Subjective Questions of Differential Equations

Q. 1. If (a + bx) $e^{y/x}$ = x, then prove that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

Solution. $(a+bx)e^{\frac{y}{x}} = x$

$\Rightarrow \quad e^{\frac{y}{x}} = \frac{x}{a+bx}$ ...(1)

Diff. w.r. to x, we get

$$e^{\frac{y}{x}} \frac{\left[(x)\frac{dy}{dx} - y\right]}{x^2} = \frac{a+bx-bx}{(a+bx)^2}$$

or $\left(x\frac{dy}{dx} - y\right)e^{\frac{y}{x}} = \frac{ax^2}{(a+bx)^2}$ ...(2)

From (1) using $e^{\frac{y}{x}} = \frac{x}{a+bx}$, we get

$$\left(x\frac{dy}{dx} - y\right) = \frac{ax}{a+bx}$$

Differentiating (3) w. r. to x, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(a+bx)a - axb}{(a+bx)^2}$$

or $x\frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2}$

$\Rightarrow \quad x^3\frac{d^2y}{dx^2} = \left(\frac{ax}{a+bx}\right)^2$ ..(4)

Comparing (3) and (4) we get

$$x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$$

Q. 2. A normal is drawn at a point P(x, y) of a curve. It meets the x–axis at Q. If PQ is of constant length k, then show that the differential equation describing such curves is $y\frac{dy}{dx}=\pm\sqrt{k^2-y^2}$

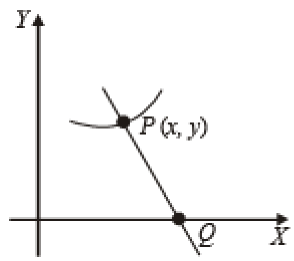
Find the equation of such a curve passing through (0, k).

Solution. The length of formula PQ to any curve y = f (x) is given by

$$y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

According to question

length of PQ = k



$$\Rightarrow\quad \left(y\frac{dy}{dx}\right)^2+y^2=k^2$$

$$\Rightarrow\quad y\frac{dy}{dx}=\pm\sqrt{k^2-y^2}$$

Which is the required differential equation of given curve. On solving this D.E. we get the eqn of curve as follows

$$\int\frac{ydy}{\sqrt{k^2-y^2}}=\int\pm dx\ \Rightarrow -\frac{1}{2}.2\sqrt{k^2-y^2}=\pm x+C$$

$$-\sqrt{k^2-y^2}=\pm x+C$$

As it passes through (0, k) we get C = 0

$\therefore$ Eq^n of curve is

$$-\sqrt{k^2 - y^2} = \pm x \quad \text{or} \quad x^2 + y^2 = k^2$$

Q. 3. Let y = f (x) be a curve passing through (1, 1) such that the triangle formed by the coordinate axes and the tangent at any point of the curve lies in the first quadrant and has area 2. Form the differential equation and determine all such possible curves.

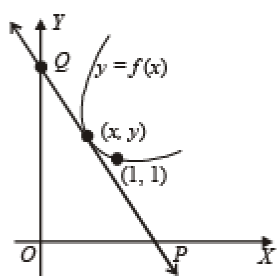
Ans. x +y = 2 and xy = 1, x,y > 0

Solution. Equation of the tangent to the curve y = f (x) at point

(x,y) is Y - y = f '(x)(X-x) ...(1)

The line (1) meets X-axis at $P\left(x - \frac{y}{f'(x)}, 0\right)$ and Y-axis in

Q (0, y - xf '(x))



Area of triangle OPQ is

$$= \frac{1}{2}(OP)(OQ) = \frac{1}{2}\left(x - \frac{y}{f'(x)}\right)(y - xf'(x))$$

$$= -\frac{(y - xf'(x))^2}{2f'(x)}$$

We are given that area of ΔOPQ = 2

$$\Rightarrow \frac{-(y - xf'(x))^2}{2f'(x)} = 2 \Rightarrow (y - xf'(x))^2 + 4f'(x) = 0$$

$\Rightarrow (y - px)^2 + 4p = 0$ ...(2)

where $p = f'(x) = \frac{dy}{dx}$

Since OQ > 0, y - xf '(x) > 0. Also note that

p = f '(x)<0

We can write (2) as $y - px = 2\sqrt{-p}$

$\Rightarrow \quad y = px + 2\sqrt{-p}$ ...(3)

