Similarity

Practice Set 1.1

Q. 1. Base of a triangle is 9 and height is 5. Base of another triangle is 10 and height is 6. Find the ratio of areas of these triangles.

Answer : We know that area of triangle = $\frac{1}{2}$ × Base× Height

⇒ Area (triangle 1) = $\frac{1}{2}$ ×9× 5

= $\frac{45}{2}$

⇒ Area (triangle 2) = $\frac{1}{2}$ ×10× 6

= 30

∴ The ratio of areas of these triangles will be = $\frac{\text{Area(triangle 1)}}{\text{Area(triangle 2)}}$

= $\frac{\frac{45}{2}}{30}$

= $\frac{45}{2} \times \frac{1}{30}$

= $\frac{3}{4}$

Q. 2. If figure 1.13 BC ⊥ AB, AD ⊥ AB, BC = 4, AD = 8, then find $\frac{A\,(\Delta ABC)}{A\,(\Delta ADB)}$.

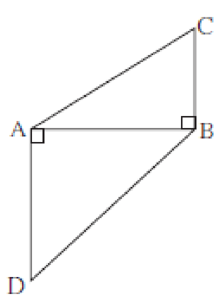


Fig. 1.13

Answer : Here, ΔABC and ΔADB has common Base.

$$\therefore \frac{Ar(\Delta ABC)}{Ar(\Delta ADB)} = \frac{\text{height of } \Delta ABC}{\text{height of } \Delta ADB}$$

(PROPERTY: Areas of triangles with equal bases are proportional to their corresponding heights.)

$$\Rightarrow \frac{Ar(\Delta ABC)}{Ar(\Delta ADB)} = \frac{BC}{AD}$$

$= \frac{4}{8}$

$= \frac{1}{2}$

Q. 3. In adjoining figure 1.14 seg PS ⊥ seg RQ, seg QT ⊥ seg PR. If RQ = 6, PS = 6 and PR = 12, then find QT.

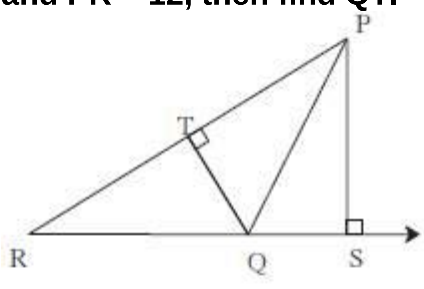


Fig. 1.14

Answer :

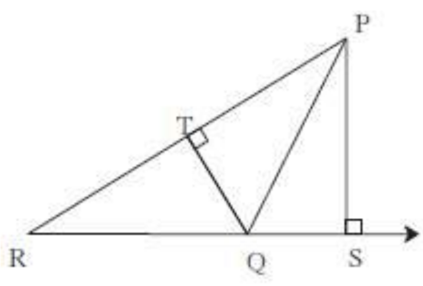


Fig. 1.14

Considering, Area of (ΔPQR) with base QR

⇒ PS will be the Height

Now, consider the Area of (ΔPQR) with base PR

⇒ QT will be the Height

∵ The triangle is the same

⇒ The area will be the same irrespective of the base taken.

And we know that area of triangle = $\frac{1}{2}$ × Base× Height

⇒ $\frac{1}{2}$×QR×PS

= $\frac{1}{2}$ × PR × QT

⇒ $\frac{1}{2}$ × 6 × 6

= $\frac{1}{2}$ × 12 × QT

⇒ QT = 3

Q. 4. In adjoining figure, AP ⊥ BC, AD || BC, then find A(ΔABC) : A (ΔBCD)

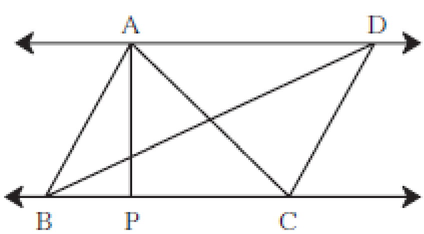
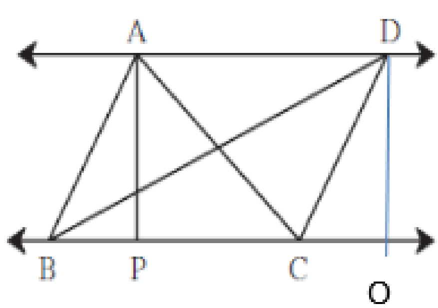


Fig. 1.15

Answer :



We can re-draw the fig. 1.15 (as shown above) where we add DO

Which will be height of ΔBCD.

Now, $\frac{A(\Delta ABC)}{A(\Delta BCD)} = \frac{AP}{DO}$

(PROPERTY: Areas of triangles with equal bases are proportional to their corresponding heights.)

$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta BCD)} = \frac{AP}{DO}$

$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta BCD)} = \frac{1}{1}$

(∵ the distance between the two parallel lines is always equal ⇒ AP = DO)

$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta BCD)}$ = 1:1

Q. 5. In adjoining figure PQ ⊥ BC, AD ⊥ BC then find following ratios.

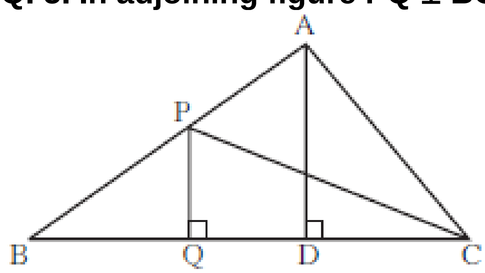


Fig. 1.16

(i) $\frac{A(\Delta PQB)}{A(\Delta PBC)}$

(ii) $\frac{A(\Delta PBC)}{A(\Delta ABC)}$

(iii) $\frac{A(\Delta ABC)}{A(\Delta ADC)}$

(iv) $\frac{A(\Delta ADC)}{A(\Delta PQC)}$

Answer : We know that area of triangle = $\frac{1}{2}$ × Base × Height

(i) $\frac{A(\Delta PQB)}{A(\Delta PBC)} = \frac{BQ}{BC}$

(PROPERTY: Areas of triangles with equal heights are proportional to their corresponding bases.)

(ii) $\frac{A(\Delta PBC)}{A(\Delta ABC)} = \frac{PQ}{AD}$

(PROPERTY: Areas of triangles with equal bases are proportional to their corresponding heights.)

(iii) $\frac{A(\Delta ABC)}{A(\Delta ADC)} = \frac{BC}{DC}$

(PROPERTY: Areas of triangles with equal heights are proportional to their corresponding bases.)

(iv) $\frac{A(\Delta ADC)}{A(\Delta PQC)} = \frac{\frac{1}{2} \times AD \times DC}{\frac{1}{2} \times PQ \times QC}$

$= \frac{AD \times DC}{PQ \times QC}$

Practice Set 1.2

Q. 1. Given below are some triangles and lengths of line segments. Identify in which figures, ray PM is the bisector of ∠OPR.

(1)

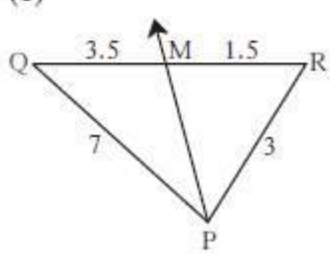


Fig. 1.33

(2)

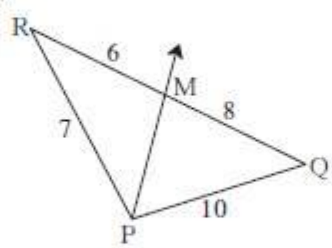


Fig. 1.34

(3)

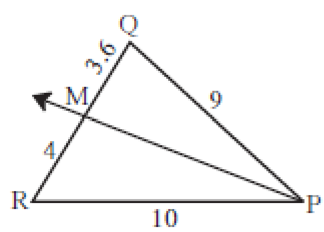


Fig. 1.35

Answer :

Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

Therefore, we'll find the ratio for all the triangle. Hence, for

(1) $\frac{QM}{MR} = \frac{3.5}{1.5}$

= 2.33

And $\frac{QP}{PR} = \frac{7}{3}$

= 2.33

⇒ $\frac{QM}{MR} = \frac{QP}{PR}$

⇒ In (1), ray PM is a bisector.

