Exercise 6.3

Answer 1E.

(a)

Consider the function $y = \log_a x$

$y = \log_a x$ is a logarithmic function with base 'a' which is the inverse function of the function $f(x) = a^x$

(b)

The Domain of the logarithmic function is $\boxed{(0,\infty)}$

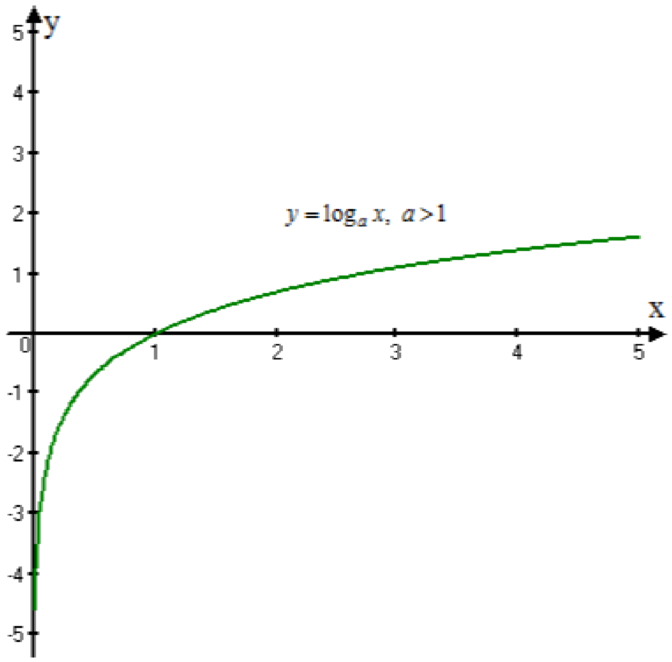
(c)

The Range of the logarithmic function is the set of real numbers.

Range of this function is the set of real number R

(d)

Sketch the general slope of graph of the function $y = \log_a x,\ a > 1$



Answer 2E.

(A)

The logarithm with base 'e' is called natural logarithm denoted as

$$\log_e x = \ln x$$

(B)

The common logarithm is the logarithm to base 10

$$\Rightarrow (\log_{10} x \text{ is called common logarithm})$$

(C)

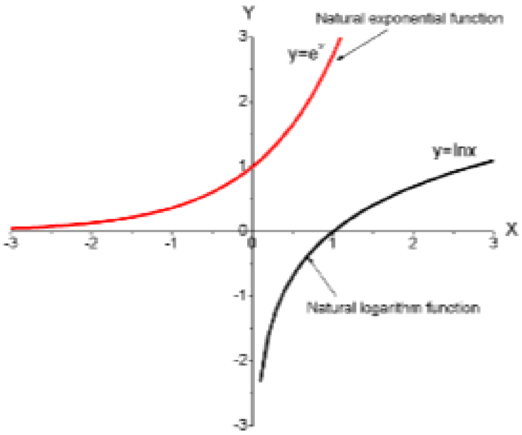


Fig.1

$y = e^x$ is the natural exponential function and $y = \ln x$ is the natural logarithm function.

Answer 3E.

(a)

Consider the expression $\log_5 125$

Recollect that

$\log_a x = y$ if and only if $a^y = x$

So that,

$\log_5 125 = 3$ Because $3^5 = 125$

Therefore, $\log_5 125 = \boxed{3}$

(b)

Consider the expression $\log_3\left(\frac{1}{27}\right)$

Recollect that

$\log_a x = y$ if and only if $a^y = x$

So that,

$\log_3\left(\frac{1}{27}\right) = -3$ Because $3^{-3} = \frac{1}{27}$

Therefore, $\log_3\left(\frac{1}{27}\right) = \boxed{-3}$

Answer 4E.

(a)

Consider the following logarithm:

$$\ln\left(\frac{1}{e}\right)$$

From the laws of Logarithms,

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y \qquad \text{......}(1).$$

Here, $x \text{ and } y$ are positive numbers.

Let, $x = 1$ and $y = e.$

Substitute $x = 1$ and $y = e$ in equation (1), to get the following results:

$$\begin{aligned}\ln\left(\frac{1}{e}\right) &= \ln 1 - \ln e \\ &= 0 - 1 \\ &= -1\end{aligned}$$

(Since, $\ln 1 = 0$ and $\ln e = 1$)

Hence, the final answer is $\boxed{-1}$.

(b)

Consider the following logarithm:

$$\log_{10}\sqrt{10}$$

Rewrite the logarithm $\log_{10}\sqrt{10}$ as follows:

$$\log_{10}\sqrt{10} = \log_{10}(10)^{\frac{1}{2}}$$

From the laws of Logarithms,

$$\log_{10} m^n = n\log_{10} m \qquad \text{......}(2)$$

Let, $m = 10$ and $n = \frac{1}{2}$.

Substitute $m = 10$ and $n = \frac{1}{2}$ in equation (2), to get the following results:

$$\begin{aligned}\log_{10}\sqrt{10} &= \log_{10}(10)^{\frac{1}{2}} \\ &= \frac{1}{2}\log_{10}(10) \\ &= \frac{1}{2} \\ &= 0.5\end{aligned}$$

Hence, the final answer is $\boxed{0.5}$.

Answer 5E.

(a)

Consider the expression $e^{\ln 4.5}$.

Note that, $e^{\ln x} = x, \quad x > 0$

Therefore,

$$e^{\ln 4.5} = \boxed{4.5}.$$

(b)

Consider the expression $\log_{10} 0.0001$

Note that, $\log_a\left(a^x\right) = x, \quad \text{for every } x \in \mathbb{R}$

Since 0.0001 can be written as

$$0.0001 = \frac{1}{10000} = 10^{-4}$$

Therefore,

$$\begin{aligned}\log_{10} 0.0001 &= \log_{10}\left(10^{-4}\right)\\ &= \boxed{-4}\end{aligned}$$

Answer 6E.

(a)

$$\begin{aligned}\ln_{1.5} 2.25 &= \ln_{1.5}(1.5)^2\\ &= 2\ln_{1.5} 1.5\\ &= 2(1)\\ &= 2\end{aligned}$$

Therefore $\boxed{\ln_{1.5} 2.25 = 2}$

(b)

$$\begin{aligned}\ln_5 4 - \ln_5 500 &= \ln_5\left(\frac{4}{500}\right)\\ &= \ln_5 5^{-3}\\ &= (-3)\ln_5 5\\ &= (-3)1\\ &= -3\end{aligned}$$

Therefore $\boxed{\ln_5 4 - \ln_5 500 = -3}$

Answer 7E.

(a)

Consider the following expression:

$$\log_2 6 - \log_2 15 + \log_2 20$$

Find the exact value of the expression.

Recollect the formula, $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$.

Use this formula rewrite the expression.

$$\log_2 6 - \log_2 15 = \log_2\left(\frac{6}{15}\right)$$

Take the complete expression.

$$\log_2 6 - \log_2 15 + \log_2 20$$

Substitute, $\log_2 6 - \log_2 15 = \log_2\left(\frac{6}{15}\right)$ in $\log_2 6 - \log_2 15 + \log_2 20$.

$$\log_2\left(\frac{6}{15}\right) + \log_2 20$$

Recollect the formula, $\log_a x + \log_a y = \log_a(xy)$.

Use this formula rewrite the expression.

$$= \log_2\left(\frac{6}{15}\cdot 20\right)$$

$= \log_2\left(\frac{120}{15}\right)$ Simplify

$$= \log_2(8)$$

$$= \log_2(2^3)$$

$= 3\log_2 2$ Since $\log x^y = y\log x$

$= 3\cdot 1$ Since $\log_2 2 = 1$

$$= 3$$

Thus, $\log_2 6 - \log_2 15 + \log_2 20 = 3$.

Therefore, $\log_2 6 - \log_2 15 + \log_2 20 = \boxed{3}$.

(b)

Consider the following expression:

$$\log_3 100 - \log_3 18 - \log_3 50$$

Find the exact value of the expression.

Recollect the formula, $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$.

Use this formula rewrite the expression.

$$\log_3 100 - \log_3 18 = \log_3\left(\frac{100}{18}\right)$$

Take the complete expression.

$\log_3 100 - \log_3 18 - \log_3 50$

Substitute, $\log_3 100 - \log_3 18 = \log_3\left(\frac{100}{18}\right)$ in $\log_3 100 - \log_3 18 - \log_3 50.$

$\log_3\left(\frac{100}{18}\right) - \log_3 50$

Recollect the formula, $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right).$

Use this formula to rewrite the expression as under:

$$\log_3\left(\frac{100}{18}\right) - \log_3 50 = \log_3\left(\frac{\left(\frac{100}{18}\right)}{50}\right)$$

$$= \log_3\left(\frac{100}{18}\cdot\frac{1}{50}\right)$$

$$= \log_3\left(\frac{2}{18}\right)$$

$= \log_3\left(\frac{1}{9}\right)$ Simplify

$$= \log_3\left(9^{-1}\right)$$

$= -1\log_3(9)$ Since $\log x^y = y\log x$

$$= -1\log_3\left(3^2\right)$$

$$= -2\log_3 3$$

$= -2\cdot 1$ Since $\log_3 3 = 1$

$$= -2$$

Thus, $\log_3 100 - \log_3 18 - \log_3 50 = -2$.

Therefore, $\log_3 100 - \log_3 18 - \log_3 50 = \boxed{-2}$.

Answer 8E.

Consider the expression, $e^{-2\ln 5}$

Recollect that

$e^{\ln x} = x, \quad x > 0$

So that,

$$e^{-2\ln 5} = e^{\ln 5^{(-2)}}$$

$$= 5^{-2}$$

$$= \frac{1}{5^2}$$

$$= \frac{1}{25}$$

Therefore, $e^{-2\ln 5} = \boxed{\frac{1}{25}}$

Consider the expression, $\ln\left(\ln e^{e^{10}}\right)$

Recollect that

$e^{\ln x} = x, \quad x > 0$

Let $e^{10} = x$ then $\ln x = 10$

So that,

$$\begin{aligned}\ln\left(\ln e^{e^{10}}\right) &= \ln\left(\ln e^{x}\right) \\ &= \ln\left(x \ln e\right) \\ &= \ln\left[x(1)\right]\end{aligned}$$

$$= \ln x$$
$$= 10$$

Therefore, $\ln\left(\ln e^{e^{10}}\right) = \boxed{10}$

Answer 9E.

$$\begin{aligned}\text{Consider } \ln\sqrt{ab} &= \ln(ab)^{1/2} \\ &= \frac{1}{2}\ln(ab)\end{aligned}$$

$$\boxed{\ln\sqrt{ab} = \frac{1}{2}[\ln a + \ln b]}$$

Answer 10E.

$$\begin{aligned}\text{Consider } \log_{10}\sqrt{\frac{x-1}{x+1}} &= \log_{10}\left(\frac{x-1}{x+1}\right)^{1/2} \\ &= \frac{1}{2}\log_{10}\left(\frac{x-1}{x+1}\right) \\ &= \frac{1}{2}\left[\log_{10}(x-1) - \log_{10}(x+1)\right]\end{aligned}$$

Therefore $\boxed{\log_{10}\sqrt{\frac{x-1}{x+1}} = \frac{1}{2}\left[\log_{10}(x-1) - \log_{10}(x+1)\right]}$

Answer 11E.

$$\begin{aligned}\text{Consider } \ln\frac{x^2}{y^3z^4} &= \ln\left(x^2\right) - \ln\left(y^3z^4\right) \\ &= \ln\left(x^2\right) - \left[\ln\left(y^3\right) + \ln\left(z^4\right)\right] \\ &= 2\ln x - 3\ln y - 4\ln z\end{aligned}$$

Therefore $\boxed{\ln\frac{x^2}{y^3z^4} = 2\ln x - 3\ln y - 4\ln z}$

Answer 12E.

$$\begin{aligned}\text{Consider } \ln\left(s^4\sqrt{t\sqrt{u}}\right) &= \ln\left(s^4\right) + \ln\left(\sqrt{t\sqrt{u}}\right) \\ &= 4\ln s + \frac{1}{2}\ln\left(t\sqrt{u}\right) \\ &= 4\ln s + \frac{1}{2}\left[\ln t + \ln\sqrt{u}\right] \\ &= 4\ln s + \frac{1}{2}\left[\ln t + \frac{1}{2}\ln u\right]\end{aligned}$$

Therefore $\boxed{\ln\left(s^4\sqrt{t\sqrt{u}}\right) = 4\ln s + \frac{1}{2}\ln t + \frac{1}{4}\ln u}$

Answer 13E.

Consider $2\ln x + 3\ln y - \ln z$

$$= \ln x^2 + \ln y^3 - \ln z \quad (\text{since} \quad m\ln a = \ln a^m)$$

$$= \ln x^2y^3 - \ln z \quad (\text{since} \quad \ln a + \ln b = \ln(ab))$$

$$= \ln\left(\frac{x^2y^3}{z}\right) (\text{since } \ln a - \ln b = \ln\frac{a}{b})$$

Therefore $\boxed{2\ln x + 3\ln y - \ln z = \ln\left(\frac{x^2y^3}{z}\right)}$

Answer 14E.