Differentiating (3) with respect to x, we get

$$\frac{dy}{dx} = p + \frac{dp}{dx}x + 2\left(\frac{1}{2}\right)(-p)^{-1/2}(-1)\frac{dp}{dx}$$

$$\Rightarrow \quad \frac{dp}{dx}x - (-p)^{-1/2}\frac{dp}{dx} = 0$$

$$\Rightarrow \quad \frac{dp}{dx}[x - (-p)^{-1/2}] = 0 \quad \Rightarrow \quad \frac{dp}{dx} = 0 \text{ or } x = (-p)^{-1/2}$$

If $\frac{dp}{dx} = 0,$ then p = c where c < 0 [∵ p < 0]

Putting this value in (3) we get

$y = cx + 2\sqrt{-c}$ ...(4)

$$1 = c + 2\sqrt{-c} \quad \Rightarrow \quad -c - 2\sqrt{-c} + 1 = 0$$

$$\Rightarrow \quad (\sqrt{-c} - 1)^2 = 0$$

$$\text{or} \quad \sqrt{-c} = 1 \quad \Rightarrow \quad -c = 1 \text{ or } c = -1$$

Putting the value of c in (4), we get

y = -x + 2, or x +y= 2

Next, putting $x = (-p)^{-1/2}$ or $-p = x^{-2}$ in (3) we get

$$y=\frac{-x}{x^2}+2\left(\frac{1}{x}\right)=\frac{1}{x}$$

⇒ x y = 1(x > 0,y > 0)

Thus, the two required curves are x +y= 2 and xy = 1, (x > 0,y> 0).

Q. 4. Determine the equation of the curve passing through the origin, in the form y = f (x), which satisfies the differential equation $\frac{dy}{dx}=\sin(10x+6y).$

Ans. $y=\frac{1}{3}\left[\tan^{-1}\left(\frac{5\tan 4x}{4-3\tan 4x}\right)-5x\right]$

Solution. Put 10 x +6 y = v

$$\therefore \quad 10+6\frac{dy}{dx}=\frac{dv}{dx} \quad \therefore \frac{dv}{dx}-10=6\sin v$$

$$\Rightarrow \quad \frac{dv}{6\sin v+10}=dx \text{ or } \frac{dv}{12\sin\frac{v}{2}\cos\frac{v}{2}+10}=dx$$

Divide numerator and denominator by $\cos^2\left(\frac{v}{2}\right)$ and put $\tan\left(\frac{v}{2}\right)=t$

$$\therefore \quad \frac{2dt}{12t+10(1+t^2)}=dx \text{ or } \frac{dt}{5t^2+6t+5}=dx$$

$$\frac{dt}{\left(t+\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2}=5dx$$

$$\text{or} \quad \frac{5}{4}\tan^{-1}\frac{5t+3}{4}=5x+5c \quad \text{or} \quad \tan^{-1}\frac{5t+3}{4}=4x+c$$

At origin x = 0,y = 0

$$\therefore \quad v=0, \ \therefore t=\tan\frac{v}{2}=0$$

Hence, from above

$$\tan^{-1}\frac{3}{4} = c \Rightarrow \tan^{-1}\frac{5t+3}{4} - \tan^{-1}\frac{3}{4} = 4x$$

$$\text{or} \quad \frac{\frac{5t+3}{4} - \frac{3}{4}}{1 + \frac{5t+3}{4}\frac{3}{4}} = \tan 4x \quad \text{or} \quad \frac{20t}{25+15t} = \tan 4x$$

or $4t = (5 + 3t) \tan 4x$ or $t (4 - 3 \tan 4 x) = 5 \tan 4x$

$$\text{or} \quad \tan\frac{v}{2} = \frac{5\tan 4x}{4 - 3\tan 4x}$$

$$\text{or} \quad \tan(5x+3y) = \frac{5\tan 4x}{4 - 3\tan 4x}$$

$$\text{or} \quad 5x + 3y = \tan^{-1}\left(\frac{5\tan 4x}{4 - 3\tan 4x}\right)$$

$$\text{or} \quad y = \frac{1}{3}\left(\tan^{-1}\left(\frac{5\tan 4x}{4 - 3\tan 4x}\right) - 5x\right)$$