(2) $\frac{RM}{MQ} = \frac{6}{8}$

= 0.75

And $\frac{RP}{PQ} = \frac{7}{10}$

= 0.7

⇒ $\frac{RM}{MQ} \neq \frac{RP}{PQ}$

⇒ In (2), ray PM is not a bisector.

(3) $\frac{RM}{MQ} = \frac{4}{3.6}$

= 1.1

And $\frac{RP}{PQ} = \frac{10}{9}$

= 1.11

$\Rightarrow \frac{RM}{MQ} = \frac{RP}{PQ}$

⇒ In (3), ray PM is a bisector.

Q. 2. In Δ PQR, PM = 15, PQ = 25, PR = 20, NR = 8. State whether line NM is parallel to side RQ. Give reason.

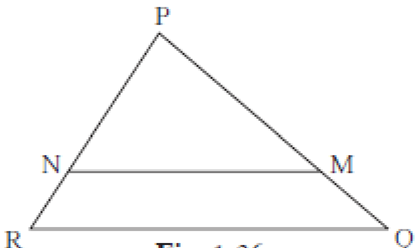


Fig. 1.36

Answer : By Converse of basic Proportionality Theorem

(Theorem: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.)

⇒ If $\frac{PN}{NR} = \frac{PM}{MQ}$, then line NM is parallel to side RQ.

∴ We'll check if $\frac{PN}{NR} = \frac{PM}{MQ}$.

$\Rightarrow \frac{PN}{NR} = \frac{PR-NR}{NR}$

$= \frac{20-8}{8}$

$= \frac{12}{8}$

$= \frac{3}{2}$

And, $\frac{PM}{MQ} = \frac{PM}{PQ-PM}$

$= \frac{15}{25-15}$

$= \frac{15}{10}$

$= \frac{3}{2}$

$\Rightarrow \frac{PN}{NR} = \frac{PM}{MQ} = \frac{3}{2}$, therefore line NM || side RQ

Q. 3. In Δ MNP, NQ is a bisector of ∠N. If MN = 5, PN = 7 MQ = 2.5 then find QP.

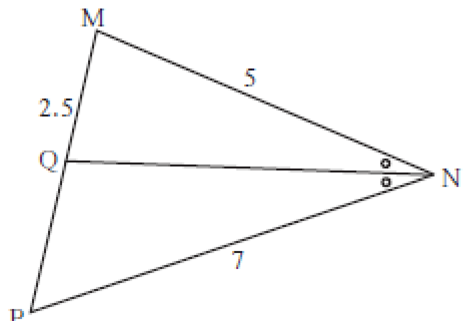


Fig. 1.37

Answer :

Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

$\Rightarrow \frac{MQ}{QP} = \frac{MN}{NP}$

$\Rightarrow \frac{2.5}{QP} = \frac{5}{7}$

$\Rightarrow QP \times 5 = 2.5 \times 7$

$\Rightarrow QP = \frac{2.5 \times 7}{5}$

$\Rightarrow QP = 3.5$

Q. 4. Measures of some angles in the figure are given. Prove that $\frac{AP}{PB} = \frac{AQ}{QC}$

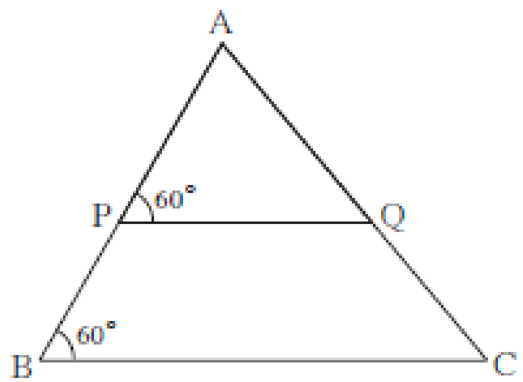


Fig. 1.38

Answer : Here, PQ||BC (∵ ∠ APQ≅ ∠ ABC)

(PROPERTY: If a transversal intersects two lines so that corresponding angles are congruent, then the lines are parallel)

∴ By Basic Proportionality Theorem

(Theorem : If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the sides in the same proportion.)

$\Rightarrow \frac{AP}{PB} = \frac{AQ}{QC}$

Q. 5. In trapezium ABCD, side AB || side PQ || side DC, AP = 15, PD = 12, QC = 14, find BQ.

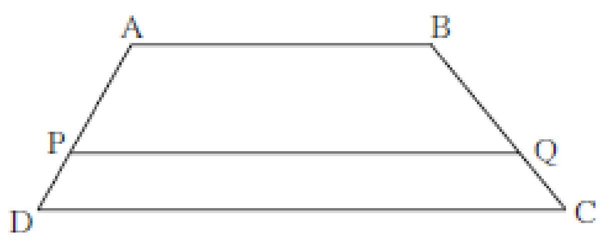


Fig. 1.39

Answer : By Basic Proportionality Theorem

(Theorem : If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the sides in the same proportion.)

