Principles of Mathematical Induction

Question1

Consider the statement: P(n) : $n^2 - n + 41$ is prime." Then which one of the following is true?
[Jan. 10, 2019 (II)]

Options:

A. Both P(3) and P(5) are true.

B. P(3) is false but P(5) is true.

C. Both P(3) and P(5) are false.

D. P(5) is false but P(3) is true.

Answer: A

Solution:

$P(n) = n^2 - n + 41$
$\Rightarrow P(3) = 9 - 3 + 41 = 47$ (prime)
$P(5) = 25 - 5 + 41 = 61$ (prime)
$\therefore$ P(3) and P(5) are both prime i.e., true.

Question2

Let $S(K) = 1 + 3 + 5 \ldots + (2K - 1) = 3 + K^2$. Then which of the following is true
[2004]

Options:

A. Principle of mathematical induction can be used to prove the formula

B. $S(K) \Rightarrow S(K + 1)$

C. $S(K)$ not $\Rightarrow S(K + 1)$

D. S(1) is correct

Answer: B

Solution:

$S(K) = 1 + 3 + 5 + \ldots + (2K - 1) = 3 + K^2$
S(1) : 1 = 3 + 1, which is not true

$\because S(1)$ is not true.
$\therefore$ P.M.I cannot be applied
Let $S(K)$ is true, i.e. $S(K) : 1 + 3 + 5.... + (2K - 1) = 3 + K^2$
Adding $2K + 1$ on both sides
$\Rightarrow 1 + 3 + 5... + (2K - 1) + 2K + 1$
$= 3 + K^2 + 2K + 1 = 3 + (K + 1)^2 = S(K + 1)$
$\therefore S(K) \Rightarrow S(K + 1)$

Question3

If $a_n = \sqrt{7 + \sqrt{7 + \sqrt{7 + ...}}}$ having n radical signs then by methods of mathematical induction which is true
[2002]

Options:

A. $a_n > 7\ \forall n \geq 1$

B. $a_n < 7\ \forall n \geq 1$

C. $a_n < 4\ \forall n \geq 1$

D. $a_n < 3\ \forall n \geq 1$

Answer: B

Solution:

For $n = 1$, $a_1 = \sqrt{7} < 7$. Let $a_m < 7$.
Then $a_{m+1} = \sqrt{7 + a_m}$
$\Rightarrow a_{m+1}^2 = 7 + a_m < 7 + 7 < 14$
$\Rightarrow a_{m+1} < \sqrt{14} < 7$; So, by the principle of mathematical induction $a_n < 7,\ \forall n$