Q. 5. Let u(x) and v(x) satisfy the differential

equation $\frac{du}{dx} + p(x)\,u = f(x)$ and $\frac{dv}{dx} + p(x)\,v = g(x)$, **where p(x) f(x) and g(x) are continuous functions. If $u(x_1) > v(x_1)$ for some x_1 and f(x) > g(x) for all $x > x_1$, prove that any point (x, y) where $x > x_1$, does not satisfy the equations y = u(x) and y = v(x).**

Solution. (i) y = u (x) and y = v(x) are solutions of given differential equations.

(ii) $u(x_1) > v(x_1)$ for some x_1

(iii) $f(x) > g(x), \forall x > x_1$

$$\frac{du}{dx} + p(x)u = f(x)$$

$$\therefore \quad \frac{d}{dx}\left[u.e^{\int p\,dx}\right] = f(x)e^{\int p\,dx}$$

Similarly, $\frac{d}{dx}\left[v.e^{\int p\,dx}\right] = g(x)e^{\int p\,dx}$

Subtracting, $\frac{d}{dx}\left[(u-v).e^{\int p\,dx}\right] = [f(x) - g(x)]e^{\int p\,dx}$

From above since $f(x) > g(x), \forall x > x_1$ and exponential function is always +ive, then R.H.S. is +ive.

$$\therefore \quad \frac{d}{dx}\left[(u-v).e^{\int p\,dx}\right] > 0 \text{ or } \frac{dF}{dx} > 0$$

Hence the function $F = (u-v)e^{\int pdx}$ is an increasing function.

$\therefore \quad F = (u-v)e^{\int pdx}$ is +ive at $x = x_1$

$\Rightarrow \quad F = (u-v)e^{\int pdx}$ is +ive $\forall x > x_1$

(F being increasing function)

$\therefore \quad u(x) > v(x), \forall x > x_1$

∴ Hence there is no point (x,y) such that $x > x_1$ which can satisfy the equations.

y = u (x) and y = v(x).

Q. 6. A curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of P from the x–axis. Determine the equation of the curve.

Ans. $x^2 + y^2 - 2x = 0, x - 1 = 0$

Solution. Equation of normal is $\frac{dx}{dy}(X - x) + Y - y = 0$

$$\therefore \quad \frac{\left|x\frac{dx}{dy} + y\right|}{\sqrt{1 + \left(\frac{dx}{dy}\right)^2}} = |y|$$

$$\Rightarrow\ x^2\left(\frac{dx}{dy}\right)^2 + y^2 + 2xy\frac{dx}{dy} = y^2 + y^2\left(\frac{dx}{dy}\right)^2$$

$$\Rightarrow\ \left(\frac{dx}{dy}\right) = 0,\ \text{or}\ \frac{dx}{dy} = \frac{2xy}{y^2 - x^2}$$

If $\frac{dx}{dy} = 0,$ then x = c, when x = 1, y = 1,c= 1.

∴ x = 1 ...(1)

When $\frac{dx}{dy} = \frac{2xy}{y^2 - x^2}$ (homogeneous)

Putting $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

$$\therefore\ v + y\frac{dv}{dy} = \frac{2v}{1 - v^2}$$

$$\Rightarrow\ y\frac{dv}{dy} = \frac{2v}{1 - v^2} - v = \frac{2v - v + v^3}{1 - v^2} = \frac{v + v^3}{1 - v^2}$$

$$\Rightarrow\ \frac{(1 - v^2)dv}{v(1 + v^2)} = \frac{dy}{y} \Rightarrow \left(\frac{1}{v} - \frac{2v}{(1 + v^2)}\right)dv = \frac{dy}{y}$$

$$\text{or}\quad \frac{v}{1 + v^2} = cy \Rightarrow \frac{xy}{x^2 + y^2} = cy$$

$$\Rightarrow\ \frac{x}{x^2 + y^2} = c,$$

Putting $x = 1, y = 1$ gives $c = \frac{1}{2}$

∴ Solution is $x^2 + y^2 - 2x = 0$ ...(2)

Hence the solutions are,

$x^2 + y^2 - 2x = 0, x - 1 = 0.$

Q. 7. A country has a food deficit of 10%. Its population grows continuously at a

rate of 3% per year. Its annual food production every year is 4% more than that of the last year.