$\Rightarrow \frac{AP}{PD} = \frac{BQ}{QC}$

$\Rightarrow \frac{15}{12} = \frac{BQ}{14}$

$\Rightarrow$ BQ = $\frac{15 \times 14}{12}$

$\Rightarrow$ BQ = 17.5

Q. 6. Find QP using given information in the figure.

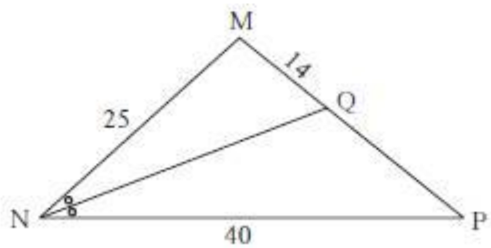


Fig. 1.40

Answer : Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

And ∵ NQ is angle bisector of ∠N

$\Rightarrow \frac{MQ}{QP} = \frac{MN}{NP}$

$\Rightarrow \frac{14}{QP} = \frac{25}{40}$

$\Rightarrow$ QP = $\frac{14 \times 40}{25}$

$\Rightarrow$ QP = 22.4

Q. 7. In figure 1.41, if AB || CD || FE then find x and AE.

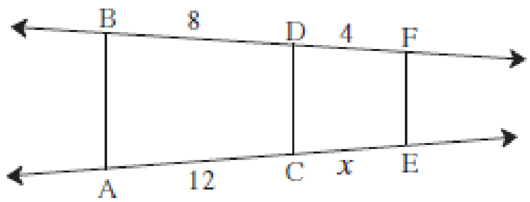


Fig. 1.41

Answer : Theorem: The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

$\Rightarrow \frac{BD}{DF} = \frac{AC}{CE}$

$\Rightarrow \frac{8}{4} = \frac{12}{x}$

$\Rightarrow x = \frac{12 \times 4}{8}$

$\Rightarrow x = 6$

Now, AE = AC + CE

= 12 + x

= 12 + 6

$\Rightarrow$ AE = 18

Q. 8. In Δ LMN, ray MT bisects ∠ LMN If LM = 6, MN = 10, TN = 8, then find LT.

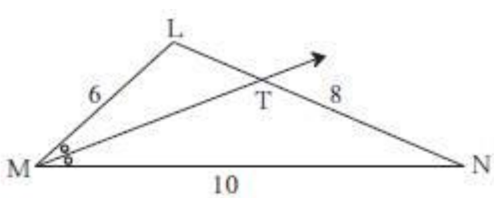


Fig. 1.42

Answer : Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

$\Rightarrow \frac{LT}{TN} = \frac{LM}{MN}$

$\Rightarrow LT = \frac{LM \times TN}{MN}$

$\Rightarrow LT = \frac{6 \times 8}{10}$

$\Rightarrow LT = 4.8$

Q. 9. In Δ ABC, seg BD bisects ∠ ABC. If AB = x, BC = x + 5, AD = x – 2, DC = x + 2, then find the value of x.

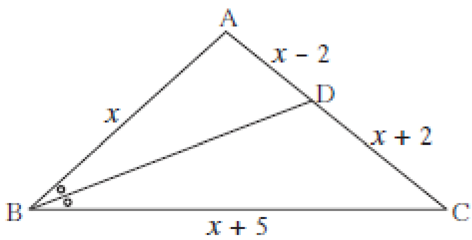


Fig. 1.43

Answer : Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

$\Rightarrow \frac{AD}{DC} = \frac{AB}{BC}$

$\Rightarrow \frac{x-2}{x+2} = \frac{x}{x+5}$

$\Rightarrow x(x + 2) = (x-2)(x + 5)$

$\Rightarrow x^2 + 2x = x^2 -2x + 5x-10$

$\Rightarrow x^2 + 2x-x^2 + 2x-5x + 10 = 0$

$\Rightarrow x = 10$

Q. 10. In the figure 1.44, X is any point in the interior of triangle. Point X is joined to vertices of triangle. Seg PQ || seg DE, seg QR || seg EF. Fill in the blanks to prove that, seg PR || seg DF.

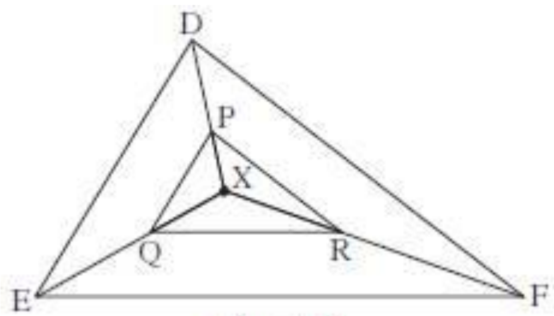


Fig. 1.44

Proof : IN $\Delta XDE, PQ \parallel DE$ .... $\square$

$$\therefore \frac{XP}{\square} = \frac{\square}{QE} \quad ....(I)$$

(Basic proportionality theorem)

In $\Delta XDE, QR \parallel EF$ .... $\square$

$$\therefore \frac{\square}{\square} = \frac{\square}{\square} \quad ....(II) \square$$

$$\therefore \frac{\square}{\square} = \frac{\square}{\square} \quad ....\text{from (I) and (II)}$$

$\therefore$ seg $PR \parallel$ seg DE ....

(Converse of basic proportionality theorem)

Answer : Proof: In ΔXDE, PQ||DE….. (Given)

$\therefore \frac{XP}{XQ} = \frac{PD}{DE}$ …..(I)

(Basic proportionality theorem)

In ΔXDE, QR||EF …….(Given)

$\therefore \frac{XR}{RF} = \frac{XQ}{QE}$ ………(II) (Basic Proportionality Theorem)

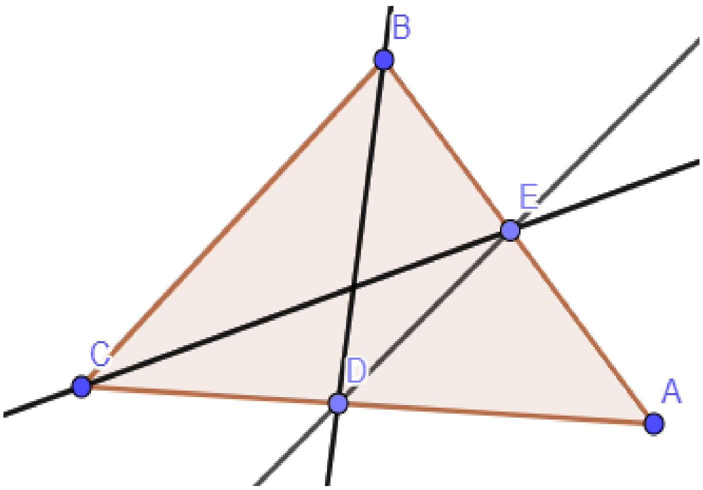
$\therefore \frac{XP}{PD} = \frac{XR}{RF}$ ……… from (I) and (II)

∴ seg PR||Seg DE ………..

(converse of basic proportionality theorem)

Q. 11. In ΔABC, ray BD bisects ∠ABC and ray CE bisects ∠ACB. If seg AB ≅ seg AC then prove that ED || BC.

Answer : PROOF:



Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

$\Rightarrow \frac{AD}{DC} = \frac{AB}{CB}$ ……(1)

And $\frac{AE}{EB} = \frac{AC}{CB}$ …..(2) (∵ BD and CE are angle bisectors of ∠B and ∠C respectively.)

Now, ∵ seg AB ≅ seg AC

⇒ AB = AC

$\Rightarrow \frac{AB}{CB} = \frac{AC}{CB}$

⇒ R.H.S of (1) & (2) are equal.

⇒ L.H.S of (1) & (2) will be equal.

∴ Equating L.H.S of (1) &(2), we get-

$\Rightarrow \frac{AD}{DC} = \frac{AE}{EB}$

⇒ ED||BC (By converse basic proportionality theorem)

Practice Set 1.3

Q. 1. In figure 1.55, ∠ABC = 75°, ∠EDC = 75° state which two triangles are similar and by which test? Also write the similarity of these two triangles by a proper one to one correspondence.

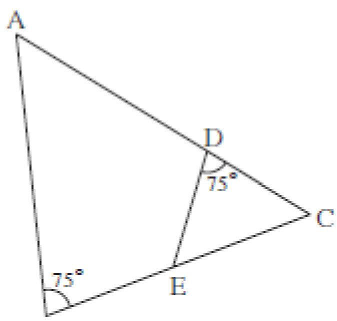


Fig. 1.55

Answer : With one- to-one correspondence ABC ↔ EDC

∵ ∠ABC ≅ ∠EDC = 75°
∠ACB ≅ ∠ECD (Is common in both the triangles ABC and EDC)

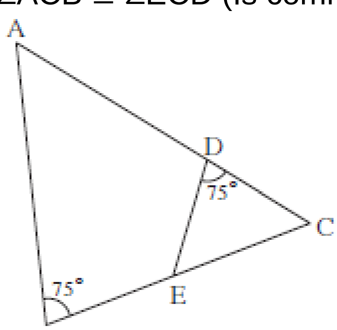


Fig. 1.55

⇒ Δ ABC~Δ EDC ………(By AA Test)

Q. 2. Are the triangles in figure 1.56 similar? If yes, by which test?

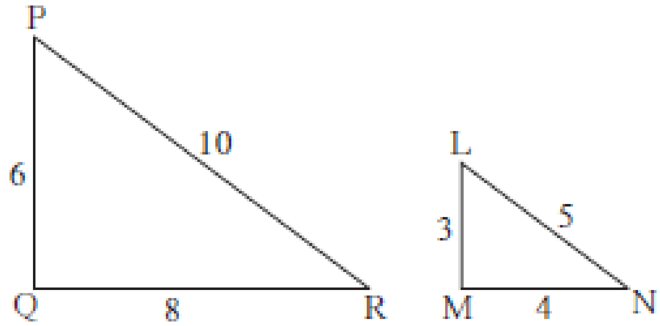


Fig. 1.56

Answer : In Δ PQR and Δ LMN

$\frac{PQ}{LM} = \frac{6}{3} = 2$

And $\frac{QR}{MN} = \frac{8}{4} = 2$

And $\frac{PR}{LN} = \frac{10}{5} = 2$

$\Rightarrow \frac{PQ}{LM} = \frac{QR}{MN} = \frac{PR}{LN} = 2$

⇒ ΔPQR~ΔLMN …………(By SSS Similarity Test)

Q. 3. As shown in figure 1.57, two poles of height 8 m and 4 m are perpendicular to the ground. If the length of shadow of smaller pole due to sunlight is 6 m then how long will be the shadow of the bigger pole at the same time?

