular to plane

**Answer**

.

### 4. Question

If two medians of a triangle are equal then the triangle will be—

A. right angled triangle

B. isosceles triangle

C. equilateral triangle

D. none of these

**Answer**

.

**5. Question**

If AB and CD are two non-parallel lines then the locus of the point P equidistant from these will be—

A. a line passing through point P and parallel to line AB.

B. The bisecting line of angles included between lines AB and CD and passing through point P.

C. a line passing through P and parallel to lines AB and CD.

D. a line passing through P and perpendicular to AB and CD.

**Answer**

.

**6. Question**

A triangle where orthocenter, circumcentre and incentre are coincident is called—

A. equilateral triangle

B. right angled triangle

C. isosceles triangle

D. none of these

**Answer**

.

**7. Question**

A triangle whose orthocentre is a vertex of the triangle is called—

A. right angled triangle

B. equilateral triangle

C. isosceles triangle

D. none of these

**Answer**

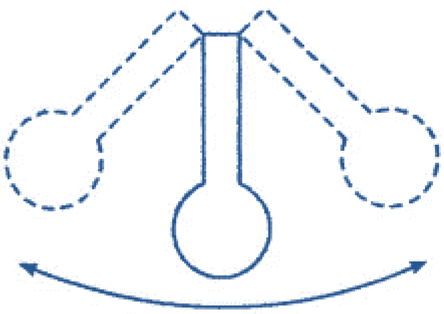
.

**8. Question**

Write the locus of the end of the pendulum of the clock.

**Answer**

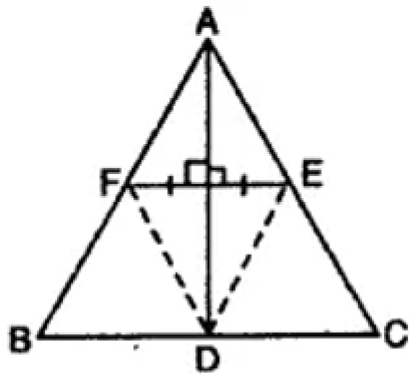
The locus of the end of the pendulum of the clock is an arc of the circle.



**9. Question**

The mid-points of the sides BC, CA and AB of a triangle ABC are respectively D, E and F. Then prove that EF bisects AD.

**Answer**




Construction: Join DE and DF.

In ΔABC, D and E are the mid points of BC and AC.

∴ DE || AB

DE = 1/2 AB

Now DE||AB

⇒ FA||DE ..... (1)

Similarly EA||DF ..... (2)

From (1) and (2),

EAFD is parallelogram.

As diagonals of ||gm bisects each other,

EF will bisect AD.