Assuming that the average food requirement per person remains constant, prove that the country will become self– sufficient in food after n years, where n is the smallest integer bigger than or equal to $\frac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}$.

Solution. Let X_0 be initial population of the country and Y_0 be its initial food production. Let the average consumption be a units. Therefore, food required initially a X_0. It is given

$$Y_0 = aX_0\left(\frac{90}{100}\right) = 0.9aX_0 \qquad \text{...(i)}$$

Let X be the population of the country in year t.

Then dX/dt = rate of change of population

$$= \frac{3}{100}X = 0.03X$$

$$\frac{dX}{X} = 0.03\,dt; \text{ Integrating, } \int\frac{dX}{X} = \int 0.03\,dt$$

$$\Rightarrow \quad \log X = 0.03t + c \quad \Rightarrow \quad X = A.e^{0.03t}$$

$$\text{At } t = 0, X = X_0, \text{ thus } X_0 = A, X = X_0e^{0.03t}$$

Let Y be the food production in year t.

$$\text{Then } Y = Y_0\left(1 + \frac{4}{100}\right)^t = 0.9\,a\,X_0(1.04)^t$$

$$\text{(since } Y = 0.9aX_0 \text{ from (i))}$$

Food consumption in the year t is a X 0e$^{0.03t}$

$$\text{Again } Y - a\,X \geq 0 \text{ (given)}$$

$$\Rightarrow \quad 0.9a(1.04)^t \geq aX_0e^{0.03t} \Rightarrow \frac{(1.04)^t}{e^{0.03t}} \geq \frac{1}{0.9} = \frac{10}{9}$$

Taking log on both sides,

$$t[\ln(1.04) - 0.03] \geq \ln 10 - \ln 9$$

$$\Rightarrow \quad t \geq \frac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}$$

Thus, the least integral value of the year n, when the country becomes self-sufficient, is the smallest integer greater than or equal to $\text{to } \frac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}$.

Q. 8. A hemispherical tank of radius 2 metres is initially full of water and has an outlet of 12 cm2 cross–sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law v(t) = 0.6 $\sqrt{2gh(t)}$, where v(t) and h(t) are respectively the velocity of the flow through the outlet and the height of water level above the outlet at time t, and g is the acceleration due to gravity. Find the time it takes to empty the tank. (Hint : Form a differential equation by relating the decrease of water level to the outflow).

Ans. $\frac{14\pi}{27\sqrt{g}}(10)^5$ units

Solution. Let the water level be at a height h after time t, and water level falls by dh in time dt and the corresponding volume of water gone out be dV.

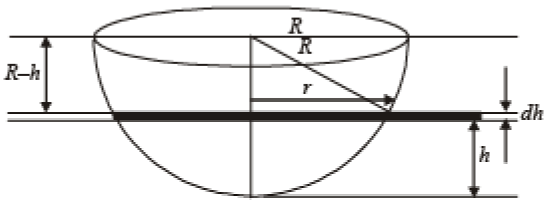
$\Rightarrow \quad |dV| = |\pi r^2 dh|$ (∵ dh is very small)

$\Rightarrow \quad \frac{dV}{dt} = -\pi r^2 \frac{dh}{dt}$ (∵ as t increases, h decreases)

Now, velocity of water, $v = \frac{3}{5}\sqrt{2gh}$

Rate of flow of water = Av (A= 12 cm ^{2})

$$\Rightarrow \quad \frac{dV}{dt} = \left(\frac{3}{5}\sqrt{2gh}A\right) = -\pi r^2 \frac{dh}{dt}$$