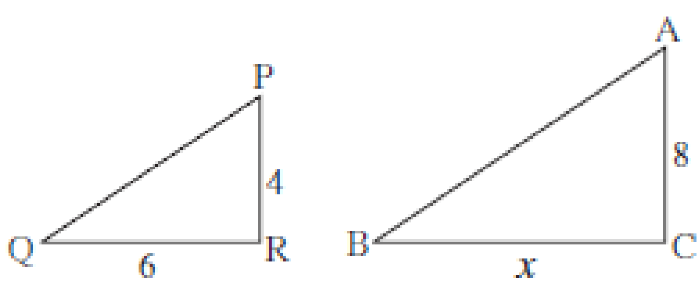


Fig. 1.57

Answer : ∵ The shadows are measured at the same time

⇒ Angle of of elevation will be equal for both the pole

⇒ Δ PQR~Δ ABC ………(By AA Test)

⇒ $\frac{PR}{AC} = \frac{QR}{BC}$

⇒ BC $= \frac{QR \times AC}{PR}$

⇒ x = $\frac{6 \times 8}{4}$

⇒ x = 12 m

Q. 4. In ΔABC, AP ⊥ BC, BQ ⊥ AC B- P-C, A-Q - C then prove that, ΔCPA ~ ΔCQB. If AP = 7, BQ = 8, BC = 12 then find AC.

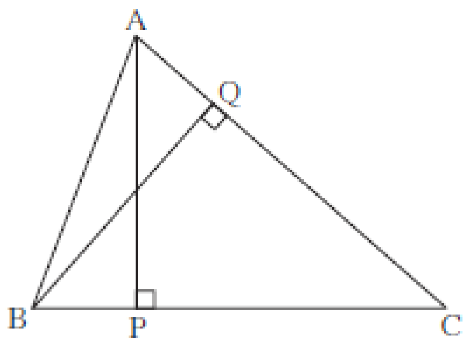


Fig. 1.58

Answer : From fig.

⇒ ∠ APC≅ ∠ BQC (∵ AP⊥ BC and BQ⊥ AC)

⇒ Also, ∠ ACP≅ ∠ BCQ (Common)

⇒ Δ CPA~Δ CQB (By AA Test)

⇒ $\frac{AP}{BQ} = \frac{AC}{BC}$

⇒ AC = $\frac{AP \times BC}{BQ}$

⇒ AC = $\frac{7 \times 12}{8}$

$\Rightarrow$ AC = 10.5

Q. 5. Given : In trapezium PQRS, side PQ || side SR, AR = 5AP, AS = 5AQ then prove that, SR = 5PQ

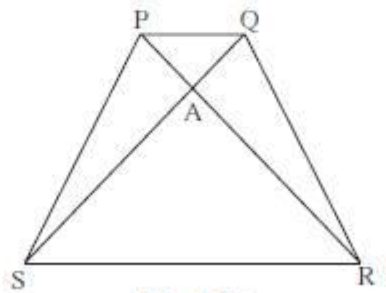


Fig. 1.59

Answer : Given that, AR = 5AP and AS = 5AQ

$\Rightarrow \frac{AR}{AP}$ = 5 ....(1)

And $\frac{AS}{AQ}$ = 5..........(2)

$\Rightarrow \frac{AR}{AP} = \frac{AS}{AQ}$

And, ∠ SAR≅ ∠ QAP ...... (Opposite angles)

$\Rightarrow$ Δ SAR ~ Δ QAP ....(SAS Test of similarity)

$\Rightarrow \frac{AS}{AQ} = \frac{AR}{AP} = \frac{SR}{QP}$ (corresponding sides are proportional)

But, $\frac{AS}{AQ} = \frac{AR}{AP}$ = 5

$\Rightarrow \frac{SR}{QP}$ = 5

$\Rightarrow$ SR = 5PQ

Q. 6. In trapezium ABCD, (Figure 1.60) side AB || side DC, diagonals AC and BD intersect in point O. If AB = 20, DC = 6, OB = 15 then find OD.

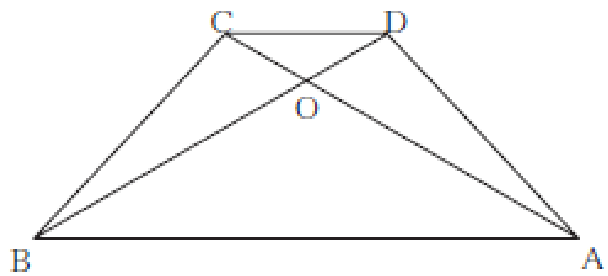


Fig. 1.60

Answer : In Δ AOB and ΔCOD

⇒ ∠ AOB≅ ∠ COD (opposite angles)

⇒ ∠ CDO≅ ∠ ABO (Alternate angles ∵ AB||DC)

⇒ Δ AOB ~ Δ COD (By AA Test)

⇒ $\frac{AB}{DC} = \frac{OB}{OD}$ (corresponding sides are proportional)

⇒ OD = $\frac{OB \times DC}{AB}$

⇒ OD = $\frac{15 \times 6}{20}$

⇒ OD = 4.5

Q. 7. ABCD is a parallelogram point E is on side BC. Line DE intersects ray AB in point T. Prove that DE × BE = CE × TE.

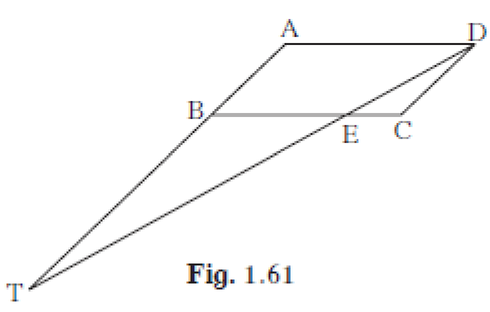


Fig. 1.61

Answer : In Δ CED and ΔBET

⇒ ∠ CED≅ ∠ BET (opposite angles)

⇒ ∠ CDE≅ ∠ BTE (Alternate angles)

(∵ AB||DC ⇒ BT||DC, as BT is extension to AB)

⇒ Δ CED ~ Δ BET (By AA Test)

⇒ $\frac{CE}{DE} = \frac{BE}{TE}$ (corresponding sides are proportional)

⇒ DE × BE = CE × TE

Q. 8. In the figure, seg AC and seg BD intersect each other in point P and $\frac{AP}{Cp} = \frac{BP}{DP}$. Prove that, ΔABP ~ ΔCDP