Consider $\log_{10} 4 + \log_{10} a - \frac{1}{3}\log_{10}(a+1)$

$$= \log_{10}(4a) - \log_{10}(a+1)^{1/3}$$

$$= \log_{10}(4a) - \log_{10}\sqrt[3]{a+1}$$

$$= \log_{10}\frac{4a}{\sqrt[3]{a+1}}$$

Therefore $\boxed{\log_{10} 4 + \log_{10} a - \frac{1}{3}\log_{10}(a+1) = \log_{10}\frac{4a}{\sqrt[3]{a+1}}}$

Answer 15E.

Consider the following limit:

$$\ln 5 + 5\ln 3$$

Use the properties of following logarithmic functions:

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a(x^r) = r\log_a x$$

The logarithm with base e is called the natural logarithm $\log_e x = \ln x$.

Therefore, the expression can be written as follows:

$\ln 5 + 5\ln 3 = \ln 5 + \ln 3^5$ Since $r\ln x = \ln x^r$

$= \ln 5 + \ln 243$ Since $\ln x + \ln y = \ln xy$

$= \ln(5 \times 243)$

$= \ln(1215)$

Therefore, the expression of a single logarithm is $\boxed{\ln(1215)}$.

Answer 16E.

$\ln 3 + \frac{1}{3}\ln 8$

$= \ln 3 + \ln(8)^{1/3}$ Since $\ln x^y = y\ln x$

$= \ln 3.(8)^{1/3}$ Since $\ln xy = \ln x + \ln y$.

$= \ln(3.2)$ Since $8^{1/3} = 2$

$= \ln 6$

Hence

$\boxed{\ln 3 + \frac{1}{3}\ln 8 = \ln 6}$

Answer 17E.

Consider $\frac{1}{3}\ln(x+2)^3 + \frac{1}{2}\left[\ln x - \ln\left(x^2+3x+2\right)^2\right]$

$$= \frac{1}{3}\times 3\ln(x+2) + \frac{1}{2}\ln x - \frac{1}{2}\times 2\ln\left(x^2+3x+2\right)$$

$$= \ln(x+2) + \frac{1}{2}\ln x - \ln\left(x^2+3x+2\right)$$

$$= \ln\left(\frac{(x+2)\sqrt{x}}{x^2+3x+2}\right)$$

$$= \ln\left(\frac{(x+2)\sqrt{x}}{(x+1)(x+2)}\right)$$

$$= \ln\left(\frac{\sqrt{x}}{x+1}\right)$$

Therefore $\boxed{\frac{1}{3}\ln(x+2)^3 + \frac{1}{2}\left[\ln x - \ln\left(x^2+3x+2\right)^2\right] = \ln\frac{\sqrt{x}}{x+1}}$

Answer 18E.

Given that $\ln(a+b)+\ln(a-b)-2\ln c$

This can be written as $\ln((a+b)(a-b))-2\ln c$

$\ln((a+b)(a-b))-\ln c^2$

$\Rightarrow \ln\left(a^2-b^2\right)-\ln c^2$

$\Rightarrow \ln\left(\frac{a^2-b^2}{c^2}\right)$

Answer 19E.

(A) $\log_{12} e = \frac{\ln e}{\ln 12}$ Since $\log_a x = \frac{\ln x}{\ln a}$

$$= \frac{1}{\ln 12}$$

$$= \frac{1}{2.484907}$$

$$\approx 0.402430$$

Hence

$\boxed{\log_{12} e \approx 0.402430}$

(C) $\log_2 \pi = \frac{\ln \pi}{\ln 2}$ Since $\log_a x = \frac{\ln x}{\ln a}$

$$= \frac{1.144730}{0.693147}$$

$$\approx 1.651496$$

Hence

$\boxed{\log_2 \pi \approx 1.651496}$

Answer 20E.

By the formula $\log_a x = \frac{\ln x}{\ln a}$ $a > 0$ and $a \neq 1$

So we have

$$y = \log_2 x = \frac{\ln x}{\ln 2}$$

$$y = \log_4 x = \frac{\ln x}{\ln 4}$$

$$y = \log_6 x = \frac{\ln x}{\ln 6}$$

And $y = \log_8 x = \frac{\ln x}{\ln 8}$

Now we sketch the curves $y=\dfrac{\ln x}{\ln 2}$, $y=\dfrac{\ln x}{\ln 4}$, $y=\dfrac{\ln x}{\ln 6}$ *and* $y=\dfrac{\ln x}{\ln 8}$ on the common screen (figure1)

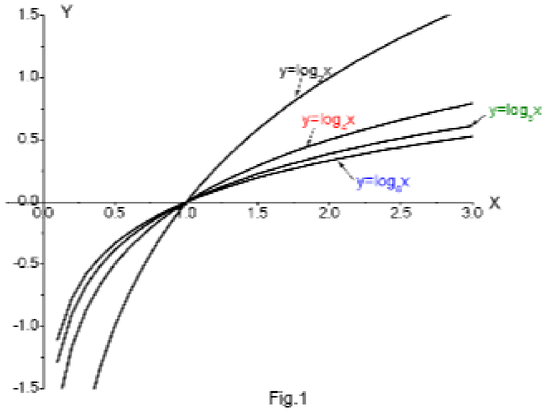


Fig.1

We see that

(1) As $x\to 0^+$, all graphs $\to -\infty$
(2) All graphs pass through (1, 0)
(3) All are increasing function
(4) As the base is increasing, the rate of increase is decreasing

Answer 21E.

By the formula $\log_a x=\dfrac{\ln x}{\ln a}$, $a>0$ and $a\neq 1$

We have

$$y=\log_{1.5} x=\frac{\ln}{\ln(1.5)}$$

$$y=\log_{10} x=\frac{\ln}{\ln 10}$$

$$y=\log_{50} x=\frac{\ln}{\ln 50}$$

Now we sketch the curves $y=\dfrac{\ln}{\ln(1.5)}$, $y=\ln x$, $y=\dfrac{\ln}{\ln 10}$ *and* $y=\dfrac{\ln}{\ln 50}$ on the common screen (Figure 1).

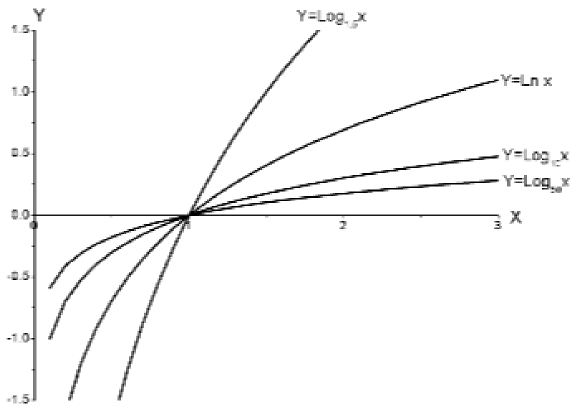


Fig.1

We see that

1 as $x\to 0^+$, all graphs $\to -\infty$
2 all graphs pass though (1,0)
3 all are increasing
4 as the base is increasing, the rate of increase is decreasing

Answer 22E.

By the formula $\log_a x = \dfrac{\ln x}{\ln a}$, $a > 0$ and $a \neq 1$

We have

$y = \log_{10} x$

$= \dfrac{\ln x}{\ln 10}$

Now we sketch the curves $y = \ln x$, $y = \log_{10} x = \dfrac{\ln x}{\ln 10}$, $y = e^x$ and $y = 10^x$ on the same screen (figure1)

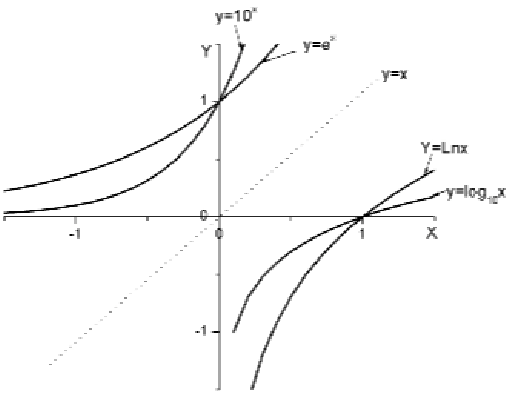


Fig.1

We see that

(1) $y = \ln x$ and $y = \log_{10} x$ Both curves pass through (1, 0)

(2) $y = \ln x$ Has more rate of increase than $y = \log_{10} x$

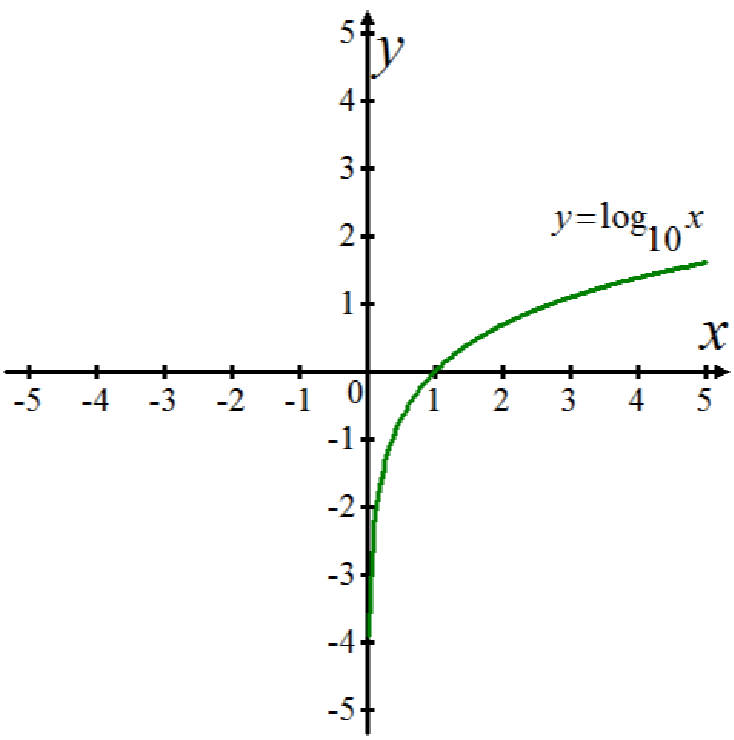
(3) $y = \ln x$ is the reflected curve of $y = e^x$ about $y = x$ and $y = \log_{10} x$ is the reflected curve of $y = 10^x$ about the line $y = x$

Answer 23E.

(a)

Consider the expression $y = \log_{10}(x+5)$

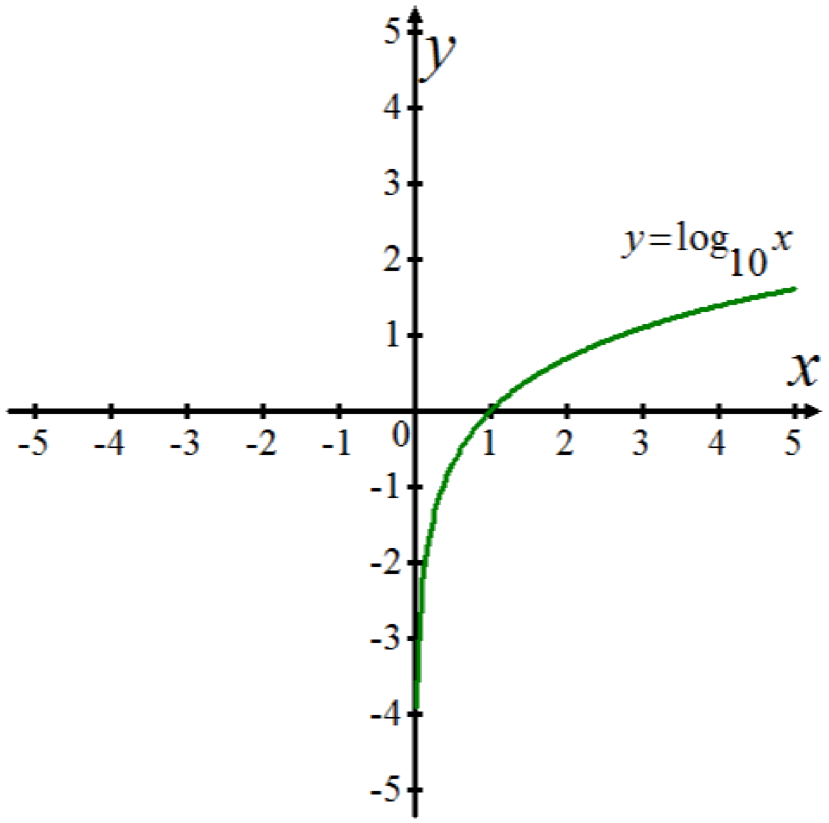
The graph of $y = \log_{10} x$



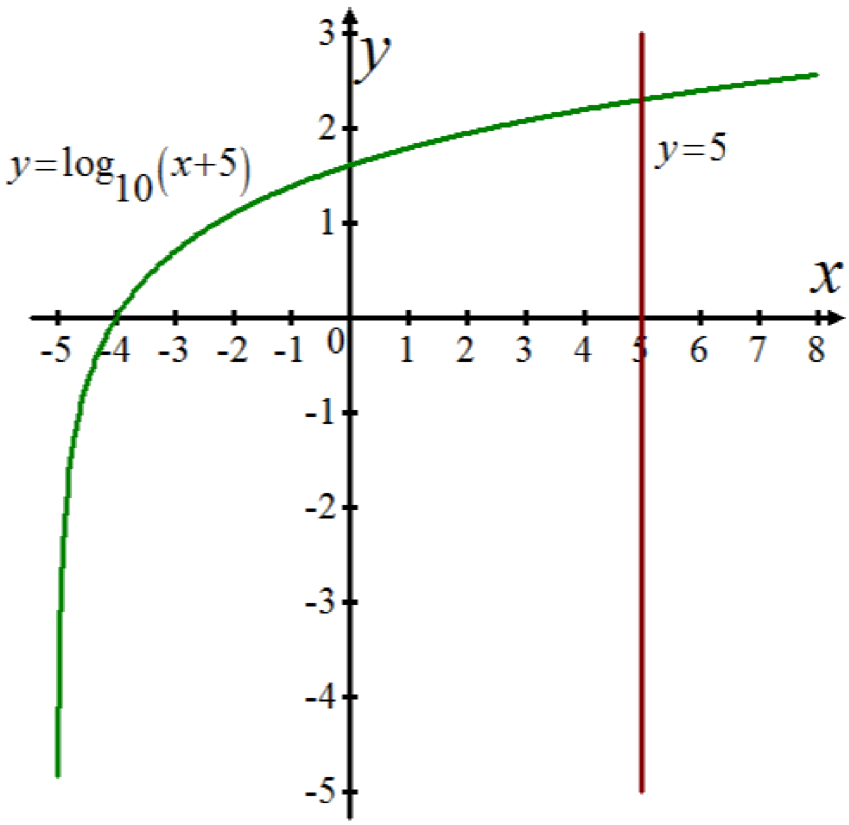
(a)