Also from figure,

$R^2 = (R - h)^2 + r^2 \Rightarrow r^2 = 2hR - h^2$

$$\text{So, } \frac{3}{5}\sqrt{2g}.\sqrt{h}A = -\pi(2hR - h^2).\frac{dh}{dt}$$

$$\Rightarrow \frac{2hR - h^2}{\sqrt{h}}dh = -\frac{3}{5\pi}\sqrt{2g}.A.dt$$

Integrating,

$$\int_R^0 (2R\sqrt{h} - h^{3/2})dh = -\frac{3\sqrt{2}g}{5\pi}.A.\int_0^T dt$$

$$\Rightarrow T = \frac{5\pi}{3A\sqrt{2g}}\left(2R.\frac{h^{\frac{3}{2}}}{\frac{3}{2}} - .\frac{h^{\frac{5}{2}}}{\frac{5}{2}}\right)_R^0$$

$$= \frac{5\pi}{3A\sqrt{2g}}\left(-\frac{2}{5}R^{5/2} + \frac{4R}{3}R^{3/2}\right) = \frac{5\pi}{3A\sqrt{2g}}.\frac{14}{15}.R^{5/2}$$

$$= \frac{56\pi}{9A\sqrt{g}}(10)^5 = \frac{56\pi}{9\times12\sqrt{g}}(10)^5 = \frac{14\pi}{27\sqrt{g}}(10)^5 \text{ units.}$$

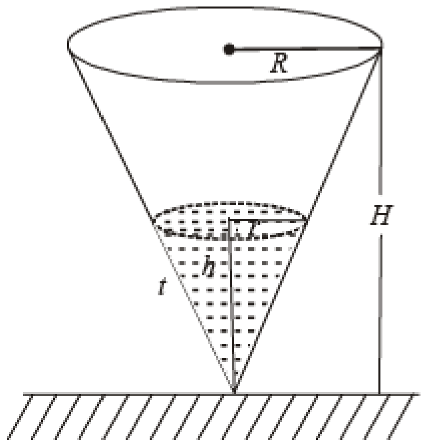
Q. 9. A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant = k > 0).

Find the time after which the cone is empty.

Ans. H/k

Solution. Let at time t, r and h be the radius and height of cone of water.

∴ At time t, surface area of liquid in contact with air $= \pi r^2$



ATQ $-\frac{dV}{dt} \propto \pi r^2$

[∵ '–'ve sign shows that V decreases with time.]

$\Rightarrow \quad \frac{dV}{dt} = -k\pi r^2 \quad \Rightarrow \quad \frac{d}{dt}\left[\frac{1}{3}\pi r^2 h\right] = -k\pi r^2$

$\Rightarrow \quad \frac{1}{3}\pi \frac{d}{dt}[r^2 h] = -k\pi r^2$

But from figure $\frac{r}{h} = \frac{R}{H}$ [Using similarity of Δ's]

$\Rightarrow \quad h = \frac{rH}{R}$

$\therefore$ We get, $\frac{1}{3}\frac{d}{dt}\left[r^2 . \frac{rH}{R}\right] = -kr^2$

$\Rightarrow \quad \frac{r^2 H}{R}\frac{dr}{dt} = -kr^2$

$\Rightarrow \quad \frac{dr}{dt} = -\frac{kR}{H} \quad \Rightarrow \quad r = \frac{-kR}{H}t + C$

But at t = 0, r = R ⇒ R = 0 + C ⇒ C = R

$\therefore \quad r = \frac{-kRt}{H} + R$

Now let the time at which cone is empty be T then at T, r = 0 (no liquid is left)

$\therefore \quad 0 = \frac{-kRT}{H} + R \Rightarrow T = \frac{H}{k}$

Q. 10. A curve 'C' passes through (2, 0) an d the slope at (x, y)

as $\frac{(x+1)^2 + (y-3)}{x+1}$. **Find the equation of the curve. Find the area bounded by curve**

and x–axis in fourth quadrant.