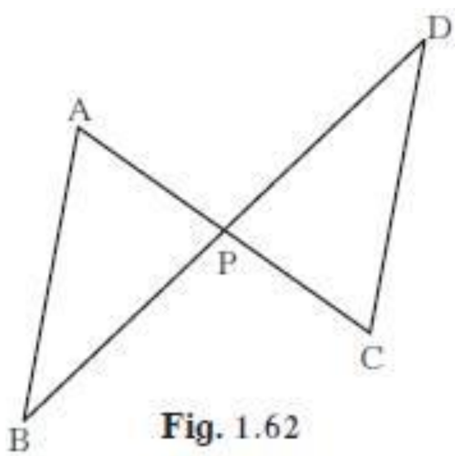


Fig. 1.62

Answer :

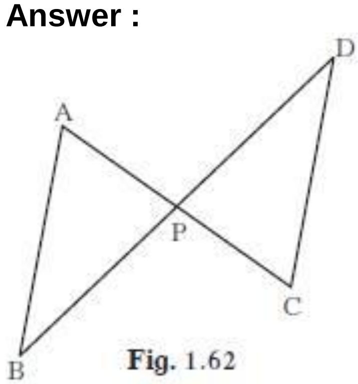


Fig. 1.62

In Δ APB & Δ CPD

$\Rightarrow \frac{AP}{CP} = \frac{BP}{DP}$ ……(Given)

And, ∠APB = ∠DPC (vertically opposite angles)

⇒ Δ APB ~ Δ CPD (By SAS Test)

Q. 9. In the figure, in ΔABC, point D on side BC is such that, ∠BAC = ∠ADC.

Prove that, $CA^2 = CB \times CD$

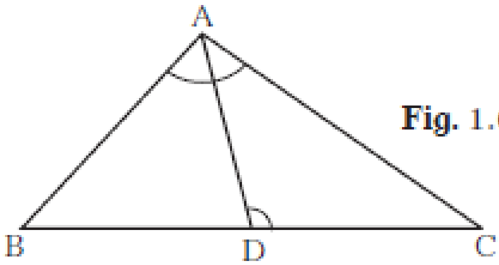


Fig. 1.63

Answer : In Δ BAC & Δ ADC

⇒ ∠ BAC ≅ ∠ ADC ……(Given)

And, ∠ACB ≅ ∠DCA ……(common)

⇒ Δ BAC ~ Δ ADC (By AA Test)

$\Rightarrow \frac{CA}{CD} = \frac{CB}{CA}$ (corresponding sides are proportional)

⇒ $CA^2 = CB \times CD$

Practice Set 1.4

Q. 1. The ratio of corresponding sides of similar triangles is 3 : 5; then find the ratio of their areas.

Answer : Theorem: When two triangles are similar, the ratio of areas of those triangles is equal to the ratio of the squares of their corresponding sides.

⇒ Ratio of areas = $3^2:5^2$

⇒ Ratio of areas = 9 : 25

Q. 2. If ΔABC ~ ΔPQR and AB: PQ = 2:3, then fill in the blanks.

$$\frac{A(\Delta\ ABC)}{A(\Delta PQR)} = \frac{AB^2}{\square} = \frac{2^2}{3^2} = \frac{\square}{\square}$$

Answer : ∵ Δ ABC~Δ PQR and AB:PQ = 2:3

$$\Rightarrow \frac{A(\Delta\ ABC)}{A(\Delta\ PQR)} = \frac{AB^2}{PQ^2} = \frac{2^2}{3^2} = \frac{4}{9}$$

Q. 3. If ΔABC ~ ΔPQR, A (ΔABC) = 80, A(ΔPQR) = 125, then fill in the blanks.

$$\frac{A(\Delta\ ABC)}{A(\Delta \ldots)} = \frac{80}{125} \qquad \therefore \frac{AB}{PQ} = \frac{\square}{\square}$$

Answer : ∵ Δ ABC~Δ PQR

$\Rightarrow \frac{A(\Delta\ ABC)}{A(\Delta\ PQR)} = \frac{80}{125}$ (∵ A(Δ PQR) = 125 is given)

$$\Rightarrow \frac{A(\Delta\ ABC)}{A(\Delta\ PQR)} = \frac{AB^2}{PQ^2}$$

$$\Rightarrow \frac{AB}{PQ} = \sqrt{\frac{A(\Delta\ ABC)}{A(\Delta\ PQR)}}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{4}{5}$$

Q. 4. ΔLMN ~ ΔPQR, 9 × A(ΔPQR) = 16 × A (ΔLMN). If QR = 20 then find MN.

Answer : ∵ Δ ABC~Δ PQR

⇒ Given that, 9 × A(Δ ABC) = 16× A(Δ PQR)

$$\Rightarrow \frac{A(\Delta\ PQR)}{A(\Delta\ LMN)} = \frac{16}{9}$$

And, $\frac{A(\Delta\ PQR)}{A(\Delta\ LMN)} = \frac{QR^2}{MN^2}$

$\Rightarrow \frac{QR^2}{MN^2} = \frac{16}{9}$

$\Rightarrow \frac{20^2}{MN^2} = \frac{16}{9}$

$\Rightarrow MN^2 = \frac{400 \times 9}{16}$

$\Rightarrow MN = 15$

Q. 5. Areas of two similar triangles are 225 sq.cm & 81 sq.cm. If a side of the smaller triangle is 12 cm, then find corresponding side of the bigger triangle.

Answer : Let area of one(bigger) triangle be 'A', other(smaller) triangle be 'B', corresponding side of smaller triangle be 'a' and bigger triangle be 'b'.

$\Rightarrow \frac{A}{B} = \frac{b^2}{a^2}$ (By theorem)

And a = 12cm, A = 225 sq.cm, B = 81 sq.cm ……(Given)

$\Rightarrow \frac{225}{81} = \frac{b^2}{12^2}$

$\Rightarrow b^2 = \frac{225 \times 144}{81}$

$\Rightarrow b = \sqrt{400}$

$\Rightarrow b = 20$ cm

Q. 6. Δ ABC and ΔDEF are equilateral triangles. If A(ΔABC) : A(ΔDEF) = 1 : 2 and AB = 4, find DE.

Answer : We know that, all the angles of an equilateral triangles are equal, i.e., 60°.

$\Rightarrow \Delta ABC \sim \Delta DEF$ ……(By AAA Similarity Test)

$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta DEF)} = \frac{AB^2}{DE^2}$

And, $\frac{A(\Delta ABC)}{A(\Delta DEF)} = \frac{1}{2}$ (Given)

$\Rightarrow \frac{AB^2}{DE^2} = \frac{1}{2}$

$\Rightarrow DE^2 = 2 \times 4^2$ ($\because$ AB = 4)

$\Rightarrow DE = \sqrt{32}$

$\Rightarrow DE = 4\sqrt{2}$

Q. 7. In figure 1.66, seg PQ || seg DE, A(Δ PQF) = 20 units, PF = 2 DP, then find A(DPQE) by completing the following activity.