Consider the expression $y = \log_{10}(x+5)$

The graph of $y = \log_{10} x$



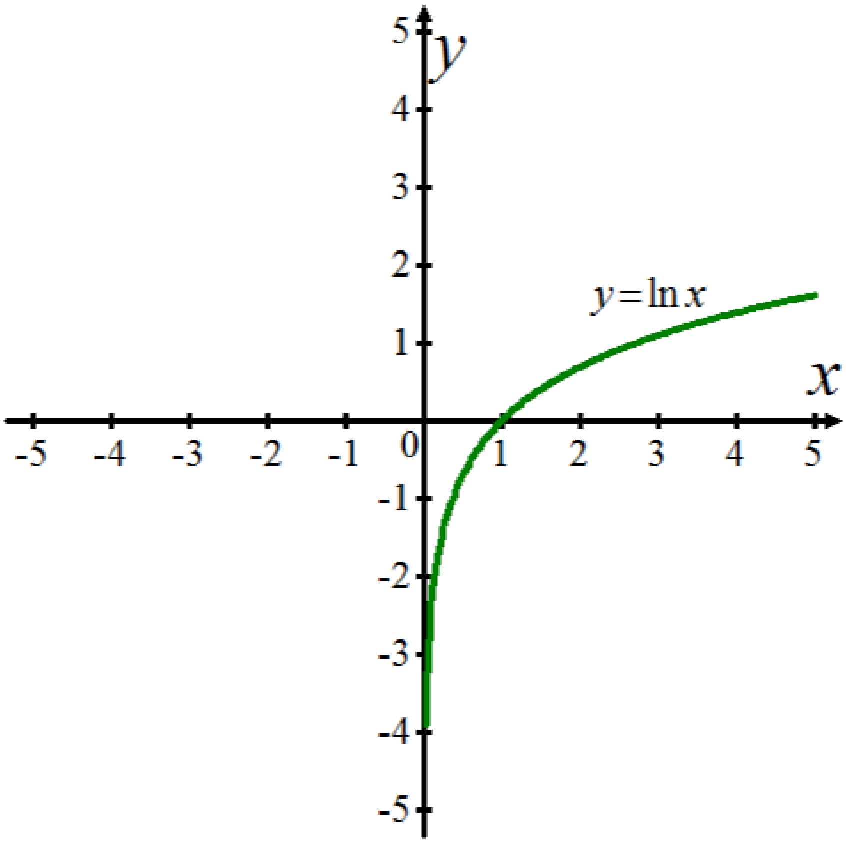
The graph of $y = \log_{10}(x+5)$



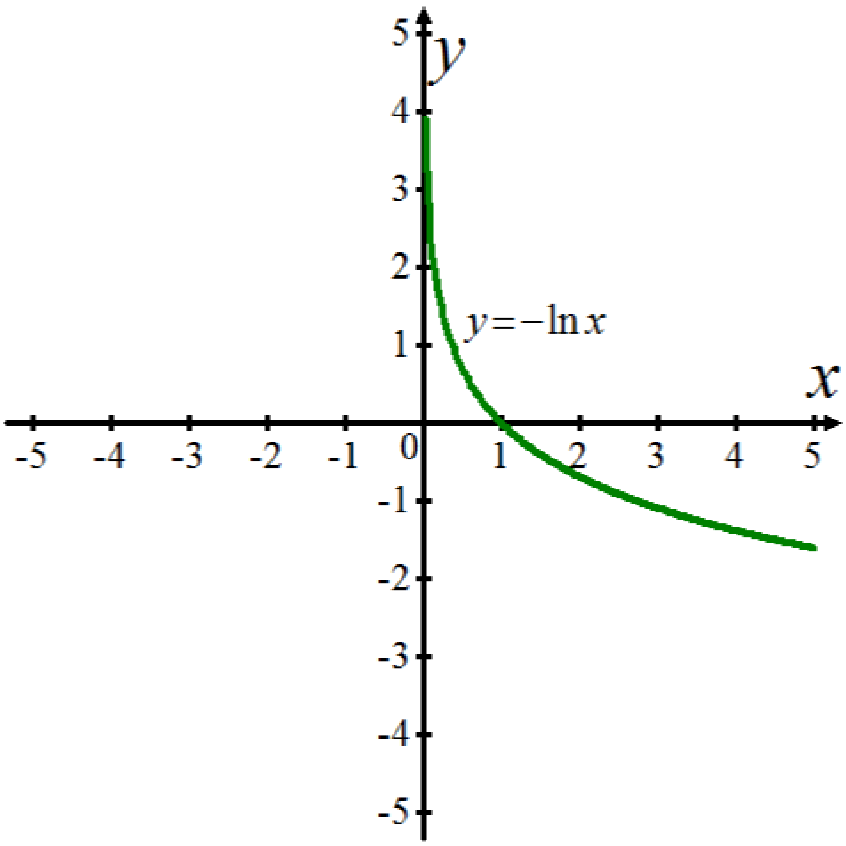
So, shift the graph of $y = \log_{10} x$ five units to the left to obtain the graph of $y = \log_{10}(x+5)$

(b)

Consider the expression $y = \ln x$



Consider the expression $y = -\ln x$



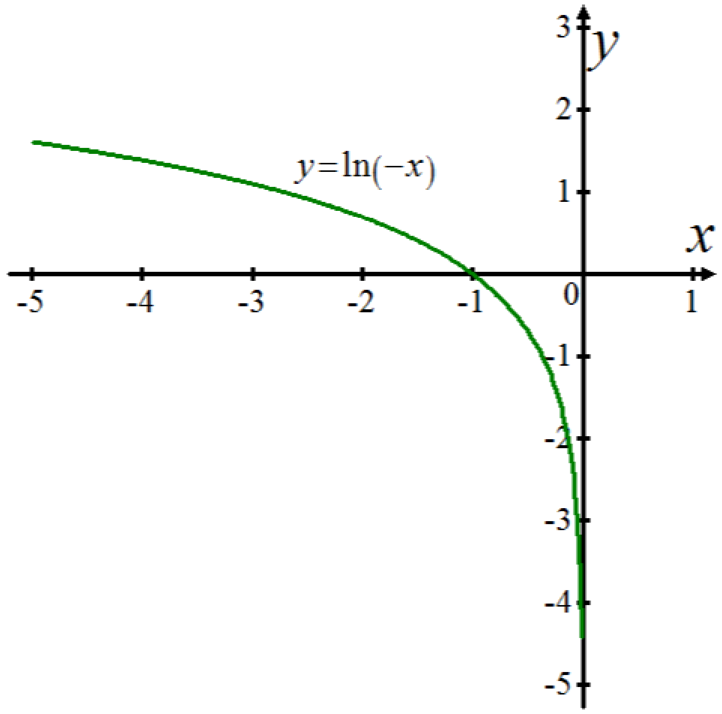
Reflect the graph of $y = \ln x$ about the x – axis to obtain the graph of $y = -\ln x$

Answer 24E.

(a)

Consider the expression $y=\ln(-x)$

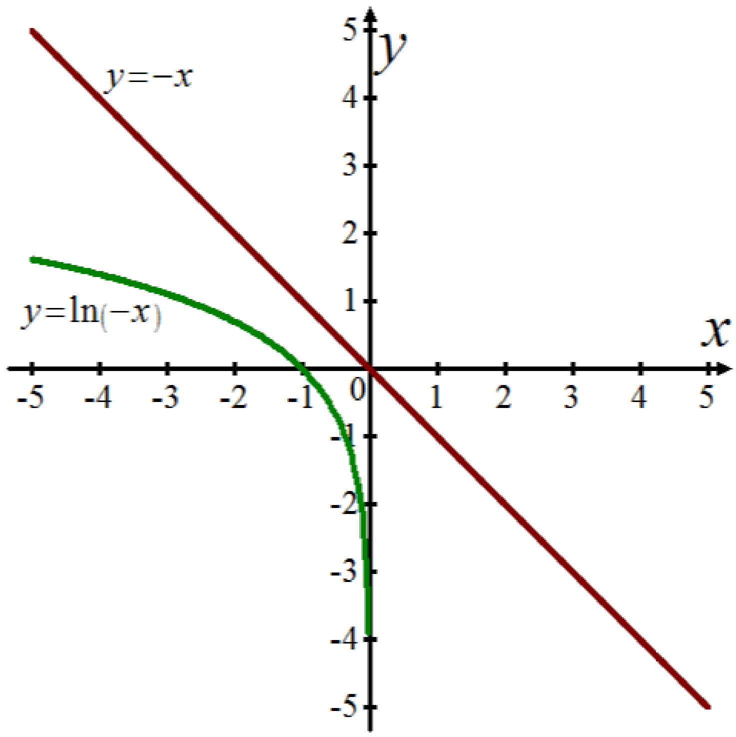
The graph of $y=\ln(-x)$



Recollect that the log function can be defined for log negative real number only so when $-x>0$ or $x<0$ $\ln(-x)$ is defined.

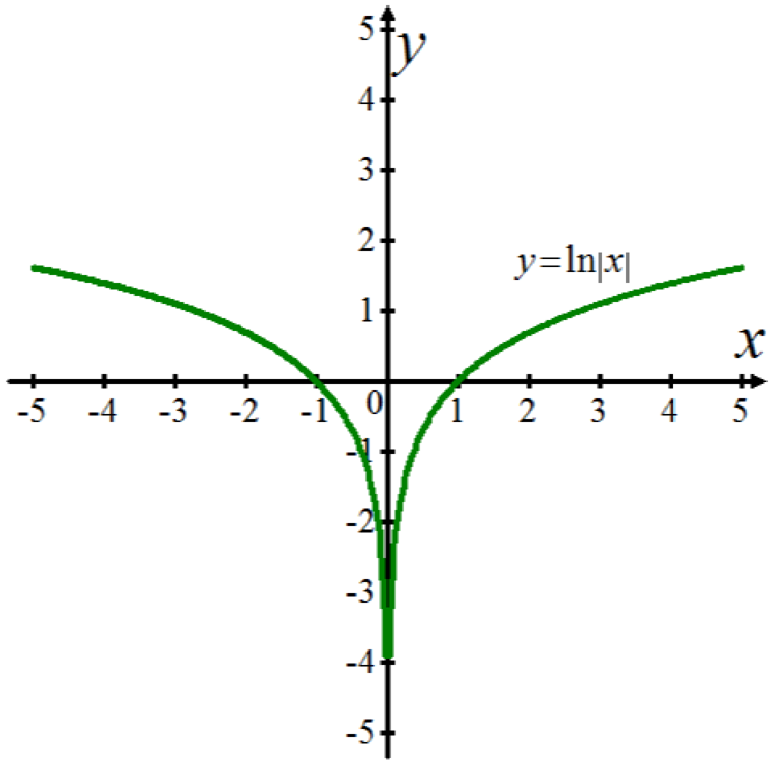
It will be exactly on the opposite side of $x-$axis or negative side of $x-$axis take the form $y=\ln(x)$ further $\ln(1)=0$ allows that when $x=-1$, $\ln(-x)$ become zero so the graph of $y=\ln(-x)$ will be below $x-$axis for $-1<x<0$ and above $x-$axis when $x<-1$

Recollect that $\ln(x)<x$ for all x and $\ln(-x)<-x$ for all $x\in\Re$. This shows that the curve of $\ln(-x)$ cannot intersection the line $y=-x$ use this information to get the possible graph of $\ln(-x)$



(b) Consider the expression $y = \ln|x|$

The graph of $y = \ln|x|$



When $\ln|x|$ is nothing but $\ln(x)$ which for negative real numbers, the log function is not defined with careful infrastructure it can be contractual that $\ln(-x)$ include meant for negative real number but due to the negative symbol of x it is required to use negative real numbers as the domain of $\ln(-x)$

Answer 25E.