Ans. $\frac{4}{3}$ sq. units

Solution. According to question

Slope of curve C at $(x, y) = \frac{(x+1)^2 + (y-3)}{(x+1)}$

$\Rightarrow \frac{dy}{dx} = (x+1) + \frac{y-3}{x+1}$

$\Rightarrow \frac{dy}{dx} - \left(\frac{1}{x+1}\right) y = x + 1 - \frac{3}{x+1}$

I.F. $= e^{-\int \frac{1}{x+1} dx} = e^{-\log(x+1)} = \frac{1}{x+1}$

$\therefore$ Sol^n is $y \frac{1}{x+1} = \int \left[1 - \frac{3}{(x+1)^2}\right] dx$

$\frac{y}{x+1} = x + \frac{3}{x+1} + C$

y = x(x + 1) + 3 + C (x+ 1) ...(1)

As the curve passes through (2, 0)

$\therefore$ 0 = 2.3 +3 + C.3

$\Rightarrow$ C =-3

$\therefore$ Eq^n. (1) becomes

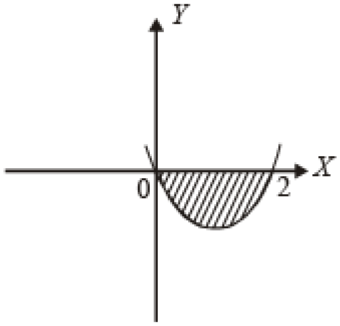
y = x(x +1) + 3 - 3x-3

$y = x^2 - 2x$ ...(2)

Which is the required eq^n of curve.

This can be written as $(x - 1)^2 = (y + 1)$

[Upward parabola with vertex at (1, –1), meeting x-axis at (0, 0) and (2, 0)]



Area bounded by curve and x-axis in fourth quadrant is as shaded region in fig. given by

$$A = \left|\int_0^2 y\,dx\right| = \left|\int_0^2 (x^2 - 2x)\,dx\right| = \left[\frac{x^3}{3} - x^2\right]_0^2$$

$$= \left|\frac{8}{3} - 4\right| = \frac{4}{3} \text{ sq. units.}$$

Q. 11. If length of tangent at any point on the curve y = f(x) intercepted between the point and the x–axis is of length 1. Find the equation of the curve.

Ans. $\log\left|\frac{1 - \sqrt{1 - y^2}}{y}\right| + \sqrt{1 - y^2} = \pm x + c$

Solution. We know that length of tangent to curve y = f (x) is given by

$$\left|\frac{y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}{\left(\frac{dy}{dx}\right)}\right|$$

As per question $\left| \dfrac{y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}}{\left(\dfrac{dy}{dx}\right)} \right| = 1$

$$\Rightarrow \quad y^2\left(1+\left(\frac{dy}{dx}\right)^2\right) = \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2 = \frac{y^2}{1-y^2} \quad \Rightarrow \quad \frac{dy}{dx} = \pm\frac{y}{\sqrt{1-y^2}}$$

$$\Rightarrow \quad \int \frac{\sqrt{1-y^2}}{y}\,dy = \int \pm dx$$

$\Rightarrow$ Put $y = \sin\theta$ so that $dy = \cos\theta\, d\theta$

$$\Rightarrow \quad \int \frac{\cos\theta}{\sin\theta}\cos\theta\, d\theta = \pm x + c$$

$$\Rightarrow \quad \int (\operatorname{cosec}\theta - \sin\theta)\,d\theta = \pm x + c$$

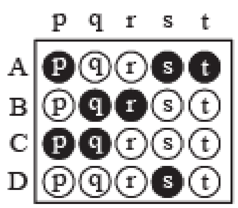
$$\Rightarrow \quad \log|\operatorname{cosec}\theta - \cot\theta| + \cos\theta = \pm x + c$$

$$\Rightarrow \quad \log\left|\frac{1-\sqrt{1-y^2}}{y}\right| + \sqrt{1-y^2} = \pm x + c$$

Match the Following of Differential Equations

Match the Following

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



Q. 1. Match the statements/expressions in Column I with the open intervals in Column II.

Column I	**Column II**
(A) Interval contained in the domain of definition of non-zero solutions of the differential equation $(x - 3)2 + y' + y = 0$	(p) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$	(q) $\left(0, \frac{\pi}{2}\right)$
(C) Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	(r) $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
(D) Interval in which tan–1 (sin x + cos x) is increasing	(s) $\left(0, \frac{\pi}{8}\right)$
	(t) $(-\pi, \pi)$

Ans. (A) → p,q,r,s,t; (B) → p,t; (C) → p,q,r,t; (D) → s

Solution.