A(Δ PQF) = 20 units, PF = 2 DP, Let us assume DP = x. ∴ PF = 2x

$$DF = DP + \square = \square + \square = 3x$$

In Δ FDE and Δ FPQ,
∠FDE ≅ ∠ .... corresponding angles
∠FED ≅ ∠ .... corresponding angles
∴ Δ FDE ~ Δ FPQ .... AA test

$$\therefore \frac{A(\Delta FDE)}{A(\Delta FPQ)} = \frac{\square}{\square} = \frac{(3x)^2}{(2x)^2} = \frac{9}{4}$$

$$A(\Delta FDE) = \frac{9}{4} A(\Delta FPQ) = \frac{9}{4} \times \square = \square$$

$$A(\square DPQE) = A(\Delta FDE) - A(\Delta FPQ)$$

$$= \square - \square$$

$$= \square$$

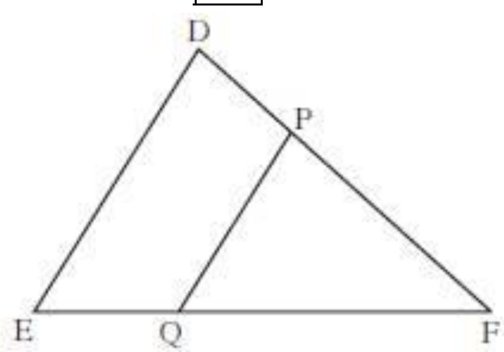


Fig. 1.66

Answer : A(Δ PQF) = 20units,PF = 2DP,Let us assume DP = x,∴ PF = 2x

⇒ DF = DP + PF = x + 2x = 3x

In Δ FDE & Δ FPQ

∠ FDE ≅ ∠ FPQ (Corresponding angles)

∠ FEP ≅ ∠ FQP (Corresponding angles)

∴ Δ FDE~ Δ FPQ (AA Test)

∴ $\frac{A(\Delta FDE)}{A(\Delta FPQ)} = \frac{DF^2}{PF^2} = \frac{(3x)^2}{(2x)^2} = \frac{9}{4}$

A(Δ FDE) = $\frac{9}{4}$ A(ΔFPQ) = $\frac{9}{4}$× 20 = 45

A(□DPQE) = A (Δ FDE) - A(ΔFPQ)

= 45 – 20

= 25 sq. unit.

Problem Set 1

Q. 1. A. Select the appropriate alternative.

In Δ ABC and ΔPQR, in a one to one correspondence $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then

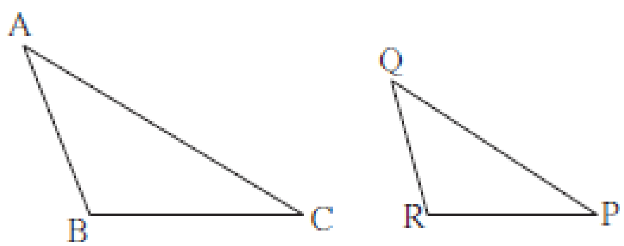


Fig. 1.67

A. Δ PQR ~ Δ ABC

B. Δ PQR ~ Δ CAB
C. Δ CBA ~ Δ PQR
D. Δ BCA ~ Δ PQR

Answer : $\because \frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$

⇒ Δ CAB~Δ PQR

(A) doesn’t match the solution.

(C) doesn’t match the solution.

(D) doesn’t match the solution.

Q. 1. B. Select the appropriate alternative.

If in ΔDEF and ΔPQR, ∠D ≅ ∠Q, ∠R ≅ ∠E then which of the following statements is false?

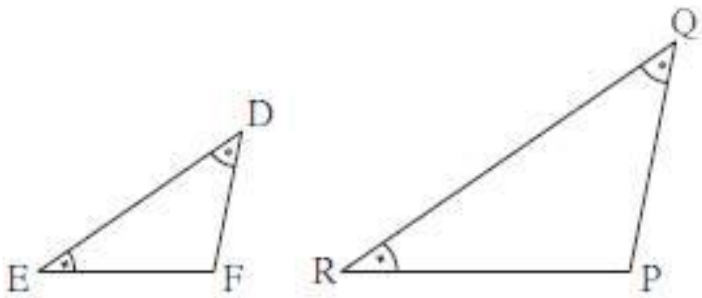


Fig. 1.68

A. $\frac{EF}{PR} = \frac{DF}{PQ}$

B. $\frac{DE}{PQ} = \frac{EF}{RP}$

C. $\frac{DE}{QR} = \frac{DF}{PQ}$

D. $\frac{EF}{RP} = \frac{DE}{QR}$

Answer : In Δ DEF & Δ PQR

∠ D≅ ∠ Q and ∠ R≅ ∠ E (Given)

⇒ Δ DEF ~ Δ PQR

$\Rightarrow \frac{DE}{PQ} = \frac{EF}{QR} = \frac{FD}{RP}$ (corresponding sides are proportional)

(A) Is matching the solution, hence can't be false.

(C) Is matching the solution, hence can't be false.

(D) Is matching the solution, hence can't be false.

Q. 1. C. Select the appropriate alternative.

In Δ and ΔDEF ∠B = ∠E, ∠F = ∠C and AB = 3DE then which of the statements regarding the two triangles is true?

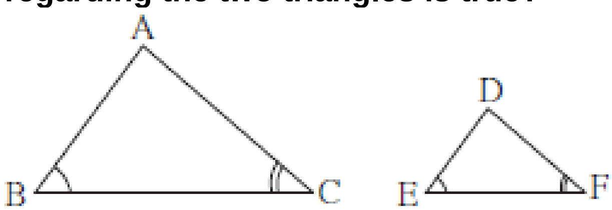


Fig. 1.69

A. The triangles are not congruent and not similar

B. The triangles are similar but not congruent.
C. The triangles are congruent and similar.
D. None of the statements above is true.

Answer : In Δ ABC & Δ DEF

∠ B≅ ∠ E and ∠ C≅ ∠ F (Given)

⇒ Δ ABC ~ Δ DEF (By AA Test)

⇒ The triangles are similar.

And, Δ ABC ≅ Δ DEF, if AB = DE.

But, given that - AB = 3DE.

⇒ The triangles are not congruent.

(A) doesn't match the solution.

(C) doesn't match the solution.

(D) doesn't match the solution.

Q. 1. D. Select the appropriate alternative.

ΔABC and ΔDEF are equilateral triangles, A(ΔABC) : A(ΔDEF) = 1 : 2. If AB = 4 then what is length of DE?

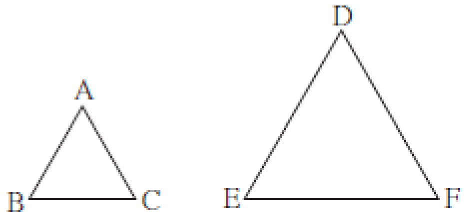


Fig. 1.70

A. $2\sqrt{2}$

B. 4

C. 8

D. $4\sqrt{2}$

Answer : Solution: We know that, all the angles of an equilateral triangles are equal, i.e., 60°.

⇒ Δ ABC~Δ DEF ……(By AAA Similarity Test)

$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta DEF)} = \frac{AB^2}{DE^2}$

And, $\frac{A(\Delta ABC)}{A(\Delta DEF)} = \frac{1}{2}$ (Given)

$\Rightarrow \frac{AB^2}{DE^2} = \frac{1}{2}$

$\Rightarrow DE^2 = 2 \times 4^2$ (∵ AB = 4)

⇒ DE = √32

⇒ DE = 4√2

(A) doesn’t match the solution.

(B) doesn’t match the solution.

(C) doesn’t match the solution.

Q. 1. E. Select the appropriate alternative.

In figure 1.71, seg XY || seg BC, then which of the following statements is true?

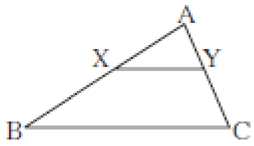


Fig. 1.71

A. $\frac{AB}{AC} = \frac{AX}{AY}$

B. $\frac{AX}{XB} = \frac{AY}{AC}$

C. $\frac{AX}{YC} = \frac{AY}{XB}$

D. $\frac{AB}{YC} = \frac{AC}{XB}$

Answer : ∵ segXY|| segBC

⇒∠ AXY≅ ∠ ABC

And, ∠ XAY≅ ∠ BAC (Common)

⇒ Δ AXY~ Δ ABC (By AA Test)

⇒ $\frac{AX}{AB} = \frac{AY}{AC} = \frac{XY}{BC}$ (corresponding sides are proportional)

⇒ $\frac{AB}{AC} = \frac{AX}{AY}$

(B) doesn't match the solution.