Given $f(x) = \ln x + 2$

(a) Since $\ln x$ is not defined for $x \le 0$

Domain of $f(x) = \ln x + 2$ is $x > 0$, i.e. $(0, \infty)$

And range of $f(x)$ is $(-\infty, \infty)$

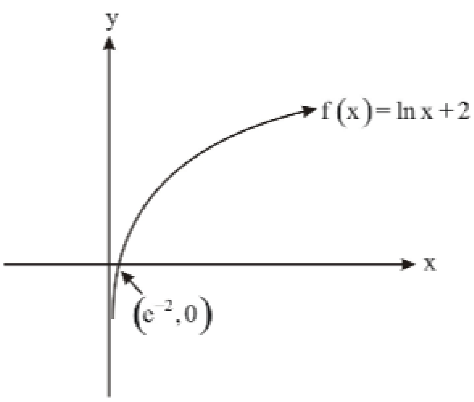
(b) To get x-intercept $f(x) = 0$

$$\Rightarrow \quad \ln x + 2 = 0$$

$$\Rightarrow \quad x = e^{-2}$$

i.e; $\boxed{x - \text{intercept} = e^{-2}}$

(c) The graph of $f(x)$ is



Answer 26E.

Given $f(x) = \ln(x-1) - 1$

(a) Since $\ln x$ is defined for all $x > 0$

Therefore $x - 1 > 0$

$\Rightarrow \quad x > 1$

i.e; $\boxed{\text{domain of } f = (1, \infty)}$

$\boxed{\text{Range of } f(x) \text{ is } \mathbb{R} = (-\infty, \infty)}$

(b) To get x-intercept $f(x) = 0$

$\Rightarrow \quad \ln(x-1) - 1 = 0$

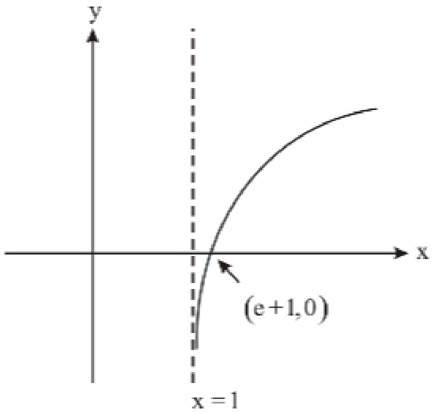
$\Rightarrow \quad e = x - 1$

$\Rightarrow \quad x = e + 1$

i.e.; x-intercept is $e + 1$

(c)

The graph of $f(x)$ is



Answer 27E.

(a) Consider $e^{7-4x} = 6$

Then taking logarithms on both sides

$7 - 4x = \ln 6$

$\Rightarrow \quad 4x = 7 - \ln 6$

$\Rightarrow \quad \boxed{x = \frac{1}{4}(7 - \ln 6)}$

(b) Consider $\ln(3x - 10) = 2$

$\Rightarrow \quad 3x - 10 = e^2$

$\Rightarrow \quad \boxed{x = \frac{1}{3}(10 + e^2)}$

Answer 28E.

(a) Given $\ln(x^2 - 1) = 3$

$\Rightarrow \quad x^2 - 1 = e^3$

$\Rightarrow \quad x^2 = 1 + e^3$

$\Rightarrow \quad \boxed{x = \pm\sqrt{1 + e^3}}$

(b) Given $e^{2x} - 3e^x + 2 = 0$

$\Rightarrow \quad e^{2x} - 2e^x - e^x + 2 = 0$

$\Rightarrow \quad e^x(e^x - 2) - 1(e^x - 2) = 0$

$\Rightarrow \quad (e^x - 1)(e^x - 2) = 0$

$$\Rightarrow \quad e^x - 1 = 0 \text{ or } e^x - 2 = 0$$

$$\Rightarrow \quad e^x = 1 \text{ or } e^x = 2$$

$$\Rightarrow \quad x = 0 \text{ or } x = \ln e^2$$

Therefore $\boxed{x = 0 \text{ or } \ln_e 2}$

Answer 29E.

(b)

Consider the expression $\ln x + \ln(x-1) = 1$

$\ln x(x-1) = 1$

$x(x-1) = e^1$ Since $\ln x = y$ then $e^y = x$

$x^2 - x = e$

$x^2 - x - e = 0$

The quadratic formula for $x^2 - x - e = 0$ with $a = 1, b = -1, c = -e$

$$x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1}{2}\left(1 \pm \sqrt{1+4e}\right)$$

But negative roots are not defined in natural logarithms so it is not defined for $x < 0$

Only defined for $x = \boxed{\frac{1}{2}\left(1 + \sqrt{1+4e}\right)}$

Answer 30E.

(A) The given equation is

$e^{3x+1} = k$

Taking natural logarithms of both sides of the equation, we get,

$\ln e^{3x+1} = \ln k$

$\Rightarrow \quad 3x + 1 = \ln k$ Since $\ln e^x = x$

$\Rightarrow \quad 3x = \ln k - 1$

$\Rightarrow \quad x = \dfrac{\ln k - 1}{3}$

Hence

$$\boxed{x = \frac{\ln k - 1}{3}}$$

(B) The given equation is

$log_2(mx) = c$

$\Rightarrow \quad mx = 2^c$ Since $log_a x = y \Rightarrow x = a^y$

$\Rightarrow \quad x = \dfrac{2^c}{m}$

Hence

$$\boxed{x = \frac{2^c}{m}}$$

Answer 31E.

Given $e - e^{-2x} = 1$

$\Rightarrow \quad e^{-2x} = e - 1$

Taking logarithms on both sides,

$-2x = \ln(e-1)$

$$\Rightarrow \quad \boxed{x = -\frac{1}{2}\ln(e-1)}$$

Answer 32E.

Taking natural logarithm on both sides, we get

$$\ln(10)(1+e^{x})^{-1} = \ln 3$$

$$\ln(10)+\ln(1+e^{-x})^{-1} = \ln 3 \text{ because } [\ln x + \ln y = \ln(x)(y)]$$

$$\ln 10 - \ln(1+e^{-x}) = \ln 3 \quad [r\ln x = \ln(x)^{r}]$$

$$\ln 10 - \ln 3 = \ln(1+e^{-x}) \text{ because } \left[\ln x - \ln y = \ln\left(\frac{x}{y}\right)\right]$$

$$\ln\frac{10}{3} = \ln(1+e^{-x})$$

$$\frac{10}{3} = 1+e^{-x}$$

$$e^{-x} = \frac{7}{3}$$

Again taking natural logarithm on both sides, we get

$$\ln e^{-x} = \ln\left(\frac{7}{3}\right)$$

$$-x\ln e = \ln\left(\frac{7}{3}\right) \text{ because } \ln e = 1$$

Therefore, the value of xis $x = -\ln\left(\frac{7}{3}\right)$

Answer 33E.

Consider the following equation:

$$\ln(\ln x) = 1$$

Apply exponential function to both sides of an equation $\ln(\ln x) = 1$.

$$e^{\ln(\ln x)} = e^{1}$$

From the property of exponential and logarithmic functions is as follows:

$$e^{\ln x} = x, x > 0.$$

Use the above property to $e^{\ln(\ln x)} = e^{1}$.

$$\ln x = e^{1}$$

Again apply the exponential function to both sides.

$$e^{\ln x} = e^{e^{1}}$$

$x = e^{e^{1}}$ Since $e^{\ln x} = x, x > 0$

$$x = e^{e}$$

Therefore, the solution of the given equation is $x = \boxed{e^{e}}$.

Answer 34E.

The given equation is

$$e^{e^x} = 10$$

Taking natural logarithms of both sides of the equation, we get,

$$\ln e^{e^x} = \ln 10$$

$\Rightarrow \quad e^x = \ln 10 \qquad\qquad$ Since $\ln e^x = x$

Again taking natural logarithms of both sides of the equation we get,

$$\ln e^x = \ln(\ln 10)$$

$$\Rightarrow \quad x = \ln(\ln 10)$$

Hence

$$\boxed{x = \ln(\ln 10)}$$

Answer 35E.

Consider the following equation:

$$e^{2x} - e^x - 6 = 0$$

Rewrite the equation,

$$(e^x)^2 - (e^x) - 6 = 0$$

The objective is to solve the equation for x.

Let $u = e^x$

Then, the equation becomes,

$$u^2 - u - 6 = 0$$
$$u^2 - 3u + 2u - 6 = 0$$
$$u(u-3) + 2(u-3) = 0$$
$$(u-3)(u+2) = 0$$
$$u - 3 = 0 \text{ or } u + 2 = 0$$
$$u = 3 \text{ or } u = -2$$

Therefore, $u = 3$ or $u = -2$

When $u = -2$, $e^x = -2 \, (u = e^x)$

So, $e^x = -2$ is not satisfied because the exponential value is negative.

When $u = 3$, $e^x = 3 \, (u = e^x)$

Apply natural logarithms on both sides of the above equation.

$$\ln(e^x) = \ln 3$$
$$x \ln e = \ln 3 \quad [\ln(a^b) = b\ln(a)]$$
$$x = \ln 3 \quad [\ln e = 1]$$

Therefore, the solved equation for x is $\boxed{x = \ln 3}$.

Answer 36E.

Given $\ln(2x+1) = 2 - \ln x$

$\Rightarrow \ln(2x+1) + \ln x = 2$

$\Rightarrow \ln x(2x+1) = 2 \qquad$ (Since $\ln(a) + \ln(b) = \ln ab$)

$\Rightarrow x(2x+1) = e^2 \qquad$ (By the definiti on of logorithm)

$\Rightarrow 2x^2 + x = e^2$

$\Rightarrow 2x^2 + x - e^2 = 0$

Then $x = \dfrac{-1 \pm \sqrt{1+8e^2}}{2 \times 2}$

$= \dfrac{-1 \pm \sqrt{1+8e^2}}{4}$

Hence $x=\dfrac{-1+\sqrt{1+8e^2}}{4}>0,\quad x=\dfrac{-1-\sqrt{1+8e^2}}{4}<0$

But logarithm is not defines for negative real numbers.

Therefore, $\boxed{x=\dfrac{-1+\sqrt{1+8e^2}}{4}}$

Answer 37E.

(a)

Consider the equation $e^{2+5x}=100$

Take natural logarithm of both sides of the equation and we have, $\ln\left(e^x\right)=x \quad x\in\mathbb{R}$,

$$e^{\ln x}=x \quad x>0$$

$$\ln\left(e^{2+5x}\right)=\ln 100$$

$$2+5x=\ln 100$$

$$5x=\ln 100-2$$

$$x=\frac{\ln 100-2}{5}$$

Since the natural logarithm is found on scientific calculator, we can approximate the solution: to four decimal places,

$$x=\frac{(\ln 100-2)}{5}$$

$$\approx \boxed{0.5210}$$

(b)

Consider the equation $\ln\left(e^x-2\right)=3$

We have, $\ln x=y$ if and only if $e^y=x$

$$\ln\left(e^x-2\right)=3$$

$$e^x-2=e^3$$

$$e^x=e^3+2$$

$$x=\ln\left(e^3+2\right)$$

Since the natural logarithm is found on scientific calculator, we can approximate the solution: to four decimal places,

$$x=\ln\left(e^3+2\right)$$

$$\approx \boxed{3.0949}$$

Answer 38E.

(a)

Consider the equation $\ln\left(1+\sqrt{x}\right)=2$

We have, $\ln x=y$ if and only if $e^y=x$

$$\left(1+\sqrt{x}\right)=e^2$$

$$\sqrt{x}=e^2-1$$

Squaring on both sides

$$\left(\sqrt{x}\right)^2=\left(e^2-1\right)^2$$

$$x=\left(e^2-1\right)^2$$

$$=\boxed{40.820038}$$

So, the approximate the solution: to four decimal places, $x=\boxed{40.8200}$

(b)

Consider the equation $3^{1/(x-4)} = 7$

Take natural logarithm of both sides of the equation

$$\ln\left(3^{1/(x-4)}\right) = \ln 7$$

$$\frac{1}{(x-4)}\ln 3 = \ln 7$$

$$\frac{1}{(x-4)} = \frac{\ln 7}{\ln 3}$$

$$\frac{1}{(x-4)} = 1.7712$$

$$\frac{1}{1.7712} = x-4$$

$$\frac{1}{1.7712} + 4 = x$$

$$x = \boxed{4.5645}$$

So, the approximate the solution: to four decimal places, $x = \boxed{4.5645}$

Answer 39E.