$$(x-3)^2 y' + y = 0$$

$$\Rightarrow \quad (x-3)^2 \frac{dy}{dx} = -y$$

$$\Rightarrow \quad \int\left(-\frac{1}{y}\right)dy = \int \frac{1}{(x-3)^2}dx$$

$$\text{or} \quad \log|y| = \frac{1}{x-3} + \log c,\ x \neq 3$$

$$\Rightarrow \quad \log\left(\frac{y}{c}\right) = \frac{1}{x-3},\ x \neq 3$$

$$\Rightarrow \quad \frac{y}{c} = e^{\frac{1}{x-3}} \quad \text{or} \quad y = ce^{\frac{1}{x-3}},\ x \neq 3$$

∴ The solution set is (- ∞, ∞)- {3}

The interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right), \left(\frac{\pi}{8}, \frac{5\pi}{4}\right), \left(0, \frac{\pi}{8}\right)$ and (–π, π) contained in the domain

∴ (A) → p, q,r , s,t

(B) $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$

Let (x - 3) = t ⇒ dx = dt

Also when x ⇒ 1,t → -2

and when x → 5,t → 2

∴ Integral becomes

$$\int_{-2}^{2} (t+2)(t+1)t(t-1)(t-2)dt$$

$$= \int_{-2}^{2} t(t^2-1)(t^2-4)dt = 0$$

as integrand is an odd function.

O is contained by $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $(-\pi, \pi)$

∴ (B) → p,t.

(C) Let f (x) = $\cos^2$ x+ sinx

⇒ f '(x) = -2 sin x cos x+ cosx

For critical point f '(x) = 0 $\Rightarrow \sin x = \frac{1}{2}$ or $\cos x = 0$

$$\Rightarrow \quad x = \frac{\pi}{6}, \frac{5\pi}{6}, \text{ or } x = \frac{\pi}{2}, -\frac{\pi}{2}$$

Now $f''(x) = -2\cos 2x - \sin x$

$$f''(x)\big|_{x=\pi/6} = -ve \quad f''(x)\big|_{x=5\pi/6} = -ve$$

$$f''(x)\big|_{x=\pi/2} = +ve \text{ and } f''(x)\big|_{x=-\pi/2} = +ve$$

∴ $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ are the points of local maxima.

Clearly all the intervals given in column II except $\left(0, \frac{\pi}{8}\right)$

Contain at least one point of local maxima.

∴ (C) → p, q, r,t

(D) Let f (x) = $\tan^{-1}$ (sin x+ cosx)

$$= \tan^{-1}\left[\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)\right]$$

$$f'(x) = \frac{1}{1 + 2\sin^2\left(x + \frac{\pi}{4}\right)} . \sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$$

For f (x) to be an increasing function, f '(x) > 0

$\Rightarrow \cos\left(x+\frac{\pi}{4}\right)>0 \Rightarrow -\frac{\pi}{2}<x+\frac{\pi}{4}<\frac{\pi}{2}$

$\Rightarrow -\frac{3\pi}{4}<x<\frac{\pi}{4}$

Clearly only $\left(0,\frac{\pi}{8}\right)\subset\left(-\frac{3\pi}{4},\frac{\pi}{4}\right)$

$\therefore \quad (D)\rightarrow s.$

Integer Value Correct Type

Q. 1. Let y'(x) + y(x) g'(x) = g(x), g'(x), y(0) = 0, x ∈ R, where f '(x) denotes $\frac{df(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on R with g(0) = g(2) = 0. Then the value of y(2) is

Ans. 0

Solution. The given equation is $\frac{dy}{dx}+g'(x)y = g(x)\,g'(x)$

$\text{I.F.} = e^{\int g'(x)dx} = e^{g(x)}$

$\therefore$ Solution is $y.e^{g(x)} = \int e^{g(x)} g(x).g'(x)dx$

put g (x) = t so that g¢(x) dx = dt

$= \int e^t t\,dt = e^t(t-1)+c$

$\therefore \quad y.e^{g(x)} = e^{g(x)}[g(x)-1]+c$

As y (0) = 0 and g (0) = 0

∴ C = 1

∴ $y.e^{g(x)} = e^{g(x)}[g(x) - 1] + 1$

As g (2) = 0, putting x = 2 we get

$y(2).e^{g(2)} = e^{g(2)}[g(2) - 1] + 1 \Rightarrow y(2) = 0$