(C) doesn't match the solution.

(D) doesn't match the solution.

Q. 2. In Δ ABC, B - D – C and BD = 7, BC = 20 then find following ratios.

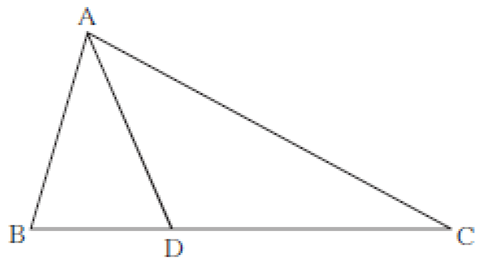


Fig. 1.72

(1) $\frac{A(\Delta ABD)}{A(\Delta ADC)}$

(2) $\frac{A(\Delta ABD)}{A(\Delta ABC)}$

(3) $\frac{A(\Delta ADC)}{A(\Delta ABC)}$

Answer : Theorem: When two triangles are similar, the ratio of areas of those triangles is equal to the ratio of the squares of their corresponding sides.

(1) $\frac{A(\Delta\ ABD)}{A(\Delta\ ADC)} = \frac{BD^2}{DC^2}$

$= \frac{BD^2}{(BC-BD)^2}$

$= \frac{7^2}{(20-7)^2}$

$= \frac{7^2}{13^2}$

(2) $\frac{A(\Delta\ ABD)}{A(\Delta\ ABC)} = \frac{BD^2}{BC^2}$

$= \frac{BD^2}{BC^2}$

$= \frac{7^2}{20^2}$

(3) $\frac{A(\Delta\ ADC)}{A(\Delta\ ABC)} = \frac{DC^2}{BC^2}$

$= \frac{(BC-BD)^2}{BC^2}$

$= \frac{(20-7)^2}{20^2}$

$= \frac{13^2}{20^2}$

Q. 3. Ratio of areas of two triangles with equal heights is 2 : 3. If base of the smaller triangle is 6 cm then what is the corresponding base of the bigger triangle?

Answer : (PROPERTY: Areas of triangles with equal heights are proportional to their corresponding bases.)

$\Rightarrow \frac{A(\text{smaller triangle})}{A(\text{bigger triangle})} = \frac{\text{base(smaller triangle)}}{\text{base(bigger triangle)}}$

$\Rightarrow \frac{2}{3} = \frac{6}{\text{base(bigger triangle)}}$

⇒ Base (bigger triangle) = 9 cm

Q. 4. In figure 1.73, ∠ABC = ∠DCB = 90° AB = 6, DC = 8 then $\frac{A(\Delta ABC)}{A(\Delta DCB)}$?

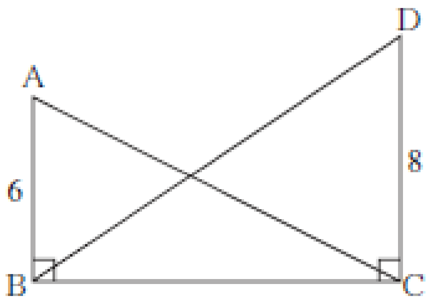


Fig. 1.73

Answer : We know that, Area of triangle = $\frac{1}{2}$ × base× height

$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{\frac{1}{2}\times BC\times AB}{\frac{1}{2}\times BC\times DC}$

$= \frac{AB}{DC}$

$= \frac{6}{8}$

$= \frac{3}{4}$

Q. 5. In figure 1.74, PM = 10 cm A(Δ PQS) = 100 sq.cm A (ΔQRS) = 110 sq.cm then find NR.

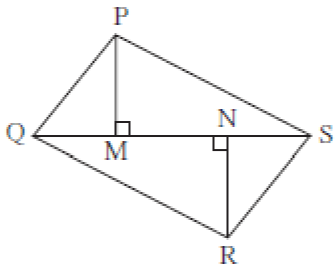


Fig. 1.74

Answer : We know that, Area of triangle = $\frac{1}{2}$ × base × height

$\Rightarrow \frac{A(\Delta\ PQS)}{A(\Delta\ QRS)} = \frac{\frac{1}{2} \times QS \times PM}{\frac{1}{2} \times QS \times NR}$

$\Rightarrow \frac{100}{110} = \frac{PM}{NR}$

$\Rightarrow \frac{100}{110} = \frac{10}{NR}$

⇒ NR = 11 cm

Q. 6. ΔMNT ~ ΔQRS. Length of altitude drawn from point T is 5 and length of altitude drawn from point S is 9. Find the ratio $\frac{A(\Delta MNT)}{A(\Delta QRS)}$.

Answer : $\frac{A(\Delta\ MNT)}{A(\Delta\ QRS)} = \frac{(\text{altitude from T})^2}{(\text{altitude frm S})^2}$

$= \frac{5^2}{9^2}$

$= \frac{25}{81}$

Q. 7. In figure 1.75, A – D – C and B – E – C seg DE || side AB If AD = 5, DC = 3, BC = 6.4 then find BE.

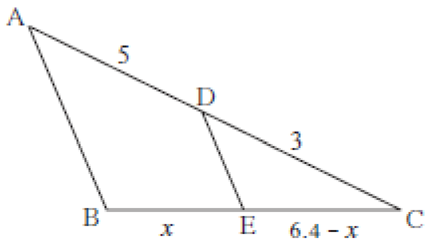


Fig. 1.75

Answer : By Basic Proportionality Theorem-

$\Rightarrow \frac{CD}{DA} = \frac{CE}{EB}$

$\Rightarrow \frac{3}{5} = \frac{6.4-x}{x}$

⇒ 3x = 32 – 5x

⇒ 8x = 32

⇒ x = 4 = BE

Q. 8. In the figure 1.76, seg PA, seg QB, seg RC and seg SD are perpendicular to line AD.

AB = 60, BC = 70, CD = 80, PS = 280 then find PQ, QR and RS.

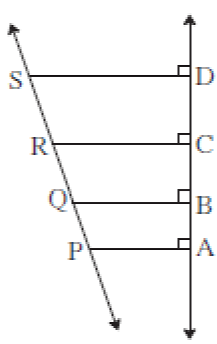


Fig. 1.76

Answer : (PROPERTY: If line AX || line BY || line CZ and line l and line m are their transversals then)

$\frac{AB}{BC} = \frac{XY}{YZ}$

$\Rightarrow \frac{AB}{BC} = \frac{PQ}{QR}$

$\Rightarrow \frac{60}{70} = \frac{PQ}{QR}$

$\Rightarrow \frac{PQ}{QR} = \frac{6}{7}$

$$\Rightarrow PQ = \frac{6}{7}QR$$ [1]

And $\frac{BC}{CD} = \frac{QR}{RS}$

$\Rightarrow \frac{70}{80} = \frac{QR}{RS}$

$\Rightarrow \frac{QR}{RS} = \frac{7}{8}$

$$\Rightarrow RS = \frac{8}{7}QR$$ [2]

And, PS = 280

⇒ PQ + QR + RS = 280 ……(3)

From [1] and [2], we have

$$\Rightarrow \frac{6}{7}QR + QR + \frac{8}{7}QR = 280$$

$$\Rightarrow \frac{14}{7}QR + QR = 280$$

$$\Rightarrow 2QR + QR = 280$$

$$\Rightarrow 3QR = 280$$

$$\Rightarrow QR = \frac{280}{3}$$

$$PQ = \frac{6}{7}QR$$

From [1], $$\Rightarrow PQ = \frac{6}{7} \times \frac{280}{3} = 80$$

From [2] $$RS = \frac{8}{7} \times \frac{280}{3} = \frac{320}{3}$$

Q. 9. In ΔPQR seg PM is a median. Angle bisectors of ∠PMQ and ∠PMR intersect side PQ and side PR in points X and Y respectively. Prove that XY || QR.