(a) Given $\ln x < 0$

Then $x < e^0 = 1$

But $\ln x$ is defined only for $x > 0$

Therefore $\boxed{\ln x < 0 \text{ is valid for } 0 < x < 1}$

(b) Given $e^x > 5$

$\Rightarrow \boxed{x > \ln 5}$

Answer 40E.

(a) Given $1 < e^{3x-1} < 2$

$\Rightarrow \ln 1 < 3x - 1 < \ln 2$

$\Rightarrow 0 < 3x - 1 < \ln 2$

$\Rightarrow 1 < 3x < 1 + \ln 2$

$\Rightarrow \boxed{\frac{1}{3} < x < \frac{1}{3}(1+\ln 2)}$

(b) Given $1 - 2\ln x < 3$

$\Rightarrow -2 < 2\ln x$

$\Rightarrow -1 < \ln x$

$\Rightarrow e^{-1} < x$ But $x > 0$

Therefore $\boxed{x > \frac{1}{e}}$

Answer 41E.

Since $1ft = 12 inches$

So $3ft = 3 \times 12 = 36\ inches$

Now according to the problem we have

$\log_2 x = 36$

Then $x = 2^{36}$ inches $\quad [\log_a x = y \Leftrightarrow a^y = x]$

Since $1\text{ ft} = \frac{1}{5280}$ miles

Then 12 inches $= \frac{1}{5280}$ miles

Therefore $2^{36}\text{inch} = \frac{2^{36}}{5280 \times 12}$ miles

$= \boxed{1084587.701 \text{ miles}} \approx 1084588$ miles

So we have to move 1084588 miles to the right before the height of the curve reaches 3ft.

Answer 42E.

(A)

$v(t) = ce^{-kt}$ Where c and k are constant

Then acceleration is

$$a(t) = \frac{dv(t)}{dt} = -kce^{-kt}$$

Or $a(t) = -kv(t)$

So $a(t) \propto v(t)$

(B) c is the dimension of velocity because e^{-kt} has no unit.

(C) Initial velocity at t = 0,

$$v(0) = ce^{0}$$

$$v(0) = c$$

According to problem

$$ce^{-kt} = \frac{1}{2}c$$

Then $e^{-kt} = \frac{1}{2}$

Then $\ln\frac{1}{2} = -kt$ $\quad [e^{y} = x \Leftrightarrow \ln x = y]$

Or $t = -\frac{1}{k}\ln\left(\frac{1}{2}\right)$

$t = -\frac{1}{k}(\ln 1 - \ln 2)$ $\quad \left[\ln\frac{x}{y} = \ln x - \ln y\right]$

$t = -\frac{1}{k}(0 - \ln 2)$ $\quad [\ln 1 = 0]$

Or $\boxed{t = \frac{\ln 2}{k}}$

Answer 43E.

Consider the magnitude of the earth quake,

$$M = \log_{10}\left(\frac{I}{S}\right)$$

The objective is to find the magnitude of the San Francisco earthquake.

Here I is intensity and S is the intensity of the standard earth quake.

Observe that the magnitude of the Loma Prieta earthquake is 7.1

And the intensity of the San Francisco earthquake is 16 times the Loma Prieta earthquake.

Observe that $M = 7.1$

$$7.1 = \log_{10}\left(\frac{I}{S}\right) \text{ ...... (1)}$$

And

$$M = \log_{10}\left(\frac{16I}{S}\right) \text{ ...... (2)}$$

Solve (1) and (2) to get,

$$7.1 - M = \log_{10}\left(\frac{I}{S}\right) - \log_{10}\left(\frac{16I}{S}\right)$$

$$7.1 - M = -\log_{10} 16$$

$$M = 7.1 + \log_{10} 16$$

$$= 7.1 + 1.2011 \qquad \text{Use } \log_{10} 16 = 1.2011$$

$$= 8.3011$$

$$\approx 8.3$$

Hence, the magnitude of the San Francisco earthquake is $M = \boxed{8.3}$.

Answer 44E.

Loudness is defined as $L = 10\log_{10}\left(\frac{I}{I_0}\right)$

Where $I_0 = 10^{-12}$ watt/m^2 (given)

According to the problem music has loudness = 120 dB

So $10\log_{10}\left(\frac{I_1}{I_0}\right) = 120$ where I_1 is the intensity of music

Or $\log_{10}\left(\frac{I_1}{I_0}\right) = 12$

Or $\frac{I_1}{I_0} = 10^{12}$ $\qquad [\log_a x = y \Leftrightarrow a^y = x]$

Or $I_1 = 10^{12} I_0$

Now the loudness of a lawn mower is = 106 dB

So $10\log_{10}\left(\frac{I_2}{I_0}\right) = 106$ where I_2 is the intensity of mower.

$$\Rightarrow \log_{10}\left(\frac{I_2}{I_0}\right) = 10.6$$

$$\Rightarrow I_2 = (10)^{10.6}.I_0$$

$$\Rightarrow I_2 = (10)^{106/10}.I_0$$

Then $\frac{I_1}{I_2} = \frac{10^{12} I_0}{(10)^{106/10} I_0}$

$$= \frac{10^{12}}{(10)^{106/10}} \approx \frac{100 \times 10^{10}}{3.98 \times 10^{10}} \approx \frac{100}{3.98}$$

Or $\frac{I_1}{I_2} \approx \frac{50}{1.99}$

If we take approximate value of 1.99 = 2 then

$$\boxed{I_1 : I_2 \approx 25 : 1}$$

Answer 45E.

(a)

Consider the equation $n = f(t) = 100 \cdot 2^{t/3}$

$$n = 100 \cdot 2^{t/3}$$

$$\frac{n}{100} = 2^{\frac{t}{3}}$$

We have $\log_a x = y$ if and only if $a^y = x$

$$\ln_2\left(\frac{n}{100}\right) = \frac{t}{3}$$

$$t = 3\ln_2\left(\frac{n}{100}\right)$$

The inverse of this function is $t = f^{-1}(n) = 3 \cdot \ln_2\left(\frac{n}{100}\right)$

This function tells us the time elapsed when there are n bacteria.

(b)

If $n = 50{,}000$ then $t = f^{-1}(n) = 3 \cdot \ln_2\left(\frac{n}{100}\right)$ is

$$f^{-1}(50000) = 3 \cdot \ln_2\left(\frac{50000}{100}\right)$$

$$= 3 \cdot \frac{\ln\left(\frac{50000}{100}\right)}{\ln 2}$$

$$= 3\left(\frac{\ln 500}{\ln 2}\right)$$

$$= 3\left(\frac{6.2146}{0.6931}\right)$$

$$= 3(8.9657)$$

$$= 26.8973$$

$$= \boxed{26.9} \text{ hours}$$

Answer 46E.

(A)

We have $Q(t) = Q_0\left(1 - e^{-t/a}\right)$

$$\Rightarrow 1 - e^{-t/a} = \frac{Q(t)}{Q_0}$$

$$\Rightarrow e^{-t/a} = 1 - \frac{Q(t)}{Q_0}$$

Taking logarithms of both sides

$$\ln e^{-t/a} = \ln\left(1 - \frac{Q(t)}{Q_0}\right)$$

$$-\frac{t}{a} = \ln\left(1 - \frac{Q(t)}{Q_0}\right)$$

Inverse is $\Rightarrow \boxed{t = -a\ln\left(1 - \frac{Q(t)}{Q_0}\right)} = Q^{-1}(t)$

This function denotes the time elapsed when electric change is $Q(t)$

(B) Given that $Q(t) = 90\% \ of \ Q_0$

So $Q(t) = \frac{9}{10} Q_0$ and $a = 2$

Then time $t = -2\ln\left(1 - \frac{9Q_0}{10Q_0}\right)$

$$t = -2\ln\left(\frac{1}{10}\right)$$

Or $t \approx -2.(-2.3)$

Or $\boxed{t \approx 4.6}$ seconds

Answer 47E.

Consider the limit,

$$\lim_{x \to 3^+} \ln\left(x^2 - 9\right).$$

The objective of the problem is to find the limit $\lim_{x \to 3^+} \ln\left(x^2 - 9\right)$.

Let $f(x) = \ln\left(x^2 - 9\right)$.

Substitute $x = 3.1, 3.01, 3.001$ into the function $f(x) = \ln\left(x^2 - 9\right)$, we have

$$\begin{aligned} f(3.1) &= \ln\left((3.1)^2 - 9\right) \\ &= \ln(0.61) \\ &\approx -0.4943 \end{aligned}$$

$$\begin{aligned} f(3.01) &= \ln\left((3.01)^2 - 9\right) \\ &= \ln(0.0601) \\ &\approx -2.8117 \end{aligned}$$

$$\begin{aligned} f(3.001) &= \ln\left((3.001)^2 - 9\right) \\ &= \ln(0.006001) \\ &\approx -5.1158 \end{aligned}$$

Observe that the function values are different and negative when $x \to 3^+$.

So it confirms that as $x \to 3^+$ then $\lim_{x \to 3^+} \ln\left(x^2 - 9\right)$ approaches to $-\infty$.

Hence, $\lim_{x \to 3^+} \ln\left(x^2 - 9\right) = \boxed{-\infty}$.

Let $f(x) = \ln\left(x^2 - 9\right)$.

Evaluate the above limit from the graph of the function.

The sketch of the function $f(x) = \ln\left(x^2 - 9\right)$ is shown below:

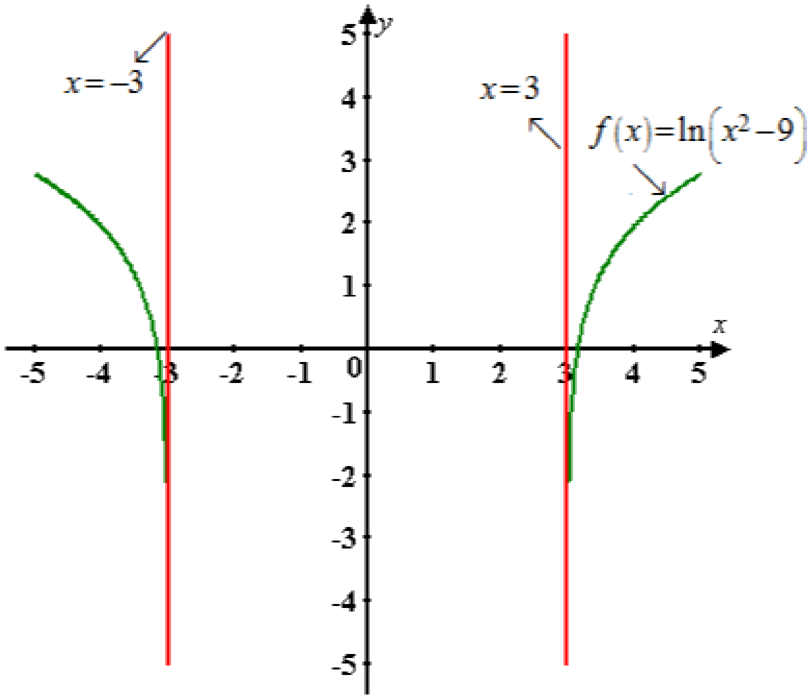


Figure 1

Observe the figure 1, it seen that the line $x=3$ and $x=-3$ are does not touch the graph of $f(x)=\ln(x^2-9)$ from the left and right side and approaches to $-\infty$.

At $x \to 3^+$ the limit of the function approaches to $-\infty$.

Hence, $\lim_{x\to 3^+} \ln(x^2-9) = \boxed{-\infty}$.

Answer 48E.

Consider the limit,

$\lim_{x\to 2^-} \log_5(8x-x^4)$.

Let $f(x)=\dfrac{\log(8x-x^4)}{\log(5)}$.

Since as $x \to 2$, implies that $8x-x^4 \to 0$.

Here, $8x-x^4$ is decreasing near 2, so if x approaches 2 from the left, $8x-x^4$ will approach 0 from the right.

Obviously as $y \to 0^+$, $\log_5 y \to -\infty$.

So the value of the limit is

$$\begin{aligned}\lim_{x\to 2^-} \log_5(8x-x^4) &= \lim_{y\to 0^+} \log_5 y \\ &= -\infty\end{aligned}$$

Hence, $\lim_{x\to 2^-} \log_5(8x-x^4) = \boxed{-\infty}$.