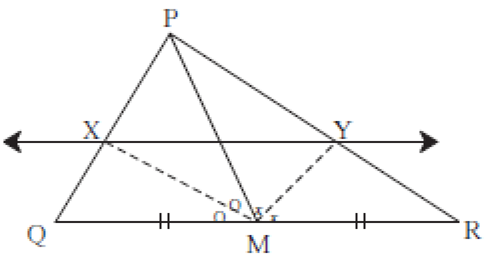


Fig. 1.77

Complete the proof by filling in the boxes. In ΔPMQ, ray MX is bisector of ∠PMQ.

$$\therefore \frac{\square}{\square} = \frac{\square}{\square}$$ **.......... (I) theorem of angle bisector.**

In ΔPMR, ray MY is bisector of ∠PMR.

$$\therefore \frac{\square}{\square} = \frac{\square}{\square}$$ **.......... (II) Theorem of angle bisector.**

But $$\frac{MP}{MQ} = \frac{MP}{MR}$$ **.......... M is the midpoint QR, hence MQ = MR.**

$$\therefore \frac{PX}{XQ} = \frac{PY}{YR}$$

∴ XY || QR .... converse of basic proportionality theorem.

Answer : $\therefore \frac{PM}{MQ} = \frac{PX}{XQ}$ .... (I) theorem of angle bisector.

And

$\therefore \frac{PM}{MR} = \frac{PY}{YR}$ .... (II) Theorem of angle bisector.

Q. 10. In fig 1.78, bisectors of ∠B and ∠C of ΔABC intersect each other in point X. Line AX intersects side BC in point Y. AB = 5, AC = 4, BC = 6 then find $\frac{AX}{XY}$.

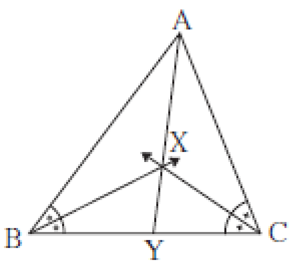


Fig. 1.78

Answer : By Bisector Theorem-

$\Rightarrow \frac{AX}{XY} = \frac{AB}{BY}$ ……(1)

$\Rightarrow$ And, $\frac{AX}{XY} = \frac{AC}{CY}$……(2)

Equating (1) & (2), we get-

$\Rightarrow \frac{AB}{BY} = \frac{AC}{CY}$

$\Rightarrow \frac{AB}{AC} = \frac{BY}{CY}$

$\Rightarrow \frac{AB + AC}{AC} = \frac{BY + CY}{CY}$

$= \frac{BC}{CY}$

$\Rightarrow \frac{5 + 4}{4} = \frac{6}{CY}$

$\Rightarrow$ CY = $\frac{24}{9}$

$\Rightarrow$ CY = $\frac{8}{3}$

Now, $\frac{AX}{XY} = \frac{AC}{CY}$

$\Rightarrow \frac{AX}{XY} = \frac{4}{\frac{8}{3}}$

$\Rightarrow \frac{AX}{XY} = \frac{3}{2}$

Q. 11. In □ABCD, seg AD || seg BC. Diagonal AC and diagonal BD intersect each other in point P. Then show that $\frac{AP}{PD} = \frac{PC}{BP}$

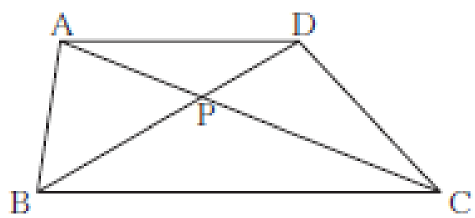


Fig. 1.79

Answer : In Δ APD and ΔCPB

⇒ ∠ APD≅ ∠ CPB (opposite angles)

⇒ ∠ ADP≅ ∠ PBC (Alternate angles ∵ AD||BC)

⇒ Δ APD ~ Δ CPB (By AA Test)

$\Rightarrow \frac{AP}{PC} = \frac{PD}{BP}$ (corresponding sides are proportional)

$\Rightarrow \frac{AP}{PD} = \frac{PC}{BP}$

Q. 12. In fig 1.80, XY || seg AC. If 2AX = 3BX and XY = 9. Complete the activity to find the value of AC.

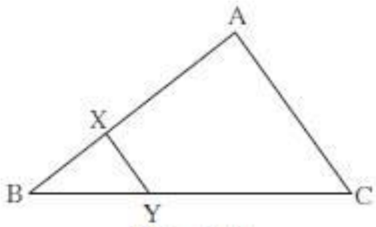


Fig. 1.80

Activity : $2AX = 3BX \therefore \frac{AX}{BX} = \frac{\square}{\square}$

$\frac{AX + BX}{BX} = \frac{\square + \square}{\square}$ .... **by componendo.**

$\frac{AB}{BX} = \frac{\square}{\square}$ .... **(I)**

$\Delta BCA \sim \Delta BYX$ .... $\square$ **test of similarity.**

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$ .... **corresponding sides of similar triangles.**

$\therefore \frac{\square}{\square} = \frac{AC}{9} \therefore AC = \square$ **...from (I)**

Answer : ACTIVITY: $2AX = 3BX \therefore \frac{AX}{BX} = \frac{3}{2}$

$\frac{AX + BX}{BX} = \frac{3 + 2}{2}$ ……(By Componendo)

$\frac{AB}{BX} = \frac{5}{3}$…….(I)

Δ BCA~ ΔBYX…… (AA test of similarity).

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$………. (Corresponding sides of similar triangles).

$\frac{5}{3} = \frac{AC}{9} \therefore AC = 15$……from (I)

Q. 13. In figure 1.81, the vertices of square DEFG are on the sides of ΔABC, ∠A = 90°. Then prove that $DE^2 = BD \times EC$

(Hint : Show that ΔGBD is similar to ΔDFE. Use GD = FE = DE.)

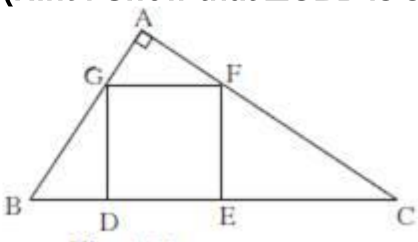


Fig. 1.81

Answer : Proof: In □ DEFG is a square

⇒ GF||DE

⇒ GF||BC

Now, In Δ AGF and Δ DBG

⇒ ∠ AGF≅ ∠ DBG (corresponding angles)

⇒ ∠ GDB≅ ∠ FAG (Both are 90°)

⇒ Δ AGF~Δ DBG ……(1) (AA similarity)

Now, In Δ AGF and Δ EFC

⇒ ∠ AFG≅ ∠ ECF (corresponding angles)

⇒ ∠ GAF≅ ∠ FEC (Both are 90°)

⇒ Δ AGF~Δ EFC ……(2) (AA similarity)

From (1) & (2), we have-

⇒ Δ EFC~Δ DBG

⇒ $\frac{EF}{BD} = \frac{EC}{DG}$

⇒ EF × DG = BD× EC

Now, ∵ DEFG is a square

$\Rightarrow DE = EF = DG$

$\Rightarrow DE \times DE = BD \times EC$

$\Rightarrow DE^2 = BD \times EC$