The sketch of the function $f(x)=\dfrac{\log(8x-x^4)}{\log(5)}$ is shown below:

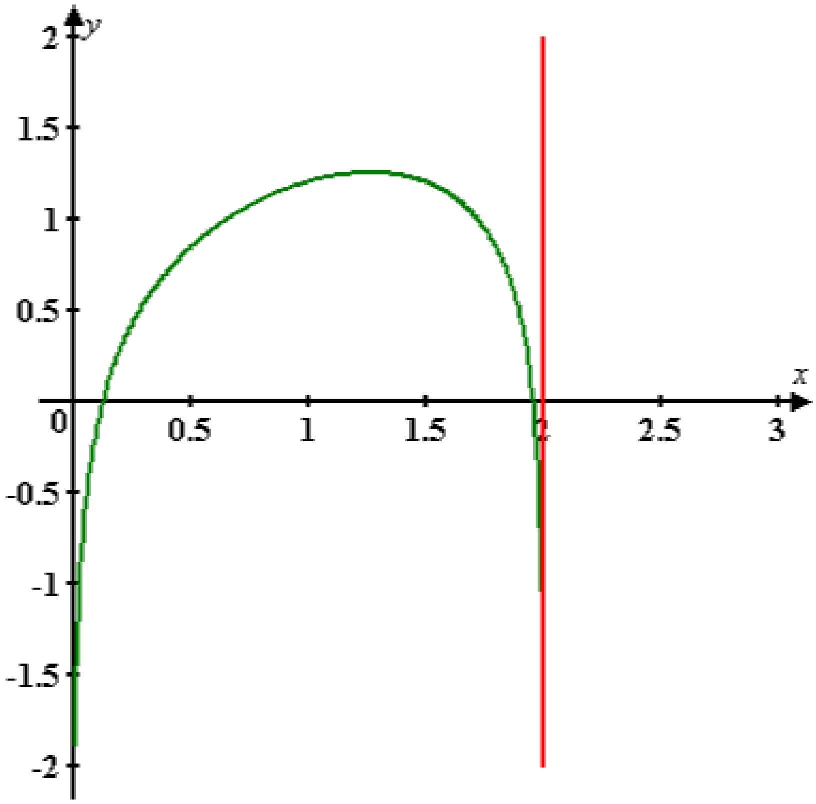


Figure 1

Observe the figure 1, it confirms that the values of the function to the right of 2 approaches to negative values and the limit value is $-\infty$.

Hence, $\lim_{x\to 2^-} \log_5\left(8x - x^4\right) = \boxed{-\infty}$.

Answer 49E.

We have to evaluate $\lim_{x\to 0} \ln\left(\cos x\right)$

$$\lim_{x\to 0} \ln\cos x = \ln\left(\cos 0\right)$$
$$= \ln 1$$
$$= \boxed{0}$$

Answer 50E.

We have to evaluate $\lim_{x\to 0^+} \ln\left(\sin x\right)$

For this let $\sin x = t$

If x is close to 0 but greater than 0 then $\sin x$ i.e. t is a small positive number close to 0.

Therefore, $\lim_{x\to 0^+} \ln\left(\sin x\right) = \lim_{t\to 0^+} \ln t$

$= -\infty$ Since $\lim_{x\to 0} \ln x = -\infty$.

Hence

$$\boxed{\lim_{x\to 0^+} \ln\left(\sin x\right) = -\infty}$$

Answer 51E.

We have to evaluate $\lim_{x\to\infty} \left[\ln(1+x^2)-\ln(1+x)\right]$

Now $\lim_{x\to\infty} \left[\ln(1+x^2)-\ln(1+x)\right]$

$= \lim_{x\to\infty} \ln\frac{(1+x^2)}{(1+x)}$ Since $\ln\frac{x}{y}=\ln x-\ln y$

$= \lim_{x\to\infty} \ln\frac{x^2\left(\frac{1}{x^2}+1\right)}{x\left(\frac{1}{x}+1\right)}$

$= \lim_{x\to\infty} \ln\frac{x\left(1+\frac{1}{x^2}\right)}{\left(1+\frac{1}{x}\right)}$

$= \lim_{x\to\infty} \ln x+\lim_{x\to\infty} \ln\frac{\left(1+\frac{1}{x^2}\right)}{\left(1+\frac{1}{x}\right)}$ Since $\ln xy=\ln x+\ln y$

$= \lim_{x\to\infty} \ln x+\lim_{x\to\infty}\ln\left(1+\frac{1}{x^2}\right)-\lim_{x\to\infty}\ln\left(1+\frac{1}{x}\right)$ Since $\ln\frac{x}{y}=\ln x-\ln y$

$=\infty+\ln 1-\ln 1$ Since $\lim_{x\to\infty} \ln x=\infty$

and $\lim_{x\to\infty}\frac{1}{x^2}=0$

$=\infty$

Hence

$$\boxed{\lim_{x\to\infty} \left[\ln(1+x^2)-\ln(1+x)\right]=\infty}$$

Answer 52E.

We have to evaluate $\lim_{x\to\infty} \left[\ln(2+x)-\ln(1+x)\right]$

Now $\lim_{x\to\infty} \left[\ln(2+x)-\ln(1+x)\right]$

$= \lim_{x\to\infty} \ln\left(\frac{2+x}{1+x}\right)$ Since $\ln\frac{x}{y}=\ln x-\ln y$

$= \lim_{x\to\infty} \ln\frac{x\left(\frac{2}{x}+1\right)}{\left(\frac{1}{x}+1\right)}$

$= \lim_{x\to\infty} \ln\left(\frac{1+\frac{2}{x}}{1+\frac{1}{x}}\right)$

$= \ln\frac{\left(1+\lim_{x\to\infty}\frac{2}{x}\right)}{\left(1+\lim_{x\to\infty}\frac{1}{x}\right)}$

$= \ln\left(\frac{1+0}{1+0}\right)$ Since $\lim_{x\to\infty}\frac{k}{x}=0$ where k is a constant.

$= \ln 1$
$= 0$

Hence

$$\boxed{\lim_{x\to\infty} \left[\ln(2+x)-\ln(1+x)=0\right]}$$

Answer 53E.

Consider the function $f(x)=\log_{10}(x^2-9)$

Let $y=(x^2-9)$

The domain of $y=(x^2-9)$ is $\{x/x^2-9>0\}$

$=\{x/|x|>3\}$
$=(-\infty,-3)\cup(3,\infty)$

The domain of $f(x)=\log_{10}(x^2-9)$ is $\boxed{(-\infty,-3)\cup(3,\infty)}$

Answer 54E.

Consider the function $f(x)=\ln x+\ln(2-x)$

$$f(x)=\ln x+\ln(2-x)$$
$$=\ln[x(2-x)]$$

Let $y=[x(2-x)]$

The domain of $y=[x(2-x)]$ is $\{x/x(2-x)>0\}$

$=\{x/x(2-x)>0\}$
$=\{x/x(x-2)<0\}$
$=\{x/(x-0)(x-2)<0\}$
$=x\in(0,2)$

The domain of $f(x)=\ln x+\ln(2-x)$ is $\boxed{x\in(0,2)}$

Answer 55E.

(A) $f(x)=\sqrt{3-e^{2x}}$

$f(x)$ is defined when

$3-e^{2x}\geq 0$

Or $e^{2x}\leq 3$

Or $\ln e^{2x}\leq\ln 3$ [Taking logarithms]

Or $2x\leq\ln 3$ $[\ln e^x = x\ \ for\ \ all\ \ x]$

Or $x\leq\frac{1}{2}\ln 3$

So domain of $f(x)$ is $=\boxed{\left(-\infty,\frac{1}{2}\ln 3\right]}$

(B) Let $y=\sqrt{3-e^{2x}}$

Then $y^2=3-e^{2x}$

Or $e^{2x}=3-y^2$

Taking logarithms of both sides

$\ln e^{2x}=\ln(3-y^2)$

Or $2x=\ln(3-y^2)$

Or $x=\frac{1}{2}\ln(3-y^2)$

Replacing x and y

$$\boxed{f^{-1}(x)=y=\frac{1}{2}\ln(3-x^2)}$$

$f^{-1}(x)$ is defined when $3-x^2>0$ or $x^2<3$ or $|x|<\sqrt{3}$

So domain of $f^{-1}(x)$ is $\boxed{=\left(-\sqrt{3},\sqrt{3}\right)}$

Answer 56E.

(A) We have $f(x) = \ln(2+\ln x)$

$f(x)$ is defined when

$2+\ln x > 0$

Or $\ln x > -2$

Or $e^{\ln x} > e^{-2}$

Or $x > e^{-2}$

So domain of $f(x)$ is (e^{-2}, ∞)

(B) Let $y = \ln(2+\ln x)$

Or $e^y = 2+\ln x$ $\quad [\ln x = y \Leftrightarrow e^y = x]$

Or $e^y - 2 = \ln x$

Or $e^{(e^y-2)} = x$

Replacing x and y $\quad y = \boxed{f^{-1}(x) = e^{(e^x-2)}}$

Domain of the function $f^{-1}(x)$ is the set of real numbers $\mathbb{R}$

Answer 57E.

Given $f(x) = \ln(e^x - 3)$

(a) $\ln x$ is defined for $x > 0$

Here $e^x - 3 > 0$

$\Rightarrow e^x > 3$

$\Rightarrow x > \ln 3$

Therefore domain $f(x)$ is $(\ln 3, \infty)$

(b) Let $y = \ln(e^x - 3)$

$\Rightarrow e^y = e^x - 3$

$\Rightarrow e^x = e^y + 3$

$\Rightarrow x = \ln(e^y + 3)$

$\Rightarrow \boxed{f^{-1}(x) = \ln(e^x + 3)}$

$\boxed{\text{And domain of } f^{-1} \text{ is } \mathbb{R} = (-\infty, \infty)}$

Answer 58E.

Consider the following functions:

$e^{\ln(300)}$ and $\ln(e^{300})$

(a)

The objective is to find the values of $e^{\ln(300)}$ and $\ln(e^{300})$

Find the value of $e^{\ln(300)}$

Use the formula $e^{(\ln x)} = x,\ x > 0$

Substitute 300 for *x*

$e^{\ln(300)} = 300$

Find the value of $\ln(e^{300})$

Use the formula $\ln(e^x) = x, x \in \mathbb{R}$

Substitute 300 for *x*.

$\ln(e^{300}) = 300$

Therefore, the values of $e^{\ln(300)}$ and $\ln(e^{300})$ is $\boxed{300}$

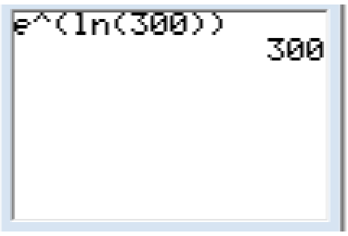
(b)

Use graphing calculator to find the value of $e^{\ln(300)}$.

Key strokes:

2nd LN ^ LN 300))

Input and output screen shots.



Therefore, the value of $e^{\ln(300)}$ is $\boxed{300}$

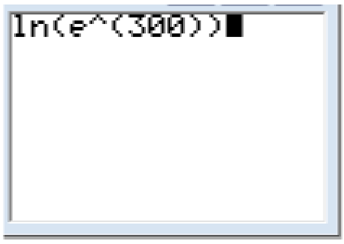
(b)

Use graphing calculator to find the value of $\ln\left(e^{300}\right)$.

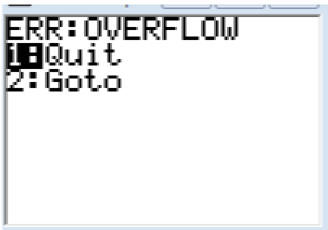
Key strokes:

LN 2nd LN ^ 300))

Input and output screen shots:



The output screen shot is



Therefore, the value of $\ln\left(e^{300}\right)$ is not obtained by graphing calculator because, the exponential value is large.

Answer 59E.

We have $y = \ln(x+3)$

Then $e^y = x+3$ by $[\ln x = y \Leftrightarrow e^y = x]$

Or $x = e^y - 3$

Replacing x and y

$y = e^x - 3$

So inverse function is $\boxed{y = e^x - 3}$

Answer 60E.

We have $y = 2^{10^x}$

Taking logarithms of both sides

$\ln y = \ln 2^{10^x}$

Or $\ln y = 10^x \ln 2$ $[\ln x^r = r \ln x]$

Or $10^x = \frac{(\ln y)}{(\ln 2)}$

Or $x = \log_{10}\left(\frac{\ln y}{\ln 2}\right)$ $\left[\log_a x = y \Leftrightarrow a^y = x\right]$

Or $x = \log_{10}(\log_2 y)$ $\left[\log_a x = \frac{\ln x}{\ln a}\right]$

Replacing x and y

Inverse function is $\boxed{y = \log_{10}(\log_2 x)}$

Answer 61E.

We have $f(x) = e^{x^3}$

Let $y = e^{x^3}$

Taking logarithms of both sides

$\ln y = \ln e^{x^3}$

Or $x^3 = \ln y$ $\left[\ln e^x = x\right]$

Or $x = \sqrt[3]{\ln y}$

Replacing x and y

The inverse function is $\boxed{f^{-1}(x) = \sqrt[3]{\ln x}}$

Answer 62E.

We have $y = (\ln x)^2$, $x \geq 1$

Then $\sqrt{y} = (\ln x)$

Here $x \geq 1$ so $\ln x \geq 0$ and $\sqrt{y} \geq 0$

Now $\sqrt{y} = \ln x$

So $e^{\sqrt{y}} = x, y \geq 0$ $\left[\ln x = y \Leftrightarrow e^y = x\right]$

Replacing x and y

The inverse function is $\boxed{f^{-1}(x) = e^{\sqrt{x}}}$, $x \geq 0$

Answer 63E.

Consider the function $y = \log_{10}\left(1 + \frac{1}{x}\right)$

Recollect that, $\log_a x = y$ if and only if $a^y = x$

$$10^y = \left(1 + \frac{1}{x}\right)$$

$$\frac{1}{x} = 10^y - 1$$

$$x = \frac{1}{10^y - 1}$$

Interchange x and y so, $y = \frac{1}{10^x - 1}$

Therefore the inverse function of $y = \log_{10}\left(1 + \frac{1}{x}\right)$ is $\boxed{y = \frac{1}{10^x - 1}}$

Answer 64E.

Consider the function $y=\frac{e^x}{1+2e^x}$.

To find the inverse of the function, solve for *x* in terms of *y*.

$y=\frac{e^x}{1+2e^x}$

$y\left(1+2e^x\right)=e^x$ Multiply by $1+2e^x$

$y+2ye^x=e^x$

$y=e^x-2ye^x$ Subtract $2ye^x$ from both sides

$y=e^x\left(1-2y\right)$ Common out e^x

$\frac{y}{1-2y}=e^x$ Divide by $1-2y$

$x=\ln\left(\frac{y}{1-2y}\right)$ Apply $\ln$ on both sides; solve for *x*

$y=\ln\left(\frac{x}{1-2x}\right)$ Interchange *x* and *y*

$f^{-1}(x)=\ln\left(\frac{x}{1-2x}\right)$ Replace *y* by $f^{-1}(x)$

Therefore,

The inverse of the function $y=\frac{e^x}{1+2e^x}$ is $\boxed{f^{-1}(x)=\ln\left(\frac{x}{1-2x}\right)}$

Answer 65E.

We have the function $f(x)=e^{3x}-e^x$

Differentiating with respect to x

$$f'(x)=\frac{d}{dx}e^{3x}-\frac{d}{dx}e^x$$

Or $f'(x)=3.e^{3x}-e^x$ [by chain rule and $\frac{d}{dx}e^x=e^x$]

Since for increasing function $f(x)$, $f'(x)>0$

So $3e^{3x}-e^x>0$

Or $3e^{2x}-1>0$

Or $e^{2x}>\frac{1}{3}$

Taking logarithms

$$\ln e^{2x}>\ln\left(\frac{1}{3}\right)$$

$$\Rightarrow 2x>\ln\left(\frac{1}{3}\right) \qquad \left[\ln e^x=x\right]$$

$$\Rightarrow x>\frac{1}{2}\ln\left(\frac{1}{3}\right)$$

$$\Rightarrow x>\frac{1}{2}\ln\left(3^{-1}\right)$$

$$\Rightarrow x>-\frac{1}{2}\ln 3 \qquad \left[\ln x^r=r\ln x\right]$$

So $f(x)$ is increasing on $\boxed{\left(-\frac{1}{2}\ln 3,\infty\right)}$

Answer 66E.

We have $f(x)=y=2e^x-e^{-3x}$

Then $y'=2e^x-(-3)e^{-3x}$ [by chain rule and $\frac{d}{dx}e^x=e^x$]

$$\Rightarrow y'=2e^x+3e^{-3x}$$

Again differentiating with respect to x

$$y'' = 2e^x + 3(-3)e^{-3x}$$

$$\Rightarrow y'' = 2e^x - 9e^{-3x}$$

Since $f(x)$ is concave downward when $y'' < 0$ *or* $f''(x) < 0$

$\Rightarrow y'' < 0$ When $2e^x - 9e^{-3x} < 0$

Or $2 - 9e^{-4x} < 0$

Or $-9e^{-4x} < -2$

Or $e^{-4x} > \frac{2}{9}$

Or $4x < \ln\left(\frac{9}{2}\right)$

$$\Rightarrow x < \frac{1}{4}\ln\left(\frac{9}{2}\right)$$

So $f(x)$ is concave downward on the interval $\boxed{\left(-\infty, \frac{1}{4}\ln\left(\frac{9}{2}\right)\right)}$

Answer 67E.

(A) The given function is $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$.

To show that $f(x)$ is odd function, we have to show that $f(-x) = -f(x)$.

Now $f(-x) = \ln\left[-x + \sqrt{(-x)^2+1}\right]$

$$= \ln\left[-x + \sqrt{x^2+1}\right]$$

$$= \ln\frac{\left(-x+\sqrt{x^2+1}\right)\left(x+\sqrt{x^2+1}\right)}{\left(x+\sqrt{x^2+1}\right)}$$

$$= \ln\frac{\left(\sqrt{x^2+1}\right)^2 - x^2}{\left(x+\sqrt{x^2+1}\right)}$$ Since $a^2 - b^2 = (a+b)(a-b)$

$$= \ln\frac{\left(x^2+1-x^2\right)}{\left(x+\sqrt{x^2+1}\right)}$$

$$= \ln\frac{1}{x+\sqrt{x^2+1}}$$

$$= \ln 1 - \ln\left(x+\sqrt{x^2+1}\right)$$ Since $\ln\frac{x}{y} = \ln x - \ln y$.

$$= 0 - \ln\left(x+\sqrt{x^2+1}\right)$$ Since $\ln 1 = 0$

$$= -\ln\left(x+\sqrt{x^2+1}\right)$$

$$= -f(x)$$

Hence $f(x) = \ln\left(x+\sqrt{x^2+1}\right)$ is an odd function.

(B) We have to find the inverse function of

$$f(x)=\ln\left(x+\sqrt{x^2+1}\right)$$

For this we put $y=f(x)$ and solve it for x. So we have

$$y=\ln\left(x+\sqrt{x^2+1}\right)$$

Applying exponential function to both sides of the equation we get,

$$e^y=e^{\ln\left(x+\sqrt{x^2+1}\right)}$$

$$\Rightarrow \quad e^y=x+\sqrt{x^2+1} \quad \ldots\ldots\ldots\ldots\ldots(i)$$ Since $e^{\ln x}=x$

Since $f(x)$ is odd function. So we have

$$f(-x)=-f(x)$$

$$\Rightarrow \quad \ln\left(-x+\sqrt{\left(-x^2\right)+1}\right)=-y$$

$$\Rightarrow \quad -y=\ln\left[-x+\sqrt{x^2+1}\right]$$

Applying exponential function to both sides of the equation, we get,

$$e^{-y}=e^{\ln\left(-x+\sqrt{x^2+1}\right)}$$

$$\Rightarrow \quad e^{-y}=-x+\sqrt{x^2+1} \quad \ldots\ldots\ldots\ldots\ldots(ii)$$

Subtracting equation (ii) from equation (i), we get

$$e^y-e^{-y}=\left(x+\sqrt{x^2+1}\right)-\left(-x+\sqrt{x^2+1}\right)$$

$$=x+\sqrt{x^2+1}+x-\sqrt{x^2+1}$$

$$=2x$$

Or $x=\frac{1}{2}\left(e^y-e^{-y}\right)$

Finally, we interchange x and y to get the inverse function, so we have

$$y=\frac{1}{2}\left(e^x-e^{-x}\right)$$

Hence

$$\boxed{\text{inverse function is } y=\frac{1}{2}\left(e^x-e^{-x}\right)}$$

Answer 68E.

The equation of line $2x-y=8$ can be written as $y=2x-8$. It's slope is 2.
Since tangent to the cure $y=e^{-x}$ is perpendicular to the line $2x-y=8$
Therefore,

$$\text{Slope of tangent } = -\frac{1}{\text{slope of line } 2x-y=8}$$

$$=\frac{-1}{2}$$

The equation of given curve is

$$y = e^{-x}$$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}e^{-x}$$

$$= e^{-x}(-1)$$

$$= -e^{-x}$$

Now slope of tangent to the given curve $= \frac{dy}{dx}$.

So $\quad \frac{dy}{dx} = -\frac{1}{2}$

$\Rightarrow \quad -e^{-x} = -\frac{1}{2}$

$\Rightarrow \quad \frac{1}{e^x} = \frac{1}{2}$

$\Rightarrow \quad e^x = 2$

Taking natural logarithms of both sides, we get,

$$\ln e^x = \ln 2$$

$\Rightarrow \quad x = \ln 2 \qquad\qquad$ Since $\ln e^x = x$

Also $y = e^{-x} = \frac{1}{2}$

Thus the point on the curve $y = e^{-x}$ at which the tangent is perpendicular to the line $2x - y = 8$ is $\left(\ln 2, \frac{1}{2}\right)$

The equation of tangent to the curve at point (x_1, y_1) is

$$y - y_1 = \left(\frac{dy}{dx}\right)(x - x_1)$$

Therefore, the equation of tangent to the curve $y = e^{-x}$ at point $\left(\ln 2, \frac{1}{2}\right)$ is

$$y - \frac{1}{2} = -\frac{1}{2}(x - \ln 2)$$

$\Rightarrow \quad 2y - 1 = -x + \ln 2$

$\Rightarrow \quad x + 2y = 1 + \ln 2$

Hence

Equation of tangent is
$x + 2y = 1 + \ln 2$

Answer 69E.

The given equation is

$$x^{1/\ln x} = 2$$

Taking logarithms on both sides we get

$\ln x^{1/\ln x} = \ln 2$

$\Rightarrow \quad \frac{1}{\ln x}\ln x = \ln 2 \qquad [\because \ln y^m = m \ln y]$

$\Rightarrow \quad \ln x = \ln x \ln 2$

$\Rightarrow \quad \ln x - \ln x \ln 2 = 0$

$\Rightarrow \quad \ln x[1 - \ln 2] = 0 \qquad$ Since, $1 - \ln 2 \neq 0$

$\Rightarrow \quad \ln x = 0$

$\Rightarrow \quad e^{\ln x} = e^0$

$\Rightarrow \quad x = 1$

But $x \neq 1$

Therefore, the equation $x^{1/\ln x}$ has no solution.

The given function is

$$f(x)=x^{1/\ln x}$$

Taking natural logarithms of both sides.
We have,

$$\ln f(x)=\ln(x)^{1/\ln x}$$

$\Rightarrow \quad \ln f(x)=\frac{1}{\ln x}\cdot\ln x \qquad$ Since $\ln x^y = y\ln x$

$\Rightarrow \quad \ln f(x)=1$

$\Rightarrow \quad e^{\ln f(x)}=e^1$

$\Rightarrow \quad f(x)=e \quad$ (constant). $\qquad$ Since, $e^{\ln x}=x$

Here e is a constant. Thus $f(x)=e$ is a constant function and its graph is a line parallel to the x – axis at a distance e units above the x – axis.

Answer 70E.

Definition:

Suppose any function of the form $f(x)=[g(x)]^{h(x)}, g(x)>0$, can be analyzed as a power of e by writing $g(x)=e^{\ln g(x)}$ so that $f(x)=e^{h(x)\ln g(x)}$.

(a)

Consider the limit $\lim_{x\to\infty} x^{\ln x}$.

To find the limit, use the definition above.

Let $f(x)=x^{\ln x}$ then by definition $x=e^{\ln x}$ so that

$$f(x)=e^{\ln x\cdot\ln x}$$
$$=e^{(\ln x)^2}$$

Now,

$\lim_{x\to\infty} x^{\ln x}=\lim_{x\to\infty} f(x)$ Since $f(x)=x^{\ln x}$

$=\lim_{x\to\infty} e^{(\ln x)^2}$ Substitute $f(x)=e^{(\ln x)^2}$

$=e^{(\ln\infty)^2}$

$=\infty$

Therefore,

$$\lim_{x\to\infty} x^{\ln x}=\boxed{\infty}.$$

(b)

Consider the limit $\lim_{x\to 0^+} x^{-\ln x}$.

Let $f(x)=x^{-\ln x}$ then by definition $x=e^{-\ln x}$ so that

$$f(x)=e^{-\ln x\cdot\ln x}$$
$$=e^{-(\ln x)^2}$$

Now,

$\lim_{x\to 0^+} x^{-\ln x}=\lim_{x\to 0^+} f(x)$ Since $f(x)=x^{-\ln x}$

$=\lim_{x\to 0^+} e^{-(\ln x)^2}$ Substitute $f(x)=e^{-(\ln x)^2}$

$=e^{-(\ln 0)^2}$

$=0$

Therefore,

$$\lim_{x\to 0^+} x^{-\ln x}=\boxed{0}$$

(c)

Consider the limit $\lim_{x\to 0^+} x^{1/x}$.

Let $f(x)=x^{1/x}$ then by definition $x=e^{1/x}$ so that

$$f(x)=e^{1/x(1/x)}$$
$$=e^{(1/x)^2}$$

Now,

$\lim_{x\to 0^+} x^{1/x}=\lim_{x\to 0^+} f(x)$ Since $f(x)=x^{1/x}$

$=\lim_{x\to 0^+} e^{(1/x)^2}$ Substitute $f(x)=e^{(1/x)^2}$

$=e^{(1/0)^2}$

$=\text{Undefined}$

Therefore,

$\lim_{x\to 0^+} x^{1/x}=\boxed{\text{Undefined}}$.

(d)

Consider the limit $\lim_{x\to\infty}(\ln 2x)^{-\ln x}$.

To find the limit, use the definition above.

Let $f(x)=(\ln 2x)^{-\ln x}$ then by definition $\ln 2x=e^{\ln(\ln 2x)}$ so that

$$f(x)=e^{-\ln x\cdot\ln(\ln 2x)}$$

Now,

$\lim_{x\to\infty}(\ln 2x)^{-\ln x}=\lim_{x\to\infty} f(x)$ Since $f(x)=(\ln 2x)^{-\ln x}$

$=\lim_{x\to\infty} e^{-\ln x\cdot\ln(\ln 2x)}$ Substitute $f(x)=e^{-\ln x\cdot\ln(\ln 2x)}$

$=e^{-\ln\infty\cdot\ln(\ln 2\infty)}$

$=0$

Therefore,

$\lim_{x\to\infty}(\ln 2x)^{-\ln x}=\boxed{0}$.

Answer 71E.

Consider the curve $f(x)=(1-x)e^{-x}$

(a) Required to find the interval of increase or decrease.

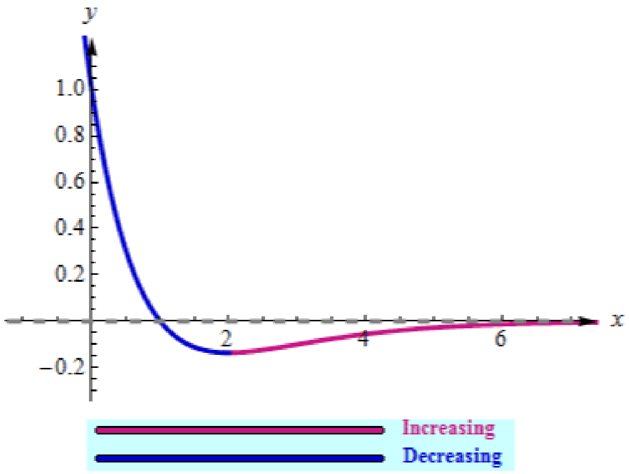
Use the I/D test. Compute $f'(x)$ and find the intervals on which $f'(x)$ is positive (*f* is increasing) and the intervals on which $f'(x)$ is negative (*f* is decreasing).

$$f(x)=(1-x)e^{-x}$$
$$f'(x)=(1-x)(-e^{-x})+e^{-x}(-1)$$
$$=e^{-x}(x-1-1)$$
$$=e^{-x}(x-2)$$

Since $f'(x)>0$ when $x>2$ and $f'(x)<0$ when $x<2$.

Therefore, f is increasing on $\boxed{(2,\infty)}$ and decreasing on $\boxed{(-\infty,2)}$.

Graph shows the interval of increase or decrease



(b) Required to find the intervals of concavity.

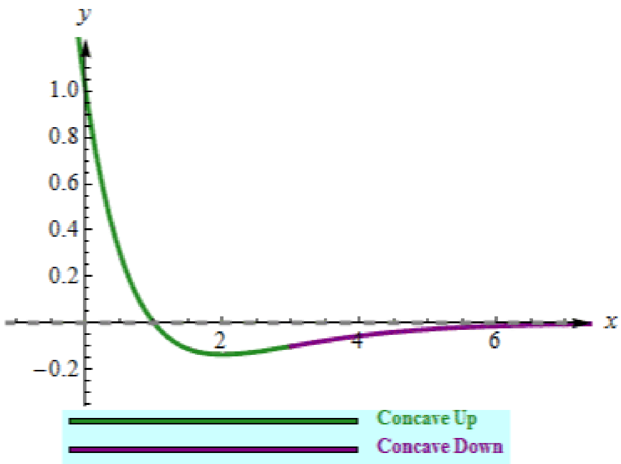
Compute $f''(x)$ and use the concavity test.the curve is concave upward where $f''(x)>0$ and concave down where $f''(x)<0$.

$$f''(x)=e^{-x}(1)+(x-2)-e^{-x}$$
$$=e^{-x}(1-x+2)$$
$$=e^{-x}(3-x)$$

Since $f''(x)>0$ when $x<3$ and $f''(x)<0$ when $x>3$.

Therefore, f is concave up on $\boxed{(-\infty,3)}$ and concave down on $\boxed{(3,\infty)}$.

Graph shows the intervals of concavity



(c) Required to find the inflection points

Inflection points occur where the direction of concavity changes.

f'' changes sign at $x=3$

Plug $x=3$ in $f(x)=(1-x)e^{-x}$

$$f(3)=(1-3)e^{-3}$$
$$=-2e^{-3}$$

So there is inflection point at $\boxed{(3,-2e^{-3})}$

Answer 72E.

Consider the following functions:

$f(x) = x^{(0.1)}$ and $g(x) = \ln x$

The objective is to compare the rates of growth of the functions $f(x)$ and $g(x)$ in several viewing rectangles.

Use maple commands.

Plot the function $f(x)$ and $g(x)$ by using plot command.

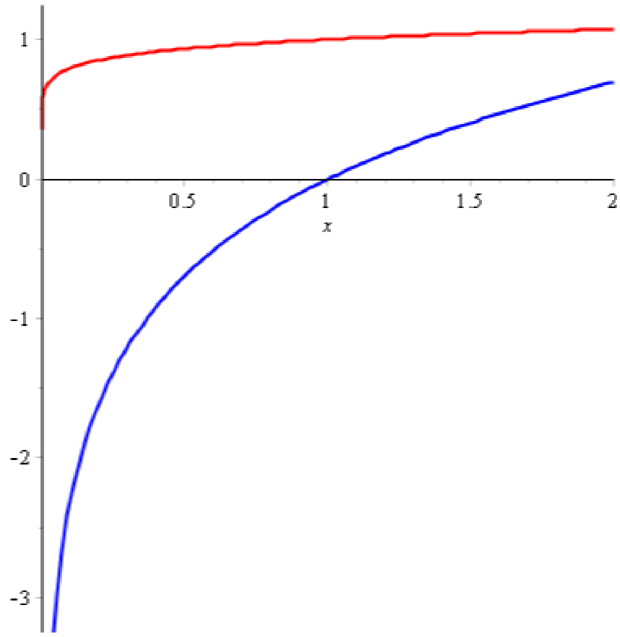
The maple command is

plot({ f, g }, x = -2..2, thickness=2, color=[red, blue]);

Key strokes:

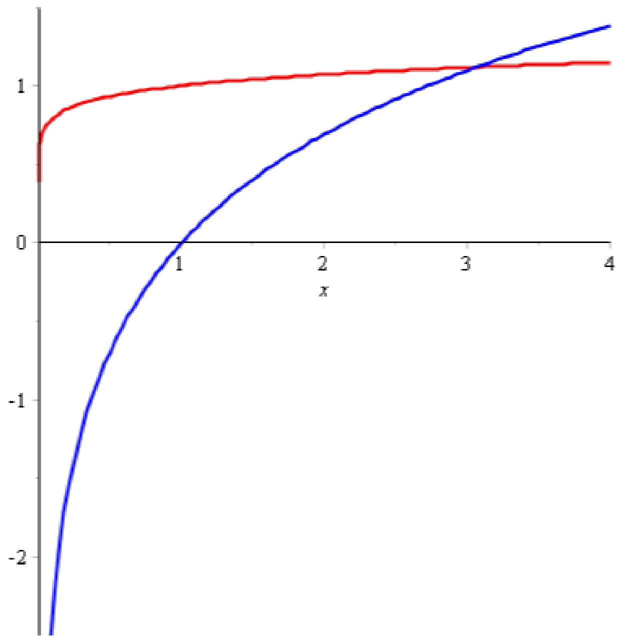
plot({x^.1, ln(x)}, x = -2 .. 2, thickness = 2, color = [red, blue])

$plot(\{x^{(0.1)}, \ln(x)\}, x=-2..2, thickness=2, color=[red, blue]);$



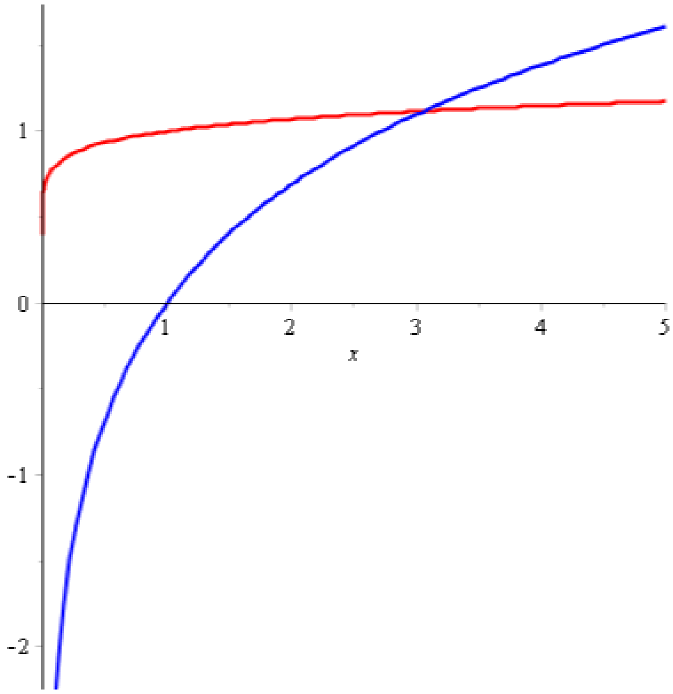
plot({x^.1, ln(x)}, x = -4 .. 4, thickness = 2, color = [red, blue])

$plot(\{x^{(0.1)}, \ln(x)\}, x=-4..4, thickness=2, color=[red, blue])$



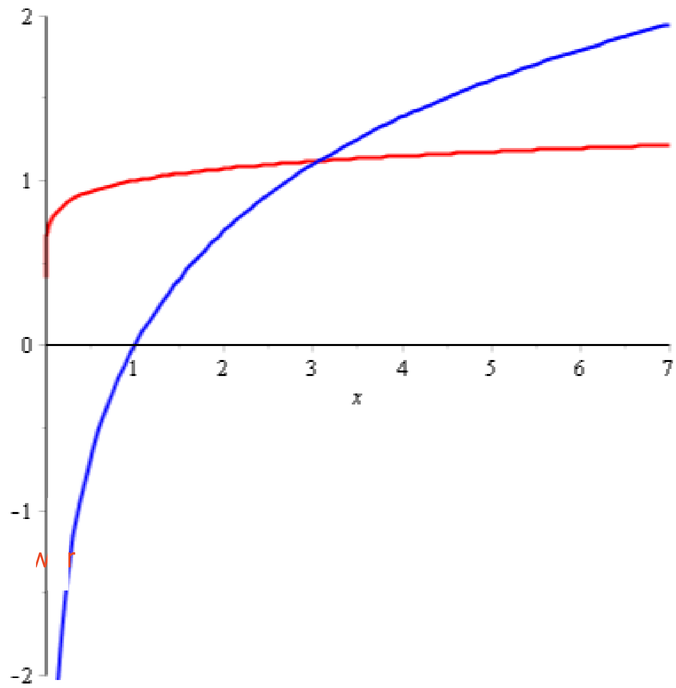
plot({x^.1, ln(x)}, x = -5 .. 5, thickness = 2, color = [red, blue])

$plot(\{x^{(0.1)}, \ln(x)\}, x=-5..5, thickness=2, color=[red, blue]);$



plot({x^.1, ln(x)}, x = -7 .. 7, thickness = 2, color = [red, blue])

$plot(\{x^{(0.1)}, \ln(x)\}, x=-7..7, thickness=2, color=[red, blue])$



Answer 73E.

Consider the inequality,

$\ln(x^2-2x-2)\le 0$

Solve the inequality as follows:

$(x^2-2x-2)\le e^0$

$x^2-2x-2\le 1$ Since $e^0=1$

$x^2-2x-2-1\le 0$ Subtract 1 from both sides

$x^2-2x-3\le 0$

$$\left.\begin{array}{r} x^2-3x+x-3\le 0 \\ x(x-3)+1(x-3)\le 0 \end{array}\right. \text{Factorize}$$

$(x-3)(x+1)\le 0$

$(x-3)\le 0 \text{ or } (x+1)\le 0$

$x\le 3 \text{ or } x\le -1$ ...... (1)

Since $\ln(x)$ is defined for all positive values of *x*.

In this case $x^2-2x-2>0$

Solve the inequality by using quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

$x>\dfrac{-(-2)\pm\sqrt{(-2)^2-4(-2)}}{2(1)}$ Substitute $a=1,b=-2,c=-2$

$=\dfrac{2\pm\sqrt{12}}{2}$

$=\dfrac{2\pm2\sqrt{3}}{2}$ Since $\sqrt{12}=2\sqrt{3}$

$=\dfrac{2(1\pm\sqrt{3})}{2}$ Common out 2 from numerator

$x>1\pm\sqrt{3}$

$1+\sqrt{3}<x \text{ or } x<1-\sqrt{3}$ ...... (2)

Therefore,

From **(1)** and **(2)** the solution set of the inequality $\ln(x^2-2x-2)\le 0$ is

$\boxed{-1\le x<1-\sqrt{3} \text{ or } 1+\sqrt{3}<x\le 3}$